BUDT 730

Data, Models and Decisions

Lecture 19

Decision Tree (2)

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Learning Objectives

- Decision Making under Uncertainty
  - Learn the concept of expected monetary value (EMV).
  - Understand the concept of risk preference/attitude
- Decision Tree
  - Understand how to construct a single stage decision tree
  - Learn how probabilities are used in the decision-making process

Decision and Risk Analysis

https://fortune.com/2014/07/15/the-art-of-predicting-business-risks-why-non-experts-do-it-better/

Decision Making under Uncertainty and Risk

- EMV maximization is a rational approach that may produce good outcomes in the long run.
- However, this decision does not take into account the risk of having undesirable outcomes
- Many business decisions are only made once
  - What if large amounts of money are at stake?
  - Should this change our decision-making approach?
  - Some decision makers sacrifice EMV to reduce risk
- We'll study how to take into account the risk in decision making under uncertainty.

Risk Analysis – Two Approaches

- We can consider two approaches.
- The choice depends on the underlying analytical method/model.
- Risk measure approach:
  - Measure the risk, and make a decision based on the EMV and the risk
  - Widely used in finance and operations management
  - We will take this approach in Simulation Analysis (Ch15)

- Utility function approach:
  - Maximize the expected utility
  - We will take this approach in Decision Tree (Ch6)

What is Risk?

Risk:

- Possibility of loss, or any other negative occurrence
- It occurs since we must make a decision for which the outcome is unknown
  - Decision rules do not eliminate risk!

- Risk measures
  - **Variance (or standard deviation): $Var\ (X), X$ = return or profit**
  - **Probability of loss: $P(X < 0)$**
  - Value at Risk (VaR)
  - Expected shortfall, and many others

Risk Attitude (Preference)

- Your risk tolerance expresses your willingness to take risk in your quest for better consequences
- Your risk tolerance depends primarily on how significant you consider the worst case compared to the best case
  - Can you afford to loose all of your retirement savings?

- Attitude towards risk
  - Risk-neutral
  - Risk-averse
  - Risk-seeking

Risk Preference

- **Risk neutral**
  - completely insensitive to risk
  - only interested in maximizing the EMV
- **Risk averse**
  - prefers a lower EMV with no/lower risk rather than a higher EMV with an unknown/higher risk
- **Risk seeking**
  - more risky decision is chosen
  - prefers a lower EMV with a greater risk rather than no-risk investment with a higher EMV,
  - but the probability of achieving high monetary value/return would be high

What is Utility?

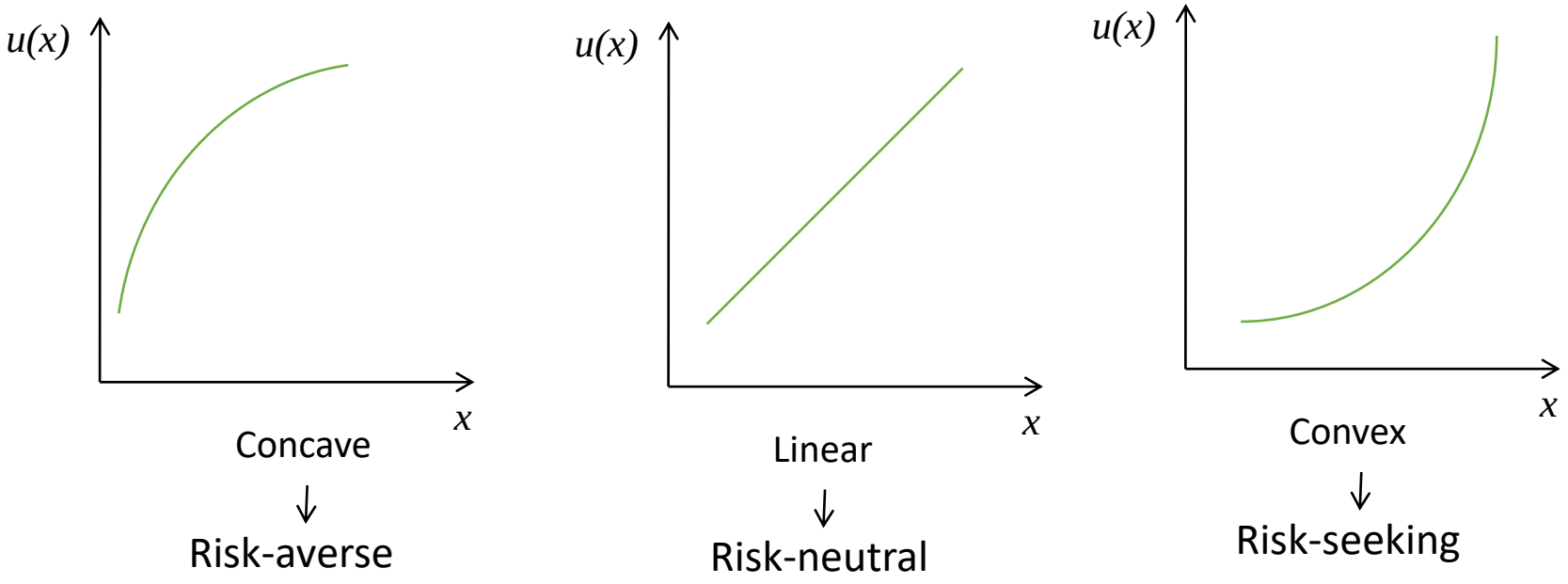
- Utility is the capacity of a commodity to satisfy some human want (i.e., happiness)
- Most researchers believe that if certain basic behavioral assumptions hold, people are **expected utility maximizers**—that is, they choose the alternative with the largest expected utility.

