Reinforcement Learning Lecture 1b

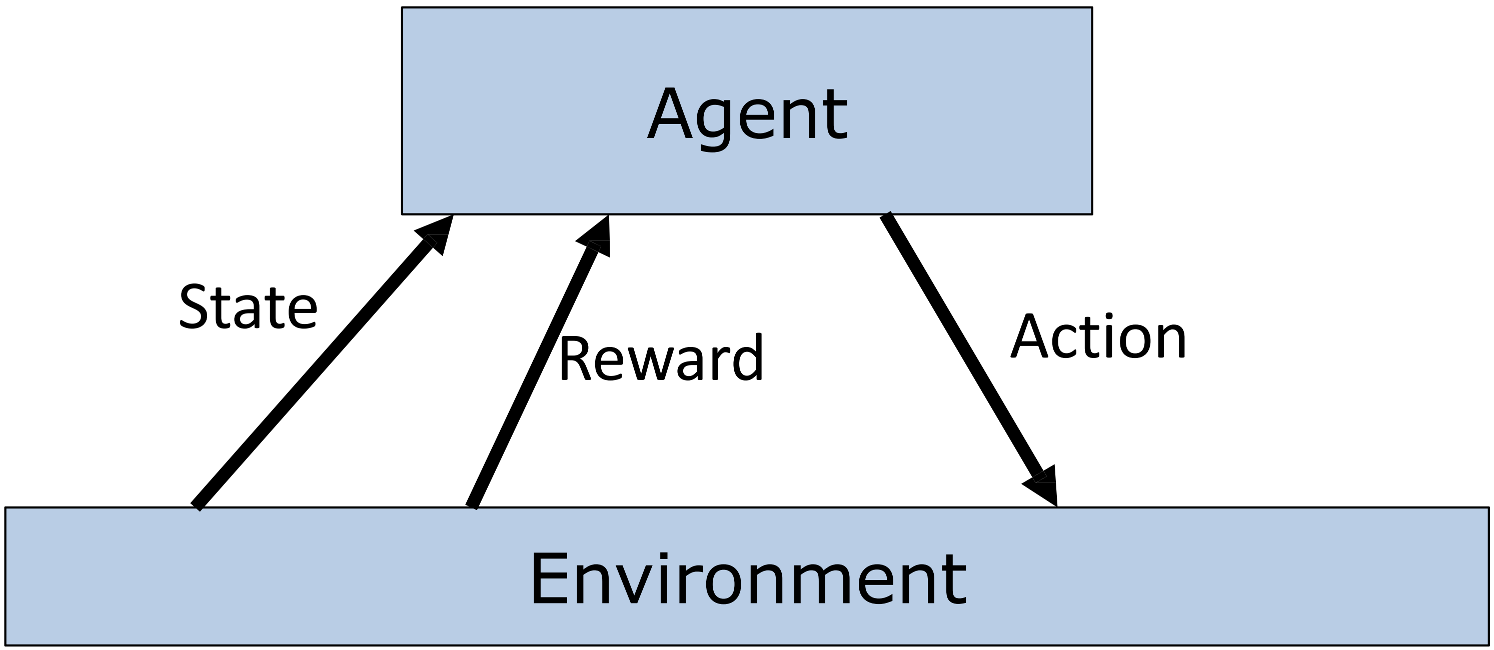
Markov Processes

[RusNor] Sec. 15.1

Outline

- Environment dynamics
- Stochastic processes
  - Markovian assumption
  - Stationary assumption

Recall: RL Problem



Goal: Learn to choose actions that maximize rewards

Unrolling the Problem

- Unrolling the control loop leads to a sequence of states, actions and rewards:

$$s_0, a_0, r_0, s_1, a_1, r_1, s_2, a_2, r_2, \ldots$$

- This sequence forms a stochastic process (due to some uncertainty in the dynamics of the process)

