

Stats531 Homework2

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Question 2.1

We investigate two ways to calculate the autocovariance function for AR and MA models. These ways are different from the two ways already demonstrated in the notes, but there is some overlap. The instructions below help you work through the case of a causal AR(1) model,

$$X_n = \phi X_{n-1} + \epsilon_n.$$

where $\{\epsilon_n\}$ is white noise with variance σ^2 , and $-1 < \phi < 1$. Assume the process is stationary, i.e., it is initialized with a random draw from its stationary distribution. Show your working for both the approaches A and B explained below. If you want an additional challenge, you can work through the AR(2) or ARMA(1,1) case instead.

A.

Using the stochastic difference equation to obtain a difference equation for the autocovariance function (ACF). Start by writing the ACF as

$$\gamma_h = \text{Cov}(X_n, X_{n+h}) = \text{Cov}(X_n, \phi X_{n+h-1} + \epsilon_{n+h}), \text{ for } h > 0.$$

Writing the right hand side in terms of γ_{h-1} leads to an equation which is formally a [first order linear homogeneous recurrence relation with constant coefficients](#). To solve such an equation, we look for solutions of the form

$$\gamma_h = A\lambda^h.$$

Substituting this general solution into the recurrence relation, together with an initial condition derived from explicitly computing γ_0 , provides an approach to finding two equations that can be solved for the two unknowns, A and λ .

B.

Via a Taylor series calculation of the $\text{MA}(\infty)$ representation. Construct a Taylor series expansion of $g(x) = (1 - \phi x)^{-1}$ of the form

$$g(x) = g_0 + g_1x + g_2x^2 + g_3x^3 + \dots$$

Do this either by hand or using your favorite math software (if you use software, please say what software you used and what you entered to get the output). Use this Taylor series to write down the $\text{MA}(\infty)$ representation of an $\text{AR}(1)$ model. Then, apply the general formula for the autocovariance function of an $\text{MA}(\infty)$ process.

C.

Check your work for the specific case of an $\text{AR}(1)$ model with $\phi_1 = 0.8$ by comparing your formula with the result of the R function `ARMAacf`.

Question 2.2

Compute the autocovariance function (ACF) of the random walk model. Specifically, find the ACF, $\gamma_{mn} = \text{Cov}(X_m, X_n)$, for the random walk model specified by

$$X_n = X_{n-1} + \epsilon_n,$$

where $\{\epsilon_n\}$ is white noise with variance σ^2 , and we use the initial value $X_0 = 0$.