Maximizing Expected Utility

- Rather than maximize EMV, we maximize expected utility (EU), which is the probability weighted average of utilities
- Utility functions transform monetary values—payoffs and costs—into utility values
  - Utility values don't have a specific meaning, but larger values are better, i.e., they make an individual happier
  - Typical shapes of utility function include log, and exponential

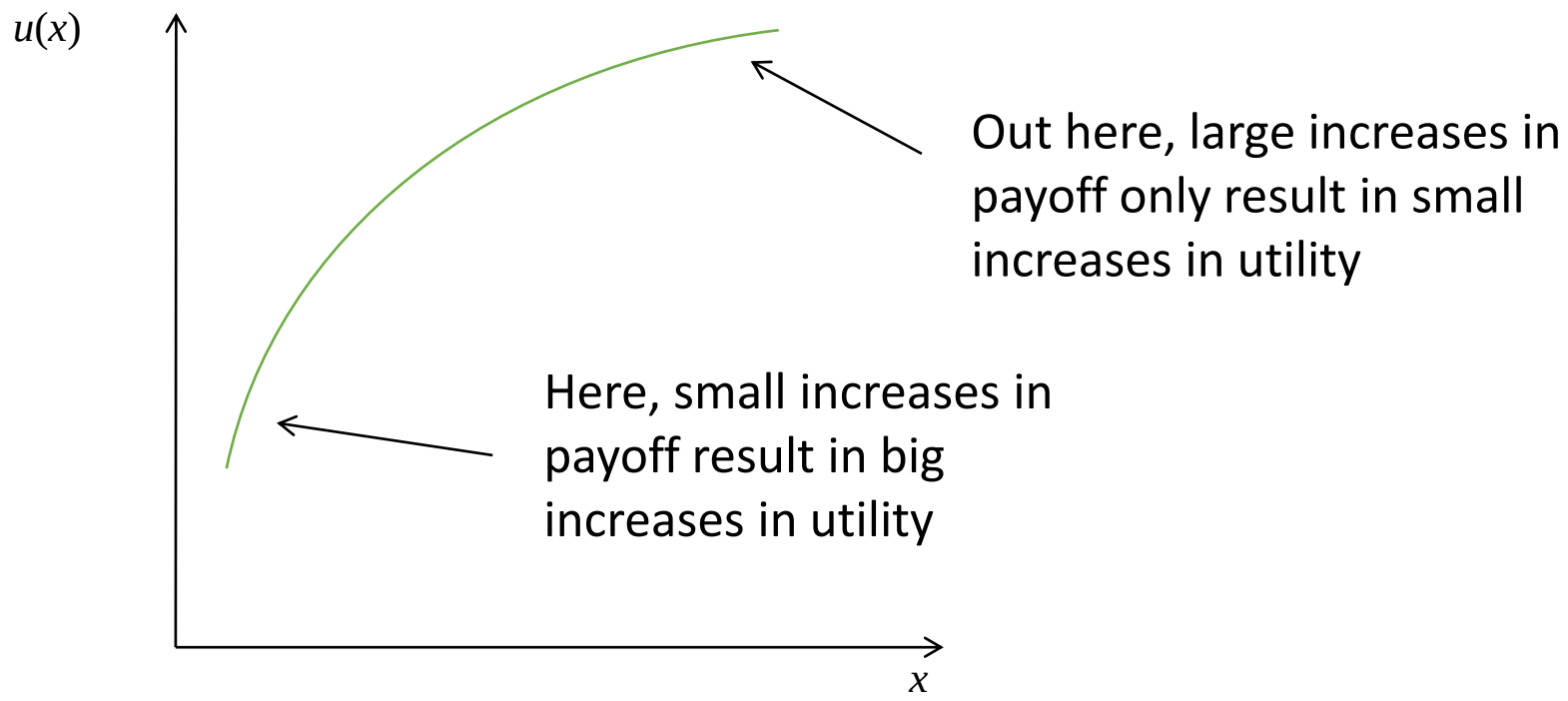
Risk Attitudes: Utility Function Approach

