Knot Theory And Topological Quantum Field Theory

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Content

Definition of Knot and Link

Knot: A knot is a particular way S^1 sit inside ordinary $\mathbb{R}^3$, or precisely, a knot is a submanifold of $\mathbb{R}^3$ that is differential to S^1.
Link: A **Link** is a submanifold of $\mathbb{R}^3$ that is diffeomorphic to a disjoint union of circles. The circles themselves are called **components** of the link.

Figure 1: Unknot

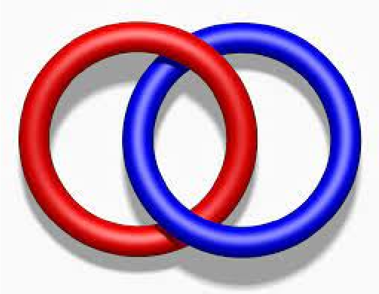


Figure 2: Hopf Link

Isotopic and Reidemeister Moves

Two knots or links in three dimension space can be transformed or deformed continuously one into the other (the usual notion of ambient isotopy) if and only if any diagram (obtained by projection to a plane) of one link can be transformed into the other link via a sequence of **Reidemeister Moves**[1].

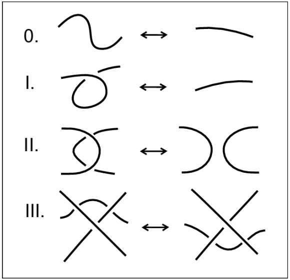
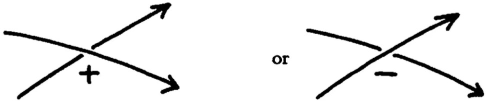


Figure 3: Reidemeister Moves

Isotopic and Reidemeister Moves

Oriented: A knot or link is said to be oriented if each arc in its diagram is assigned a direction so that at each crossing the orientations appear either as



Linking number: Let $L = \{\alpha, \beta\}$ be a link of two components α and β, then the linking number

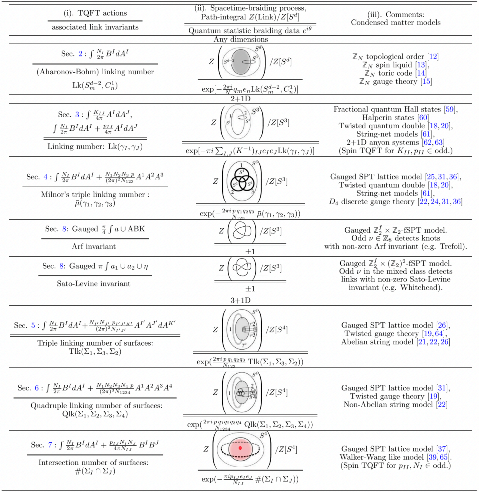
$$\ell k(\alpha,\beta) = \frac{1}{2}\sum_{p\in\alpha\sqcap\beta} \varepsilon(p) \tag{1}$$

