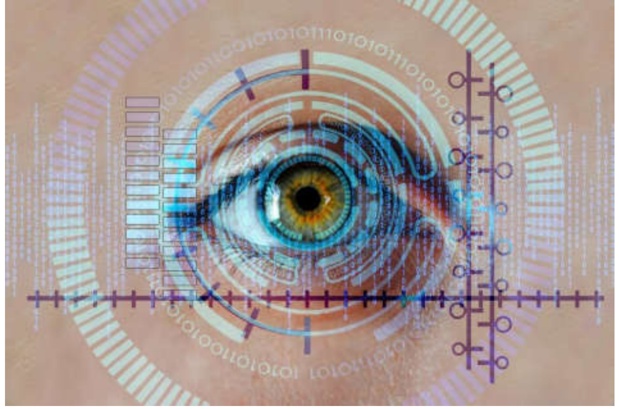

Computer Vision

# Lecture 6 Line Detection

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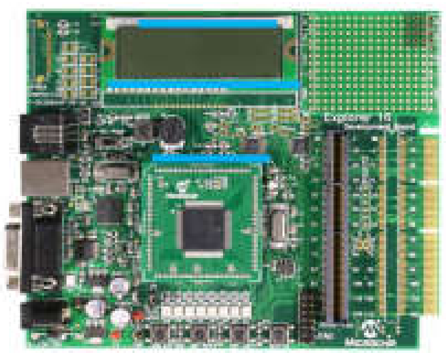
# Line Detection

- Why detect lines? Many objects characterized by presence of straight lines

- Wait, why aren’t we done just by running edge detection?

# Outline

- Hough transform
- RANSAC

# Intro to Hough transform

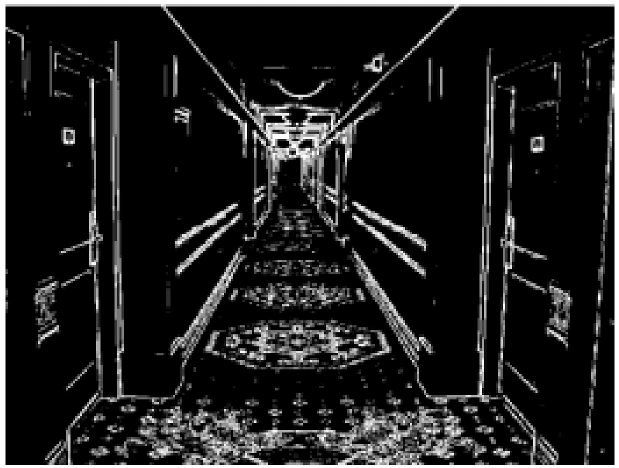
- The Hough transform (HT) can be used to detect lines.
- It was introduced in 1962 (Hough 1962) and first used to find lines in images a decade later (Duda1972).
- Our goal with the Hough Transform is to find the location of lines in images.
- Hough transform can detect lines, circles and other structures ONLY if their parametric equation is known.
- It can give robust detection under noise and partial occlusion

# Prior to Hough transform

- Assume that we have performed edge detection, for example, by thresholding the gradient magnitude image.
- Thus, we have some pixels that may partially describe the boundary of some objects.

Input Image

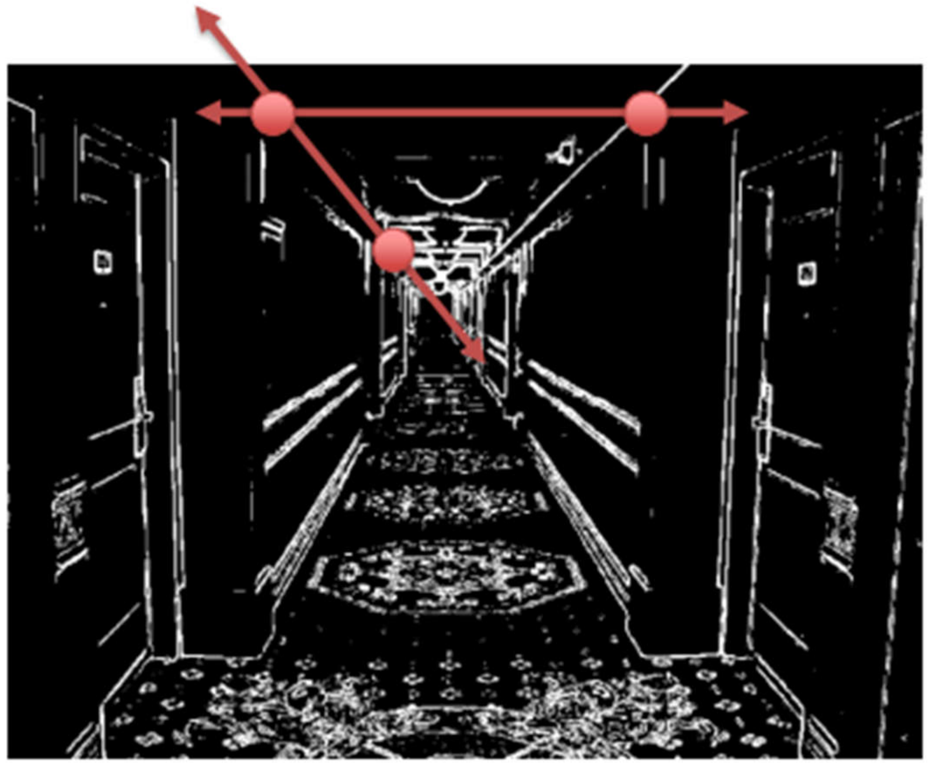
Image Gradients



Edge map(binary image)

# Naïve Line Detection

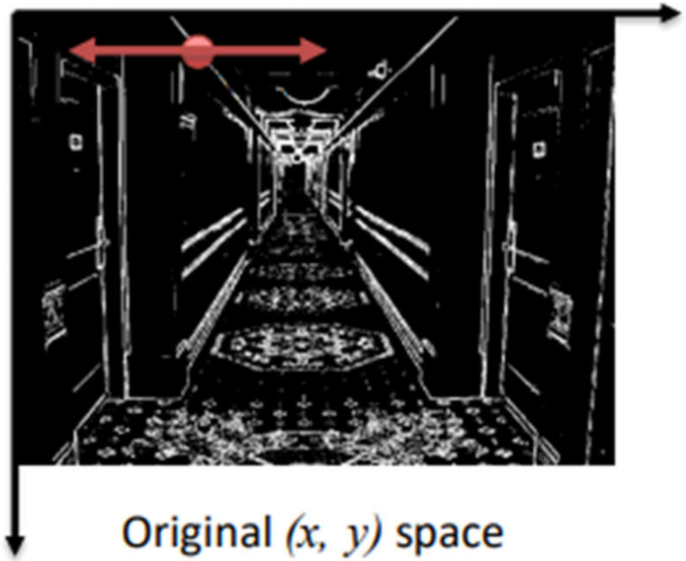
- For every pair of edge pixels
  - Compute equation of line
  - Check if other pixels satisfy equation
- Complexity?
  - O($N^2$) for an image with N edge pixels
- We can do better with the Hough Transform!



