BUDT 730
Data, Models and Decisions

Lecture 21

Decision Trees (IV)

Prof. Sujin Kim

Final Exam

Thursday, December 16, 1:30-3:30pm, VMH 1212

- A **seat map** will be posted in the morning of December 16.
- Coverage: Lecture 7-21 (from hypothesis test to decision tree), IA4, IA5, IA6 and TA2. Relevant quizzes.
- You are allowed to have one sheet of paper with notes (double-sided).
- Scratch papers for calculations will be given in the exam.
- Both notes and scratch papers will be collected after the exam.
- You are NOT allowed to use the book or other notes.
- You need to use Respondus lockdown browser to take the exam.
- You can also use a scientific calculator. You can also use the calculator in lockdown browser.
- The practice final exam will be posted under ‘Module’ later this week.
- Extra office hours: Tuesday, Dec 14, 10 am-12pm VMH - 1333 - ATK Classroom

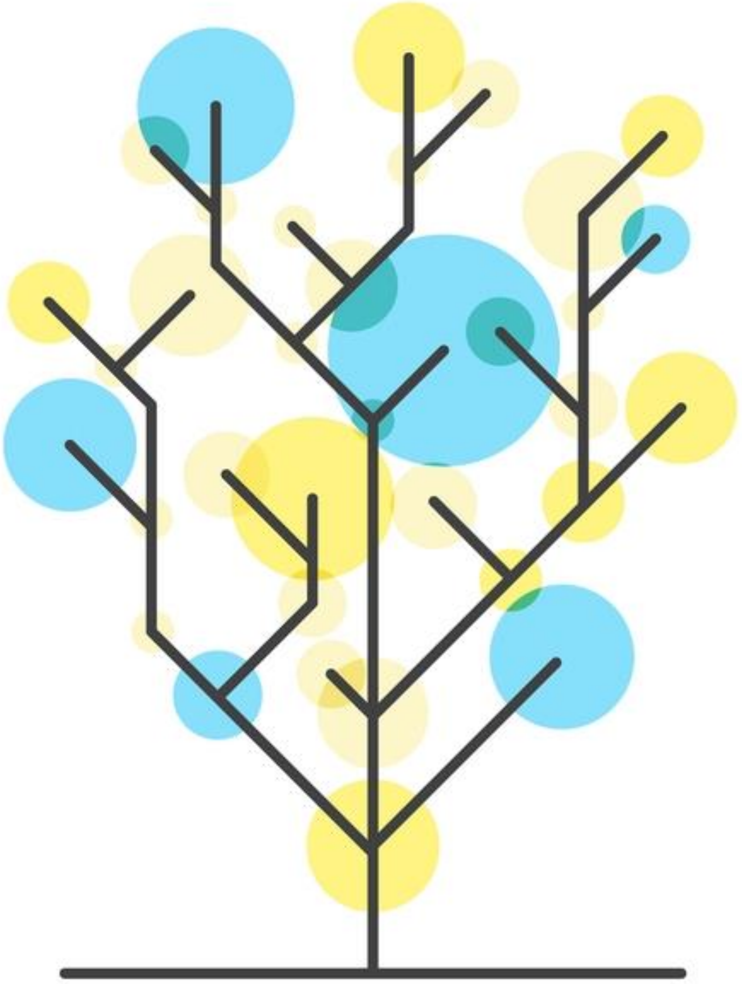
Learning Objectives

- Multi-stage problem
    - Construct a multi-stage decision tree
    - Learn how conditional probabilities are used in the decision-making process
    - Sensitivity Analysis
    - Examples:
        - Ann’s Auto Insurance Part B
        - Multi-Stage New Product Problem

Decision Tree

Multi-Stage

Decision Problem



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Multistage Decision Problems

- Many real-world decision problems evolve through time in stages.
- The objective is again to maximize EMV (or EU)
- We search for an EMV-maximizing strategy, often called a **contingency plan**, that specifies which decision to make at each stage.
  - A contingency plan tells the company which decision to make at the first stage, but the company won’t know which decision to make at the second stage until the information from the first uncertain outcome is known.

Multistage Decision Problems

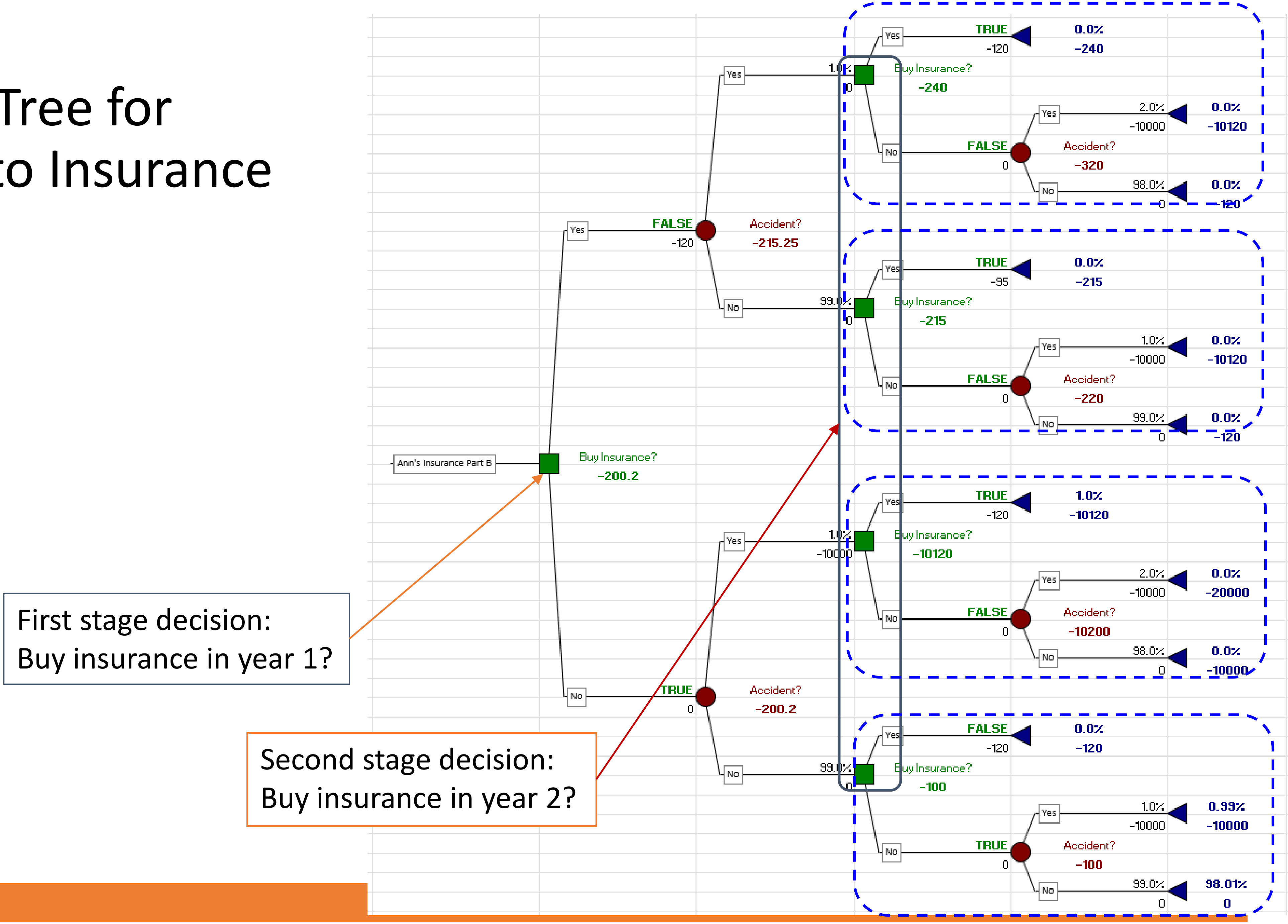
- An important aspect of multistage decision problems is that **probabilities can change through time**.
    - Specifically, after you receive the information from the first-stage uncertain outcome, you might need to reassess the probabilities of future uncertain outcomes.
- Another important aspect of multistage decision problems is **the value of information**.
    - Sometimes the first-stage decision is to buy information that will help in making the second-stage decision. The question then is how much this information is worth.
- We will use **conditional probability distributions** to update future outcomes and the corresponding probabilities.

Example of Two-Stage Decision Problem: Ann’s Auto Insurance - Part B

- Part B
  - Ann is now faced with purchasing an insurance policy for two consecutive years
  - To incentivize safe driving, she is offered a **$25 discount** for Year 2 **if she does not have any accident**
    - Only available if she purchases the insurance in Year 1
  - If Ann had an automobile accident in Year 1, she has a **2% chance** of being in an accident in Year 2 that will cost $10,000. Else, the chance of being in an accident in Year 2 is 1%.

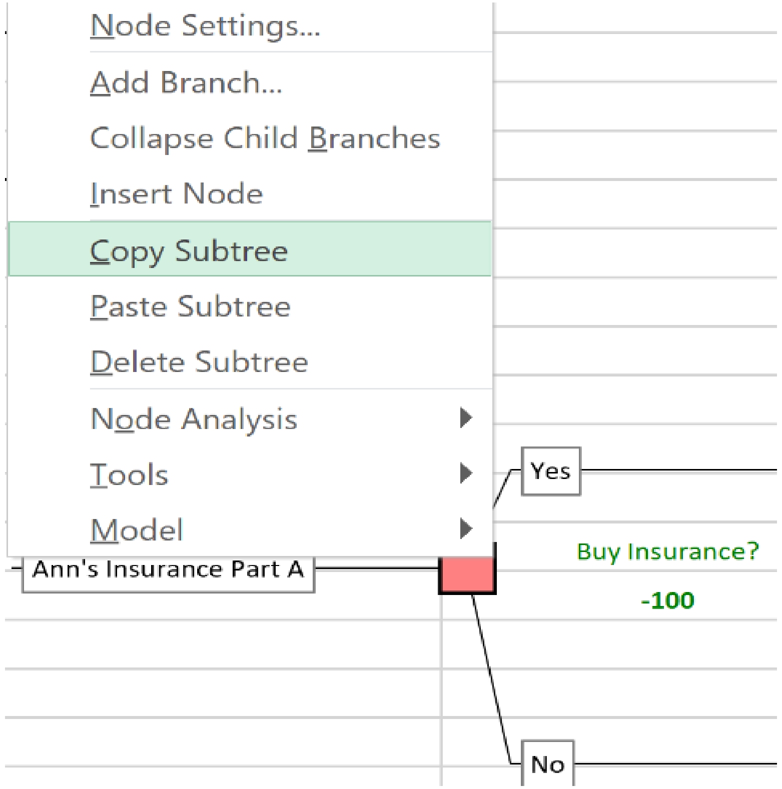
- How will this discount affect Ann’s decision?