Knot Invariant

Figure 4: $\ell k(L) = 1$

Figure 5: $\ell k(L) = 1$

TQFT Action And Link Invariants



(i). TQFT actions / associated link invariants	(ii). Spacetime-braiding process, Path-integral $Z(\text{Link})/Z[S^d]$ / Quantum statistic braiding data $e^{i\theta}$	(iii). Comments: Condensed matter models
	Any dimensions	
Sec. 2 : $\int \frac{N_I}{2\pi} B^I dA^I$ / (Aharonov-Bohm) linking number $\text{Lk}(S_m^{d-2}, C_n^1)$	$Z(\;)/Z[S^d]$ / $\exp[-\frac{2\pi i}{N} q_m e_n \text{Lk}(S_m^{d-2}, C_n^1)]$	$\mathbb{Z}_N$ topological order [12] $\mathbb{Z}_N$ spin liquid [13], $\mathbb{Z}_N$ toric code [14] $\mathbb{Z}_N$ gauge theory [15]
	2+1D	
Sec. 3 : $\int \frac{K_{IJ}}{4\pi} A^I dA^J$, $\int \frac{N_I}{2\pi} B^I dA^I + \frac{p_{IJ}}{4\pi} A^I dA^J$ / Linking number: $\text{Lk}(\gamma_I, \gamma_J)$	$Z(\;)/Z[S^3]$ / $\exp[-\pi i \sum_{I,J} (K^{-1})_{IJ} e_I e_J \text{Lk}(\gamma_I, \gamma_J)]$	Fractional quantum Hall states [59], Halperin states [60] Twisted quantum double [18, 20], String-net models [61], 2+1D anyon systems [62, 63] (Spin TQFT for $K_{II}, p_{II} \in$ odd.)
Sec. 4 : $\int \frac{N_I}{2\pi} B^I dA^I + \frac{N_1 N_2 N_3\, p}{(2\pi)^2 N_{123}} A^1 A^2 A^3$ / Milnor's triple linking number : $\bar{\mu}(\gamma_1, \gamma_2, \gamma_3)$	$Z(\;)/Z[S^3]$ / $\exp(-\frac{2\pi i\, p\, q_1 q_2 q_3}{N_{123}}\, \bar{\mu}(\gamma_1, \gamma_2, \gamma_3))$	Gauged SPT lattice model [25, 31, 36], Twisted quantum double [18, 20], String-net models [61], D_4 discrete gauge theory [22, 24, 31, 36]
Sec. 8: Gauged $\frac{\pi}{4} \int a \cup \text{ABK}$ / Arf invariant	$Z(\;)/Z[S^3]$ / ± 1	Gauged $\mathbb{Z}_2^f \times \mathbb{Z}_2$-fSPT model. Odd $\nu \in \mathbb{Z}_8$ detects knots with non-zero Arf invariant (e.g. Trefoil).
Sec. 8: Gauged $\pi \int a_1 \cup a_2 \cup \eta$ / Sato-Levine invariant	$Z(\;)/Z[S^3]$ / ± 1	Gauged $\mathbb{Z}_2^f \times (\mathbb{Z}_2)^2$-fSPT model. Odd ν in the mixed class detects links with non-zero Sato-Levine invariant (e.g. Whitehead).
	3+1D	
Sec. 5 : $\int \frac{N_I}{2\pi} B^I dA^I + \frac{N_{I'} N_{J'}\, p_{I'J'K'}}{(2\pi)^2 N_{I'J'}} A^{I'} A^{J'} dA^{K'}$ / Triple linking number of surfaces: $\text{Tlk}(\Sigma_1, \Sigma_3, \Sigma_2)$	$Z(\;)/Z[S^4]$ / $\exp(\frac{2\pi i\, p\, q_1 q_2 q_3}{N_{123}}\, \text{Tlk}(\Sigma_1, \Sigma_3, \Sigma_2))$	Gauged SPT lattice model [26], Twisted gauge theory [19, 64], Abelian string model [21, 22, 26]
Sec. 6 : $\int \frac{N_I}{2\pi} B^I dA^I + \frac{N_1 N_2 N_3 N_4\, p}{(2\pi)^3 N_{1234}} A^1 A^2 A^3 A^4$ / Quadruple linking number of surfaces: $\text{Qlk}(\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4)$	$Z(\;)/Z[S^4]$ / $\exp(\frac{2\pi i\, p\, q_1 q_2 q_3 q_4}{N_{1234}}\, \text{Qlk}(\Sigma_1, \Sigma_2, \Sigma_3, \Sigma_4))$	Gauged SPT lattice model [31], Twisted gauge theory [19], Non-Abelian string model [22]
Sec. 7 : $\int \frac{N_I}{2\pi} B^I dA^I + \frac{p_{IJ} N_I N_J}{4\pi N_{IJ}} B^I B^J$ / Intersection number of surfaces: $\#(\Sigma_I \cap \Sigma_J)$	$Z(\;)/Z[S^4]$ / $\exp(-\frac{\pi i p_{IJ} e_I e_J}{N_{IJ}}\, \#(\Sigma_I \cap \Sigma_J))$	Gauged SPT lattice model [37], Walker-Wang like model [39, 65]. (Spin TQFT for $p_{II}, N_I \in$ odd.)