Edge map (binary image)

# Detecting lines using Hough transform

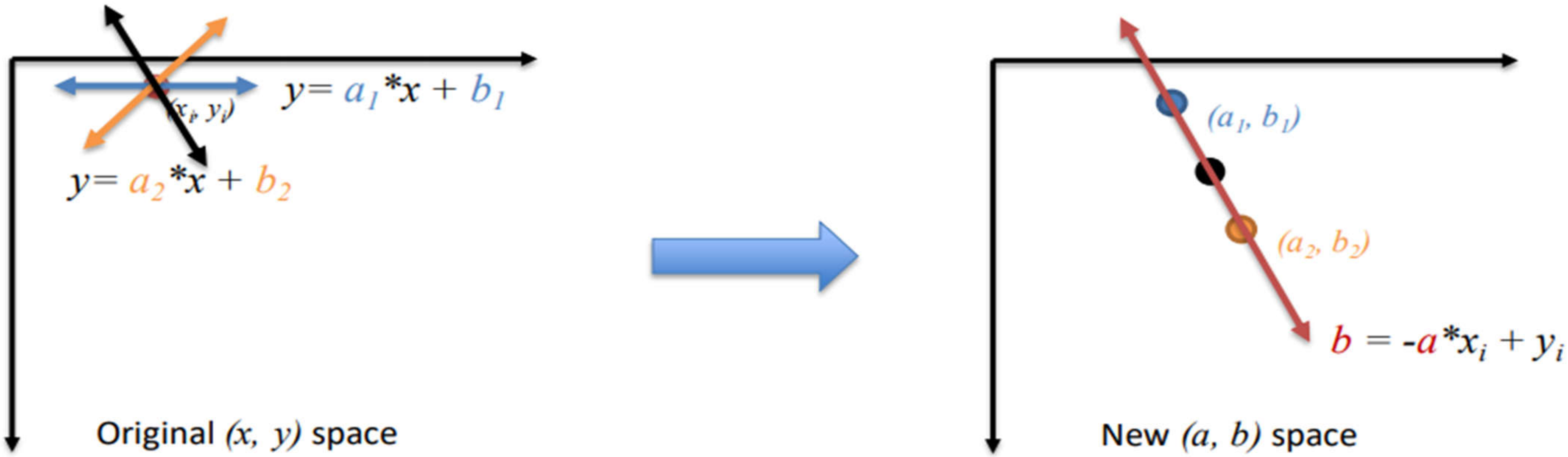
- We wish to find sets of pixels that make up straight lines.
- First step is to transform edge points into a new space.
- Consider an edge point of known coordinates $(x_i, y_i)$:
  - –There are many potential lines passing through the point $(x_i, y_i)$.
- This family of lines have the form $y_i = a * x_i + b$.



Original $(x,\ y)$ space

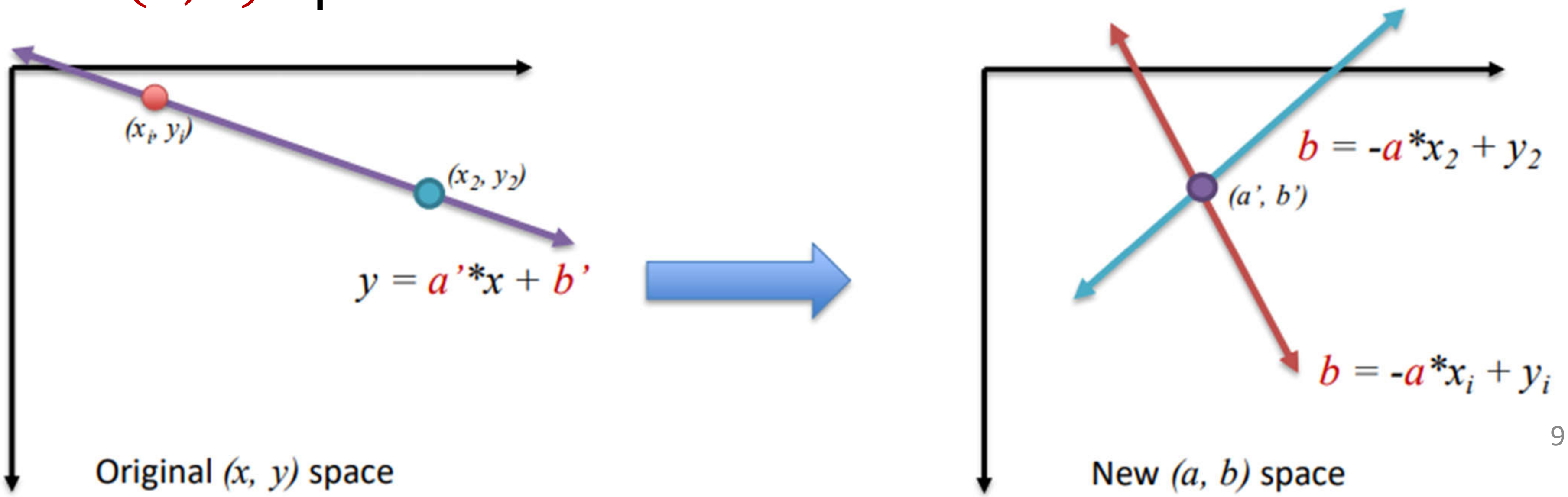
# Detecting lines using Hough transform

- This family of lines have the form $y_i = a * x_i + b$.
- Note $(x_i, y_i)$ are constants, while $(a, b)$ can change. This gives rise to a new space where $(a, b)$ are the variables.
- That means, a point $(x_i, y_i)$ transforms into a line in the $(a, b)$ space: $b = -x_i * a + y_i$.



# Detecting lines using Hough transform

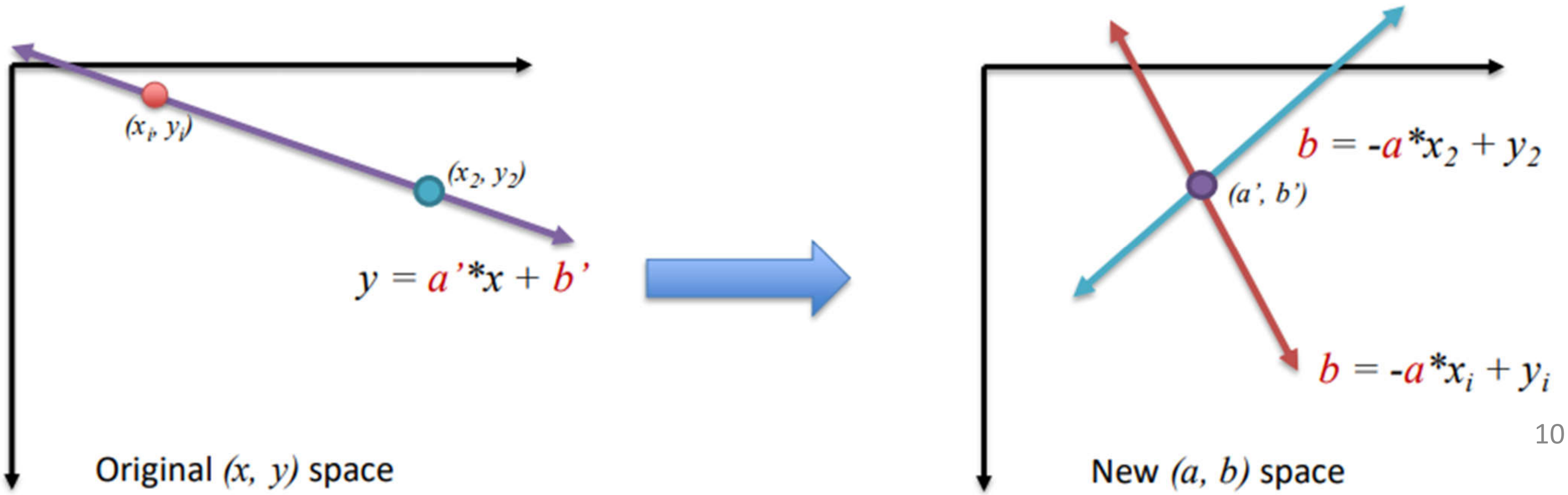
- This family of lines have the form $y_i = a * x_i + b$.
- Note $(x_i, y_i)$ are constants, while $(a, b)$ can change. This gives rise to a new space where $(a, b)$ are the variables.
- That means, a point $(x_i, y_i)$ transforms into a line in the $(a, b)$ space: $b = -x_i * a + y_i$.
- Another edge point $(x_2, y_2)$ will give rise to another line in the $(a, b)$ space.



