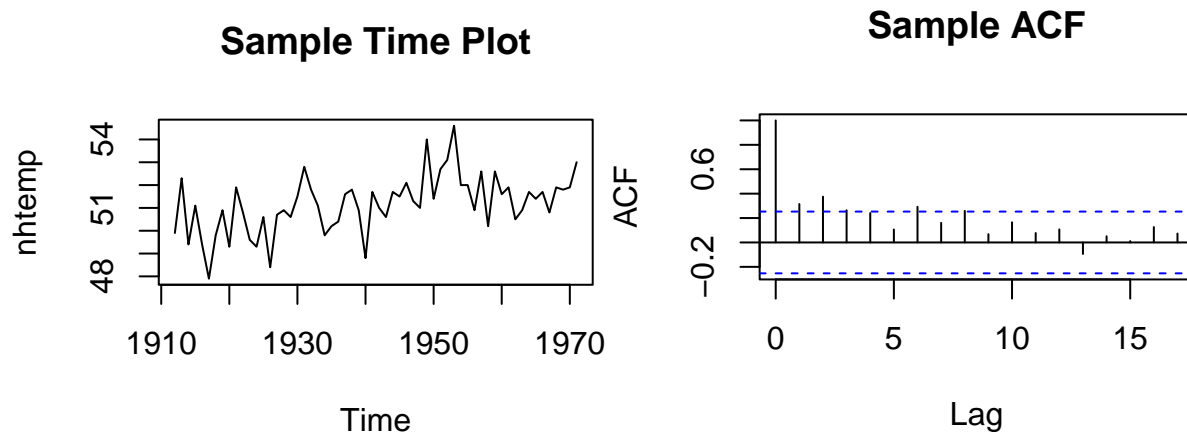# Time series CW2

Liangxiao LI,2024-04-10

Q1: nhtemp

Part1: Check Stationarity and Seasonality

First we produce the time plot and ACF plot from the given data:


From the plots above, we conclude that the series is non-stationary and non-seasonal due to following reasons:

   1) Time plot: the mean of the series appears higher between 1940-1970 to the period between 1910-1940.

   2) Sample ACF plot: doesn't decline rapidly, therefore it's not stationary.

Part2: Remove non-stationarity through first difference

To remove non-stationarity, we take the first difference of the time series **nhtemp** as **nhtemp_diff**:
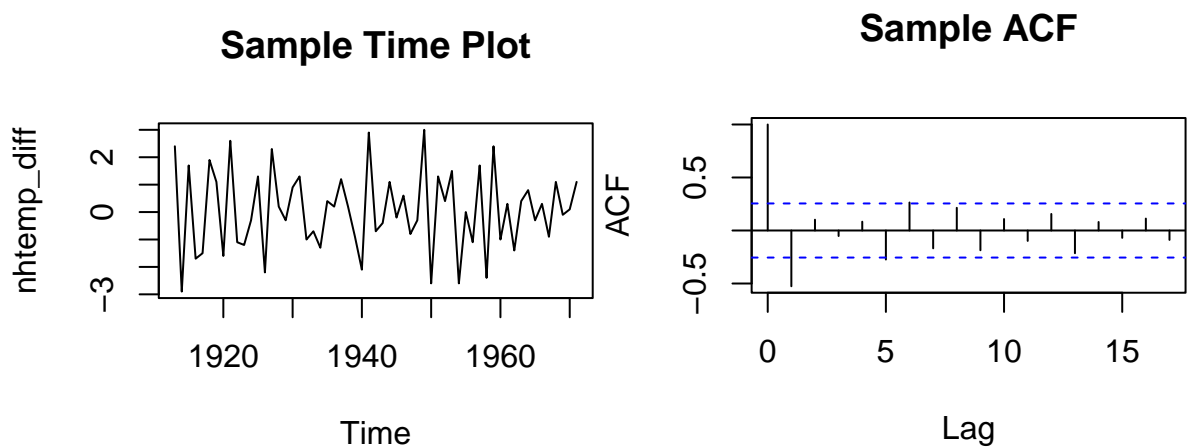
Therefore we conclude the series is (weakly) stationary without seasonality due to following reasons:

1) Time plot: has a mean equal to zero and shows constant variability over time.

2) Sample ACF plot: declines rapidly to zero as the lag increases, cut off after lag 1

In conclusion, we'll explore models with $d = 1$ in the following section.