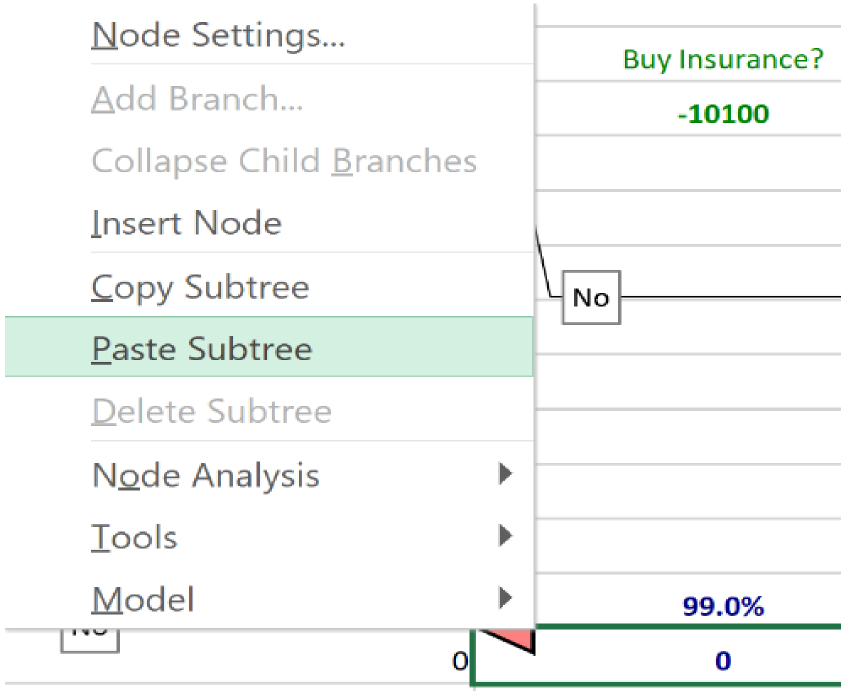
Decision Tree for Ann's Auto Insurance (Part B)



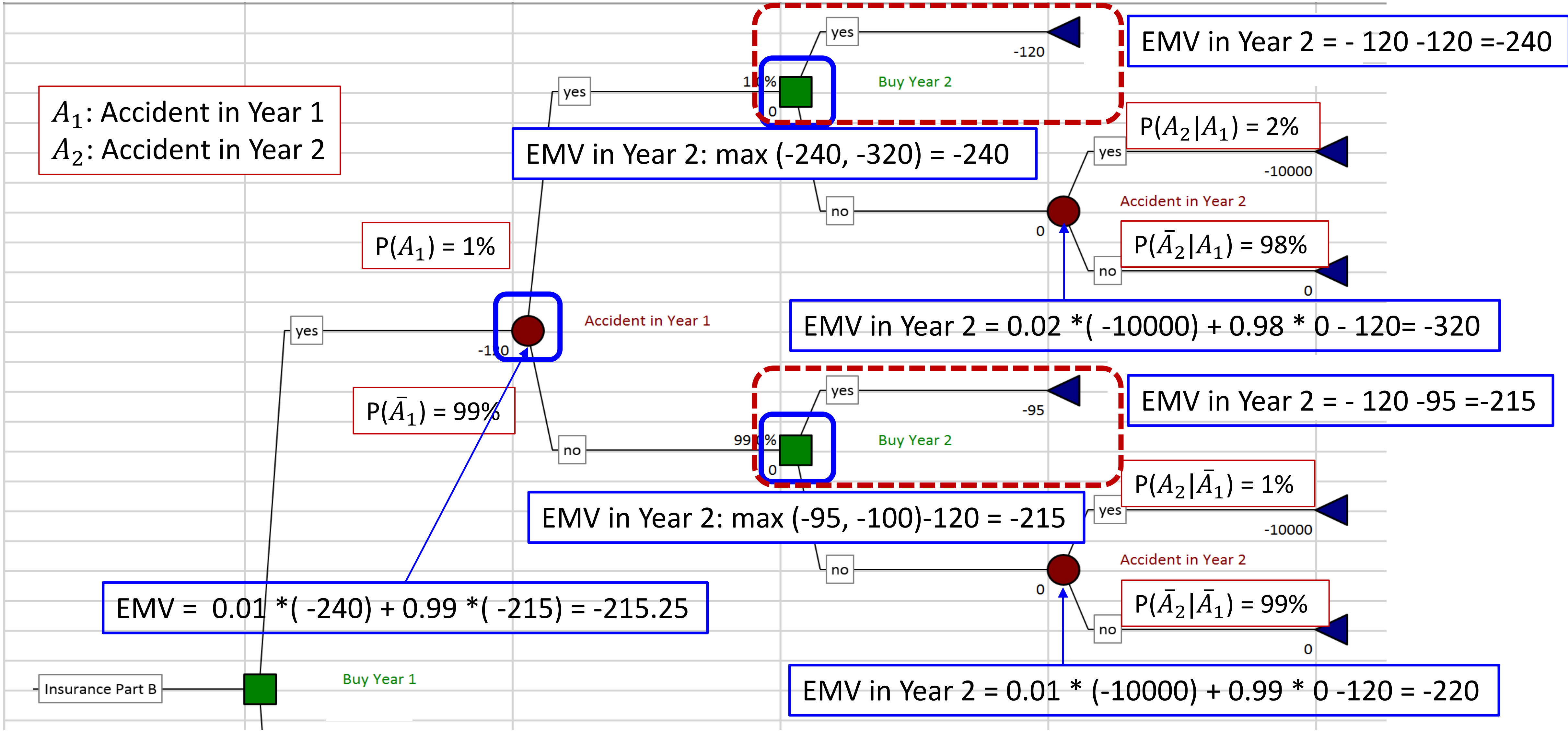
Exercise: Complete the decision tree for Part B

Note: Do not need to build this three from scratch; copy and paste Part A tree.

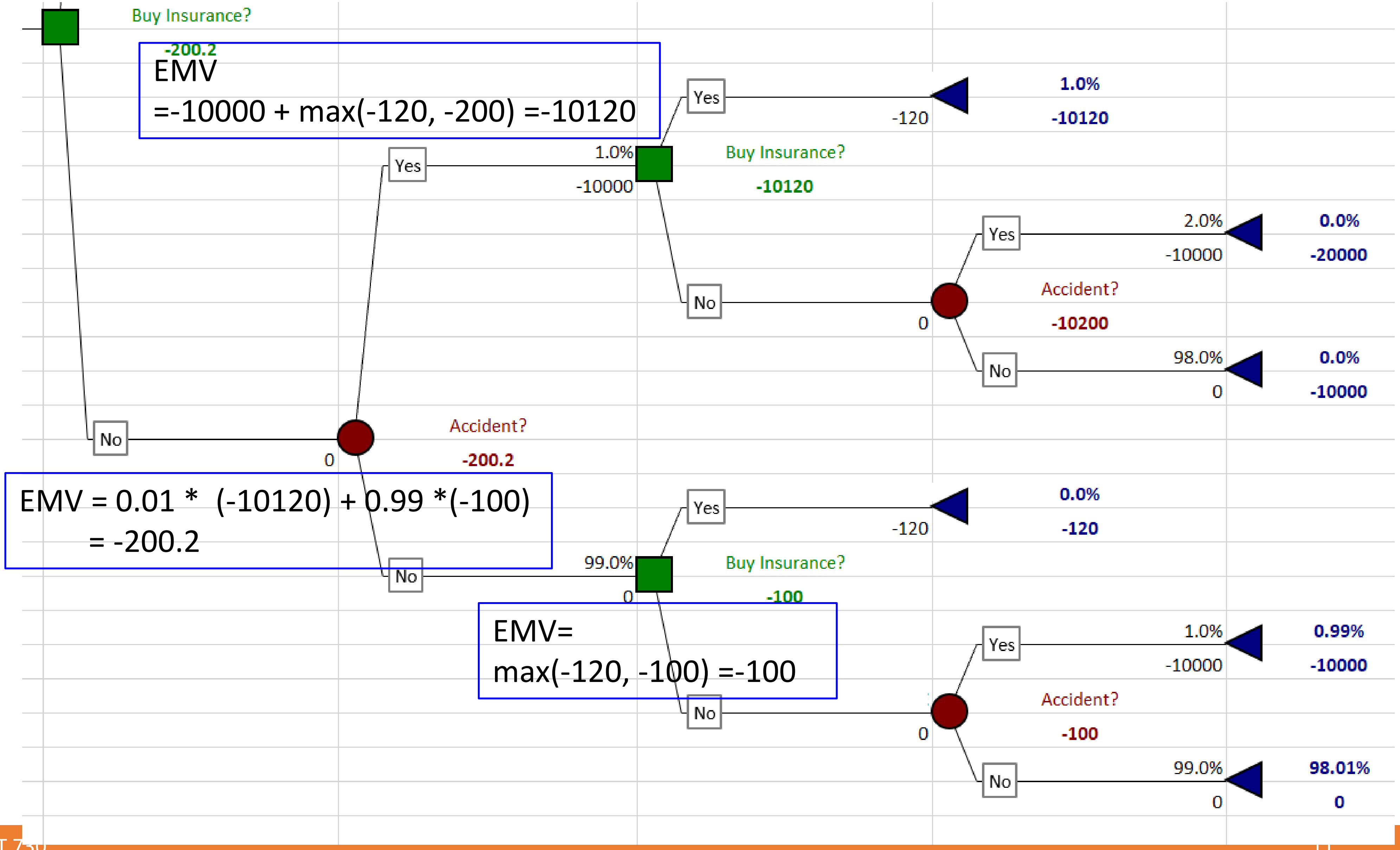




Decision Tree for Ann's Auto Insurance (Part B)



Decision Tree for Ann's Auto Insurance (Part B)



Ann’s Auto Insurance Part B

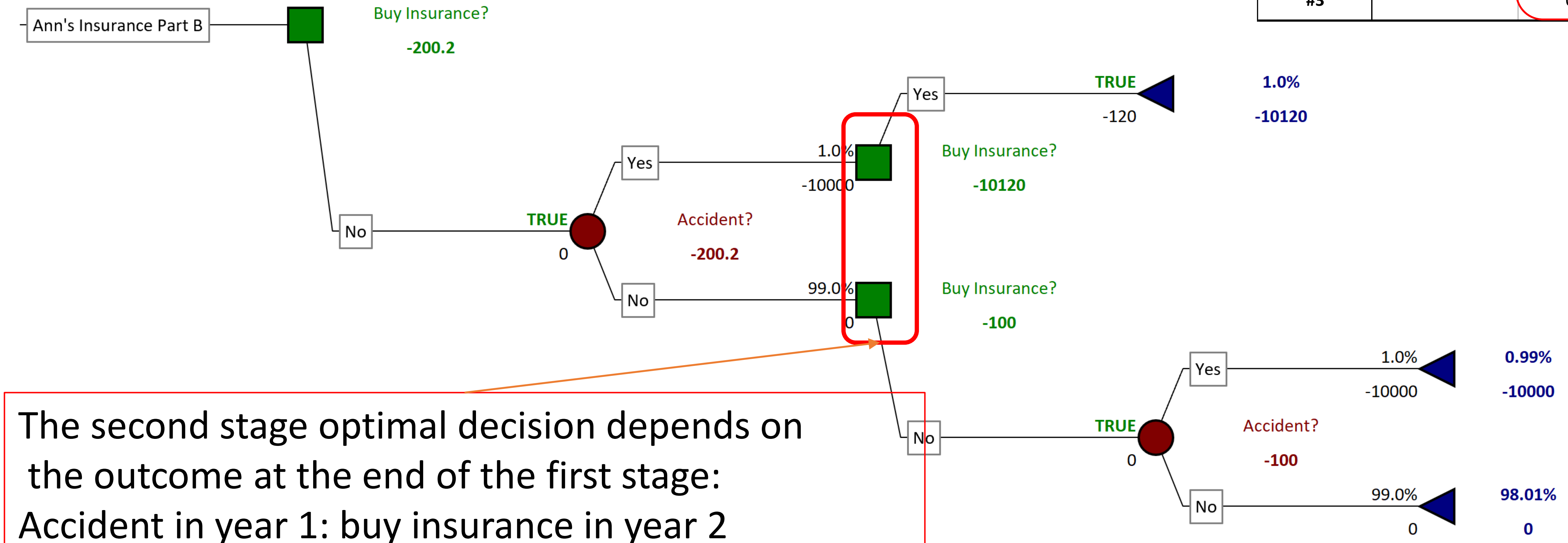
- EMV calculation
  - Buying insurance in year 1: EMV = -215.25
  - Not buying insurance in year 1: EMV = -200.2