# Detecting lines using Hough transform

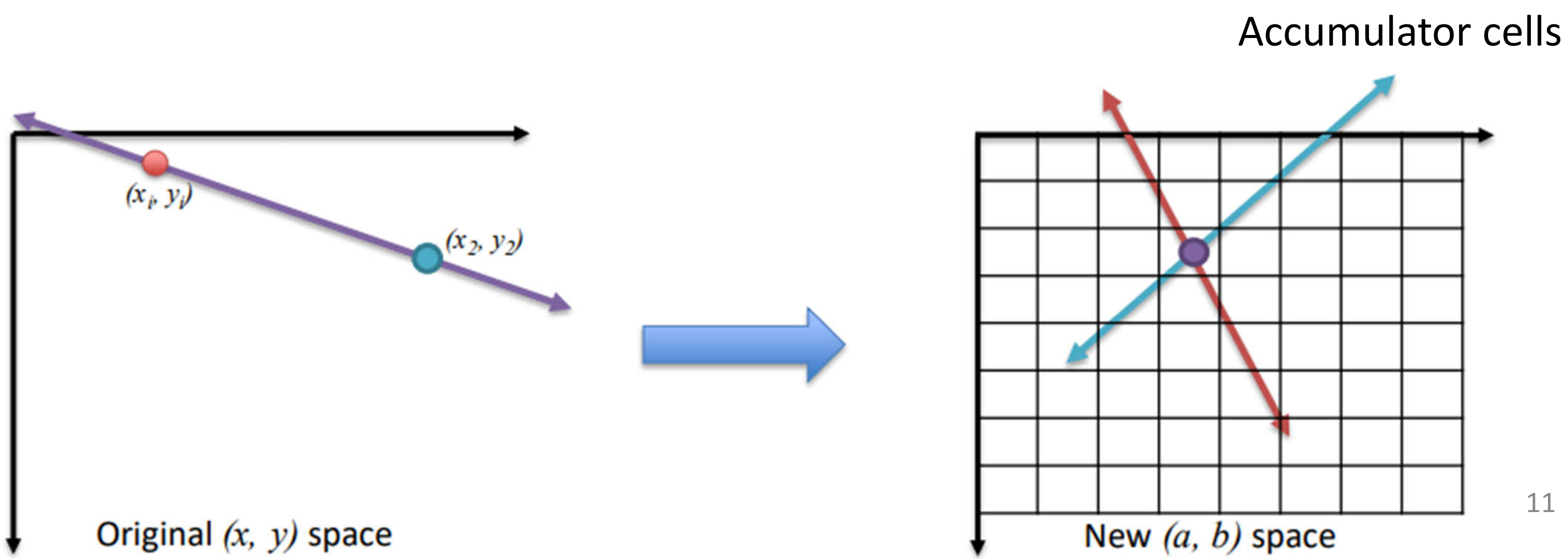
- Colinear points in the $(x, y)$ space transform into lines in the $(a, b)$ space that intersect at a single point $(a', b')$.
- We can detect lines by finding such intersection points $(a', b')$ in the $(a, b)$ space.
- Our resulting line equation in the original space is

$$y = a' * x + b'.$$



# Detecting lines using Hough transform

- We efficiently find the intersection points in the $(a, b)$ space by quantizing it into cells.
- Instead of transforming a point to an explicit line, we vote on the discrete cells that are 'activated' by the transformed line in $(a, b)$ .



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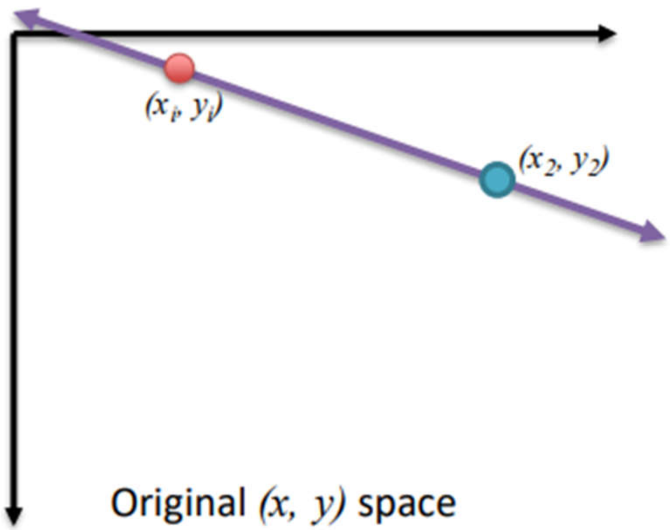
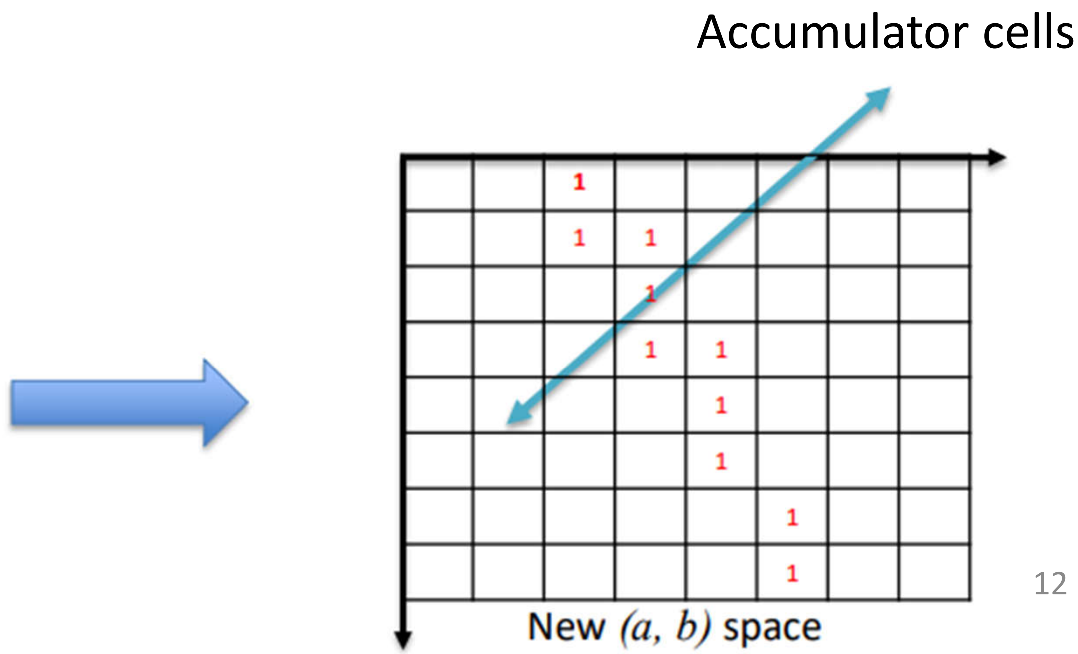
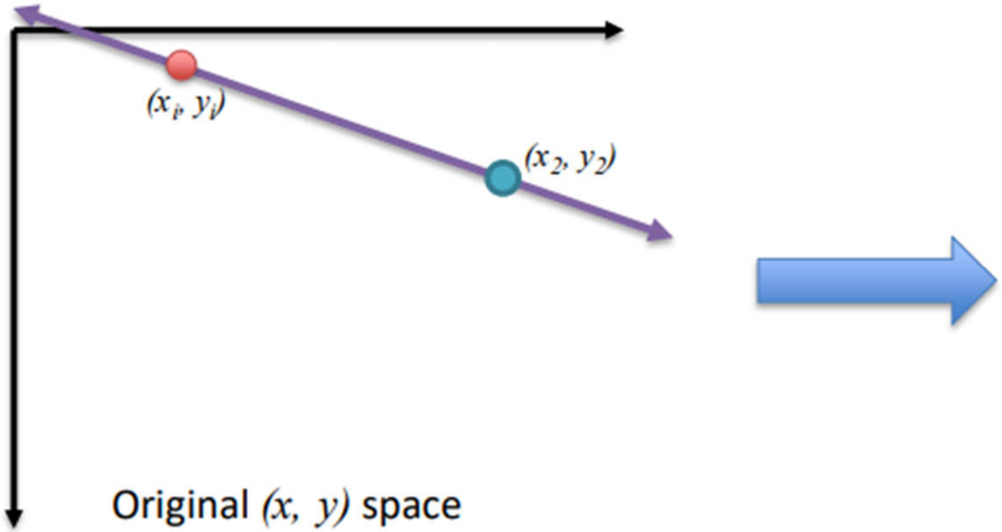