Figure 6: TQFT Action And Link Invariants

The Observable And The Path Integral

We consider the level-κ chern-simons theory with an action $\int \frac{\kappa}{2\pi} BdA$ in d dimension(not 4 π because of the wedge product form,if we expand it we will get a factor $1/2$), where κ is quantized to be an integer. Consider the following action on any closed $d-dim$ manifold M^d.

$$S[A,B] = \int_{M^d} \frac{\kappa}{2\pi} B \wedge dA \tag{2}$$

where A is a 1-form gauge field on M and B is a $(d-2)$-form gauge field on M^2.

The Observable And The Path Integral

The partition function is

$$
\begin{aligned}
Z &= \int DADBexp[iS[A,B]] \\
&= \int DADBexp[i\int_{M^d} \frac{\kappa}{2\pi} B \wedge dA]
\end{aligned} \tag{3}
$$

Let Φ be a gauge invariant functional $\Phi(A,B)$, specially , an observable of the fields A and B. The expectation of it is

$$
\begin{aligned}
<\Phi> &= \frac{1}{Z}\int DADB\Phi(A,B)exp[iS[A,B]] \\
&= \frac{1}{Z}\int DADB\Phi(A,B)exp[i\int_{M^d} \frac{\kappa}{2\pi} B \wedge dA]
\end{aligned} \tag{4}
$$

The Observable And The Path Integral

If the observable function is a product of the wilson loops around the one-dimensional loops $\{\gamma_n^1\}$ separarte and disjoint from $\{S_m^{d-2}\}$ such that

$$\Phi_0(A) = \prod_n exp[ie_n \int_{\gamma_n^1} A] \tag{5}$$

with the electric charge $e_n \in \mathbb{Z}$ associated to each loop, then

$$\begin{aligned} <\Phi> &= \frac{1}{Z}\int DADBexp[iS[A,B]]\,exp[i\sum_n e_n \int_{\gamma_n^1} A] \\ & exp[i\sum_m q_m \int_{S_m^{d-2}} B] \\ &= exp[-\frac{2\pi i}{N}\sum_{m,n} q_m e_n LK(S_m^{d-2},\gamma_n^1)] \end{aligned} \tag{6}$$

Rewrite The First Pair of Maxwell Equation

$$\nabla \cdot \mathbf{B} = 0 \tag{7}$$

$$\nabla \times \mathbf{E} + \frac{\partial B}{\partial t} = 0 \tag{8}$$

And we know that

$$\mathbf{B} = B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy \tag{9}$$

$$\mathbf{E} = E_x dx + E_y dy + E_z dz \tag{10}$$

The first pair of the Maxwell equations then become

$$dE = 0, dB = 0 \tag{11}$$

Uniform Form

We can define a unified electromagnetic field F, a 2-form on $\mathbb{R}^4$,as follows:

$$F = B + E \wedge dt \tag{12}$$

In view of the components,

$$F = \frac{1}{2} F_{\mu\nu} dx^{\mu} \wedge dx^{\nu} \tag{13}$$

We can write them as a matrix:

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \tag{14}$$

The Second Pair

$$*d*F=J \tag{15}$$

where we use the Minkowski metric,

$$J=j-\rho\,dt \tag{16}$$

Berry Curvature

In order to avoid introducing Fiber Bundle in the speech, we try to introduce Berry phase and Berry curvature to have a first glance at these concept.
Considering an adiabatic process, the Hamiltonian is $H(R(t))$,where $R(t)$ is the parameter vetor in the configuration space (varied slowly), which satisfied

$$H(R)|n(R) >= E_n(R)|n(R) > \tag{17}$$

Berry Phase and Curvature

$$\begin{aligned} \gamma_n &= -Im \int_C < n(R)|\nabla_R|n(R) > \cdot dR \\ &= -Im \int ds \cdot (< \nabla n(R) \times \nabla n(R) >) \end{aligned} \tag{18}$$

where $\nabla n(R) \times \nabla n(R)$ is the Berry connection.
We can also regard the Berry coonection as the magnetic potential $\mathbf{A}$ and the berry curvature is the $\mathbf{B}$

