Data Processing and Analysis in Python
Lecture 18
Optimization – Linear Programming

UNIVERSITY OF MARYLAND

ROBERT H. SMITH SCHOOL OF BUSINESS

DR. ADAM LEE

Optimization – Linear Programming (LP)

- **Optimization** – we all have finite resources and time and we want to make the most of them
- **Linear programming** – we depict complex relationships through linear functions and then find the optimum points
  - Depict – the real relationships might be much more complex, but we try simplifying them to linear relationships
- Examples:
  - When you are driving from home to school and want to take the shortest route
  - When you have a project delivery, you make strategies to make your team work efficiently for on-time delivery

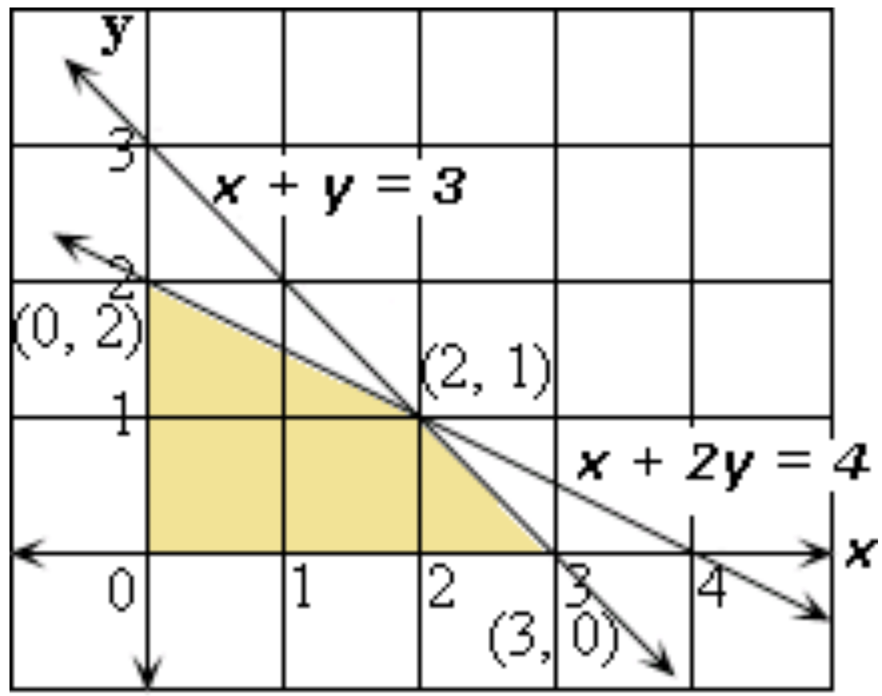
Common Terminologies and Procedure

- First need to identify the **Decision Variables**: the variables that will decide the output
- **Objective Function**: defined as the objective of making decisions
- **Constraints**: the restrictions or limitations on the decision variables
- **Non-Negativity Restriction**: for all linear programs, the decision variables should always take non-negative values

```
>>> from scipy.optimize import linprog
```

Linear Programming

- Cupcake Bakery
  - Two kinds of cupcakes: lower-priced *x*; higher-priced *y*
  - Three hours to make: each kind takes one hour
  - Four sets of ingredients: *x* takes one; *y* takes two
  - To maximize sale: x can be sold for \$4: y can be sold for \$5
  - **To make 2 lower-priced and 1 higher-priced cupcakes: \$4 x 2 + \$5 x 1 = \$13**



UNIVERSITY OF MARYLAND
ROBERT H. SMITH SCHOOL OF BUSINESS

SciPy Linear Programming

```
# model generation
# decision variables: x, y
# objective function was: max 4x + 5y
# objective function now: min -4x -5y
c = [-4, -5]
# time constraint: x + y <= 3
# resource constraint: x + 2y <= 4
a = [[1, 1], [1, 2]]
b = [3, 4]

from scipy.optimize import linprog
# linear programming
res = linprog(c, a, b)
print(res)
# x = 2, y = 1
```

SciPy Linear Programming

```
import numpy as np
import matplotlib.pyplot as plt

# create figure and display unit grids
fig, ax = plt.subplots()
plt.axis([-0.5, 4.5, -0.5, 4.5])
plt.axis("square")
ax.grid(True, which="both", color='g', linestyle=':')

# move x and y axes passing through (0, 0)
ax.spines["left"].set_position("zero")
ax.spines["bottom"].set_position("zero")
ax.spines["right"].set_color("none")
ax.spines["top"].set_color("none")
```