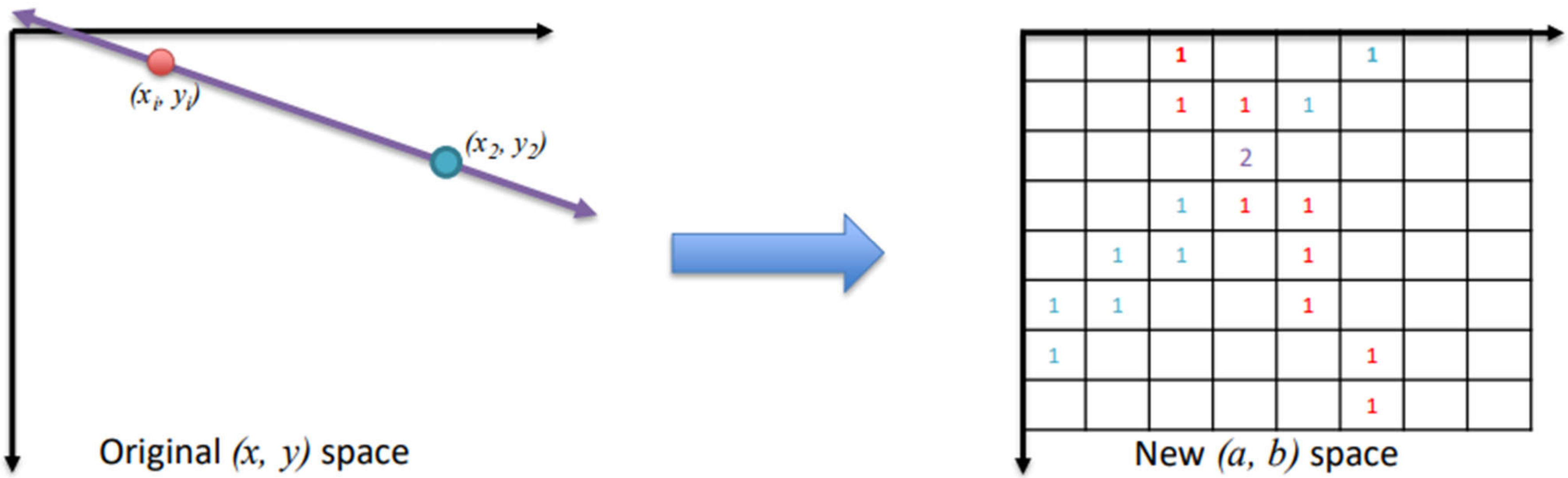
# Detecting lines using Hough transform

- We efficiently find the intersection points in the $(a, b)$ space by quantizing it into cells.
- Instead of transforming a point to an explicit line, we vote on the discrete cells that are 'activated' by the transformed line in $(a, b)$.
- Cells that receive more than a certain number of votes are assumed to correspond to lines in $(x, y)$ space.



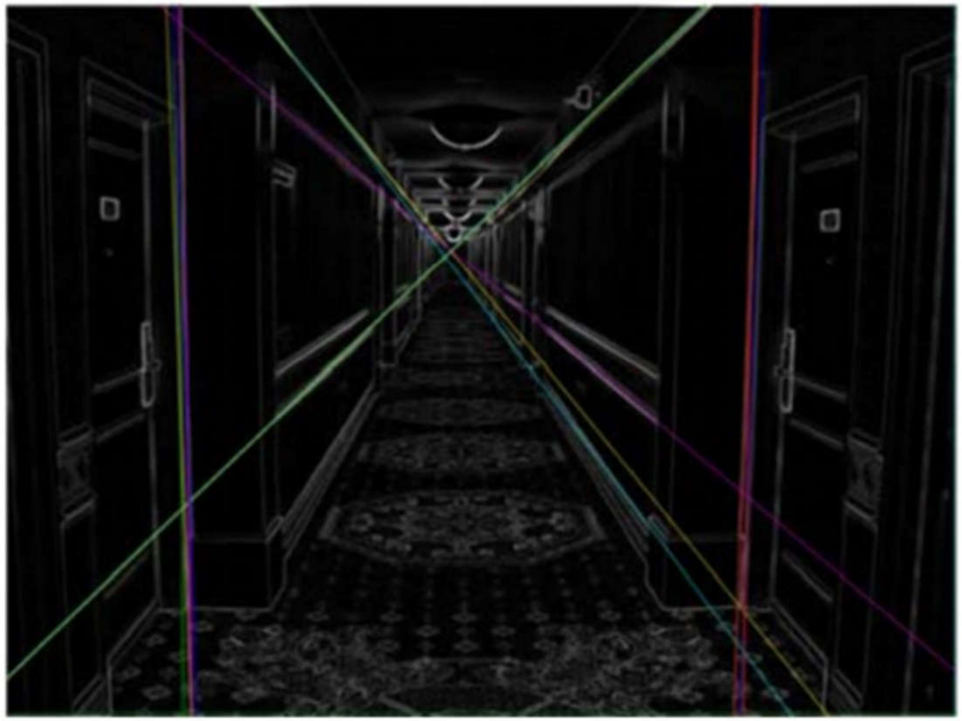

# Hough Transform Algorithm

- For each $(x, y)$ edge point:
  - Vote on cells that satisfy the corresponding $(a, b)$ line equation
- Find cells with more votes than threshold.
- Complexity?
  - Linear on number of edge points
  - Linear on number of accumulator cells