Berry Curvature

We can also rewrite the berry curvature as

$$F_{vw} = \partial_v A_w - \partial_w A_v \tag{19}$$

which is exactly the definition of the curvature of sections of E (E is a G bundle and the connection A is a G-connection, plus, G=U(1))

Yang-Mills Form

$$d_D F = 0 \tag{20}$$

(Sorry for the brutalness that I put the conclusion out here without the demonstration, because the main task is to gain the Chern-Simons term!!) By which, we can rewrite the two pairs of the Maxwell equation as

$$d_D F = 0, *d_D * F = J \tag{21}$$

which is the famous Yang-Mills equation.

Chern-Simons Form

Considering the Yang-mills action involve $*$, which means that it has relation with metric. If we consider A is self-dual, that is ,$F = *F$,we then have the chern form action

$$S(A) = \int_M tr(F \wedge F) \tag{22}$$

$$\begin{aligned} tr(F \wedge F) &= \int_0^1 \frac{d}{ds} tr(F_s \wedge F_s)\, ds \\ &= dtr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \wedge A) \end{aligned} \tag{23}$$

we call the 3-form

$$tr(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \wedge A) \tag{24}$$

the chern-simons form.

Frational Statistics

Consider a system of two identical particles and we suppose that there is a short range, infinitely strong repulse force of two identical particles, or simply thinking that we are in a low energy system!

The definition of Statistics: let$\psi(1,2)$ be the wavefunction describing two identical hard-core particles with definite angular momentum , and let us assume that when we move particle 2 around particle 1 by an azimuthal angle $\Delta\phi$,the wave function changes according to

$$\psi(1,2) \to \psi'(1,2) = e^{i\nu\Delta\phi}\psi(1,2) \tag{25}$$

The phase acquired by the wavefunction depends on a parameter which is usually called $statistics$[3].

A Coarse Picture

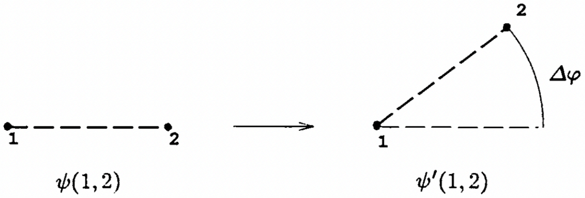


Figure 7: Particle 2 moves around particle 1

Considering two ways of exchange the particles:

1. Moving particle 2 around particle 1 by an angle $\Delta\phi = \pi$
2. Moving particle 2 around particle 1 by an angle $\Delta\phi = -\pi$

We will view these process in the center of momentum frame. As we mentioned above, the frame is a $\mathbb{R}^d - \{0\}$ space.

d≥3

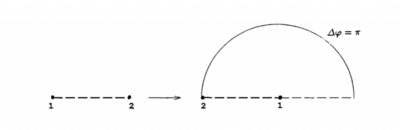


Figure 8: $\Delta\phi = \pi$

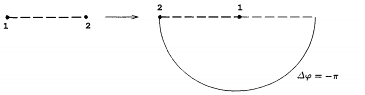


Figure 9: $\Delta\phi = -\pi$

$$e^{i\pi\nu} = e^{-i\pi\nu} \tag{26}$$

from which we obtain that $\nu = 0,1$. If $\nu = 0$, then the particles subject to the bosonic statistics while $\nu = 1$ fermionic statistics.

2-d

The situation changes drastically in two dimensions where the first way can not deform continuously to the second way.

