Question 1 (8 points)

Kate is a security analyst and has identified a gold ETF (ticker symbol GEF) as a particularly attractive investment. Currently GEF is trading at $100 per share. Kate notes that gold has underperformed in the last decade and believes that the time is ripe for a large increase in gold prices. Kate has constructed seven scenarios for the price of GEF one month from now. These scenarios and corresponding probabilities are shown below:

	Scen. 1	**Scen. 2**	**Scen. 3**	**Scen. 4**	**Scen. 5**	**Scen. 6**	**Scen. 7**
Probability	0.05	0.10	0.20	0.30	0.20	0.10	0.05
GEF price	150	130	110	100	90	80	70

a. (4 pts) What is the probability that the price of GEF is higher than $100 one month from now?

Solution:

Let X be the price of GEF one month from now, and use X_i to represent that the price shows as in Scene i.

$$\begin{aligned} P(X > 100) &= P(X_1) + P(X_2) + P(X_3) \\ &= 0.05 + 0.10 + 0.20 = 35\% \end{aligned}$$

b. (4 pts) What is the expected price of GEF one month from now?

Solution:

$$\begin{aligned} E(X) &= \sum_{i=1}^{7} P(X = X_i) E(X_i) \\ &= 0.05 * (70 + 150) + 0.10 * (80 + 130) + 0.20 * (90 + 110) + 0.30 * 100 \\ &= 102 \end{aligned}$$

Question 2 (12 points)

A family is considering a move from a midwestern city to a city in California. The distribution of housing costs where the family currently lives is normal, with mean $105,000 and standard deviation $18,200. The distribution of housing costs in the California city is normal with mean $235,000 and standard deviation $30,400. The family's current house is valued at $110, 000.

a. (4 pts) What percentage of houses in the family's current city cost less than theirs?

Solution:

Let M be one family's housing cost in the midwestern city and C be in California.

$$\begin{aligned} P(M \leq 110000) &= \mathrm{NORM.DIST}(110000, \mu_M, \sigma_M, 1) \\ &= \mathrm{NORM.DIST}(110000, 105000, 18200, 1) = 60.82\% \end{aligned}$$

b. (4 pts) If the family buys a $200,000 house in the new city, what percentage of houses there will cost less than theirs?

Solution:

$$\begin{aligned} P(C \leq 200000) &= \mathrm{NORM.DIST}(200000, \mu_C, \sigma_C, 1) \\ &= \mathrm{NORM.DIST}(200000, 235000, 30400, 1) = 12.48\% \end{aligned}$$

c. (4 pts) What price house will the family need to buy to be in the same percentile (of housing costs) in the new city as they are in the current city?

Solution:

Let X be the price of house that the family will need to buy.

$$\begin{aligned} P(C \leq X) &= P(M \leq 110000) \\ \mathrm{NORM.DIST}(X, \mu_C, \sigma_C, 1) &= \mathrm{NORM.DIST}(111000, \mu_M, \sigma_M, 1) \\ X &= \mathrm{NORM.INV}(\mathrm{NORM.DIST}(111000, \mu_M, \sigma_M, 1), \mu_C, \sigma_C) \\ &= \mathrm{NORM.INV}(\mathrm{NORM.DIST}(111000, 105000, 18200, 1), 235000, 30400) \\ &= 243352 \end{aligned}$$

Question 3 (10 points)

The amount of a soft drink that goes into a typical 12- ounce can varies from can to can. It is normally distributed with an adjustable mean and a fixed standard deviation of 0.05 ounce. (The adjustment is made to the filing machine)

a. (7 pts) If regulations require that cans have at least 11.9 ounces, what is the smallest mean that can be used so that at least 99.5% of all cans meet the regulation?