# Output of Hough transform

- Here are the top 20 most voted lines in the image

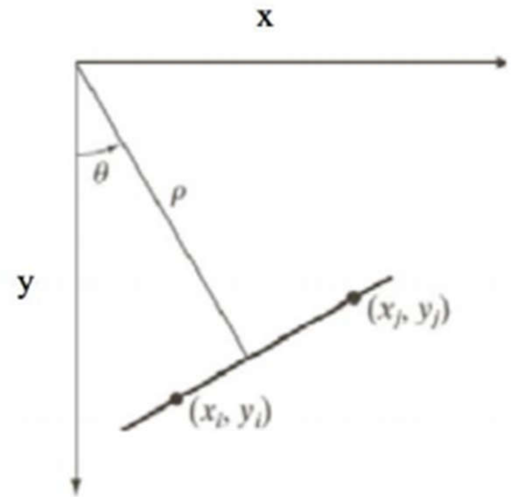


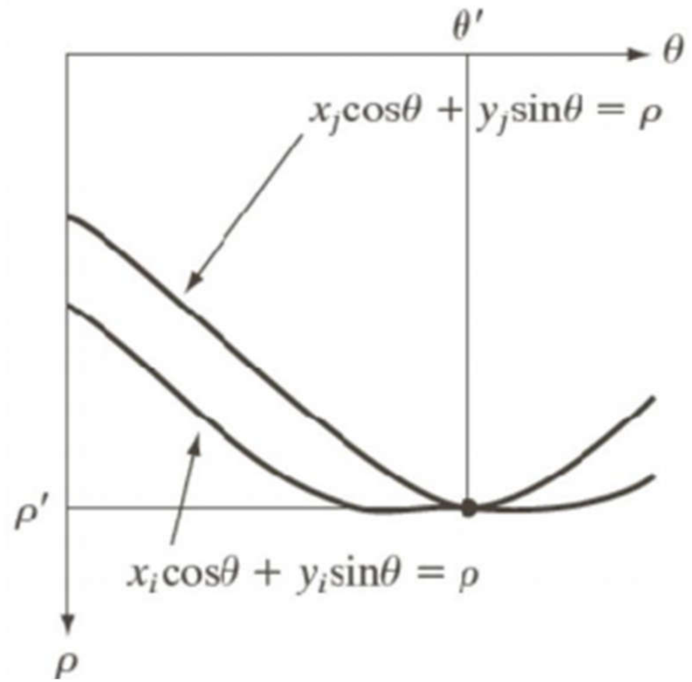
# Other Hough transformations

- We can represent lines as polar coordinates instead of $y = a * x + b$
- Polar coordinate representation:

$$x * \cos\theta + y * \sin\theta = \rho$$

- A vertical line will have $\theta = 90^{\circ}$ and $\rho$ equal to the intercept with the $x$-axis.
- A horizontal line will have $\theta = 0^{\circ}$ and $\rho$ equal to the intercept with the $y$-axis.
- Note that lines in $(x, y)$ space are not lines in $(\rho, \theta)$ space, unlike $(a, b)$ space



# Problems with parameterization

- Euclidean coordinate

*How big does the accumulator need to be for the parameterization (a,b)   ?*

A($a$,$b$)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | | | | | | 1 | | |
| | | 1 | | | | 1 | | | |
| | | | 1 | | 1 | | | | |
| | | | | 2 | | | | | |
| | | | 1 | | 1 | | | | |
| | | 1 | | | | 1 | | | |
| | 1 | | | | | | 1 | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

The space of a is huge!

$$-\infty \leq a \leq \infty$$

The space of b is huge!

$$-\infty \leq b \leq \infty$$

# Better Parameterization

- Polar coordinate

$$x\cos\theta + y\sin\theta = \rho$$

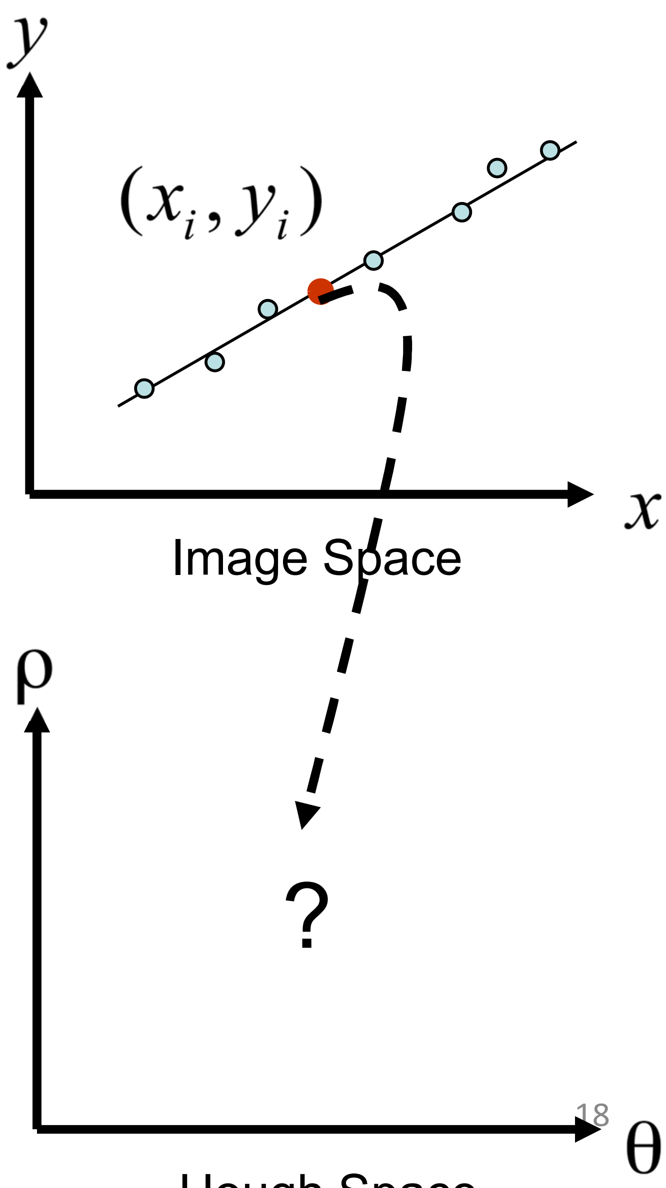
Given points $(x_i, y_i)$ find $(\rho, \theta)$