Part3: Model fitting - Parameter analysis

The analysis begins by analyzing the sample ACF and PACF plot for **nhtemp_diff**:
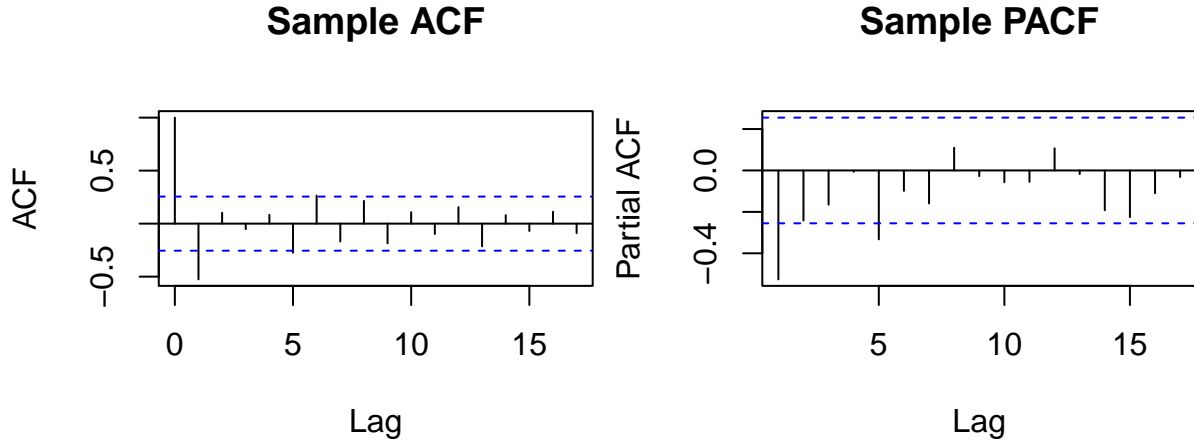
1) Since ACF cut off after lag 1, this suggest that we should begin by fitting an ARIMA(0,1,1) model

2) Since PACF doesn't cut off, this suggest the time series doesn't contain an AR component.

Part4: Model fitting - ARIMA(0,1,1)

We begins by fitting ARIMA(0,1,1), and we perform goodness of fit on the model based on following plots:
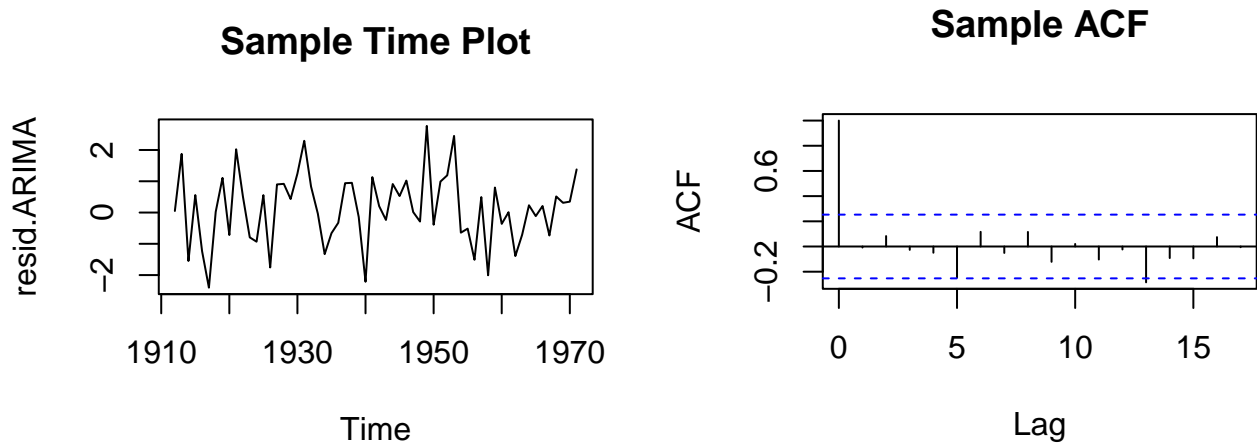
Ljung–Box test P–values
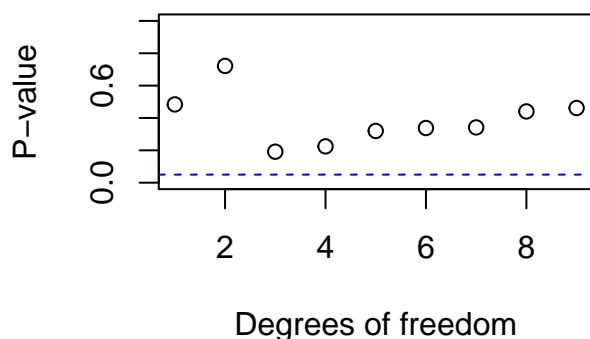
From the plots above, we conclude that ARIMA(0,1,1) is a good fit due to following reasons:

1) Time plot of the model residuals:

The time plot of the residuals looks similar to white noise, with mean zero and constant variance.

2) A plot of the sample ACF of the model residuals:

For all lags > 0, the sample ACF are all close to zero. This suggests that the residuals are independent(uncorrelated).

3) A plot of the first ten P-values for the Ljung-Box test:

All p-values are greater than 0.05(non-significant), this suggests the ARIMA(0,1,1) is a good fit to the data.