Solution:

$$
\begin{aligned}
P(X \geq 11.9) &\geq 99.5\% \\
P(Z \geq \frac{11.9-\mu}{\sigma}) &\geq \mathrm{NORM.S.INV}(0.995) \\
P(Z \leq \frac{11.9-\mu}{\sigma}) &\leq \mathrm{NORM.S.INV}(1-0.995) \\
\mu &\geq 11.9 - \sigma * \mathrm{NORM.S.INV}(0.005) \\
\mu &\geq 12.0288
\end{aligned}
$$

b. (3 pts) If the mean setting from part a is used, what is the probability that a typical can has at least 12 ounces?

Solution:

$$
\begin{aligned}
P(X \geq 12) &= 1 - \mathrm{NORM.DIST}(12, \mu, \sigma, 1) \\
&= 1 - \mathrm{NORM.DIST}(12, 12.03, 0.05, 1) = 71.76\%
\end{aligned}
$$

Question 4 (12 points)

SuperDrugs is a chain of drugstores with thirty similar-size stores in a given city. The sales in a given week for any of these stores in normally distributed with mean \$15,000 and standard deviation \$3,000.

a. (4 pts) What will the expected value of the **total** sales of the thirty stores be?

Solution:

$$
\begin{aligned}
\mu = E(\sum_{1\leq i\leq 30} X_i) &= \sum_{1\leq i\leq 30} E(X_i) \\
&= \sum_{1\leq i\leq 30} \mu = 30 * 15000 = 450,000
\end{aligned}
$$

b. (4 pts) What will the standard deviation of the **total** sales of the thirty stores be?

Solution:

$$\begin{aligned}\sigma = Stdev(\sum_{1\le i\le 30} X_i) &= \sqrt{Var(\sum_{1\le i\le 30} X_i)} \\ &= \sqrt{\sum_{1\le i\le 30} Var(X_i)} \\ &= \sqrt{\sum_{1\le i\le 30} Stdev^2(X_i)} \\ &= \sqrt{30 * 3000^2} \\ &= 16431.7\end{aligned}$$

c. (4 pts) What is the probability that the **total** sales of the thirty stores is at least \$500,000?

Solution:

As 30 drugstores are i.i.d. and the sales in a given week for any of these stores are normally distributed, the **total** sales of the thirty stores should also be normally distributed, with μ, σ calculated in **a, b**.

$$\begin{aligned}P(X \ge 500000) &= 1 - P(X \le 500000) \\ &= 1 - \mathrm{NORM.DIST}(500000, \mu, \sigma, 1) \\ &= \mathrm{NORM.DIST}(500000, 450000, 16431.7, 1) = 0.1172\%\end{aligned}$$

Question 5 (8 points)

Suppose that the delay times (i.e., differences between scheduled and actual arrival times) for a major airline are normally distributed with a mean of 10 minutes and a standard deviation of 20 minutes.

a. (4 pts) What is the probability that a randomly selected flight is not late?

Solution:

$$\begin{aligned}P(X \le 0) &= \mathrm{NORM.DIST}(0, \mu, \sigma, 1) \\ &= \mathrm{NORM.DIST}(0, 10, 20, 1) = 30.85\%\end{aligned}$$

b. (4 pts) Assume that 20 flights are randomly selected. Let X represent the number of flights in this sample that are not late. What is the expected value of X, and what is the probability that at least half of the flights are not late?

Solution:

From **a**, as 20 flights are i.i.d. and their individual delay times are normally distributed, we can say that $P(X_i \le 0) = \mathrm{NORM.DIST}(0, 10, 20, 1),\ i \in [1, 20]$

$$
\begin{aligned}
E(\sum_{1 \le i \le 20} P(X_i \le 0)) &= \sum_{1 \le i \le 20} E(P(X_i \le 0)) \\
&= 20 * \mathrm{NORM.DIST}(0, 10, 20, 1) = 6.17
\end{aligned}
$$

$$
\begin{aligned}
P(\sum_{1 \le i \le 20} P(X_i \le 0) \ge 10) &= 1 - P(\sum_{1 \le i \le 20} P(X_i \le 0) \le 9) \\
&= 1 - \mathrm{BINOM.DIST}(9, 20, \mathrm{NORM.DIST}(0, 10, 20, 1), 1) \\
&= 5.733\%
\end{aligned}
$$