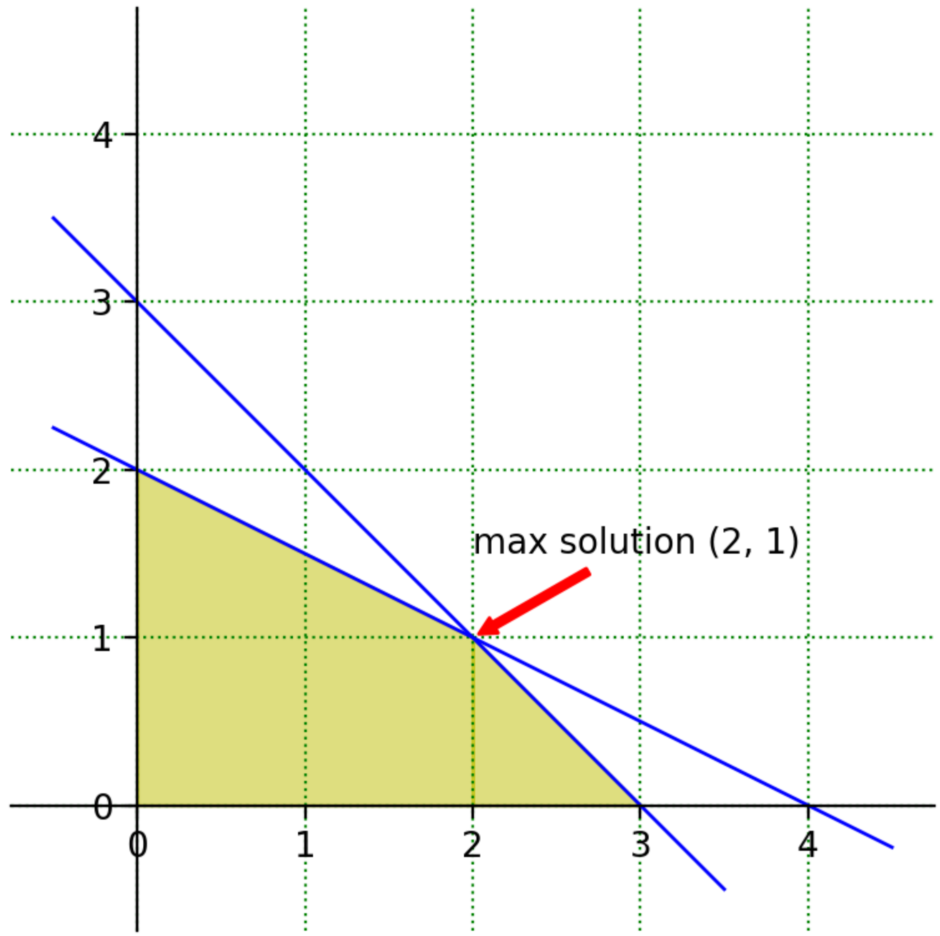

UNIVERSITY OF MARYLAND
ROBERT H. SMITH SCHOOL OF BUSINESS

SciPy Linear Programming

```
# plot model
plt.plot([-0.5, 3.5], [3.5,  -0.5], '-b', linewidth=1)
plt.plot([-0.5, 5.0], [2.25, -0.5], '-b', linewidth=1)
plt.fill_between([0.0, res.x[0]], [2.0, res.x[1]],
                 color='y')
plt.fill_between([res.x[0], 3.0], [res.x[1], 0.0],
                 color='y')
# plot solution
plt.annotate("max solution (%.0f, %.0f)" %
    (res.x[0], res.x[1]) ,
    xy=(res.x[0], res.x[1]),
    xytext=(res.x[0], res.x[1] + 0.5),
    arrowprops={"arrowstyle":"simple", "color":'r'})
plt.show()
```


UNIVERSITY OF MARYLAND
ROBERT H. SMITH SCHOOL OF BUSINESS

SciPy Linear Programming



Capital Budgeting

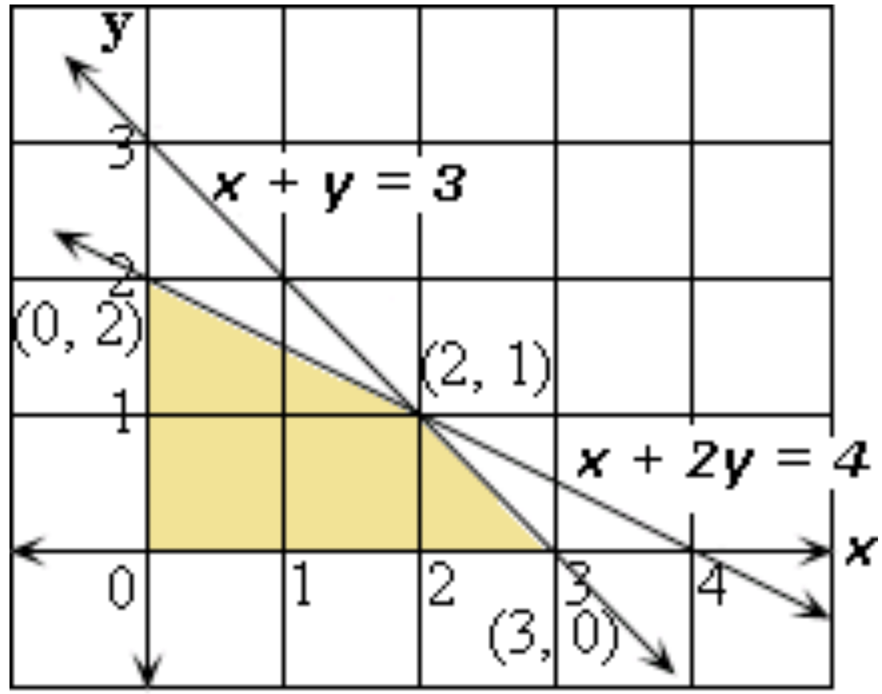
- a.k.a. investment appraisal
  - Investment decisions have a greater impact on a business' future than any other decisions it makes
- Process:
  - Identification of opportunities
  - Evaluation of opportunities
  - Selection
  - Implementation
  - Post audit