The curvature of the utility function determines the decision-maker's attitude towards risk



Risk Aversion

- Most individuals are ***risk averse***: common in investment strategies.
- The value of an additional dollar of payoff declines with the level of payoff



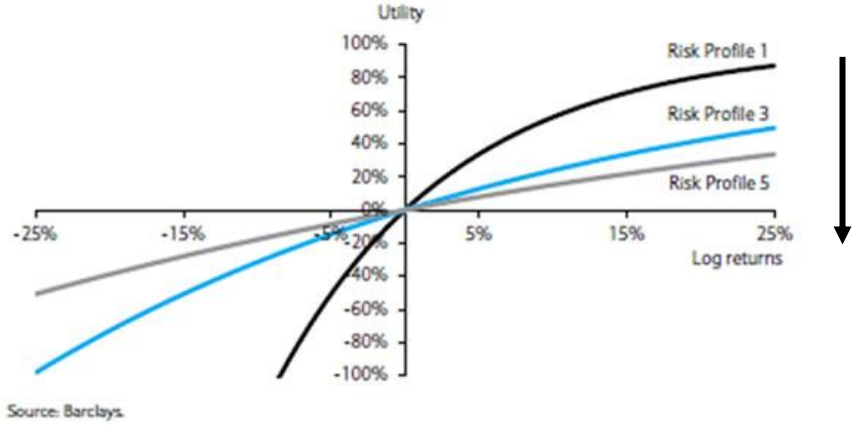
Exponential Utility

- **Exponential Utility** is a most widely used risk averse utility function
- An exponential utility function has the following form:

$$u(x) = 1 - e^{-\frac{x}{R}}$$

  - R is called the **risk tolerance**
  - It affects the shape of the exponential curve, making it more or less concave
  - The higher the risk tolerance, the less concavity, thus the less risk averse
  - It is a unitless quantity

