Reinforcement Learning
Lecture 4a:

Deep Neural Networks

[GBC] Chap. 6, 7, 8

Quick recap

- Markov Decision Processes: value iteration

$$V(s) \leftarrow \max_a R(s) + \gamma \sum_{s'} \Pr(s'|s,a)\, V(s')$$

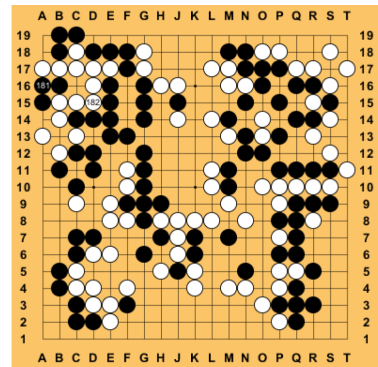
- Reinforcement Learning: Q-Learning

$$Q(s,a) \leftarrow Q(s,a) + \alpha[r + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

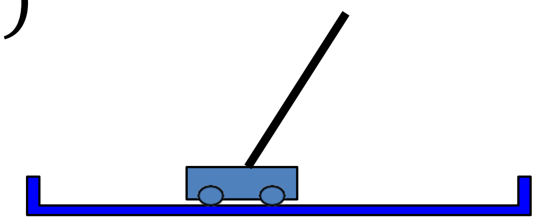
- Complexity depends on number of states and actions

Large State Spaces

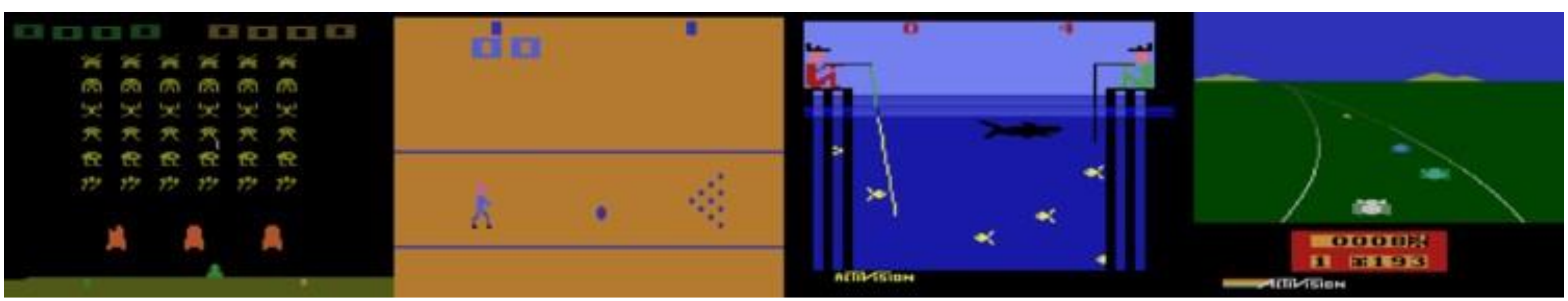
- Computer Go: 3^{361} states



- Inverted pendulum: (x, x', θ, θ')
  - 4-dimensional continuous state space



- Atari: 210x160x3 dimensions (pixel values)



Functions to be Approximated

- Policy: $\pi(s) \rightarrow a$
- Q-function: $Q(s, a) \in \Re$
- Value function: $V(s) \in \Re$