Capital Budgeting Methods

Method	Description	Equation	Decision Criteria
Payback Period (PB)	Number of years required to recapture initial investment	$= \dfrac{\textit{Initial investment}}{\textit{Annual cash flow}}$	None
Net Present Value (NPV)	The present value (PV) of all cash flows	$= PV\ (\textit{cash inflows}) - PV\ (\textit{cash outflows})$	Accept if greater than or equal to zero
Profitability Index (PI)	The ratio of the present value of the cash inflows to outflows	$= \dfrac{PV\ (\textit{inflows})}{PV\ (\textit{outflows})}$	Accept if greater than or equal to 1
Internal Rate of Return (IRR)	The interest rate that sets the present value of the cash inflows equal to the present value of the outflows	Calculator or Spreadsheet	Accept if greater than or equal to cost of capital
Modified Internal Rate of Return (MIRR)	The interest rate that sets the present values of the outflows equal to the future values of the inflows, computed at the firm's cost of capital	Calculator or Spreadsheet	Accept if greater than or equal to cost of capital

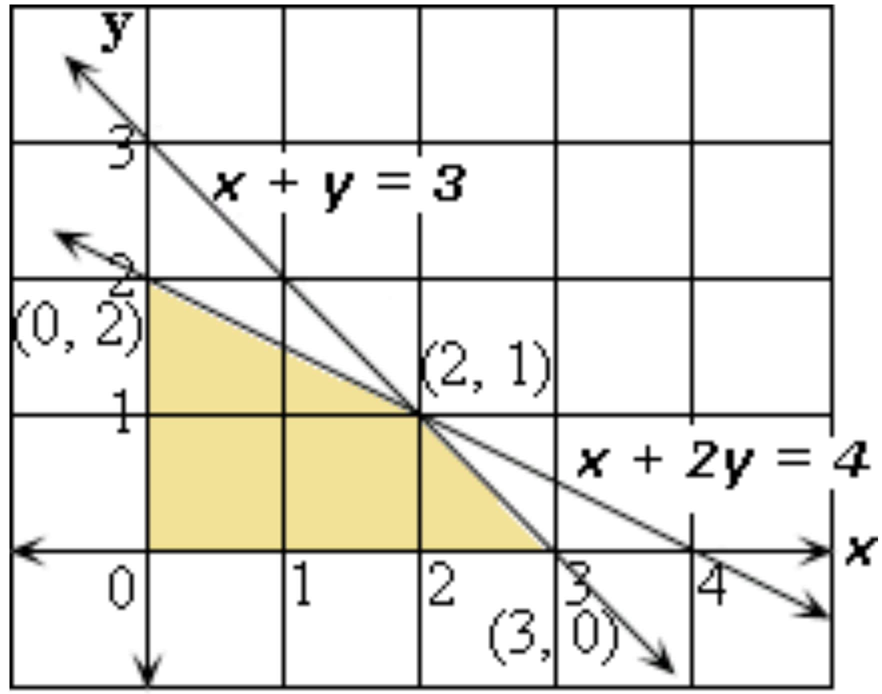
Linear Programming

- Cupcake Bakery
  - Two kinds of cupcakes: lower-priced *x*; higher-priced *y*
  - Three hours to make: each kind takes one hour
  - Four sets of ingredients: *x* takes one; *y* takes two
  - To maximize sale: x can be sold for $4: y can be sold for $5
  - To make 2 lower-priced and 1 higher-priced cupcakes: $4 x 2 + $5 x 1 = $13



Linear Programming

- Capital Budgeting
  - Two groups of projects: lower-profit *x*; higher-profit *y*
  - Three project managers: each manages one project
  - $4M of initial budgets: *x* costs $1M; *y* costs $2M
  - To maximize cash flow: x can generate $4M: y can generate $5M
  - To undertake 2 lower-profit and 1 higher-profit projects: $4M x 2 + $5M x 1 = $13M



Integer Programming

- Integer Programming (IP) ≈ Linear Programming (LP) + all variables are restricted to be integers
- Capital Budgeting
  - Two groups of projects: lower-profit x_1; higher-profit x_2
  - \$200K of initial budget: x_1 takes \$15K; x_2 takes \$30K
  - 40 hours per week: x_1 takes 8 hrs; x_2 takes 4 hrs
  - To maximize cash flow: x_1 for \$100K; x_2 for \$150K
  - To make 1 x_2 and 6 x_2: \$100K x 1 + \$150K x 6 = \$1M