R increases



Decision Tree

Single-Stage

Decision Problem

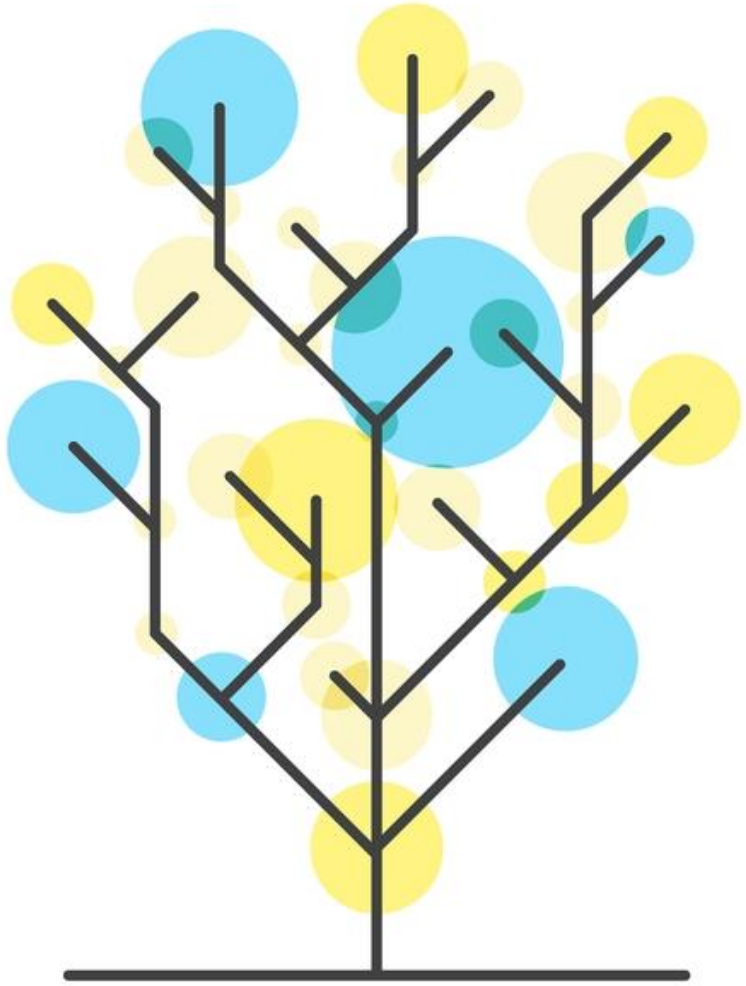


Image Credit: Boo-Tique / Shutterstock.com

Example: Ann's Auto Insurance (Part A)

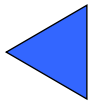
- Ann has a 1% chance of being in an automobile accident during the year that will cost $10,000
- She is offered an insurance policy for $120 (no deductible)
- **If Ann is an EMV maximizer, will she purchase the policy?**

- This is a **single stage (one-stage) decision problem**: one stage decision is made, right now.

Decision Trees

- A graphical tool called a **decision tree** has been developed to represent decision problems.
  - It is particularly useful for more complex decision problems
  - It clearly shows the sequence of events (decisions and outcomes), as well as probabilities and monetary values.
  - But, it can handle problems only with finite discrete random variables (limitation of decision tree)