Hough Space Sinusoid

$$0 \le \theta \le 2\pi$$

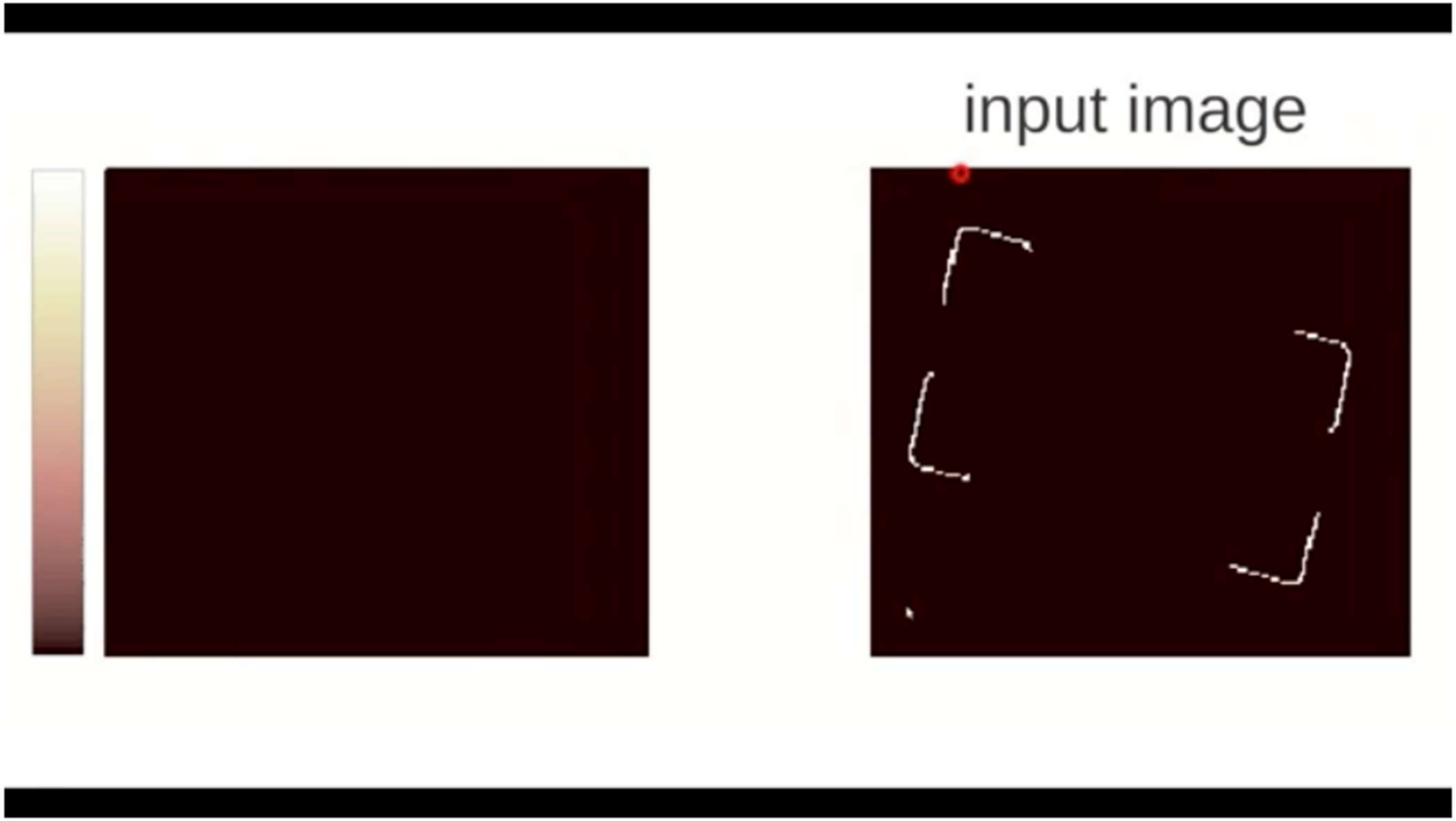
$$0 \le \rho \le \rho_{max}$$

(Finite Accumulator Array Size)



# Example video

- [Demo]



# Remarks

- Advantages:
  - Conceptually simple.
  - Easy implementation.
  - Handles missing and occluded data very gracefully.
  - Can be adapted to many types of forms, not just lines.
- Disadvantages:
  - Computationally complex for objects with many parameters.
  - Looks for only one single type of object.
  - Co-linear line segments cannot be separated.
    - Can be “fooled” by “apparent lines”.
    - The length and the position of a line segment cannot be determined.

# Outline

- Hough transform
- RANSAC

# RANSAC [Fischler & Bolles 1981]

- RANdom SAmple Consensus
- Approach: we want to avoid the impact of outliers, so let's look for "inliers", and use only those
- Intuition: if an outlier is chosen to compute the current fit, then the resulting line won't have much support from rest of the points.

# RANSAC loop

Repeat for k iterations:

① Randomly select a seed group of points on which to perform a model estimate (e.g., a group of edge points)

② Compute model parameters from seed group

③ Calculate distances and find inliers to this model

④ If the number of inliers is sufficiently large, re-compute least-squares estimate of model on all of the inliers

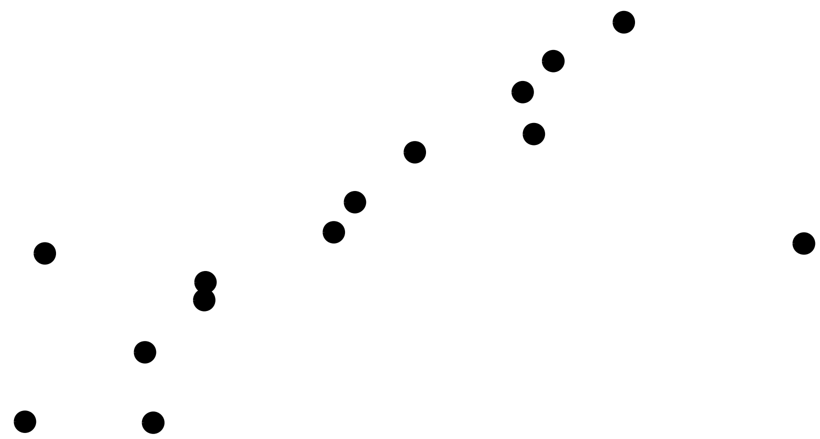
–Keep the model with the largest number of inliers



https://zhuanlan.zhihu.com/p/36301702

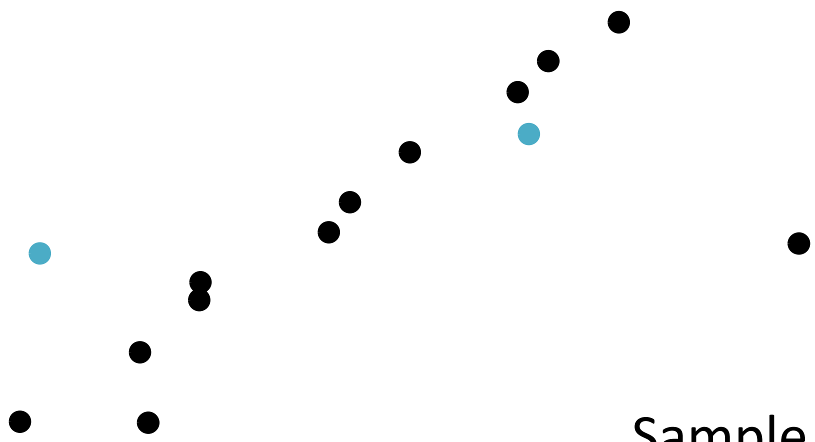
# RANSAC Line Fitting Example

- Task: Estimate the best line
  - How many points do we need to estimate the line?