- Does this scenario change Ann’s decision?
  - **No!**
  - The EMV for not buying insurance is -$200.2, which is greater than the EMV for buying insurance, which is -$215.25

Ann’s Auto Insurance Part B – Optimal Tree

Risk Profile

Chart Data	Yes		No	
	Value	Probability	Value	Probability
#1	-240	1.0000%	-10120	1.0000%
#2	-215	99.0000%	-10000	0.9900%
#3			0	98.0100%

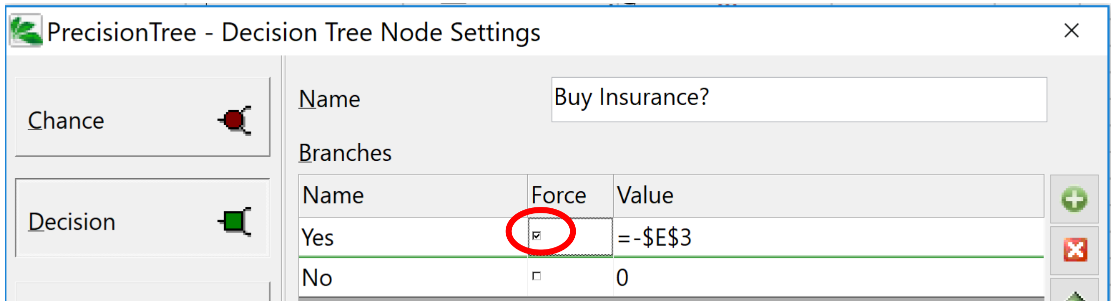


The second stage optimal decision depends on the outcome at the end of the first stage:

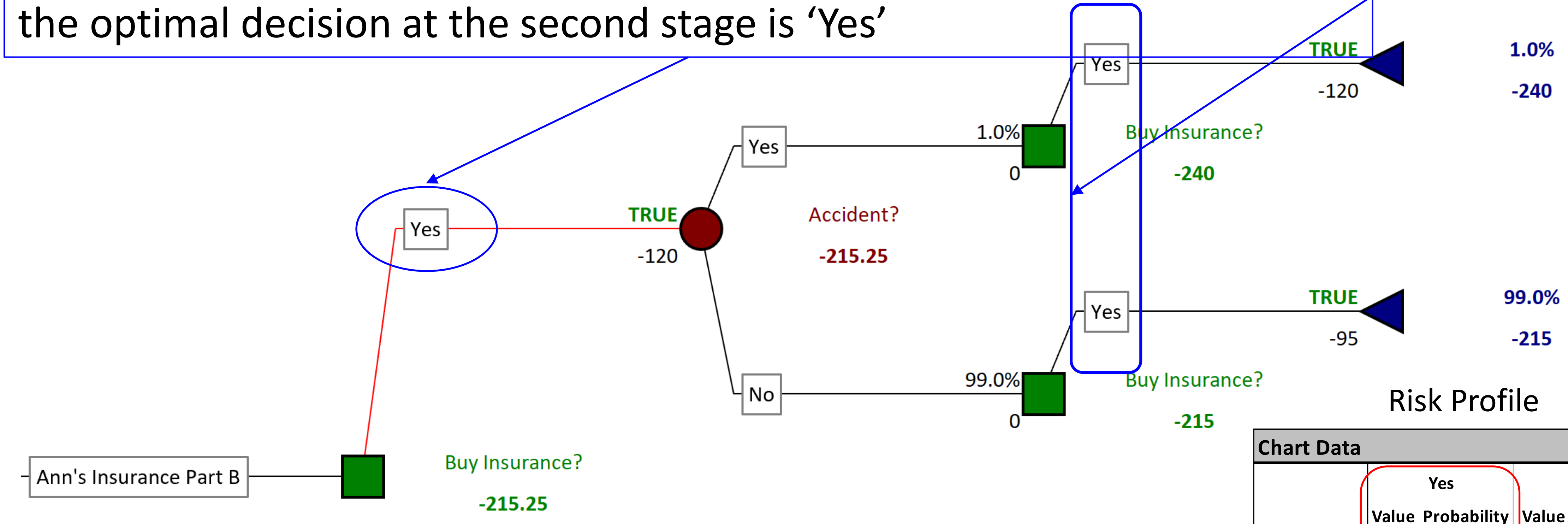
Accident in year 1: buy insurance in year 2

No accident in year 1: do not buy insurance in year 2

What-if Analysis



If the first stage decision is ‘Yes’ (this is not an optimal decision though), the optimal decision at the second stage is ‘Yes’



Risk Profile

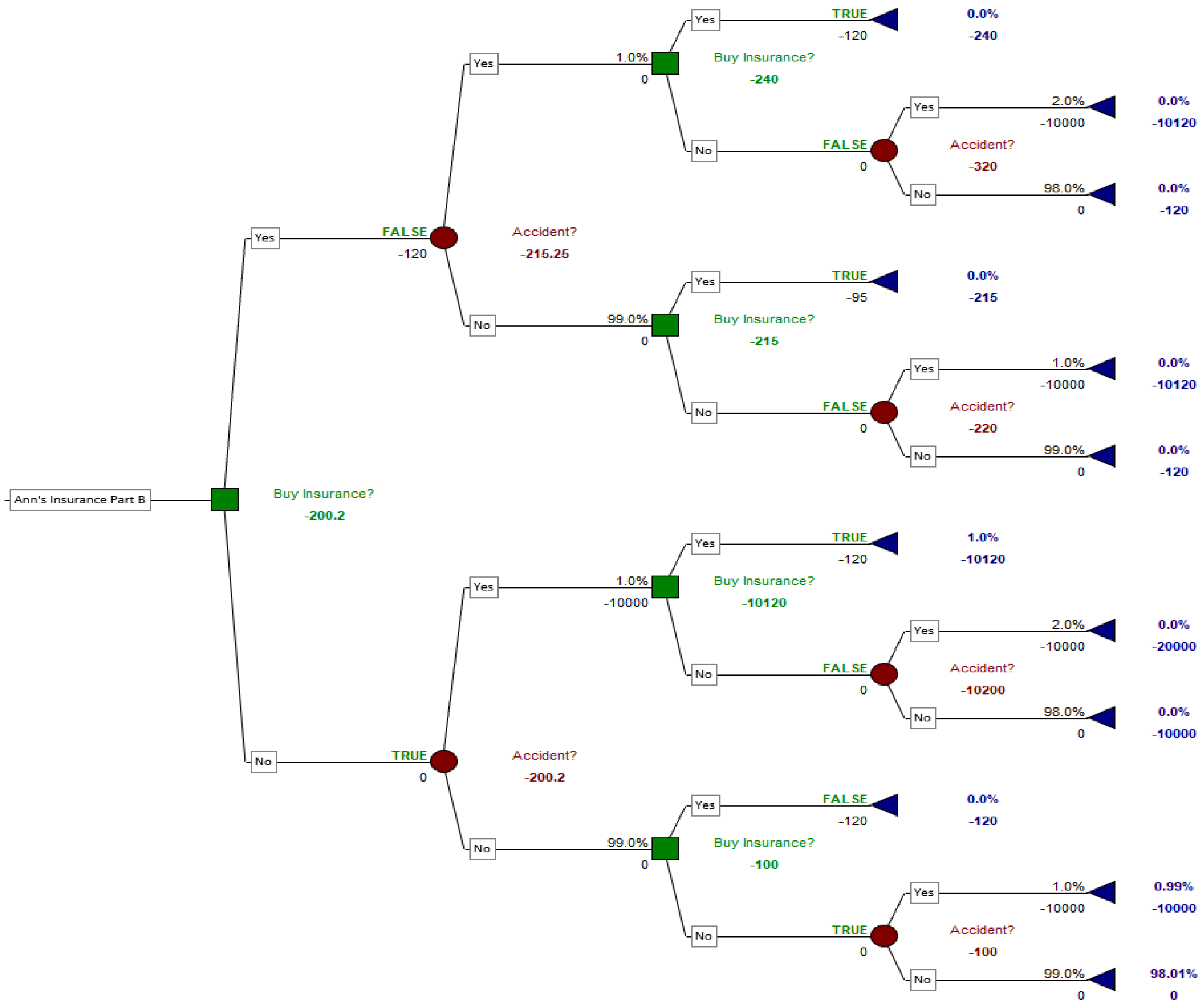
Chart Data

	Yes		No	
	Value	**Probability**	**Value**	**Probability**
#1	-240	1.0000%	-10120	1.0000%
#2	-215	99.0000%	-10000	0.9900%
#3			0	98.0100%

Risk Analysis

Risk Profile

Chart Data				
	Yes		No	
	Value	Probability	Value	Probability
#1	-240	1.0000%	-10120	1.0000%
#2	-215	99.0000%	-10000	0.9900%
#3			0	98.0100%



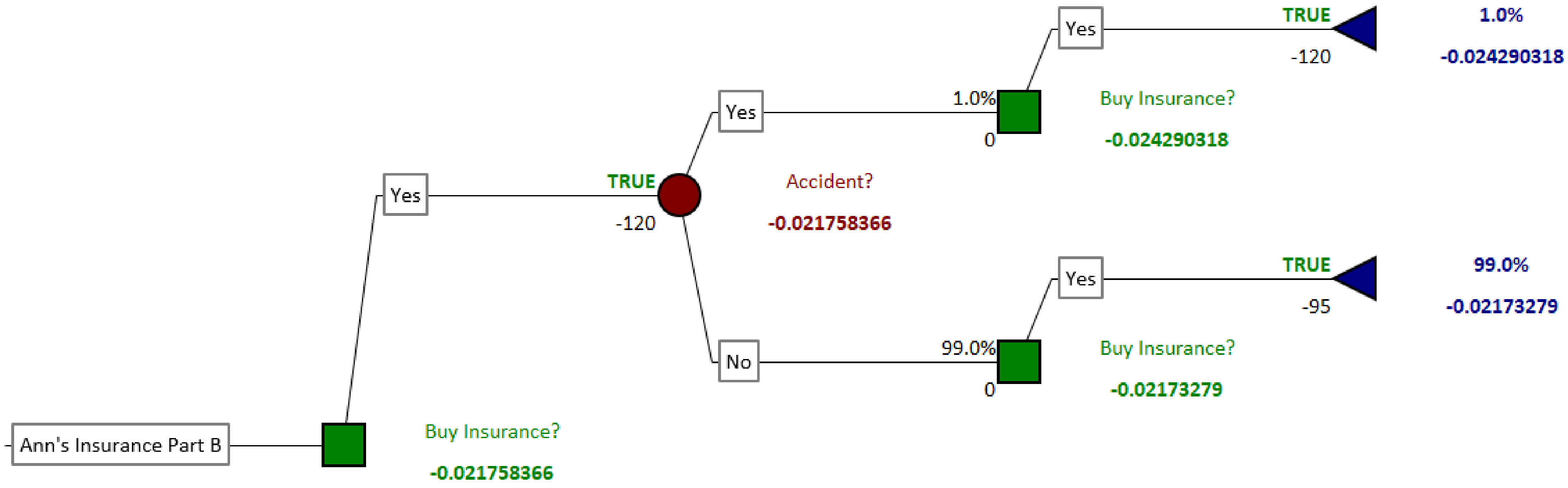
Ann’s Auto Insurance - Part B

EU Maximizer

Suppose Ann’s risk attitude is best represented by an exponential utility function with a risk tolerance R = $10,000.