Q-function Approximation

- Let $s = (x_1, x_2, \ldots, x_n)^T$
- Linear

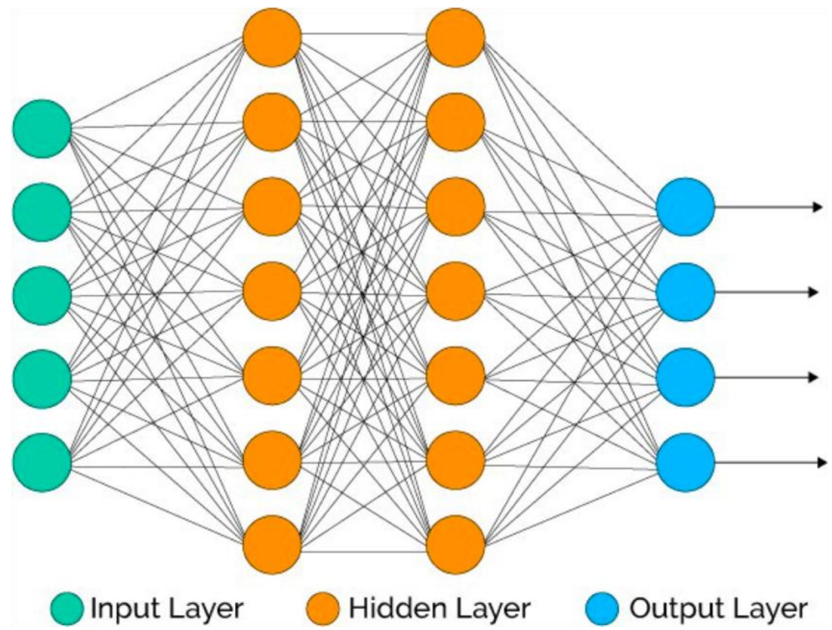
$$Q(s,a) \approx \sum_i w_{ai} x_i$$

- Non-linear (e.g., neural network)

$$Q(s,a) \approx g(\boldsymbol{x}; \boldsymbol{w})$$

Traditional Neural Network

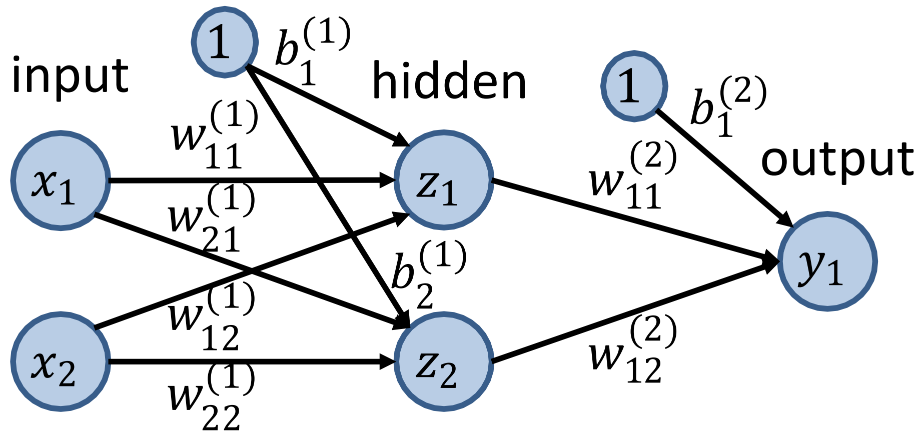
- Network of units (computational neurons) linked by weighted edges



- Each unit computes:
  $z = h(\boldsymbol{w}^{\boldsymbol{T}}\boldsymbol{x} + b)$
  - Inputs: $\boldsymbol{x}$
  - Output: z
  - Weights (parameters): $\boldsymbol{w}$
  - Bias: b
  - Activation function (usually non-linear): h

One hidden Layer Architecture

- Feed-forward neural network



- Hidden units: $z_j = h_1(\boldsymbol{w_j}^{(1)}\boldsymbol{x} + b_j^{(1)})$
- Output units: $y_k = h_2(\boldsymbol{w_k}^{(2)}\boldsymbol{z} + b_k^{(2)})$
- Overall: $y_k = h_2(\sum_j w_{kj}^{(2)} h_1\left(\sum_i w_{ji}^{(1)} x_i + b_j^{(1)}\right) + b_k^{(2)})$

Traditional activation functions h

- Threshold: $h(a) = \begin{cases} 1 & a \geq 0 \\ -1 & a < 0 \end{cases}$
- Sigmoid: $h(a) = \sigma(a) = \frac{1}{1+e^{-a}}$
- Gaussian: $h(a) = e^{-\frac{1}{2}\left(\frac{a-\mu}{\sigma}\right)^2}$
- Tanh: $h(a) = \tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}$
- Identity: $h(a) = a$

Universal function approximation

- **Theorem:** Neural networks with at least one hidden layer of sufficiently many sigmoid/tanh/Gaussian units can approximate any function arbitrarily closely.

- Picture:

Minimize least squared error

- Minimize error function

$$E(\boldsymbol{W}) = \frac{1}{2}\sum_n E_n(\boldsymbol{W})^2 = \frac{1}{2}\sum_n \left|\left|f(\boldsymbol{x_n}, \boldsymbol{W}) - y_n\right|\right|_2^2$$

where f is the function encoded by the neural net

- Train by gradient descent (a.k.a. backpropagation)
  – For each example (x_n, y_n), adjust the weights as follows:

$$w_{ji} \leftarrow w_{ji} - \eta \frac{\partial E_n}{\partial w_{ji}}$$

Deep Neural Networks

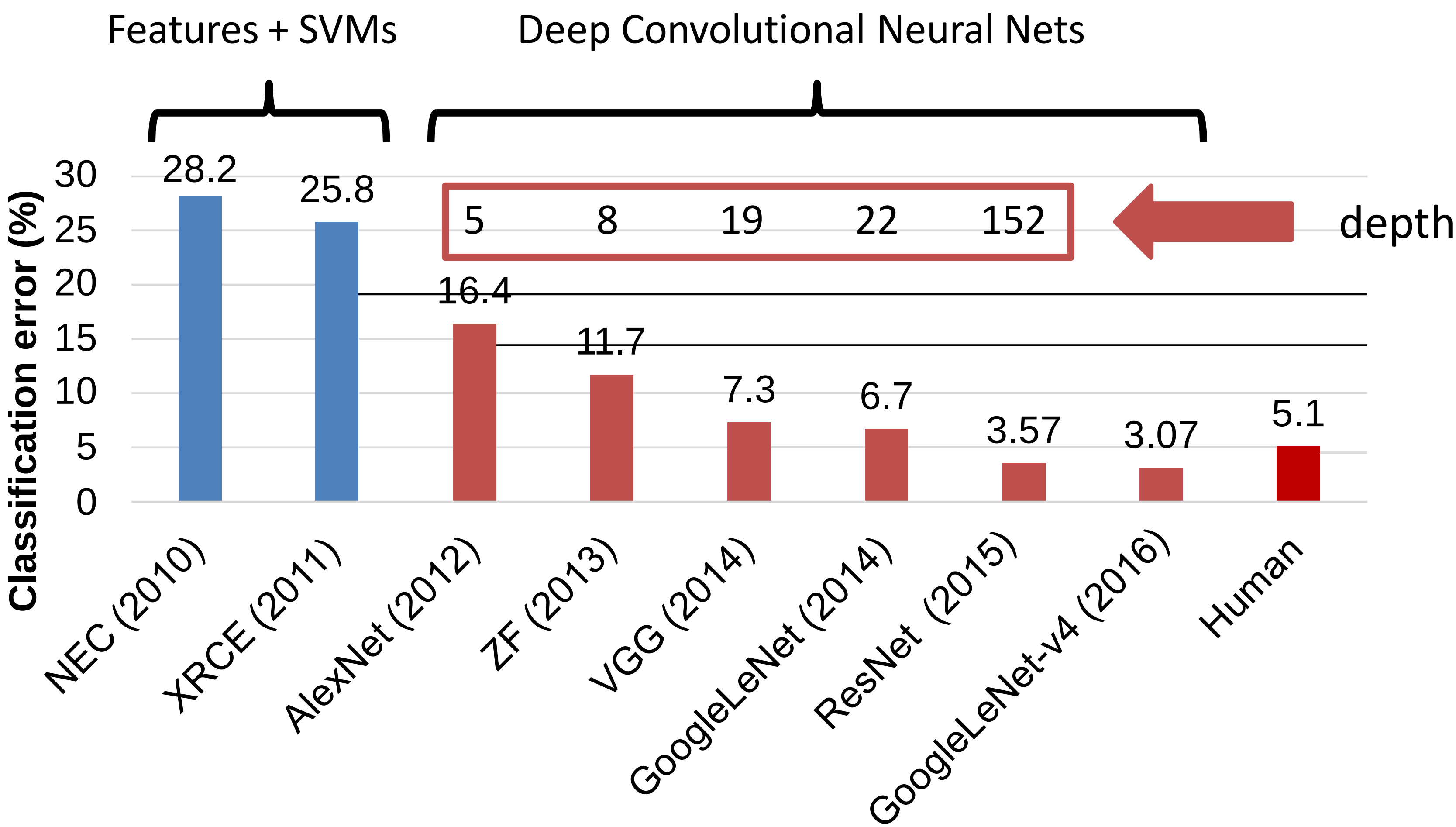
- Definition: neural network with many hidden layers
- Advantage: high expressivity
- Challenges:
  - How should we train a deep neural network?
  - How can we avoid overfitting?