Part5: ARIMA(0,1,1) vs. ARIMA(1,1,1)

However, it's still worth checking if adding AR(p) component would be a better fit. Therefore we fit the model again with ARIMA(1,1,1)

```
##
## Call:
## arima(x = nhtemp, order = c(0, 1, 1), method = "ML")
##
## Coefficients:
##           ma1
##       -0.7983
## s.e.   0.0956
##
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 187.52

##
## Call:
## arima(x = nhtemp, order = c(1, 1, 1), method = "ML")
##
## Coefficients:
##          ar1      ma1
##       0.0073  -0.8019
## s.e.  0.1802   0.1285
##
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 189.52
```

From the summary above, we conclude that ARIMA(0,1,1) is better than ARIMA(1,1,1) due to following reasons:

1) AIC for ARIMA(0,1,1) is 187.12 is less than AIC for ARIMA(1,1,1), which is 189.52.

2) Perform hypothesis test: $H_0 : \phi_1 = 0$ vs. $H_1 : \phi_1 \neq 0$. The test statistic $= \frac{0.0073}{0.1802} < 2$, therefore we don't reject the null hypothesis and thus ARIMA(0,1,1) is better than ARIMA(1,1,1) model.

3) Overall we'd prefer a parsimonious model, thus ARIMA(0,1,1) is better than ARIMA(1,1,1)

Part6: ARIMA(0,1,1) vs. ARIMA(0,1,2)

We check further whether adding an adittional MA(q) component would be a better fit. Therefore we fit the model again with ARIMA(0,1,2)

```
## 
## Call:
## arima(x = nhtemp, order = c(0, 1, 2), method = "ML")
## 
## Coefficients:
##           ma1       ma2
##       -0.7956   -0.0042
## s.e.   0.1224    0.1221
## 
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 189.52
```

From the summary above, we conclude ARIMA(0,1,1) is better than ARIMA(0,1,2) due to following reasons:

1) AIC for ARIMA(0,1,1) is 187.12 is less than AIC for ARIMA(0,1,2), which is 189.52.

2) Perform hypothesis test: $H_0 : \theta_1 = 0$ vs. $H_1 : \theta_1 \neq 0$. The test statistic $= |\frac{-0.0042}{0.1221}| < 2$, therefore we don't reject the null hypothesis and thus ARIMA(0,1,1) is better than ARIMA(0,1,2) model.

3) Overall we'd prefer a parsimonious model, thus ARIMA(0,1,1) is better than ARIMA(0,1,2)
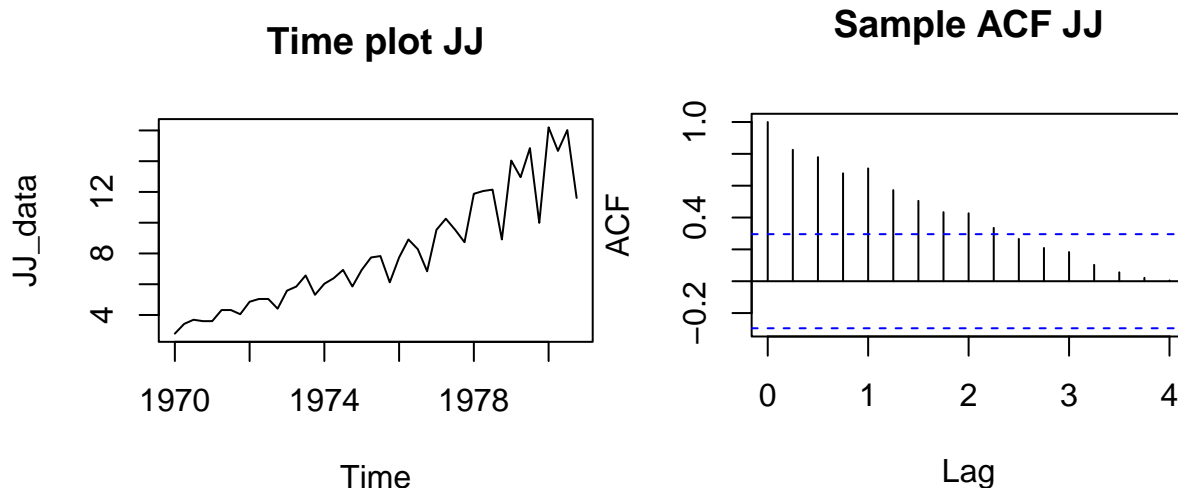
Part7: Conclusion: ARIMA(0,1,1) best