Does this scenario change Ann’s decision?

EU Maximizer – Optimal Tree

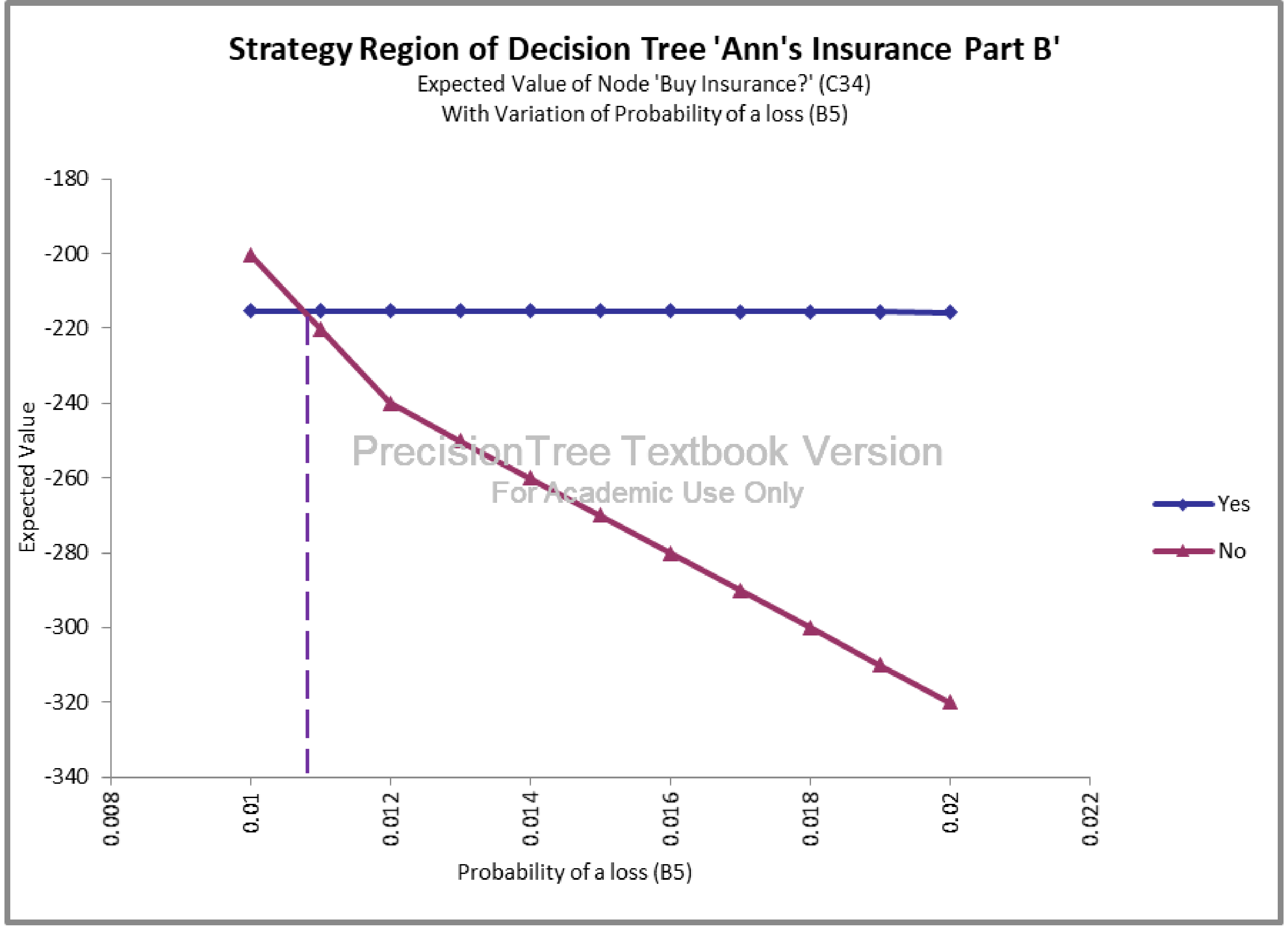


Ann's Auto Insurance Part B – Sensitivity Analysis

- Perform one-way sensitivity analysis, varying **probability of accident** from 1% to 2% (with 11 steps), and show the strategy region
  - As the probability <u>increases</u>, the EMV _____decreases___________
  - How about the optimal decision? No-> Yes

- Perform two-way sensitivity analysis, varying **cost of policy** from $80 - $200 (with 13 steps), and show the strategy region
  - As the cost of policy <u>decreases</u>, the EMV ___ increases ___________
  - How out the optimal decision? No-> Yes

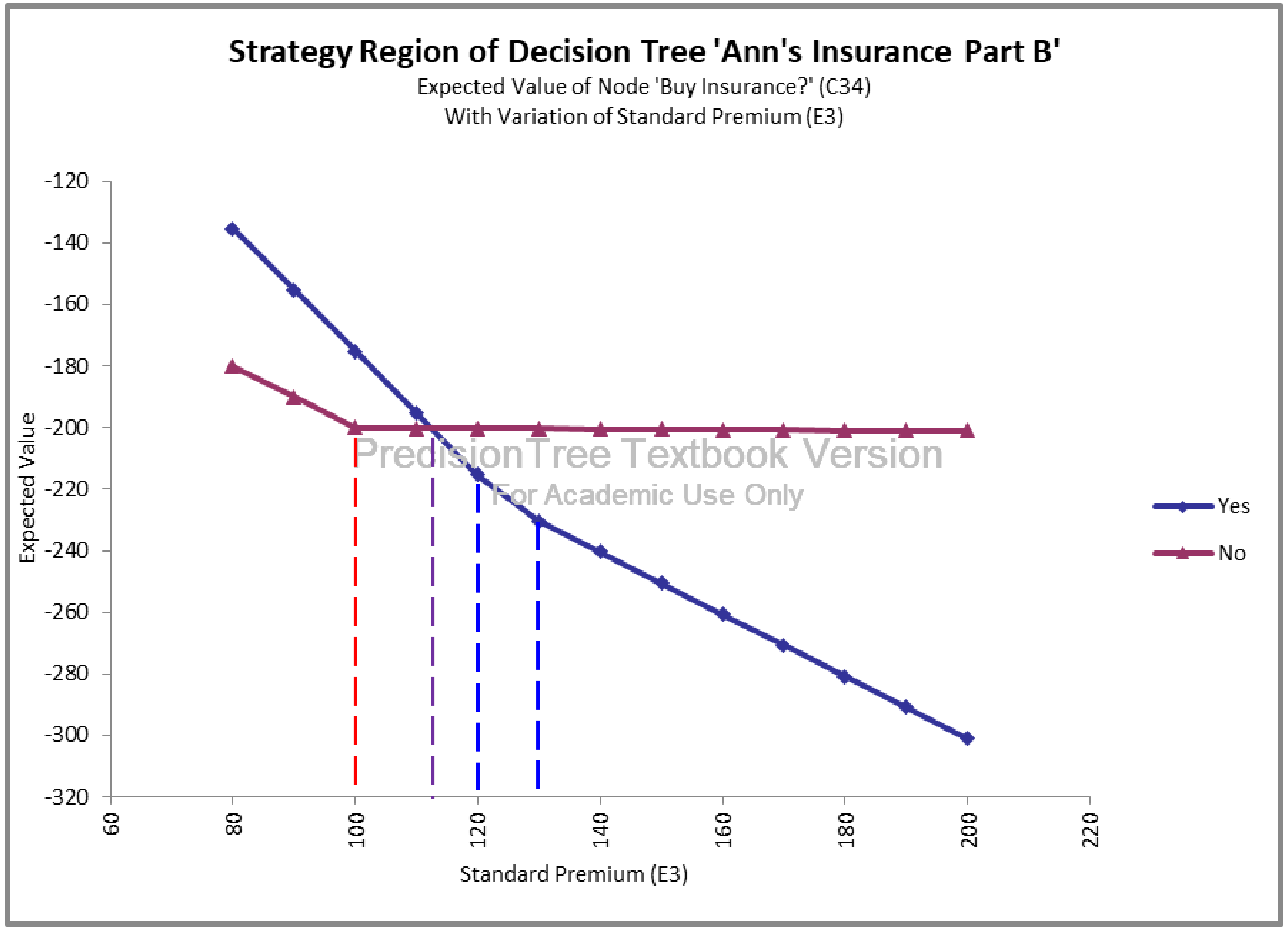
Ann's Auto Insurance Part B – Sensitivity Analysis on **probability of accident**