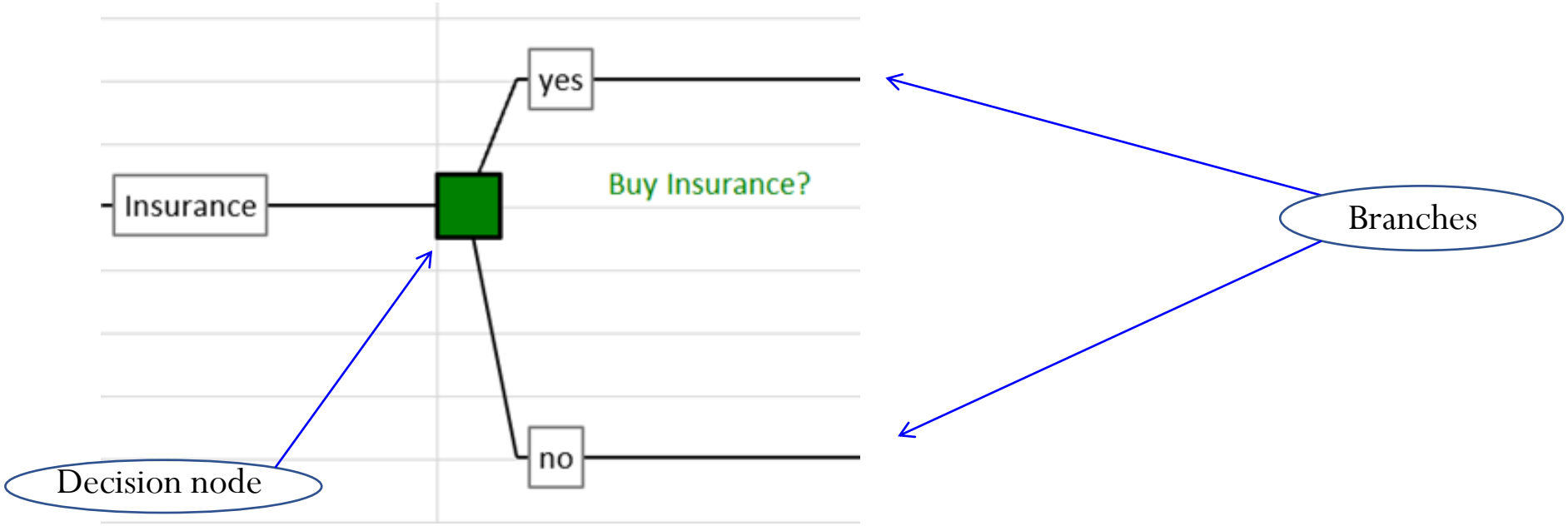
Decision Trees

- Decision trees are composed of *nodes* (squares, circles, and triangles) and *branches* (lines).
  -  **Decision node**
  -  **Chance (probability) node**
  -  **End node**
- The nodes represent points in time and time proceeds *from left to right*

Decision Tree Basics

Decision node

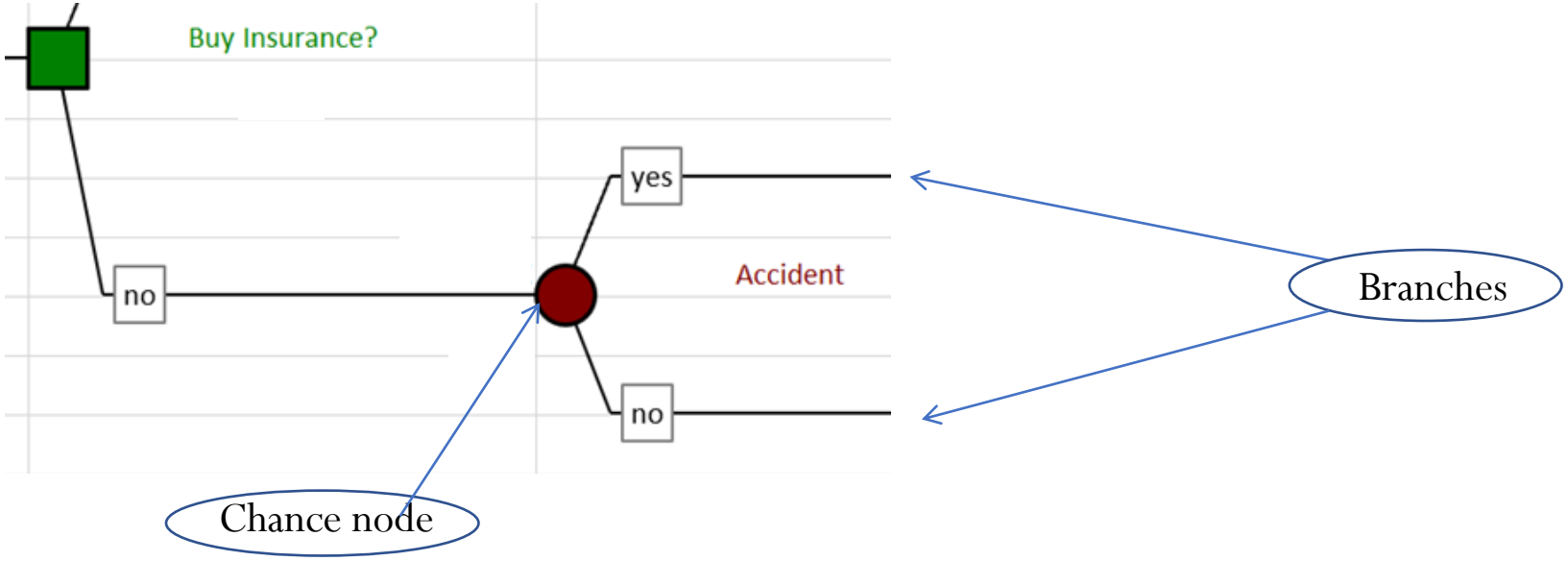
- A decision node represents a time when the decision maker makes a decision.
- Each possible decision represented as a branch emanating out of the decision node