For question 1, the equation for the final fitted model is included below:

$$(1 - B)X_t = (1 - 0.7983B)Z_t$$

Q2: JJ_data

Part1: Check Stationarity and Seasonality

First we produce the time plot and ACF plot from the given data:
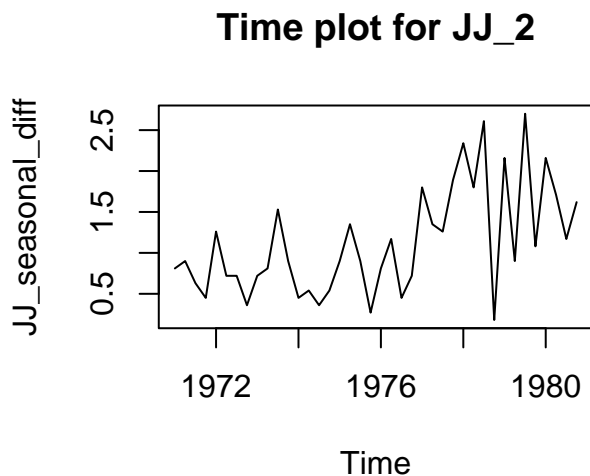
Time plot JJ

Sample ACF JJ

From the plots above, we conclude that the series is non-stationary and seasonal due to following reasons:

  1) Time plot: both the mean and variance of the series appears to increase overtime, which indicate non-stationarity.

  2) Sample ACF plot: doesn't decay rapidly, therefore it's not stationary.

 3) Time plot: the data shows seasonality, as the earnings are higher in Qtr 2,3 and lower in Qtr 1,4

Therefore would need to apply a SARIMA model for JJ_data.

Part2: Apply Seasonal difference on JJ_1

According to the data description, JJ is a time series of the quarterly earnings between years, so the seasonal difference lag should be set to $h = 4$. There fore if JJ_1 denotes our original time series, we define the lag 4 difference time series JJ_2 as $JJ_2 = \nabla_4 JJ_1 = (1 - B^4)JJ_1$
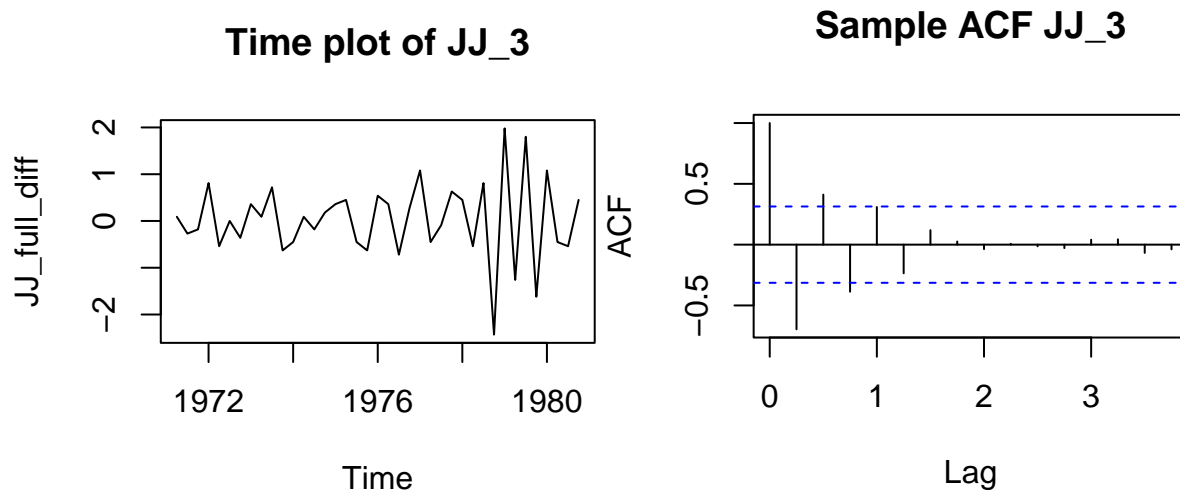
Time plot for JJ_2

From the time plot, it seems that the seasonality has been removed in JJ_2.