Strategy Region Data

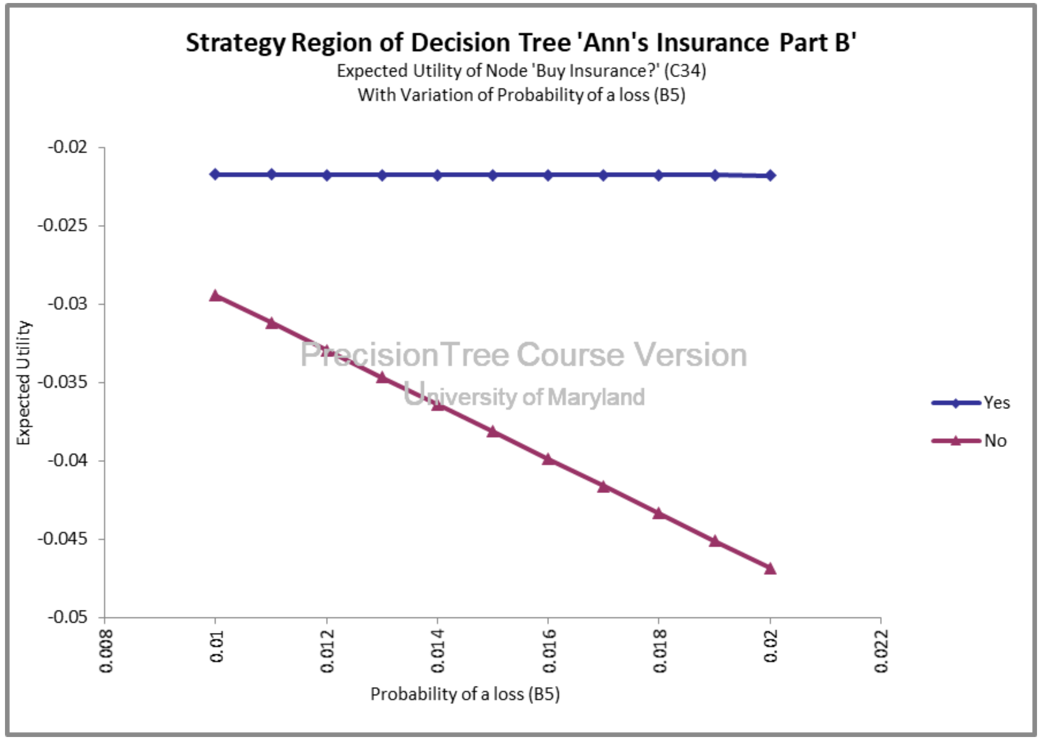
	Input		Yes		No	
	Value	Change (%)	Value	Change (%)	Value	Change (%)
#1	0.01	0.00%	-215.25	-7.52%	-200.2	0.00%
#2	0.011	10.00%	-215.275	-7.53%	-220.11	-9.95%
#3	0.012	20.00%	-215.3	-7.54%	-240	-19.88%
#4	0.013	30.00%	-215.325	-7.55%	-250	-24.88%
#5	0.014	40.00%	-215.35	-7.57%	-260	-29.87%
#6	0.015	50.00%	-215.375	-7.58%	-270	-34.87%
#7	0.016	60.00%	-215.4	-7.59%	-280	-39.86%
#8	0.017	70.00%	-215.425	-7.60%	-290	-44.86%
#9	0.018	80.00%	-215.45	-7.62%	-300	-49.85%
#10	0.019	90.00%	-215.475	-7.63%	-310	-54.85%
#11	0.02	100.00%	-215.5	-7.64%	-320	-59.84%

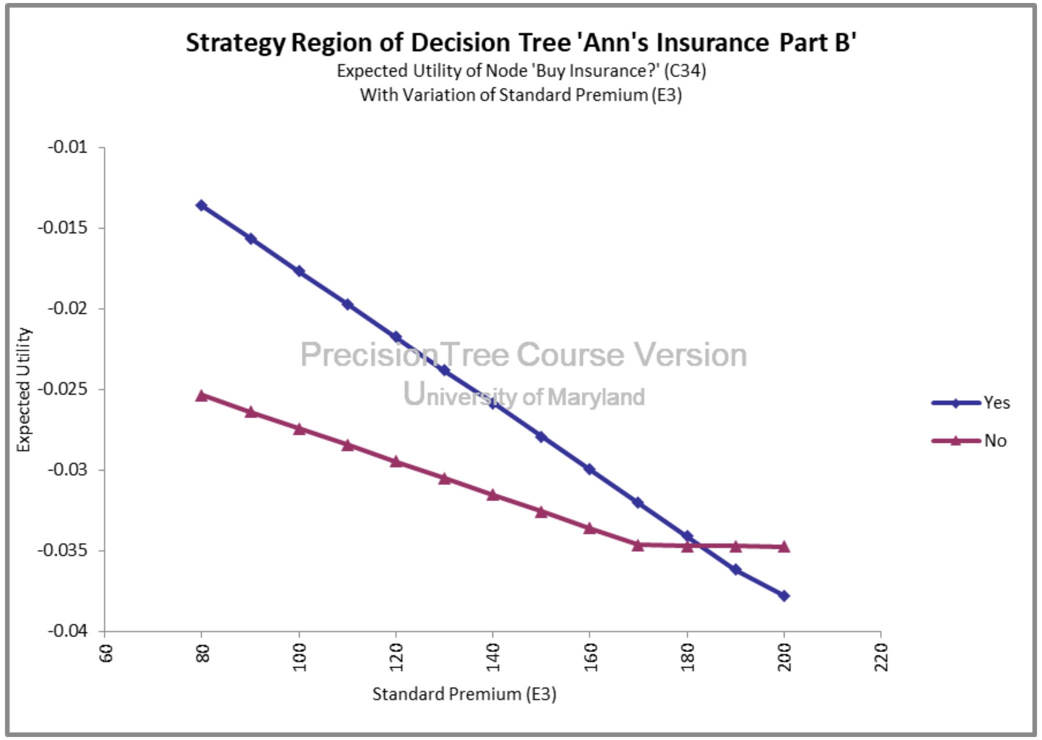
Ann’s Auto Insurance Part B – Sensitivity Analysis on **cost of policy**



Strategy Region Data						
	Input		Yes		No	
	Value	Change (%)	Value	Change (%)	Value	Change (%)
#1	80	-33.33%	-135.25	32.44%	-180	10.09%
#2	90	-25.00%	-155.25	22.45%	-190	5.09%
#3	100	-16.67%	-175.25	12.46%	-200	0.10%
#4	110	-8.33%	-195.25	2.47%	-200.1	0.05%
#5	120	0.00%	-215.25	-7.52%	-200.2	0.00%
#6	130	8.33%	-230.3	-15.03%	-200.3	-0.05%
#7	140	16.67%	-240.4	-20.08%	-200.4	-0.10%
#8	150	25.00%	-250.5	-25.12%	-200.5	-0.15%
#9	160	33.33%	-260.6	-30.17%	-200.6	-0.20%
#10	170	41.67%	-270.7	-35.21%	-200.7	-0.25%
#11	180	50.00%	-280.8	-40.26%	-200.8	-0.30%
#12	190	58.33%	-290.9	-45.30%	-200.9	-0.35%
#13	200	66.67%	-301	-50.35%	-201	-0.40%

EU Maximizer – Sensitivity Analysis





Example: New Product Decisions

With Option to Buy Information

(Example 6.3 in text)

New Product Decisions with Option to Buy Information -Two stage problem

- If the company decides to market the product, it will incur fixed marketing cost of $**2 million**

(Note: This marketing cost is much higher than the fixed cost for the single stage problem ($6,000)).

- There are two possible outcomes, bad or good
  - Bad with probability 0.6: sales volume = 100,000
  - Good with probability 0.4: sales volume = 600,000
- Unit margin = $18
- All monetary values are in $1000s, and all sales volumes are in 1000s of units.

Market	Sales volume	Net revenue
Good	600	$10,800
Bad	100	$1,800

New Product Decisions with Option to Buy Information -Two stage problem

- Acme has an option to **hire a marketing research firm** for $150,000 before making the ultimate decision

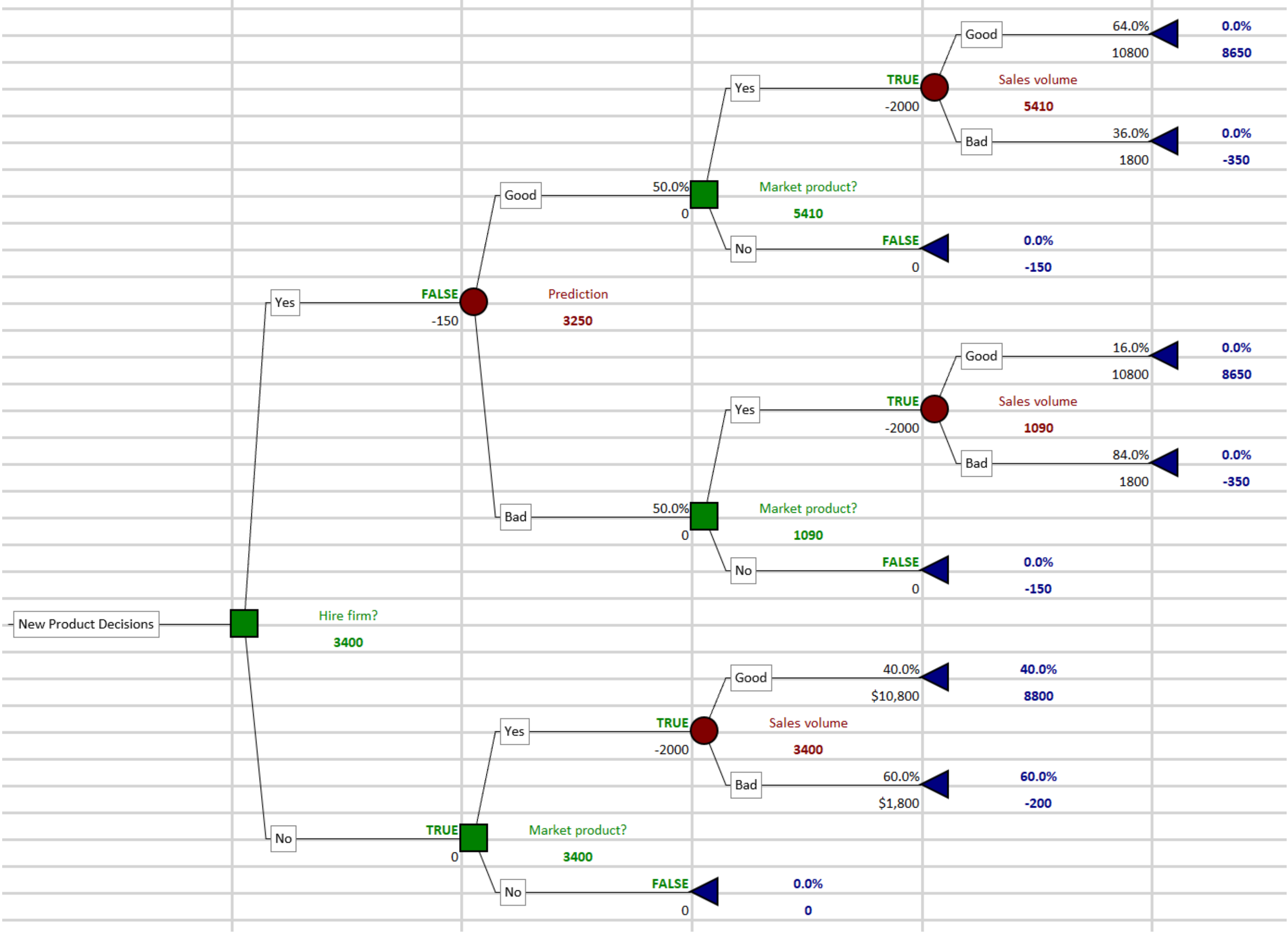
- Prediction Accuracy of Marketing Research Firm (based on historical data):
  - Conditional Probabilities: **P(Prediction|Actual)**

Actual/Predicted	Good	Bad
Good	$P(P_{Good}\|A_{Good}) = 0.8$	$P(P_{Bad}\|A_{Good}) = 0.2$
Bad	$P(P_{Good}\|A_{Bad}) = 0.3$	$P(P_{Bad}\|A_{Bad}) = 0.7$

New Product Decisions with Option to Buy Information

- **Objective**: To use a decision tree to see whether the marketing research firm is worth its cost and whether the product should be marketed
- **Two-Stage Decision Problem**
  - First stage Decision: Acme must decide whether to hire the marketing research firm.
  - Second stage Decision
    - If it decides not to, it can then immediately decide whether to market the product.
    - If it decides to hire the firm, it must then wait for the firm's prediction. After the prediction is received, Acme can then make the ultimate decision on whether to market the product.