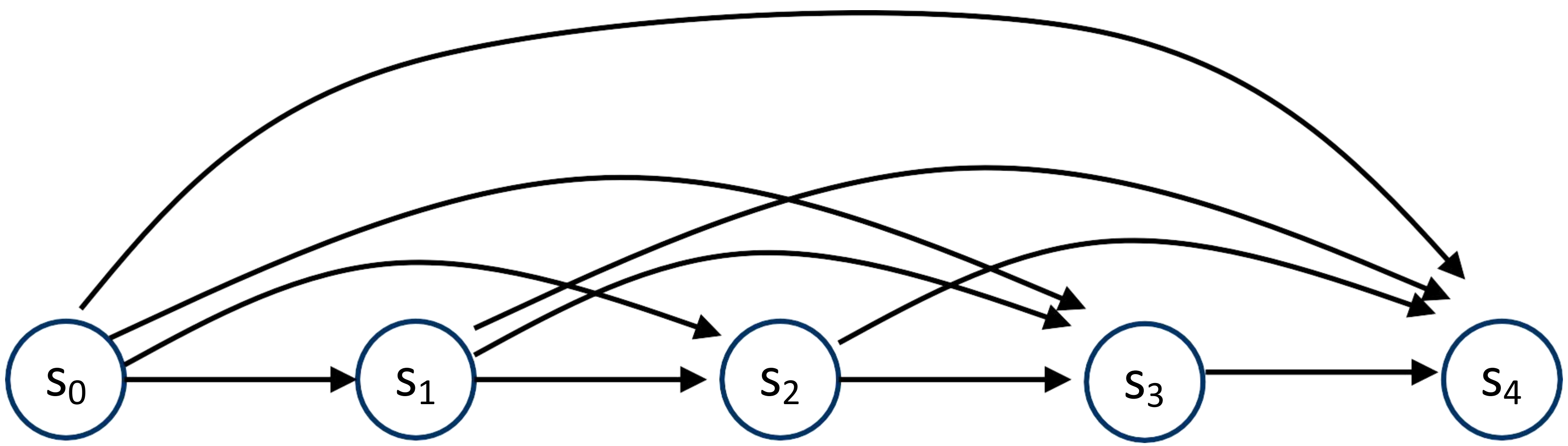
Common Properties

- Processes are rarely arbitrary
- They often exhibit some structure
  - Laws of the process do not change
  - Short history sufficient to predict future

- **Example**: weather prediction
  - Same model can be used everyday to predict weather
  - Weather measurements of past few days sufficient to predict weather.

Stochastic Process

- Consider the sequence of states only
- Definition
  - Set of States: S
  - Stochastic dynamics: $\Pr(s_t | s_{t-1}, \ldots, s_0)$

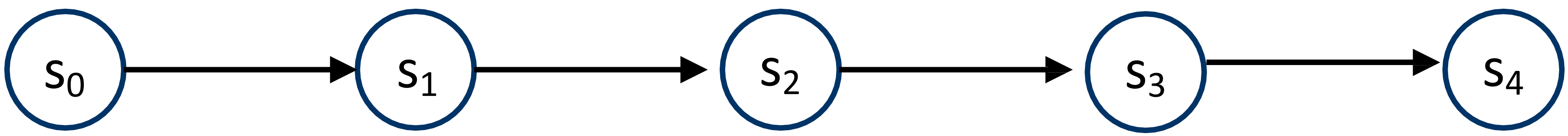


Stochastic Process

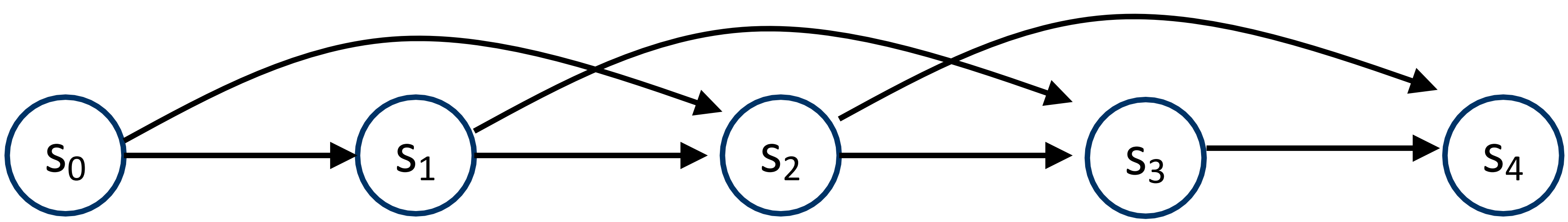
- Problem:
  - Infinitely large conditional distributions
- Solutions:
  - Stationary process: dynamics do not change over time
  - Markov assumption: current state depends only on a finite history of past states