Sample ACF JJ_2
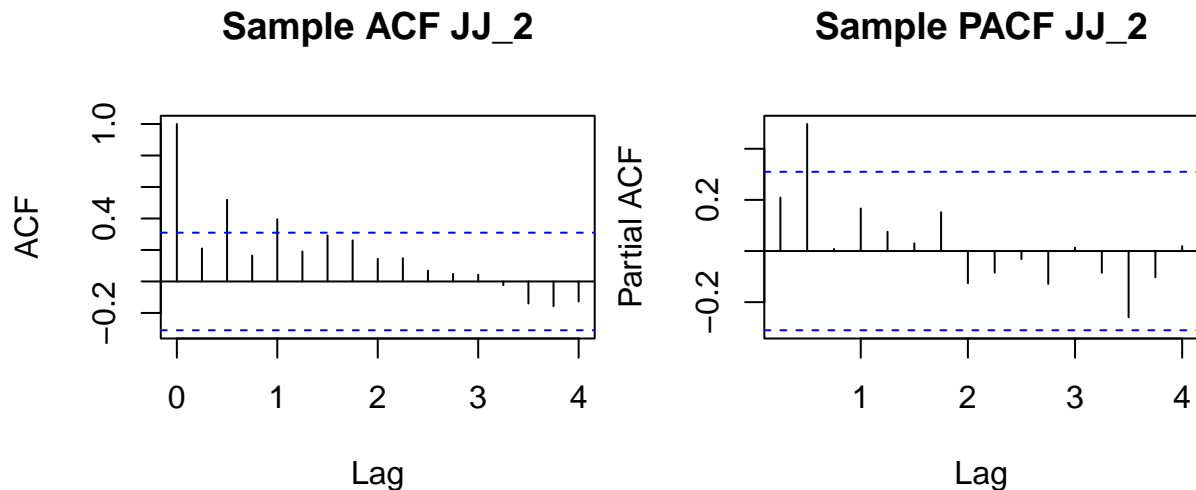
Sample PACF JJ_2

However, according to the sample ACF and sample PACF for the seasonally differnecd data, it suggest non-stationarity, because the ACF decays slowly.

Part3: Apply First difference on JJ_2

Therefore we'll take the first difference of JJ_2 and obtain $JJ_3 = \nabla^1 JJ_2 = (1 - B)JJ_2$

Time plot of JJ_3

Sample ACF JJ_3

Now JJ_3 appear to be stationary without seasonality.

However, the Time plot for JJ_3 shows a trend of non-constant variance, as the final part of the time series has greater variance compared with earlier part. Therefore we applied transformation to tackle with this problem.

## Part4: Apply Box-Cox transformation on JJ_1

I've applied both log and sqrt transformation on JJ_3 but the final fitted model but the final model doesn't perform well compared with fitting a SARIMA(1,1,1)x(0,1,0)[4] on non-transformed data.

By Googling I found box-cox transformation is a nice way to solve the problem of non-constant variance, and I implemented it on JJ_1, with optimal lambda set to be -0.305.

$$JJ_{tran1} = boxcox(JJ_1)$$

Part5: Remove non-stationarity and seasonality from JJ_tran1

Then we carry on the same process to remove the non-stationarity and seasonality. We first difference JJ_{tran1} with a seasonal difference lag $h = 4$ and gain $JJ_{tran2} = \nabla_4(JJ_{tran1}) = (1 - B^4)(JJ_{tran1})$, then we take the first difference on JJ_{tran2} and obtain $JJ_{tran3} = \nabla^1 JJ_{tran2} = (1 - B)JJ_{tran2}$. Below is the time plot for JJ_{tran2} and JJ_{tran3}

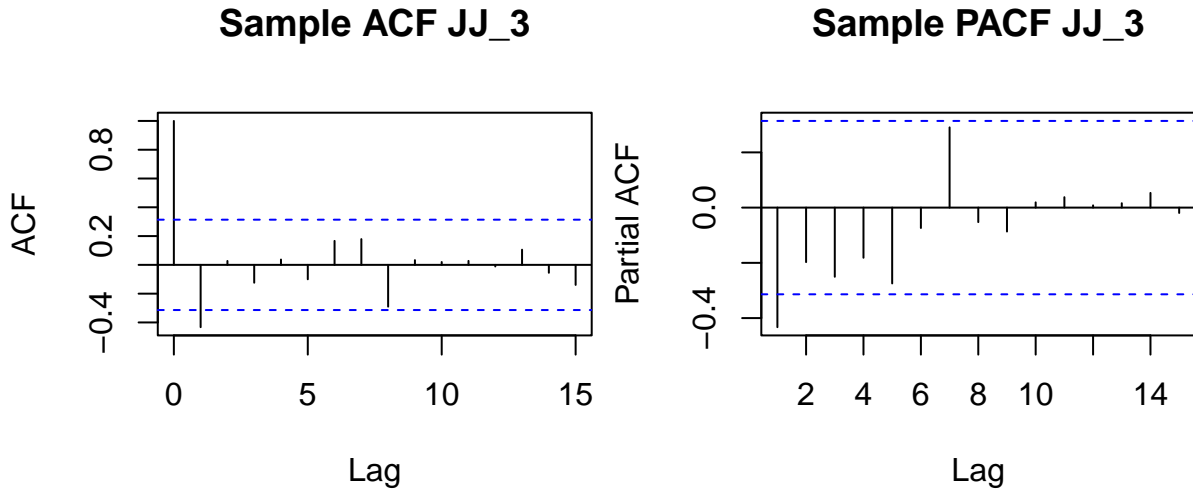### Time plot of JJ_{tran2}   Time plot of JJ_{tran3}
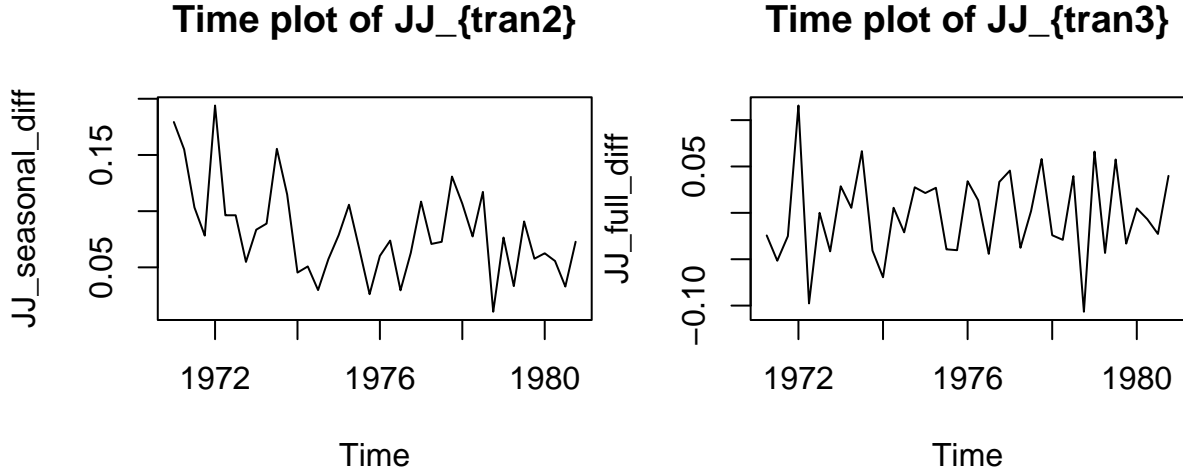
From the time plot, it seems that both non-stationarity and seasonality has been removed from JJ_{tran1}.