Decision Tree for Two-Stage ACME Problem



Posterior Probabilities

- Acme needs to update probabilities as new information becomes available.
- The original probabilities are called **prior probabilities**. Then information is observed, and the prior probabilities needs to be updated to **posterior probabilities**.

Posterior Probabilities

- Given information
  - Prior probabilities: $P(\mathrm{A}_{Good}) = 0.4, P(\mathrm{A}_{Bad}) = 0.6$
  - Conditional Probabilities (based on historical data): **P(Prediction|Actual)**

Actual/Predicted	Good	Bad
Good	$P(P_{Good}\|A_{Good}) = 0.8$	$P(P_{Bad}\|A_{Good}) = 0.2$
Bad	$P(P_{Good}\|A_{Bad}) = 0.3$	$P(P_{Bad}\|A_{Bad}) = 0.7$

- We would like to **compute posterior probabilities**:
  - $P(P_{Good})$ and $P(P_{Bad})$
  - $P(A_{Good}|P_{Good}), P(A_{Bad}|P_{Good}), P(A_{Good}|P_{Bad}), P(A_{Bad}|P_{Bad})$

Posterior Probabilities

- Given information
  - Prior probabilities: $P(\mathrm{A}_{Good}) = 0.4, P(\mathrm{A}_{Bad}) = 0.6$
  - Conditional Probabilities (based on historical data): **P(Prediction|Actual)**

Actual/Predicted	Good	Bad
Good	$P(P_{Good}\|A_{Good}) = 0.8$	$P(P_{Bad}\|A_{Good}) = 0.2$
Bad	$P(P_{Good}\|A_{Bad}) = 0.3$	$P(P_{Bad}\|A_{Bad}) = 0.7$

- We would like to **compute posterior probabilities**:
  - $P(P_{Good})$ and $P(P_{Bad})$
  - $P(A_{Good}|P_{Good}), P(A_{Bad}|P_{Good}), P(A_{Good}|P_{Bad}), P(A_{Bad}|P_{Bad})$
  - We can compute posterior probabilities using Bayes' Rule: refer to Text 6.6.a

$$\mathrm{P}(P_{Good}) = P(P_{Bad}) = \frac{1}{2}, \mathrm{P}(A_{Good}|P_{Good}) = 0.64,\ P(A_{Good}|P_{Bad}) = 0.16$$

Bayes' Rule and Posterior Probabilities (You may skip this step)

- Apply the law of total probability

$$P(P_{Good}) = P(P_{Good} \cap A_{Good}) + P(P_{Good} \cap A_{Bad})$$

$$= P(P_{Good}|A_{Good})P(A_{Good}) + P(P_{Good}|A_{Bad})P(A_{Bad}) = \frac{1}{2}$$

$$P(P_{Bad}) = 1 - P(P_{Good}) = \frac{1}{2}$$

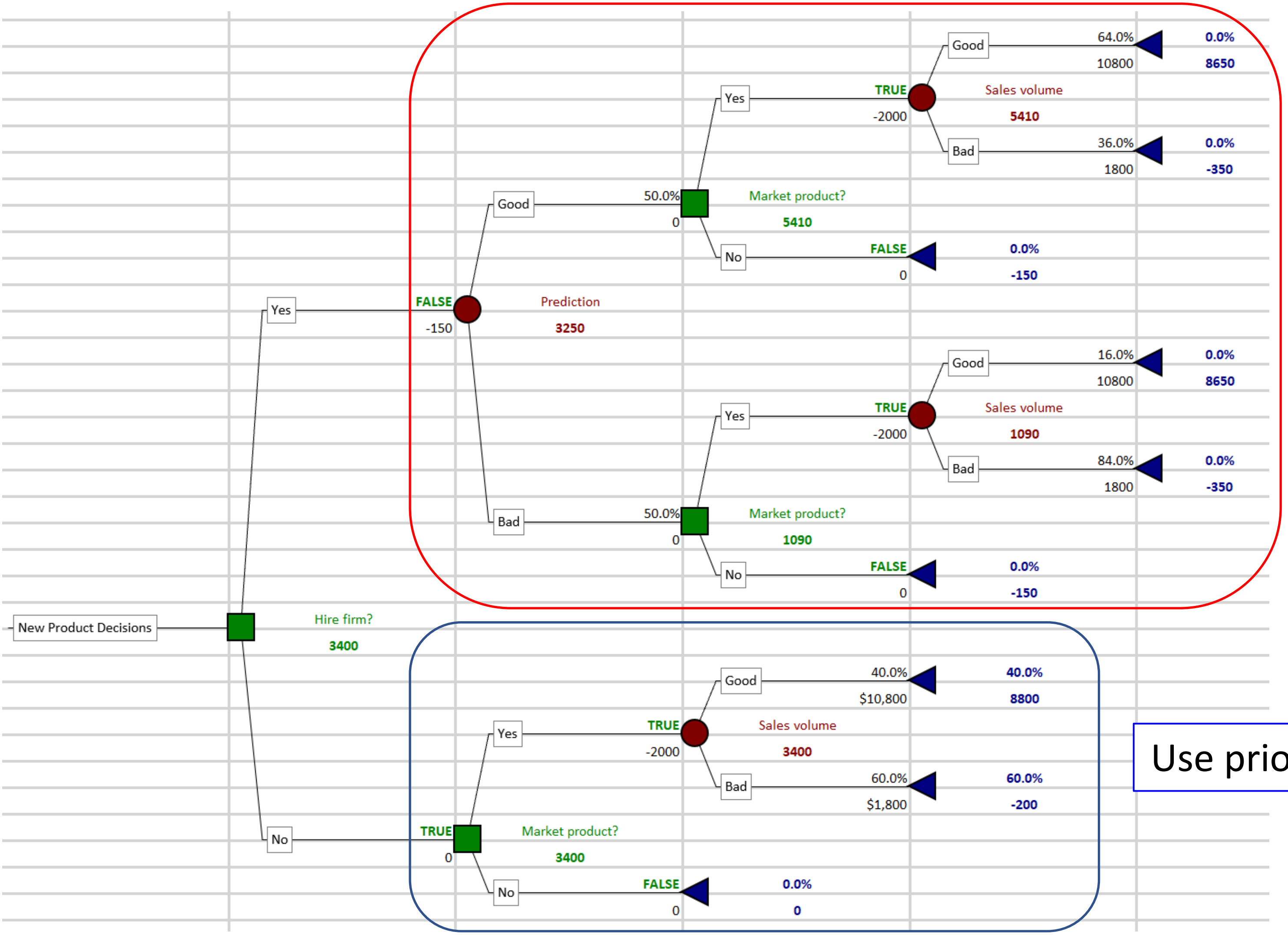
- Apply Bayes' rule:

$$P(A_{Good}|P_{Good}) = \frac{P(A_{Good} \cap P_{Good})}{P(P_{Good})}$$

$$= \frac{P(P_{Good}|A_{Good})P(A_{Good})}{P(P_{Good})} = 0.64$$

$$P(A_{Good}|P_{Bad}) = \frac{P(P_{Bad}|A_{Good})P(A_{Good})}{P(P_{Bad})} = 0.16$$

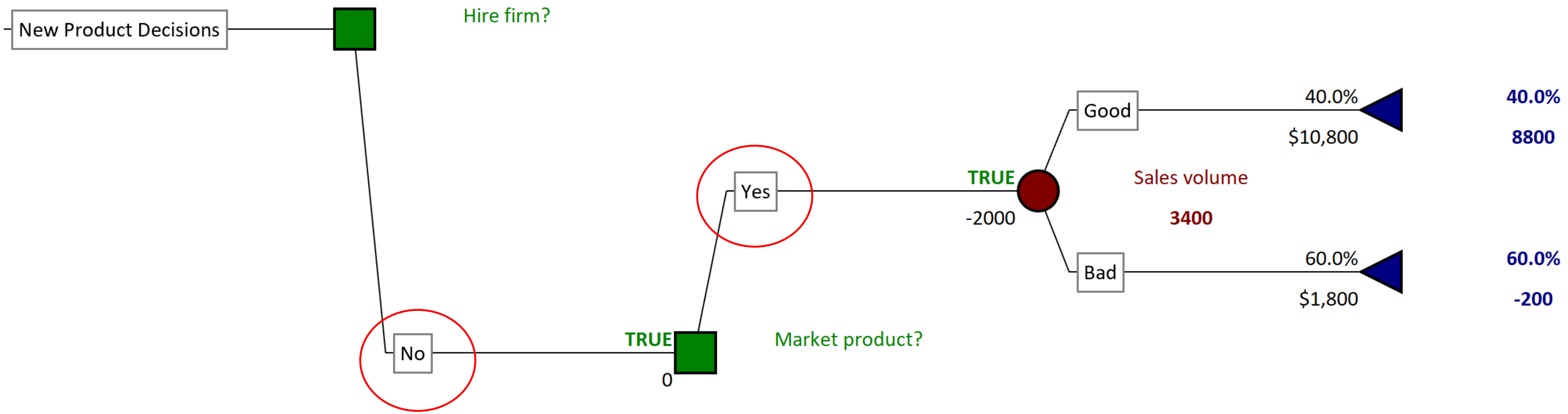
Decision Tree for Two-Stage ACME Problem



Use posterior probabilities

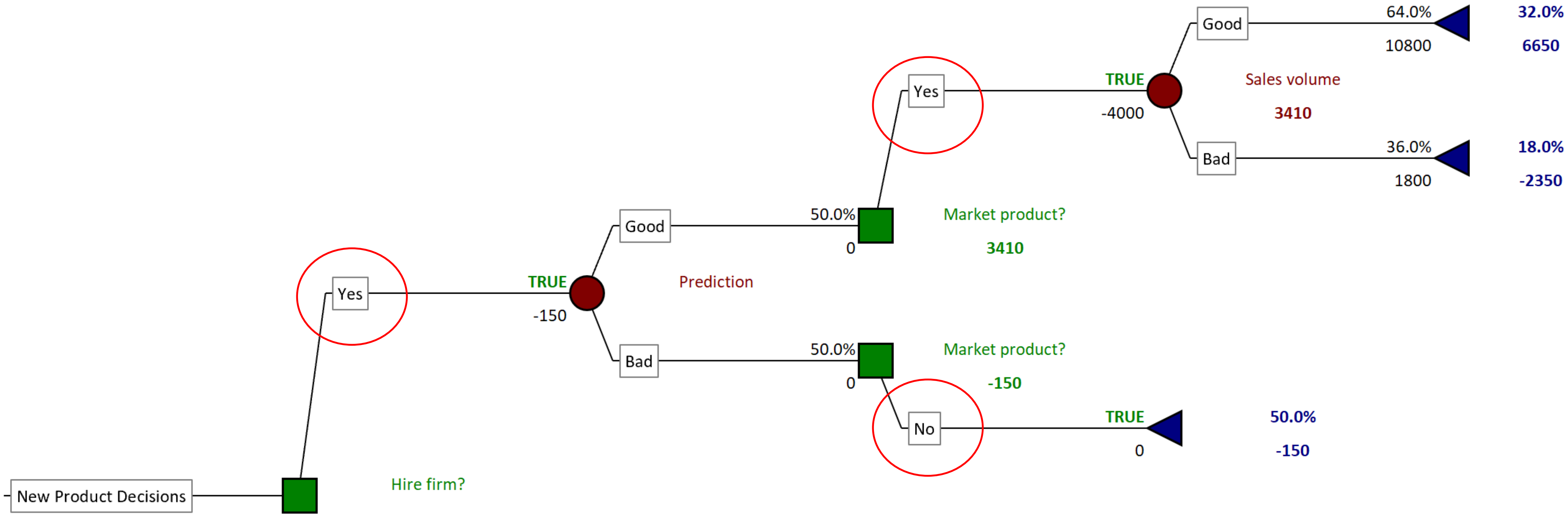
Use prior probabilities

Optimal Decision Tree:
Fixed marketing cost = $2 million



- What are the possible outcomes?
- What is the probability that Acme earns a profit?
- What is the EMV of this decision?