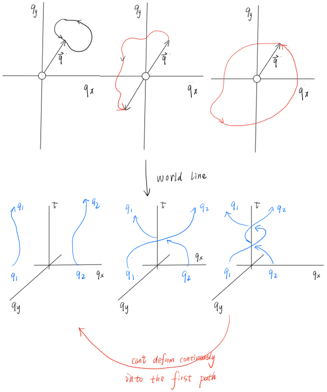


Figure 10: 2 dimension

Braid Group and Fractional Statistics

Let M_N^d be the configuration space of a collection of N identical hard-core particles in d dimensions, and let q, q' be two arbitrary points in M_N^d. The amplitude for the system to evolve from the q at time t to q' at time t' is given by the kernel

$$K(q', t'; q, t) =< q', t'|q, t >= \int_{q(t)=q; q(t')=q'} D_q e^{\frac{i}{\hbar}\int_t^{t'} d\tau \mathscr{L}[q(\tau), \dot{q}(\tau)]} \quad (27)$$

The kernel $K(q', t'; q, t)$ evolves the single-valued wavefunction $\psi(q, t)$ according to

$$\psi(q', t') = \int_{M_N^d} dq K(q', t'; q, t)\psi(q, t) \quad (28)$$

where we have define a configuration space M_N^d of a collection of N particles.

Braid Group and Fractional Statistics

All homotopic loops are grouped into one class and the set of all such classes is called the fundamental group and denoted as π_1. With this category in mind, we can organize the sum over the all loops in(28) into s sum over homotopic classes α into a PI in each class.Thus,

$$K(q,t';q,t) = \sum_{\alpha\in\pi_1(M_N^d)} K_\alpha(q',t';q,t) \tag{29}$$

This formula can be interpreted as a decompostion of the amplitude K into a sum of subamplitudes K_α
It is clear that we can assign different weights to different subamplitudes $K_\alpha(q,t';q,t)$

$$K(q,t';q,t) = \sum_{\alpha\in\pi_1(M_N^d)} \chi(\alpha) \int_{q(t)=q;q(t')=q'} D_{q_\alpha} e^{\frac{i}{\hbar}\int_t^{t'} d\tau \mathscr{L}[q(\tau),\dot{q}(\tau)]} \tag{30}$$

Braid Group and Fractional Statistics

$$M_N^d = \frac{(\mathbb{R}^d)^N - \Delta}{S_N} \tag{32}$$

where

$$\Delta = \{\mathbf{r_1}, ...\mathbf{r_N} \in (\mathbb{R}^d)^N : \mathbf{r_i} = \mathbf{r_j} for \quad i \neq j\} \tag{33}$$

To find the fundamental group of such a space is a standard problem in algebraic topology, which was addressed in 1962. Here we simply quote the results: the fundamental group of M_N^d is given by

$$\pi_1(M_N^d) = \begin{cases} S_N & if \quad d \geq 3 \\ B_N & if \quad d = 2 \end{cases} \tag{34}$$

where B_N is the Braid group and S_N is the permutation group.

Braid Group

Definition of the braid group: The braid group B_N is the group whose elements are isotopy classes of n 1-dimensional braids running vertically in 3-dimensional Cartesian space, the group operation being their concatenation.
The braid group of N strands B_N is an infinite group which is generated by $N-1$ elementary moves $\sigma_1,...\sigma_{N-1}$ satisfying

$$\sigma_I\sigma_{I+1}\sigma_I = \sigma_{I+1}\sigma_I\sigma_{I+1} \tag{35}$$

for $I = 1,2,...N-2$ and

$$\sigma_I\sigma_J = \sigma_J\sigma_I \tag{36}$$

for $|I-J| \geq 2$.
The inverse of $\sigma_I$is denoted by σ_I^{-1}, the identity by $\mathbb{1}$.