Part6: SARIMA Parameter analysis for JJ_3 and JJ_{tran3}

Now we start our fitting attempt with SARIMA(p,1,q)x(P,1,Q)[4].

Part 6.1: Non-transformed data JJ_3

Sample ACF JJ_3   Sample PACF JJ_3

The best model should be SARIMA(1,1,1) x (0,1,0)[4] due to following reasons:
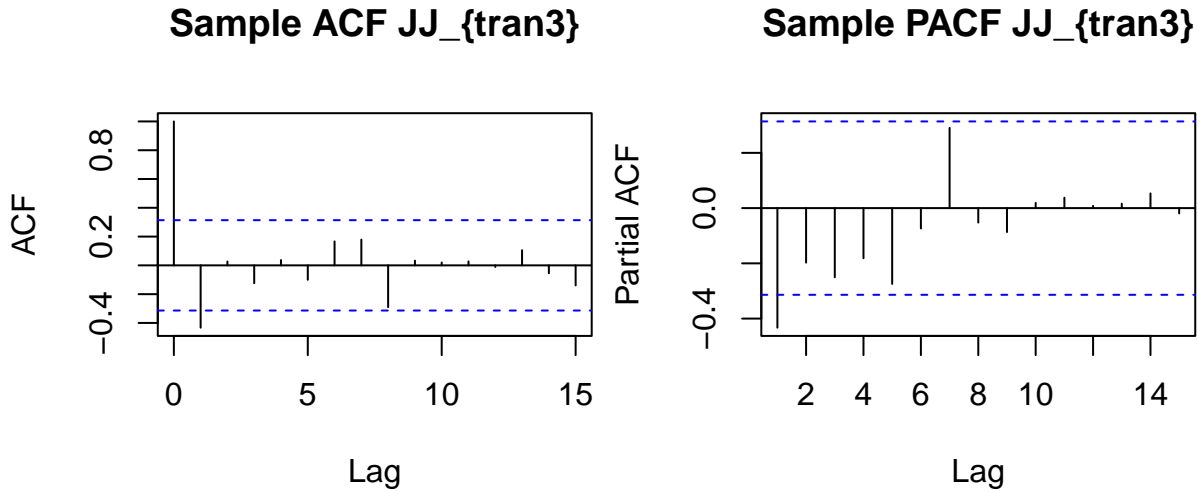
Seasonal components: (P,Q)

  1) P: Check PACF at lag = 4,8,12 ... PACF cut off already at lag = 4, therefore we choose P = 0.

  2) Q: Check ACF at lag = 4,8,12 ... ACF cut off already at lag = 4, therefore we choose Q = 0

Non Seasonal components : (p,q)

  3) p: PACF cut off after lag = 1, therefore we choose p = 1

4) q: ACF cut off after lag = 1, therefore we choose q = 1.