Decision Tree Basics

Chance node

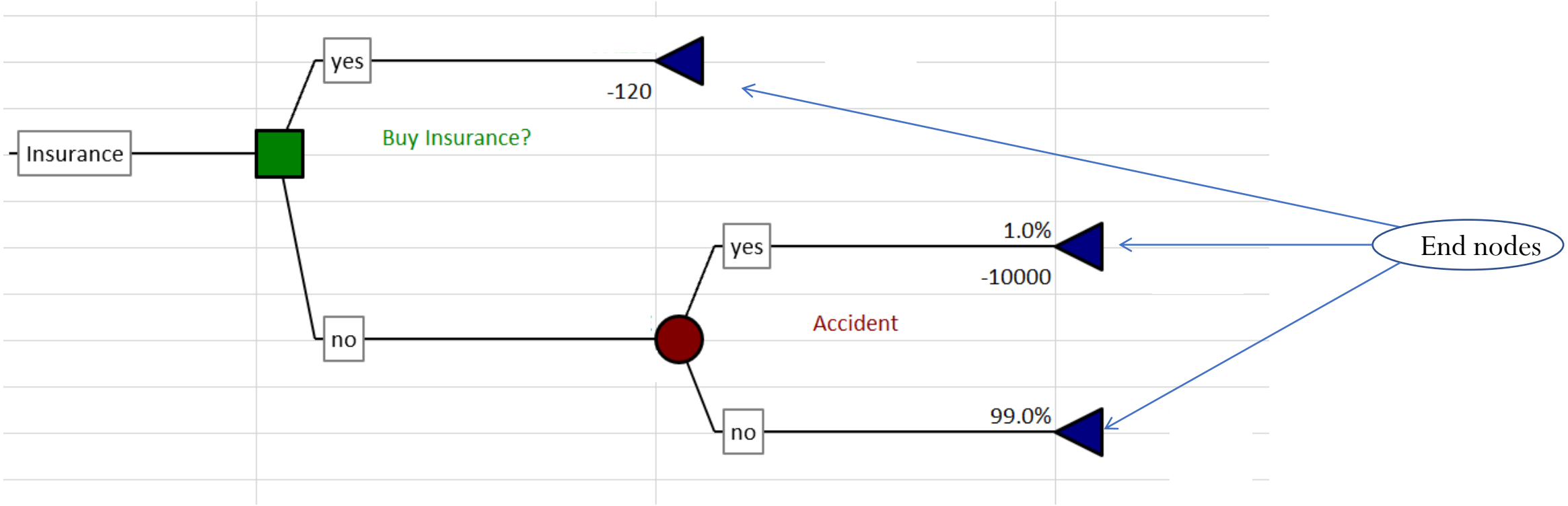
- A Chance node represents a time when the result of an uncertain outcome becomes known.
- Each possible outcome is represented as a branch that emanates out of the chance node
- All events must be mutually exclusive (no more than one outcome can occur at a time), and collectively exhaustive (probabilities sum to 1)