Optimal Decision Tree:
Fixed marketing cost = $4 million



- What are the possible outcomes?
- What is the probability that Acme earns a profit?
- What is the EMV of this decision?

Expected Value of Information (EVI)

- The value of information
  - In a decision-making context, information is usually bought to reduce the uncertainty about some outcome.
  - The **expected value of information** is the amount a firm would be willing to pay for information and is given by the formula:

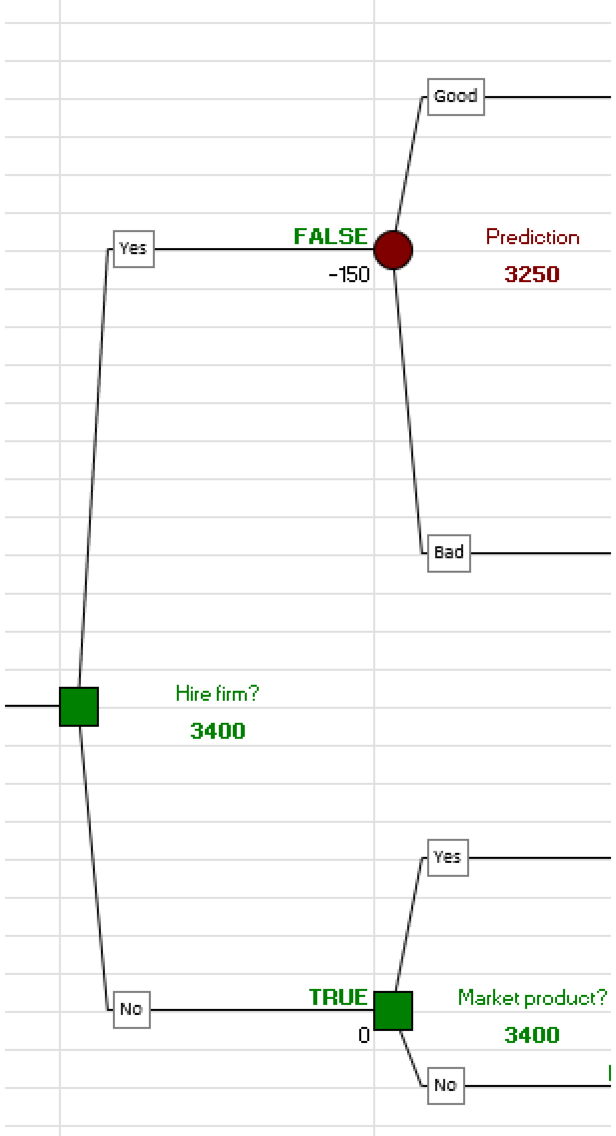
EVI

= EMV with (free) information – EMV without information

- Example: fixed marketing cost = **2 mil**

EVI = (3,250+150) – 3,400 = 0

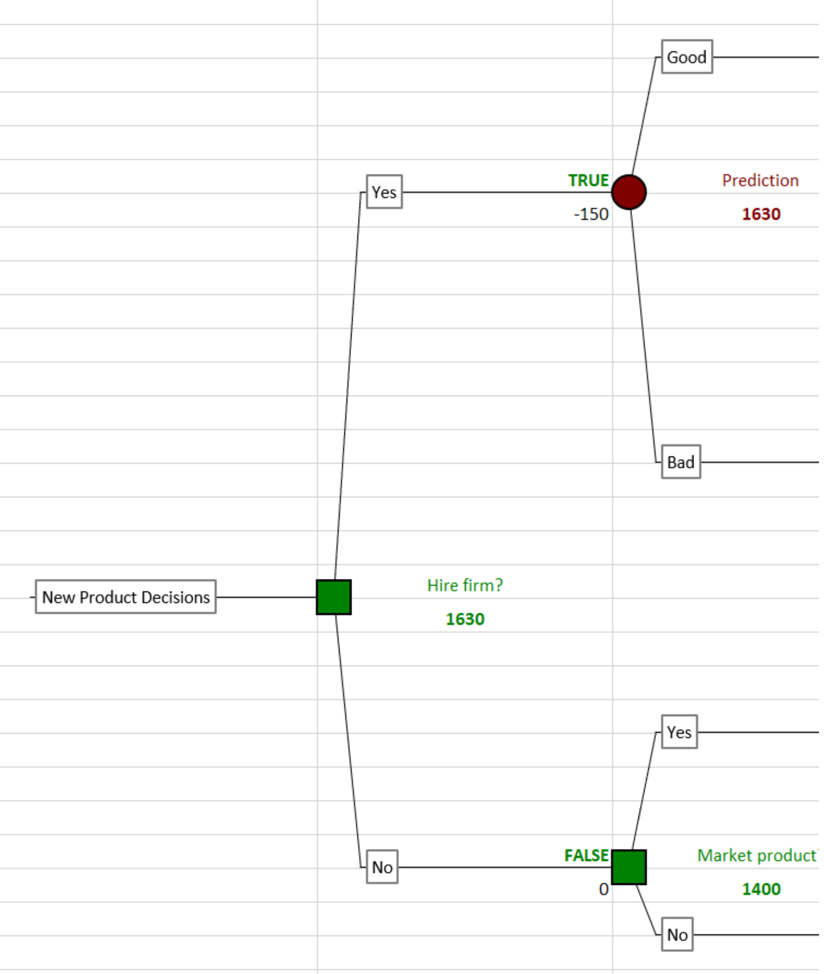
Acme might be willing to pay up to $ 0 to the marketing research firm.



Expected Value of Information (EVI)

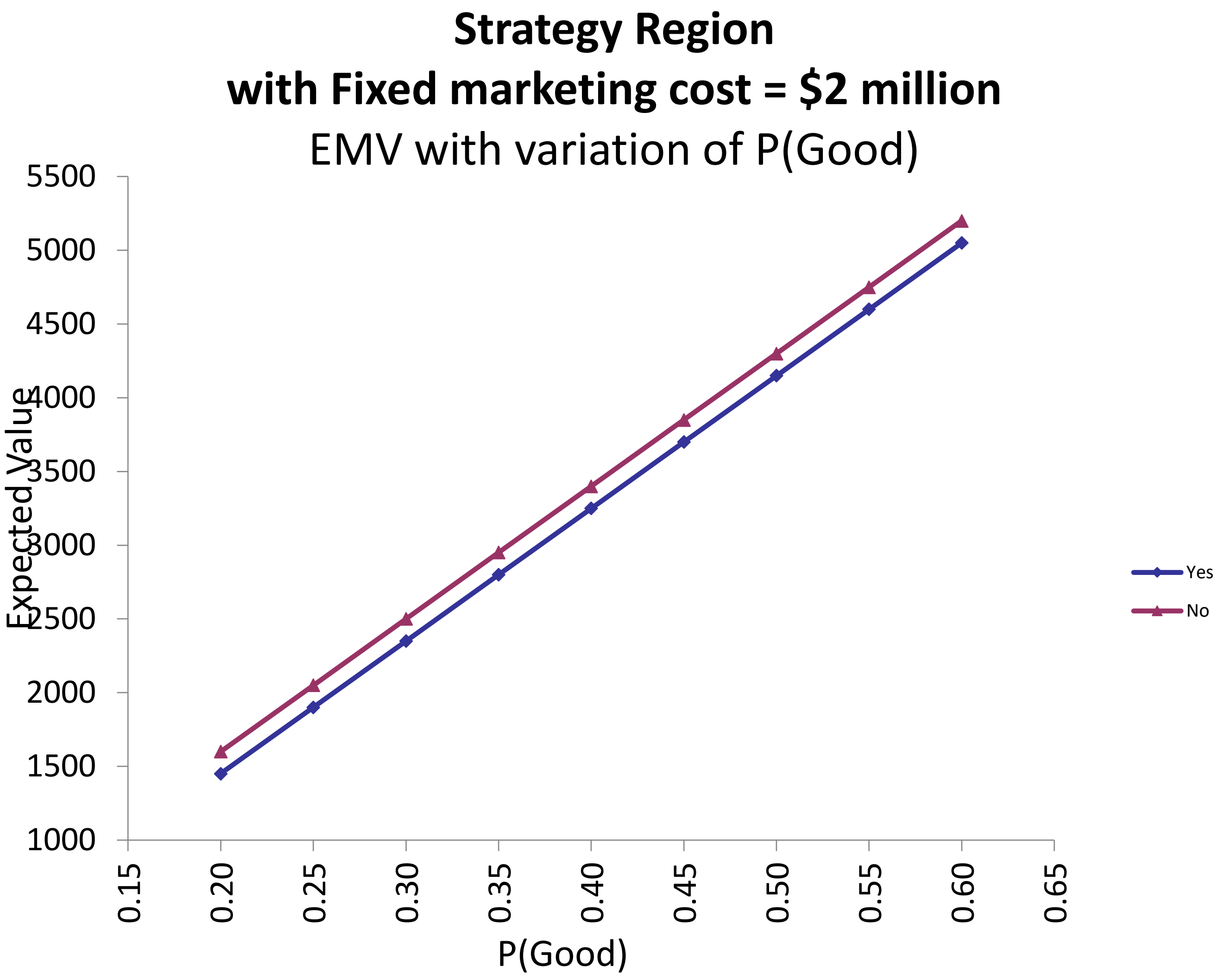
- Example: fixed marketing cost = **4 mil**

  EVI = ?