Braid Group

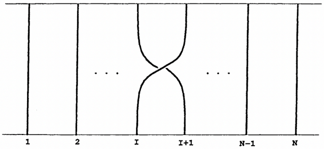


Figure 11: Elementary moves σ_I

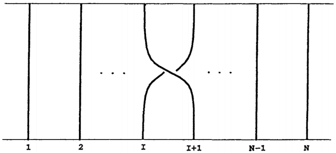


Figure 12: Inverse

Configuration of Three Particles

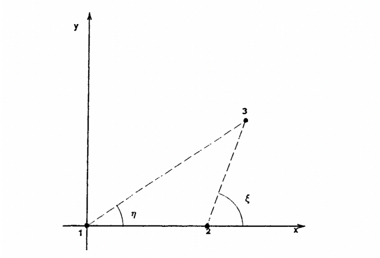


Figure 13: Configuration of Three Particles at Time t

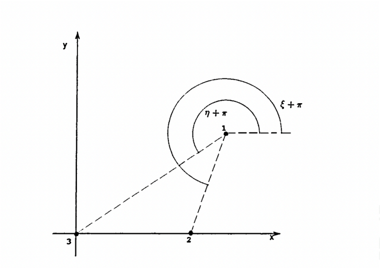


Figure 14: Configuration of Three Particles at Time t'

World-lines as the strands

$$\phi_{12}(t)=0,\phi_{13}(t)=\eta,\phi_{23}(t)=\varepsilon \tag{37}$$

Let us suppose that at time t' the particles reach the positions shown in (Fig.), now the winding angles are

$$\phi_{12}(t')=\varepsilon+\pi,\phi_{13}(t')=\eta+\pi,\phi_{23}(t')\pi \tag{38}$$

It is always true that

$$\sum_{I<J}\phi_{IJ}(t')-\sum_{I<J}\phi_{IJ}(t)=n\pi \tag{39}$$

where n is an integer(in our example $n=3$).We see that indeed there are $n=3$ generators($\sigma_1\sigma_2\sigma_1$), $i.e$ n=3 exchanges.(The dynamical process is important!)

The propagator and Braid group

One dimensional representation $\chi(\alpha)$ satisfied the definition of the braid group has the form as follows:

$$\chi(\sigma_K) = e^{-i\nu\pi} \tag{40}$$

for any $K = 1,...,N-1$, where ν is a real parameter defined modulo 2.
We can rewrite the formula

$$\chi(\sigma_K) = exp[-i\nu\Delta\phi_{K,K+1}] = exp[-i\nu\sum_{I<J}\Delta\phi_{IJ}^{(K)}] \tag{41}$$

where we have

$$\Delta_{IJ}^{(K)} \equiv \phi_{IJ}(t')^{(K)} - \phi_{IJ}(t)^{(K)} = \pi\delta_{I,K}\delta_{J,K+1} \tag{42}$$

The propagator and Braid group

We can generalised the formula 41

$$\chi(\alpha) = exp[-i\nu \sum_{I<J} \int_{t}^{t'} d\tau \frac{d}{d\tau} \phi_{IJ}^{(\alpha)}(\tau)] \tag{43}$$

$$\chi(\alpha) = exp[-i\nu \sum_{I<J} \int_{t}^{t'} d\tau \frac{d}{d\tau} \phi_{IJ}^{(\alpha)}(\tau)] \tag{44}$$

Thus we can rewrite the propagator as follows:

$$K(q, t'; q, t) = \sum_{\alpha \in \pi_1(M_N^d)} \int_{q(t)=q; q(t')=q} D_{q\alpha} \; e^{\frac{i}{\hbar} \int_t^{t'} d\tau \{ \mathscr{L}[q(\tau), \dot{q}(\tau)] - \hbar\nu \sum_{I<J} \frac{d}{d\tau} \phi_{IJ}^{(\alpha)}(\tau) \}} \tag{45}$$

The Propagator and Braid Group

Now we define

$$\mathscr{L}' = \mathscr{L} - \hbar\nu \sum_{I<J} \frac{d}{d\tau} \phi^{\alpha}_{IJ}(\tau) \tag{46}$$

then we see that the kernel $K(q, t'; q, t)$ is decomposed with respect to $\mathscr{L}'$ into subamplitudes each of which is weighted equally as if we are describing bosons, that is

$$K(q', t'; q, t) = \int_{q(t)=q; q(t')=q'} D_q e^{\frac{i}{\hbar}\int_t^{t'} d\tau \mathscr{L}'[q(\tau), \dot{q}(\tau)]} \tag{47}$$

Where is the fraction?