Decision Tree Basics

End node

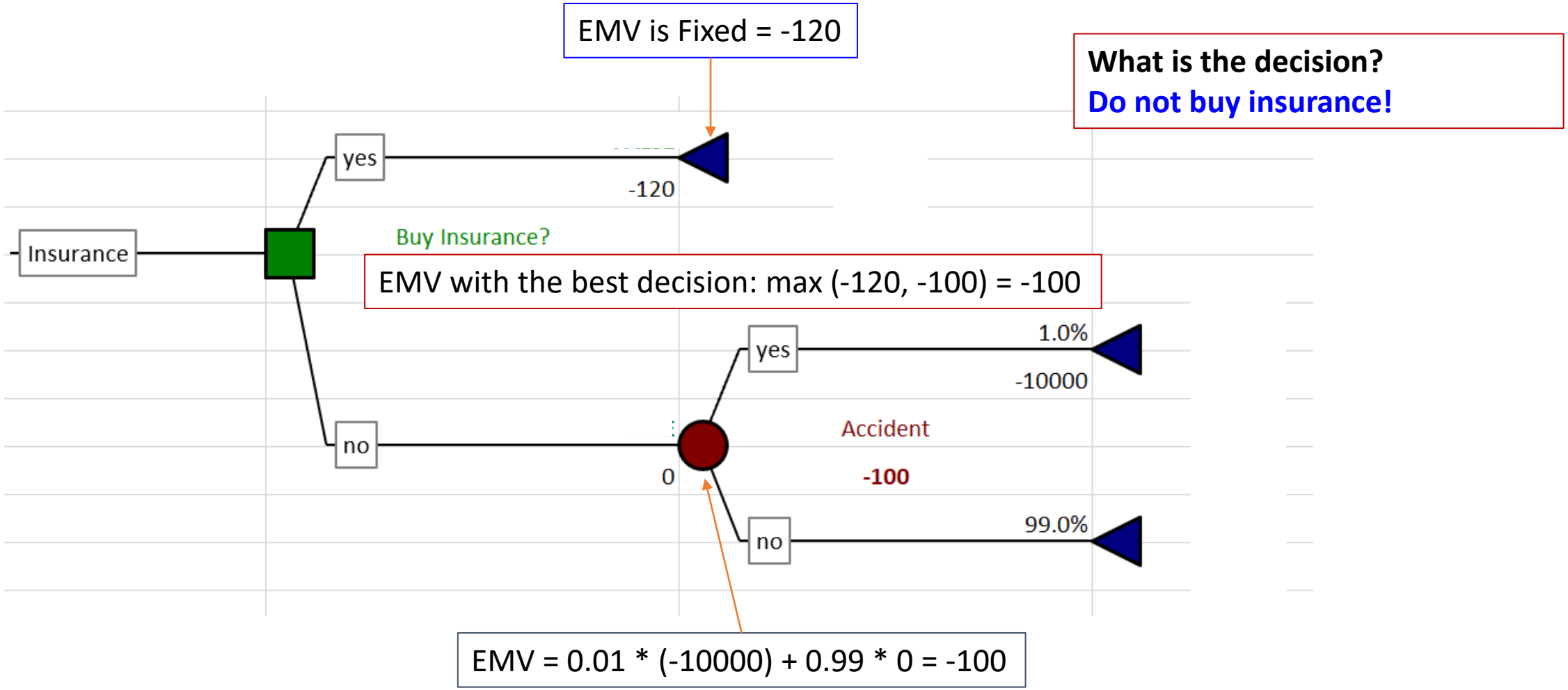
- An end node indicates that the problem is completed—all decisions have been made, all uncertainty has been resolved, and all payoffs and costs have been incurred.



Finding the ‘Best’ Decision

- The best decision is found using *the* ***rollback (folding-back) procedure***
- Start at the end and work your way backwards (to the left)
  - For a chance node, compute the EMV using the probabilities and payoffs of each branch
  - For a decision node, choose the branch with the highest EMV

