Reinforcement Learning Lecture 7b:

Actor Critic Algorithms
[SutBar] Sec. 13.4-13.5,
[Sze] Sec. 4.4, [SigBuf] Sec. 5.3

Outline

- Policy gradient with a baseline
- Actor Critic algorithms
- Deterministic policy gradient

Actor Critic

- Q-learning
  - **Model-free value-based method**
  - No explicit policy representation
- Policy gradient
  - **Model-free policy-based method**
  - No explicit value function representation
- Actor Critic
  - **Model-free policy and value based method**

Stochastic Gradient Policy Theorem

- Stochastic Gradient Policy Theorem

$$\boxed{\nabla V_\theta(s_0) \propto \sum_s \mu_\theta(s) \sum_a \nabla \pi_\theta(a|s) Q_\theta(s,a)}$$

- Equivalent Stochastic Gradient Policy Theorem with a baseline $b(s)$

$$\boxed{\nabla V_\theta(s_0) \propto \sum_s \mu_\theta(s) \sum_a \nabla \pi_\theta(a|s)[Q_\theta(s,a) - b(s)]}$$

since $\sum_a \nabla \pi_\theta(a|s) b(s) = b(s) \nabla \sum_a \pi_\theta(a|s) = b(s)\, \nabla 1 = 0$

Baseline

- Baseline often chosen to be $b(s) \approx V^{\pi}(s)$

- Advantage function: $A(s, a) = Q(s, a) - V^{\pi}(s)$

- Gradient update:

$$\theta \leftarrow \theta + \alpha\gamma^{n} A(s_n, a_n) \nabla log \pi_{\theta}(a_n | s_n)$$

- Benefit: faster empirical convergence

REINFORCE Algorithm with a baseline

REINFORCEwithBaseline(s_0, π_θ)

Initialize π_θ to anything

Initialize V_w to anything

Loop forever (for each episode)

Generate episode with $s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_T, a_T, r_T$ with π_θ

Loop for each step of the episode $n = 0, 1, \ldots, T$

$G_n \leftarrow \sum_{t=0}^{T-n} \gamma^t r_{n+t}$

$\delta \leftarrow G_n - V_w(s_n)$

Update value function: $w \leftarrow w + \alpha_w \gamma^n \delta \nabla V_w(s_n)$

Update policy: $\theta \leftarrow \theta + \alpha_\theta \gamma^n \delta \nabla log \pi_\theta (a_n | s_n)$

Return π_θ

REINFORCE Algorithm (stochastic policy)

REINFORCE(s_0, π_θ)

Initialize π_θ to anything

Loop forever (for each episode)

Generate episode $s_0, a_0, r_0, s_1, a_1, r_1, \ldots, s_T, a_T, r_T$ with π_θ

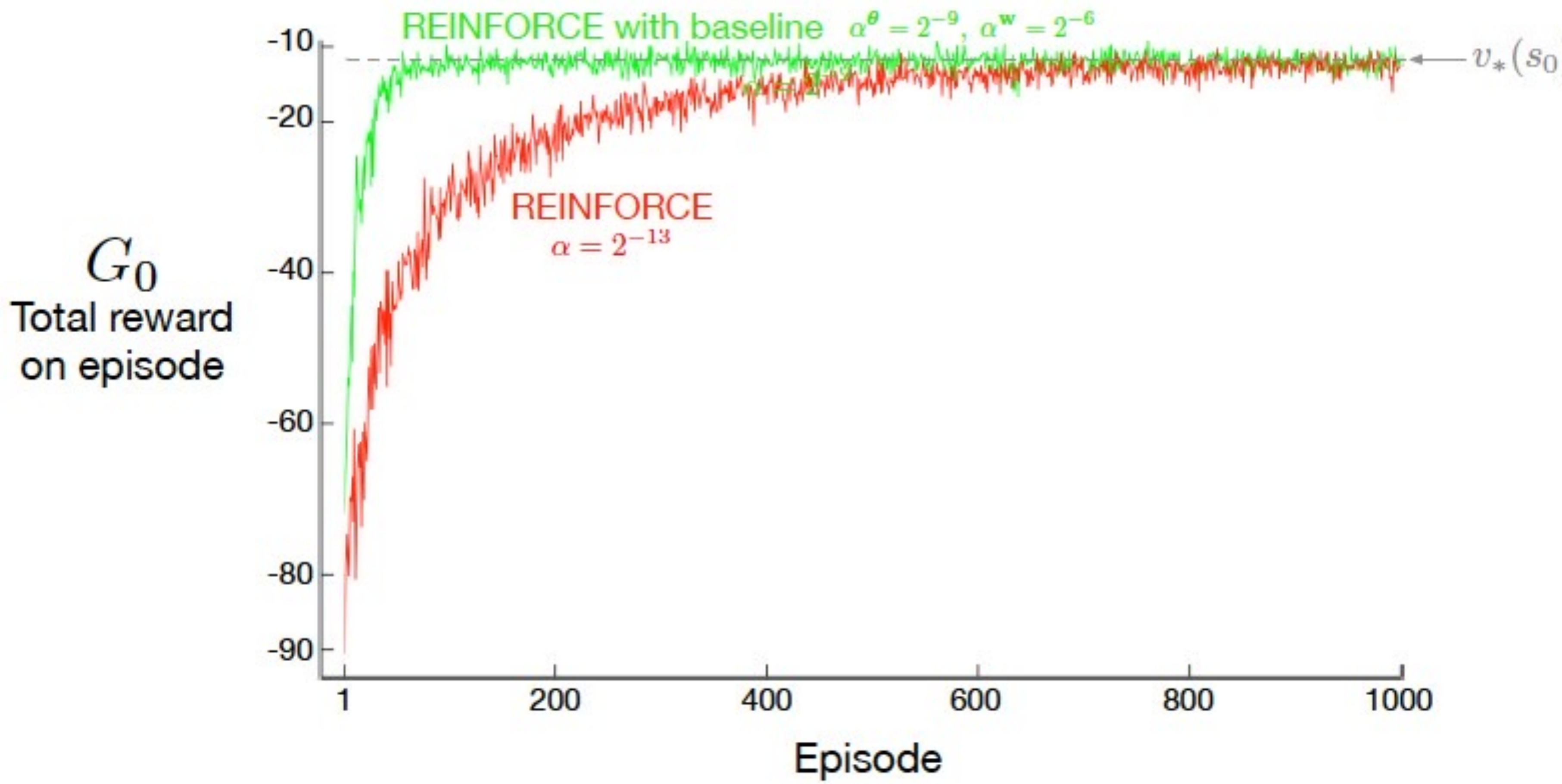
Loop for each step of the episode $n = 0, 1, \ldots, T$