Consider the case of the sphere $(\Sigma = S^2)$. The braid group $B_N(S^2)$ is generated by σ_I with $I = 1, .., N-1$ which satisfy 35 and 36 plus an additional constraint(prove this!)

$$\sigma_1\sigma_2...\sigma_{N-1}^2...\sigma_2\sigma_1 = 1 \tag{48}$$

Then we can obtain

$$e^{-i2(N-1)\nu\pi} = 1 \tag{49}$$

which immediately restricts ν to be rational, that is $\nu = \frac{p}{q}$.

CS Term

We have[4]

$$
\begin{aligned}
S_{CS} &= \frac{\kappa}{2}\int_S A \wedge dA \\
&= \frac{\kappa}{2}\int d^3x \varepsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma
\end{aligned}
\tag{50}
$$

If we consider the source then we have,

$$
\mathscr{L} = \frac{\kappa}{2}\varepsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma - j^\alpha A_\alpha \tag{51}
$$

$$
\frac{\partial L}{\partial A_\alpha} = \partial_\beta \left(\frac{\partial L}{\partial(\partial_\beta A_\alpha)}\right) \tag{52}
$$

CS Term

$$j^{\alpha} = \kappa \varepsilon^{\alpha\beta\gamma} \partial_{\beta} A_{\gamma} \tag{53}$$

$$\begin{aligned} j^{0} &= \sum_{n=1}^{N} q_n \delta(x - x_n) \\ &= \kappa \varepsilon^{0\beta\gamma} \partial_{\beta} A_{\gamma} \\ &= \kappa dA \end{aligned} \tag{54}$$

where we can see that

$$e = \kappa B \tag{55}$$

or in other words, where there is charge, there is magnetic flux!!

Charge And Flux

$$e = \kappa B \tag{56}$$

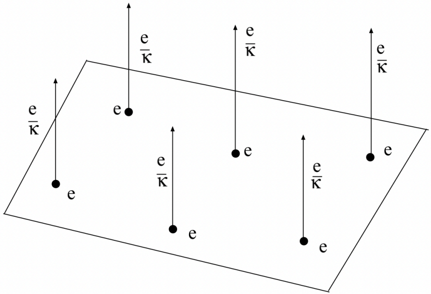


Figure 15: Charge with magnetic flux

Relation To Anyons

Consider, for example, non relativistic point charged particles moving in the plane, with magnetic flux lines attached to them. The charged density

$$\rho(\mathbf{x},t) = e\sum_{\alpha=1}^{N}\delta(\mathbf{x}-\mathbf{x}_\alpha(t)) \tag{57}$$

describes N particles, with the a^{th} particle following the trajectory $\mathbf{x}_\alpha(t)$. The corresponding current density is $\mathbf{j}(\mathbf{x},t) = e\sum_{\alpha=1}^{N}\mathbf{v}_\alpha\delta(\mathbf{x}-\mathbf{x}_\alpha)$.

$$\mathbf{B}(\mathbf{x},t) = \frac{1}{\kappa}e\sum_{\alpha=1}^{N}\delta((\mathbf{x})-\mathbf{x}_\alpha(t)) \tag{58}$$

which follows each particle throughout its motion.
Thus through a process of complicated but worthwhile calculation, we will have

Relation To Anyons

The solution can be written also as

$$A_I^i(\mathbf{r_1},...,\mathbf{r_N}) = -\frac{e}{2\pi\kappa}\frac{\partial}{\partial r_I^j}\sum_{I\neq J}\phi_{IJ} \tag{60}$$

Finally we managed to have the Lagrangian

$$\mathscr{L}' = \sum_{I=1}^{N}(\frac{1}{2}m\mathbf{v_I^2}) - \frac{e^2}{2\pi\kappa}\sum_{I<J}\frac{d}{d\tau}\phi_{IJ}(\tau) \tag{61}$$

So we immediately deduce that our particle are generally anyons of statistics

$$\nu = \frac{e^2}{2\pi\hbar\kappa} \tag{62}$$

Some Trivial Things ?

Reference

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Thank you!