Example: Ann's Auto Insurance Part A - EMV



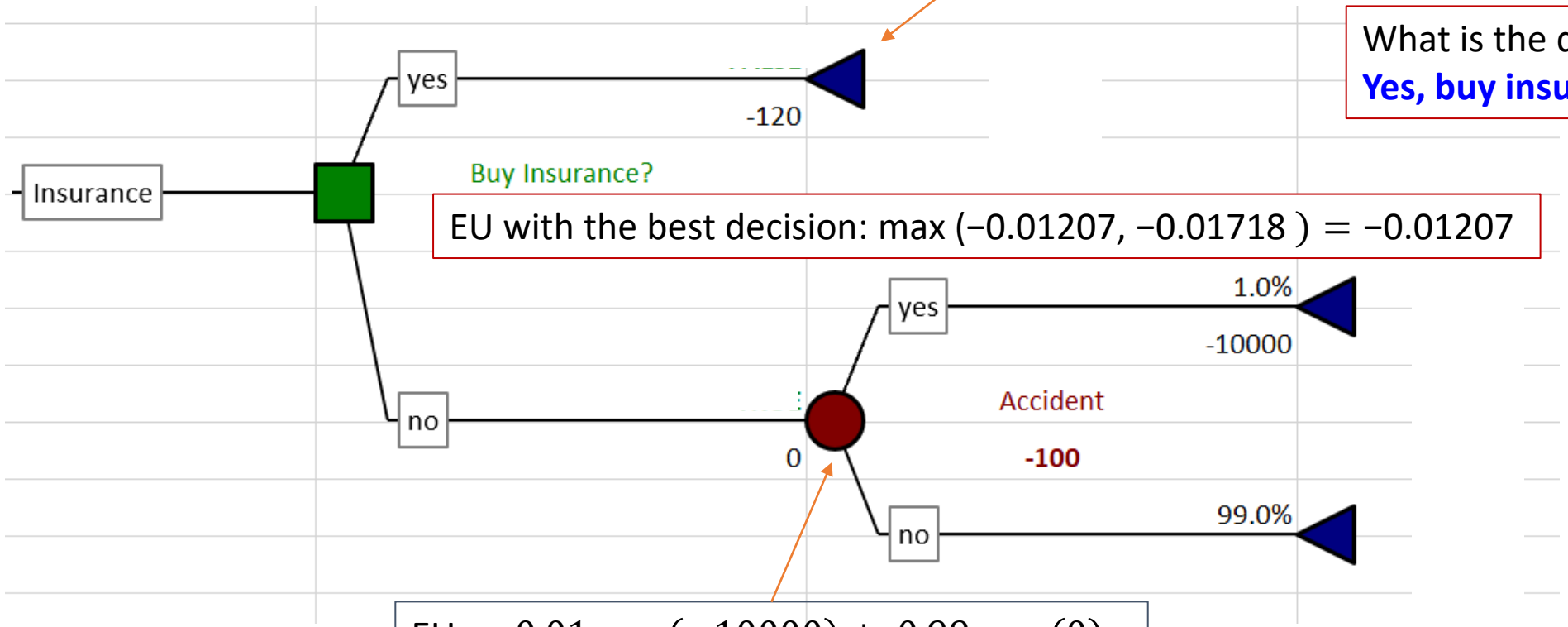
Example: Ann's Auto Insurance Part A – Risk Averse

- Part A
  - Ann has a 1% chance of being in an automobile accident during the year that will cost $10,000
  - She is offered an insurance policy for $120 (no deductible)
  - The **risk neural decision** was "**do not buy insurance**"

- Suppose Ann's risk attitude is best represented by an exponential utility function with a risk tolerance R = $10,000. How does this utility model affect Ann's decision?
  - Note: How to determine the value of R? This is not a trivial question. Based on a market research, the company may estimate it. In this class, we assume that it is given.

Example: Ann's Auto Insurance Part A – Risk Averse
$u(x) = 1 - e^{-x/10000}$

EU is Fixed = $u(-120) = 1 - e^{-120/10000} = -0.01207$



What is the decision?
Yes, buy insurance!

EU with the best decision: max (−0.01207, −0.01718) = −0.01207

$$EU = 0.01 * u(-10000) + 0.99 * u(0)$$
$$= 0.01 * -1.718 + 0.99 * 0 = -0.01718$$

In-Class Exercise

New Product Decision at ACME (Example 6-1 in Text)

New Product Decision at ACME

- ACME's cost accountants estimate the monetary inputs: the fixed costs ($6,000) and the unit margin ($18).
- The uncertain sales volume is really a continuous variable but, as in many decision problems, Acme has replaced the continuum by three representative possibilities: great (45%), fair (35%) and awful (20%)
- The company estimates that the corresponding sales volumes (in thousands of units sold) are 600, 300, and 90, respectively.
- Each sales volume is multiplied by the unit margin to obtain the net revenues.

New Product Decision at ACME

Q2: Suppose ACME's risk attitude is best represented by an exponential utility function with a risk tolerance R = $5,000. Compute the EU for each decision. How does this utility model affect ACME's decision?

Next …

- More on decision tree
  - Precision Tree
  - Sensitivity analysis
  - Multi-stage problem