K-order Markov Process

- Assumption: last k states sufficient
- First-order Markov Process
  - $\Pr(s_t | s_{t-1}, \ldots, s_0) = \Pr(s_t | s_{t-1})$



- Second-order Markov Process
  - $\Pr(s_t | s_{t-1}, \ldots, s_0) = \Pr(s_t | s_{t-1}, s_{t-2})$



Markov Process

- By default, a Markov Process refers to a
  - First-order process

$$\Pr(s_t|s_{t-1}, s_{t-2}, \dots, s_0) = \Pr(s_t|s_{t-1}) \ \ \forall t$$

  - Stationary process

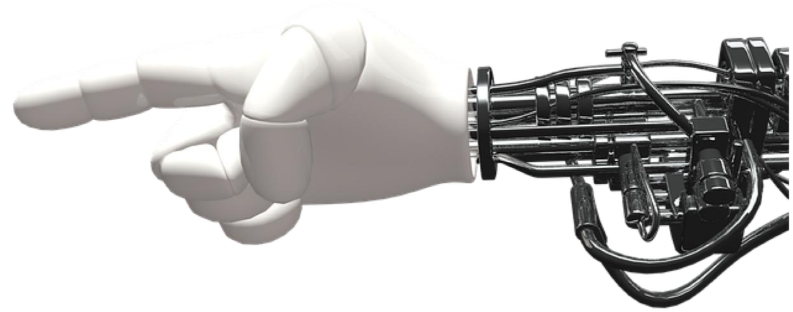
$$\Pr(s_t|s_{t-1}) = \Pr(s_{t'}|s_{t'-1}) \ \ \forall t'$$

- Advantage: can specify the entire process with a single concise conditional distribution

$$\Pr(s'|s)$$

Examples

- Robotic control
  - **States:** $\langle x, y, z, \theta \rangle$
    coordinates of joints
  - **Dynamics:** constant motion



- Inventory management
  - **States:** inventory level
  - **Dynamics:** constant (stochastic) demand

Non-Markovian and/or non-stationary processes

- What if the process is not Markovian and/or not stationary?
- Solution: add new state components until dynamics are Markovian and stationary
  - Robotics: the dynamics of $\langle x, y, z, \theta \rangle$are not stationary when velocity varies…
  - Solution: add velocity to state description e.g.
  - $\left\langle x, y, z, \theta, \dot{x}, \dot{y}, \dot{z}, \dot{\theta} \right\rangle$
  - If acceleration varies… then add acceleration to state
  - Where do we stop?

Markovian Stationary Process

- **Problem:** adding components to the state description to force a process to be Markovian and stationary may significantly increase computational complexity

- **Solution:** try to find the smallest state description that is self-sufficient (i.e., Markovian and stationary)

Inference in Markov processes

- Common task:
  - Prediction: $\Pr(s_{t+k}|s_t)$

- Computation:
  - $\Pr(s_{t+k}|s_t) = \sum_{s_{t+1}\ldots s_{t+k}} \prod_{i=1}^{k} \Pr(s_{t+i}|s_{t+i-1})$

- Discrete states (matrix operations):
  - Let T be a $|S| \times |S|$ matrix representing $\Pr(s_{t+1}|s_t)$
  - Then $\Pr(s_{t+k}|s_t) = T^k$
  - Complexity: $O(k|S|^3)$

Decision Making

- Predictions by themselves are useless
- They are only useful when they will influence future decisions

- Hence the ultimate task is decision making
- How can we influence the process to visit desirable states?
  - Model: Markov Decision Process