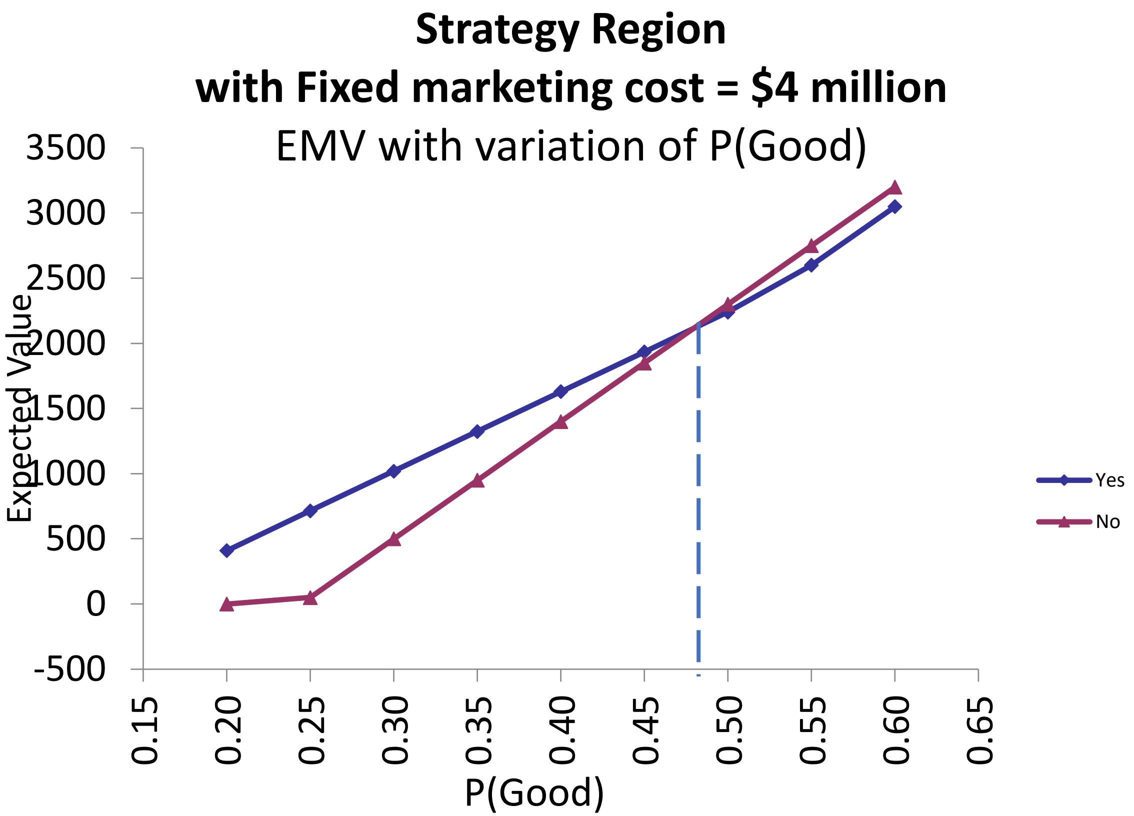
Sensitivity Analysis

- A **strategy region graph** is useful for seeing whether the optimal decision *changes* over the range of the input variable.
- It does so only if the two lines cross.



Sensitivity Analysis

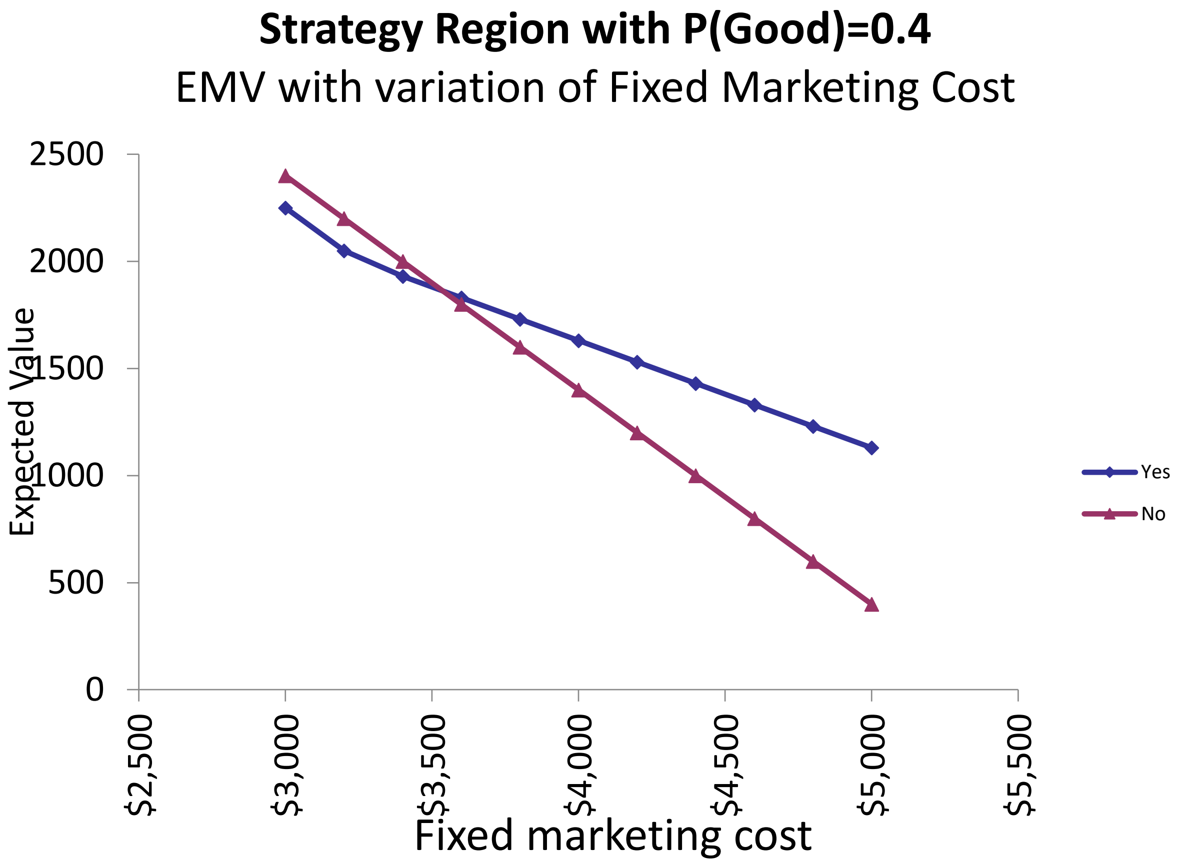
Question: Explain how the optimal decision changes over the range of the probability.



Strategy Region Data						
	Input		Yes		No	
	Value	Change (%)	Value	Change (%)	Value	Change (%)
#1	0.20	-50.00%	410	-74.85%	0	-100.00%
#2	0.25	-37.50%	715	-56.13%	50	-96.93%
#3	0.30	-25.00%	1020	-37.42%	500	-69.33%
#4	0.35	-12.50%	1325	-18.71%	950	-41.72%
#5	0.40	0.00%	1630	0.00%	1400	-14.11%
#6	0.45	12.50%	1935	18.71%	1850	13.50%
#7	0.50	25.00%	2240	37.42%	2300	41.10%
#8	0.55	37.50%	2600	59.51%	2750	68.71%
#9	0.60	50.00%	3050	87.12%	3200	96.32%

Sensitivity Analysis

Q: Explain how the optimal decision changes over the range of the marketing cost.



Strategy Region Data

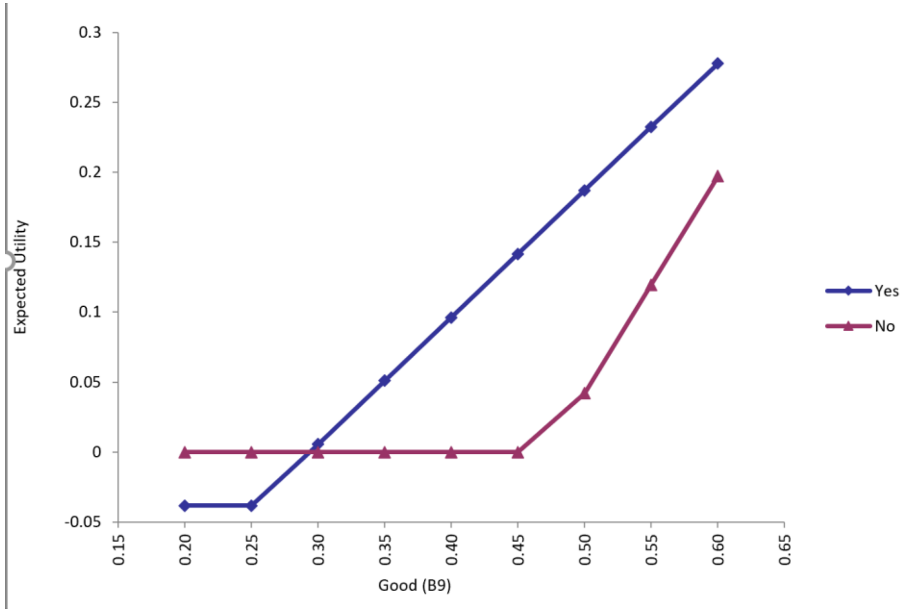
	Input		Yes		No	
	Value	Change (%)	Value	Change (%)	Value	Change (%)
#1	$3,000	-25.00%	2250	38.04%	2400	47.24%
#2	$3,200	-20.00%	2050	25.77%	2200	34.97%
#3	$3,400	-15.00%	1930	18.40%	2000	22.70%
#4	$3,600	-10.00%	1830	12.27%	1800	10.43%
#5	$3,800	-5.00%	1730	6.13%	1600	-1.84%
#6	$4,000	0.00%	1630	0.00%	1400	-14.11%
#7	$4,200	5.00%	1530	-6.13%	1200	-26.38%
#8	$4,400	10.00%	1430	-12.27%	1000	-38.65%
#9	$4,600	15.00%	1330	-18.40%	800	-50.92%
#10	$4,800	20.00%	1230	-24.54%	600	-63.19%
#11	$5,000	25.00%	1130	-30.67%	400	-75.46%

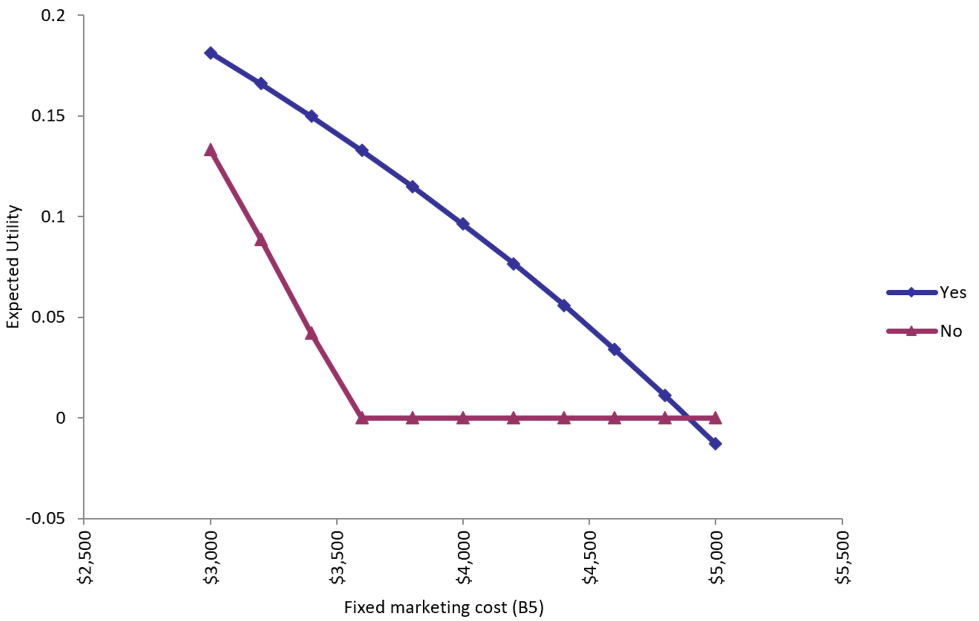
Sensitivity Analysis:
Expected Exponential Utility with R= 4000

Rerun the model with the exponential utility function with R= 4000

Q1: Explain how the optimal decision changes over the range of the probability.

Q2: Explain how the optimal decision changes over the range of the marketing cost.





Next …

- Final Exam