Mixture of Gaussians

- Deep neural network (hierarchical mixture)
- Shallow neural network (flat mixture)

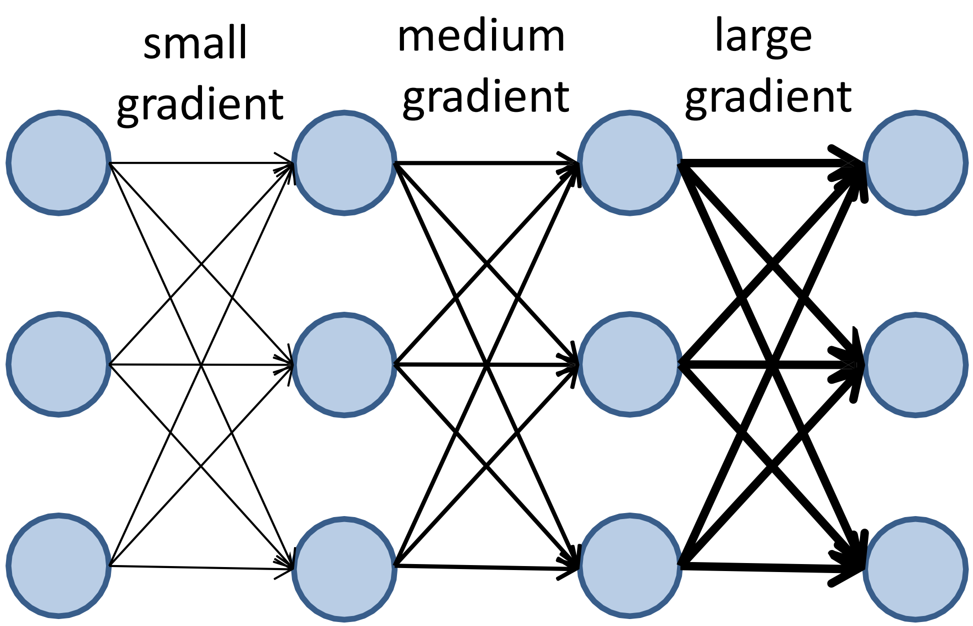
Image Classification

- ImageNet Large Scale Visual Recognition Challenge



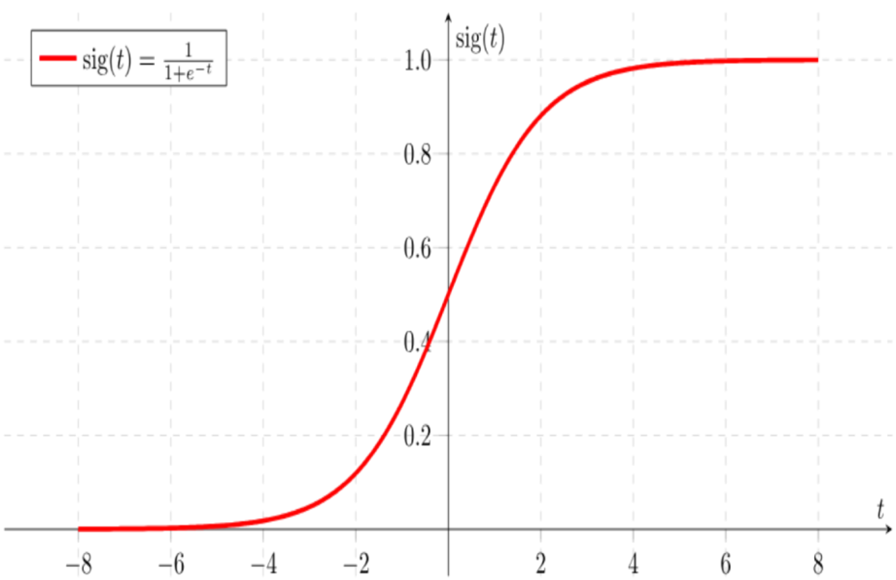
Vanishing Gradients

- Deep neural networks of sigmoid and hyperbolic units often suffer from vanishing gradients