Part 6.2: Transformed data JJ_{tran3}

Sample ACF JJ_{tran3}   **Sample PACF JJ_{tran3}**

The best model should be SARIMA(0,1,1) x (0,1,0)[4] due to following reasons:
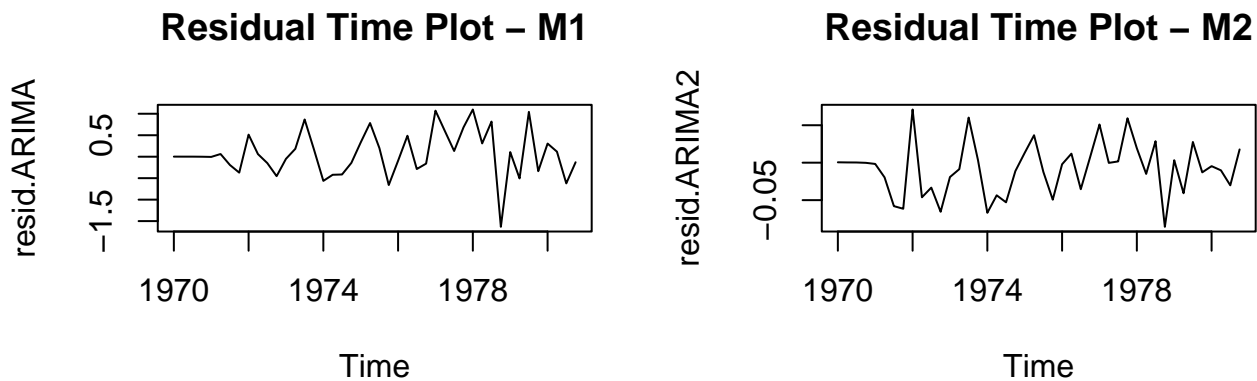
Seasonal components: (P,Q)

  1) P: Check PACF at lag = 4,8,12 ... PACF cut off already at lag = 4, therefore we choose P = 0.

  2) Q: Check ACF at lag = 4,8,12 ... ACF cut off already at lag = 4, therefore we choose Q = 0
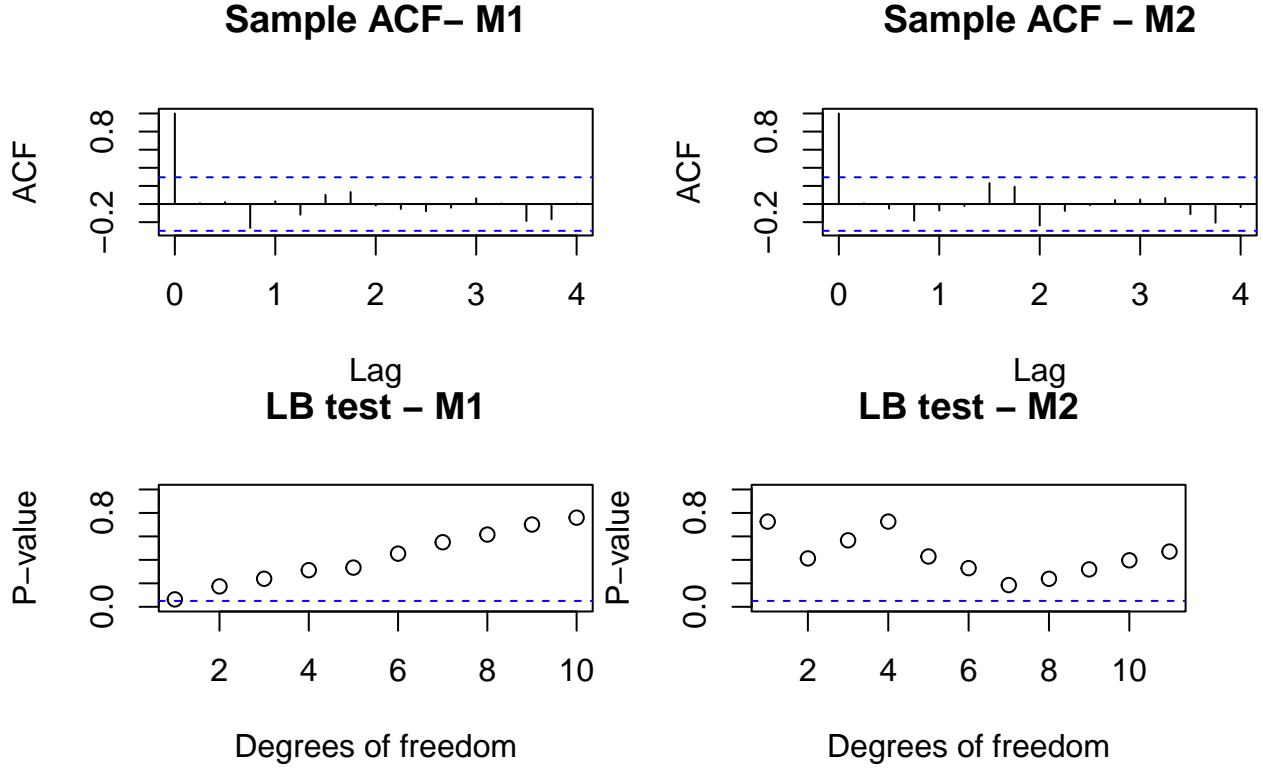
Non Seasonal components : (p,q)

  3) p: PACF cut off after lag = 0, therefore we choose p = 0

  4) q: ACF cut off after lag = 1, therefore we choose q = 1.