# RANSAC Line Fitting Example

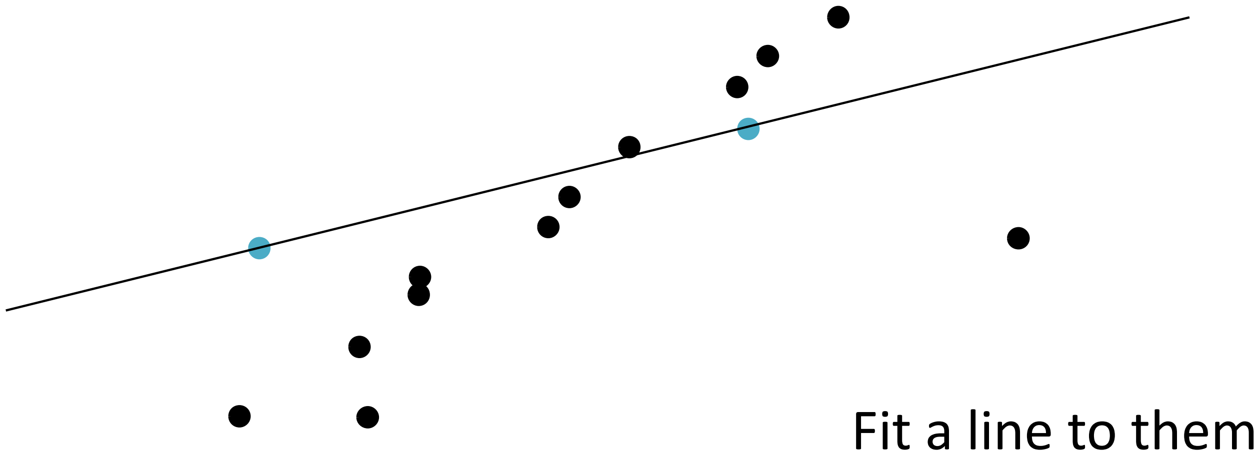
- Task: Estimate the best line



Sample two points

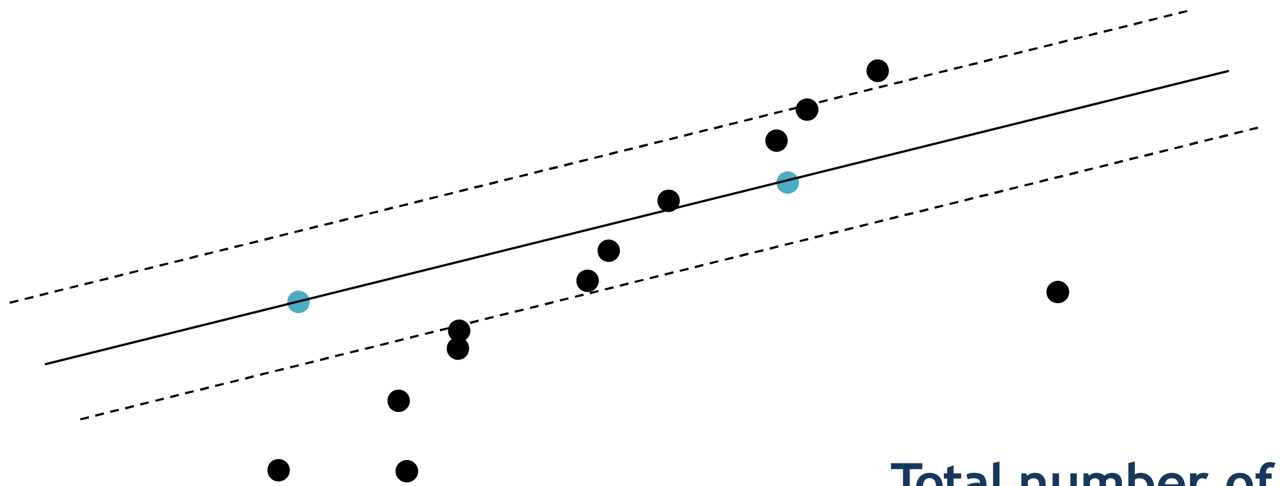
# RANSAC Line Fitting Example

- Task: Estimate the best line



# RANSAC Line Fitting Example

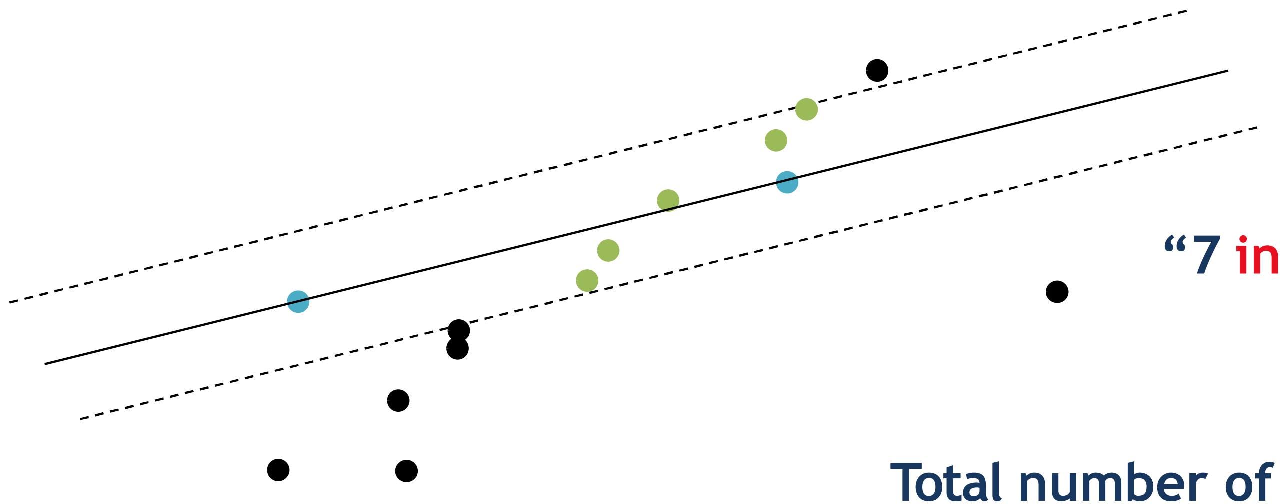
- Task: Estimate the best line



Total number of points within a threshold of line.

# RANSAC Line Fitting Example

- Task: Estimate the best line

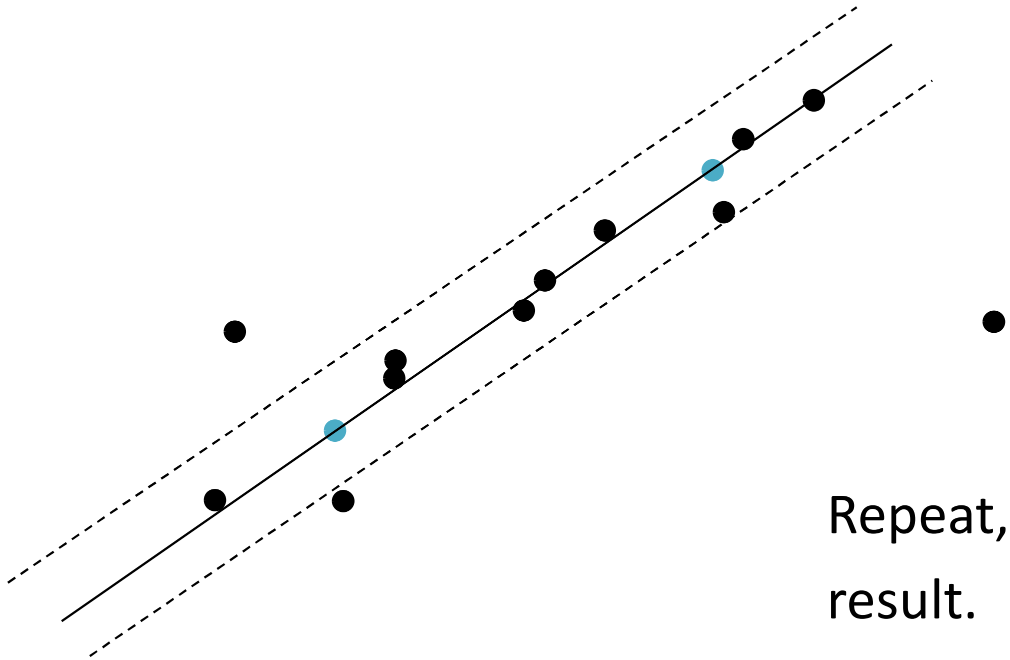


"7 inlier points"

Total number of points within a threshold of line.