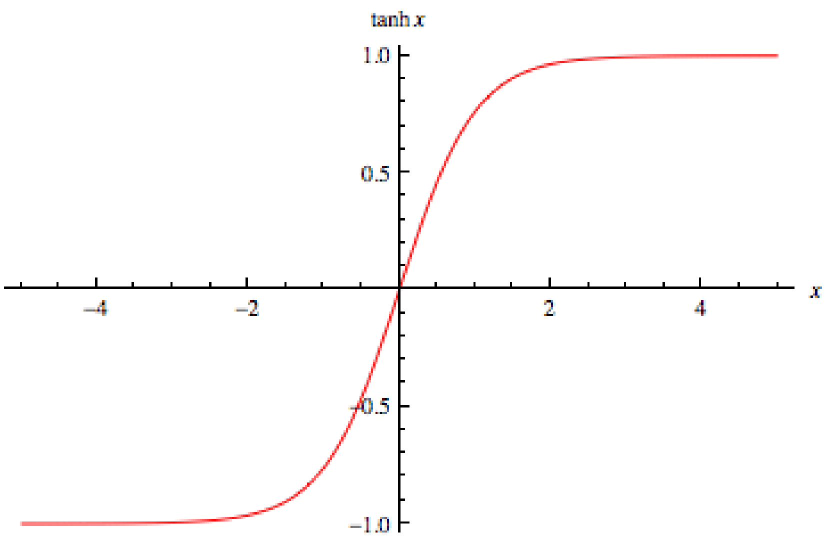


Sigmoid and hyperbolic units

- Derivative is always less than 1



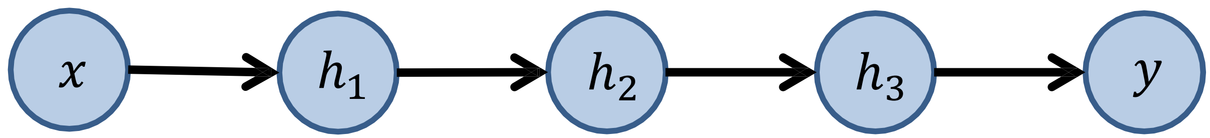
sigmoid



hyperbolic

Simple Example

- $y = \sigma(w_4\sigma(w_3\sigma(w_2\sigma(w_1 x))))$



- Common weight initialization in (-1,1)
- Sigmoid function and its derivative always less than 1
- This leads to vanishing gradients:

$$\frac{\partial y}{\partial w_4} = \sigma'(a_4)\sigma(a_3)$$

$$\frac{\partial y}{\partial w_3} = \sigma'(a_4)w_4\sigma'(a_3)\sigma(a_2) \le \frac{\partial y}{\partial w_4}$$

$$\frac{\partial y}{\partial w_2} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)\sigma(a_1) \le \frac{\partial y}{\partial w_3}$$

$$\frac{\partial y}{\partial w_1} = \sigma'(a_4)w_4\sigma'(a_3)w_3\sigma'(a_2)w_2\sigma'(a_1)x \le \frac{\partial y}{\partial w_2}$$

Mitigating Vanishing Gradients

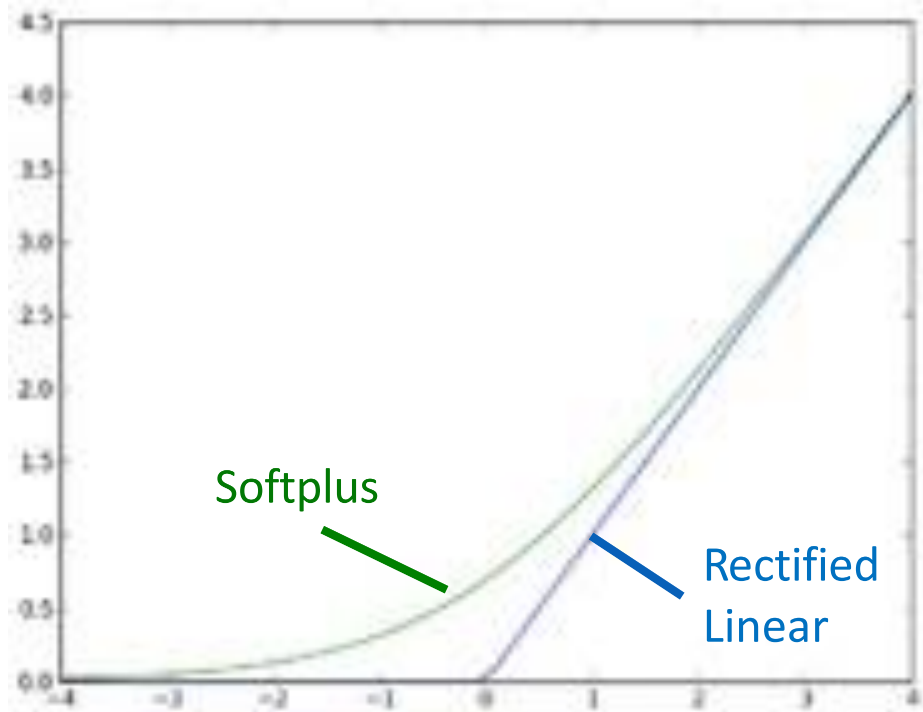
- Some popular solutions:
  - Pre-training
  - **Rectified linear units**
  - Batch normalization
  - Skip connections

Rectified Linear Units

- Rectified linear: $h(a) = \max(0, a)$
  - Gradient is 0 or 1
  - Sparse computation

- Soft version ("Softplus") :

  $h(a) = \log(1 + e^a)$



- Warning: softplus does not prevent gradient vanishing (gradient < 1)