$G_n \leftarrow \sum_{t=0}^{T-n} \gamma^t r_{n+t}$

Update policy: $\theta \leftarrow \theta + \alpha \gamma^n G_n \nabla log \pi_\theta(a_n | s_n)$

Return π_θ

Performance Comparison



Temporal difference update

- Instead of updating V(s) by Monte Carlo sampling
$$\delta \leftarrow G_n - V_w(s_n)$$
  Bootstrap with temporal difference updates
$$\delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)$$

- Benefit: reduced variance (faster convergence)

Actor Critic Algorithm

ActorCritic(s_0, π_θ)

Initialize π_θ to anything

Initialize V_w to anything

Loop forever (for each episode)

Initialize s_0 and set $n \leftarrow 0$

Loop while s is not terminal (for each time step n)

Sample $a_n \sim \pi_\theta(a|s_n)$

Execute a_n, observe s_{n+1}, r_n

$\delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)$

Update value function: $w \leftarrow w + \alpha_w \gamma^n \delta \nabla V_w(s_n)$

Update policy: $\theta \leftarrow \theta + \alpha_\theta\, \gamma^n \delta \nabla log \pi_\theta(a_n|s_n)$

$n \leftarrow n + 1$

Return π_θ

Advantage update

- Instead of doing temporal difference updates

$$\delta \leftarrow r_n + \gamma V_w(s_{n+1}) - V_w(s_n)$$

- Update with the advantage function

$$A(s_n, a_n) \leftarrow r_n + \gamma \max_{a_{n+1}} Q(s_{n+1}, a_{n+1}) - \sum_a \pi_\theta(a|s_n) Q(s_n, a)$$

$$\theta \leftarrow \theta + \alpha_\theta \, \gamma^n A(s_n, a_n) \nabla log \pi_\theta(a_n|s_n)$$

- Benefit: faster convergence

Advantage Actor Critic (A2C)

A2C()

Initialize π_θ to anything

Loop forever (for each episode)

Initialize s_0 and set $n \leftarrow 0$

Loop while s is not terminal (for each time step n)

Select a_n

Execute a_n, observe s_{n+1}, r_n

$\delta \leftarrow r_n + \gamma \max_{a_{n+1}} Q(s_{n+1}, a_{n+1}) - Q_w(s_n, a_n)$

$A(s_n, a_n) \leftarrow r_n + \gamma \max_{a_{n+1}} Q(s_{n+1}, a_{n+1})$
$- \sum_a \pi_\theta(a|s_n) Q(s_n, a)$

Update Q: $w \leftarrow w + \alpha_w \gamma^n \delta \nabla_w Q_w(s_n, a_n)$

Update π: $\theta \leftarrow \theta + \alpha_\theta\, \gamma^n A(s_n, a_n) \nabla log \pi_\theta(a_n|s_n)$

$n \leftarrow n + 1$

Continuous Actions

- Consider a deterministic policy $\pi_\theta(s) \rightarrow a$

- Deterministic Gradient Policy Theorem

$$\boxed{\nabla V_\theta(s_0) \propto E_{s\sim\mu_\theta(s)}[\nabla_\theta \pi_\theta(s) \nabla_a Q_\theta(s,a)|_{a=\pi_\theta(s)}]}$$

Proof: see Silver et al. 2014

- Stochastic Gradient Policy Theorem

$$\boxed{\nabla V_\theta(s_0) \propto \sum_s \mu_\theta(s) \sum_a \nabla_\theta \pi_\theta(a|s) Q_\theta(s,a)}$$

Deterministic Policy Gradient (DPG)

DPG(s, π_θ)

Initialize π_θ to anything

Loop forever (for each episode)

Initialize s_0 and set $n \leftarrow 0$

Loop while s is not terminal (for each time step n)

Select $a_n = \pi_\theta(s_n)$

Execute a_n, observe s_{n+1}, r_n

$\delta \leftarrow r_n + \gamma Q_w\big(s_{n+1}, \pi_\theta(s_{n+1})\big) - Q_w(s_n, a_n)$

Update Q: $w \leftarrow w + \alpha_w \gamma^n \delta \nabla_w Q_w(s_n, a_n)$

Update π: $\theta \leftarrow \theta + \alpha_\theta\, \gamma^n \nabla_\theta \pi_\theta(s_n) \nabla_a Q_w(s_n, a_n)|_{a_n=\pi_\theta(s_n)}$

$n \leftarrow n + 1$

Return