# RANSAC Line Fitting Example

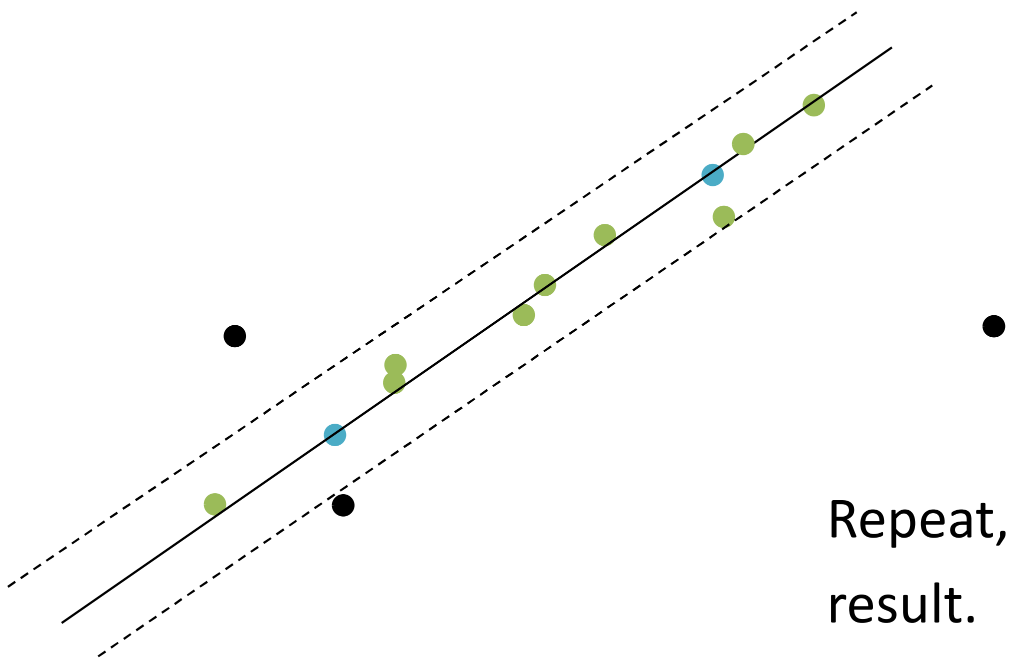
- Task: Estimate the best line



Repeat, until we get a good result.

# RANSAC Line Fitting Example

- Task: Estimate the best line
  - How many points do we need to estimate the line?





Repeat, until we get a good result.

# RANSAC algorithm

**Algorithm 15.4:** RANSAC: fitting lines using random sample consensus

Determine:
    $n$ — the smallest number of points required
    $k$ — the number of iterations required
    $t$ — the threshold used to identify a point that fits well
    $d$ — the number of nearby points required
      to assert a model fits well
Until $k$ iterations have occurred
    Draw a sample of $n$ points from the data
      uniformly and at random
    Fit to that set of $n$ points
    For each data point outside the sample
      Test the distance from the point to the line
        against $t$; if the distance from the point to the line
        is less than $t$, the point is close
    end
    If there are $d$ or more points close to the line
      then there is a good fit. Refit the line using all
      these points.
end
Use the best fit from this collection, using the
  fitting error as a criterion

# RANSAC: How many iterations " $k$ "?

- How many samples are needed?
  - Suppose $w$ is fraction of inliers (points from line).
  - $n$ points needed to define hypothesis (2 for lines)
  - $k$ samples chosen.
- Prob. that a single sample of $n$ points is correct: $w^n$
- Prob. that a single sample of $n$ points fails: $1 - w^n$
- Prob. that all $k$ samples fail is: $(1 - w^n)^k$
- Prob. that at least one of the $k$ samples is correct: $1 - (1 - w^n)^k$

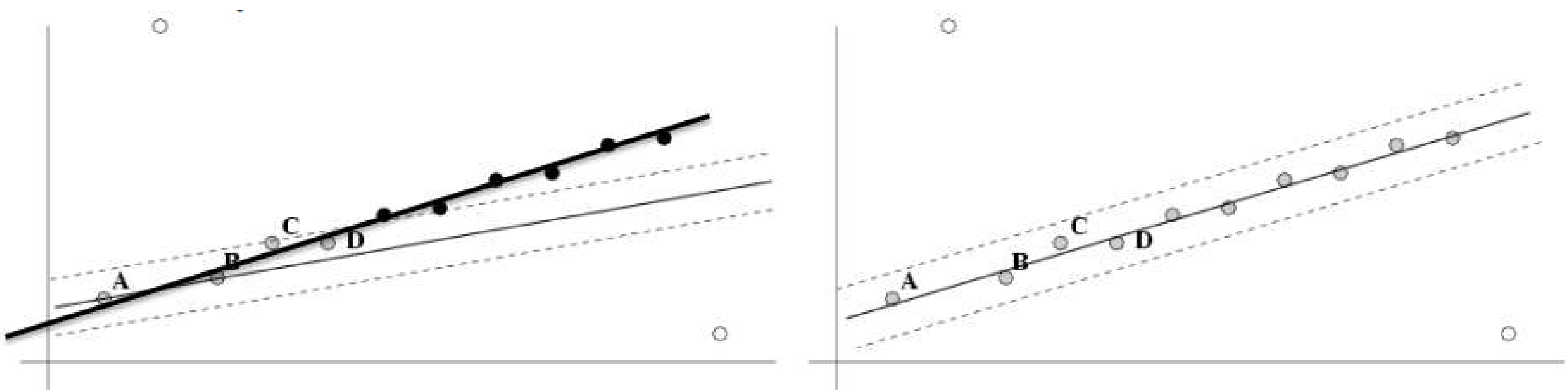
=> Choose $k$ high enough to keep this below desired failure rate.

# RANSAC: Computed k(p=0.99)

| Sample size n | Proportion of outliers | | | | | | |
|---|---|---|---|---|---|---|---|
| | 5% | 10% | 20% | 25% | 30% | 40% | 50% |
| 2 | 2 | 3 | 5 | 6 | 7 | 11 | 17 |
| 3 | 3 | 4 | 7 | 9 | 11 | 19 | 35 |
| 4 | 3 | 5 | 9 | 13 | 17 | 34 | 72 |
| 5 | 4 | 6 | 12 | 17 | 26 | 57 | 146 |
| 6 | 4 | 7 | 16 | 24 | 37 | 97 | 293 |
| 7 | 4 | 8 | 20 | 33 | 54 | 163 | 588 |
| 8 | 5 | 9 | 26 | 44 | 78 | 272 | 1177 |

# Refining RANSAC estimate

- RANSAC computes its best estimate from a minimal sample of $n$ points, and divides all data points into inliers and outliers using this estimate.
- We can improve this initial estimate by estimation over all inliers (e.g. with standard least-squares minimization).
- But this may change inliers, so alternate fitting with re-classification as inlier/outlier.



# RANSAC: Pros and Cons

- Pros:
  - General method suited for a wide range of model fitting problems
  - Easy to implement and easy to calculate its failure rate
- Cons:
  - Only handles a moderate percentage of outliers without cost blowing up
  - Many real problems have high rate of outliers (but sometimes selective choice of random subsets can help)
- A voting strategy, the Hough transform, can handle high percentage of outliers

# References

- Basic reading:
  - Szeliski textbook, Chapter 3.2, 4,1