Part7: Model Diagonostic JJ_1 **vs.** JJ_{tran1}

To make expression concise, I'll use 'M1' to refer the SARIMA(1,1,1)x(0,1,0)[4] model for JJ_1 and 'M2' to refer the SARIMA(0,1,1)x(0,1,0)[4] model for JJ_{tran1}

Residual Time Plot – M1   **Residual Time Plot – M2**

Sample ACF– M1
Sample ACF – M2
LB test – M1
LB test – M2

From the plots above, we conclude that both M1 and M2 are good fits, while M2 is slightly better due to following reasons:

1) Time plot of the model residuals:

The time plot of the residuals for M2 looks to be white noise, with mean zero and constant variance. (However for M1 the variance isn't constant)

2) A plot of the sample ACF of the model residuals

For all lags > 0, the sample ACF are all close to zero except at lag = 1. This suggests that the residuals are almost independent(uncorrelated).

3) A plot of the first ten P-values for the Ljung-Box test

All p-values for M2 are greater than 0.05(non-significant).

However for M1 the first p-value is significant

Part8: Conclusion

Let X_t denote the original JJ_data

For the non-transformed data(not included in the report due to page limit), the best model is SARIMA(1,1,1)x(0,1,0)[4], and the equation is:

$$(1 + 0.3465B)(1 - B)(1 - B^4)X_t = (1 - 0.6308B)Z_t$$

For the boxcox transformed data, the best model is SARIMA(0,1,1)x(0,1,0)[4], and the equation is:
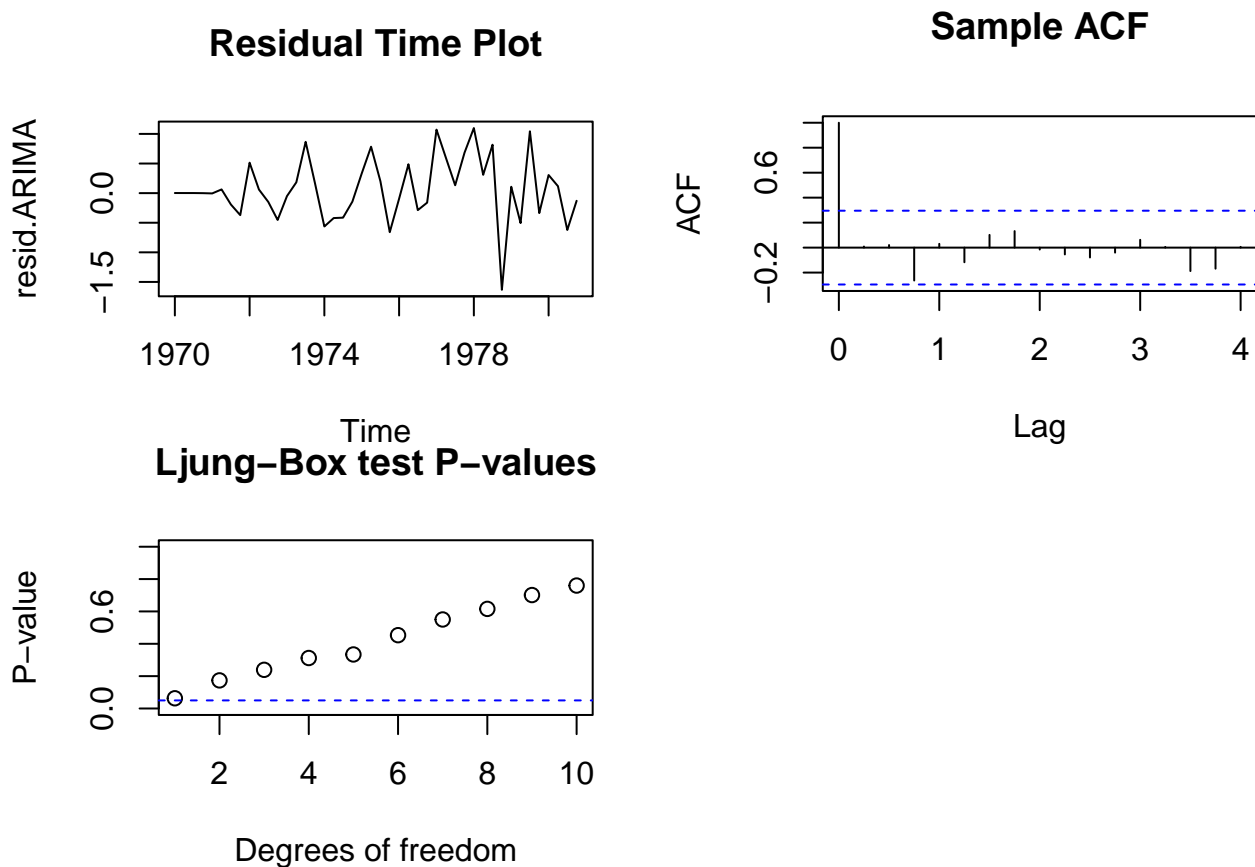
$$(1 - B)(1 - B^4)boxcox(X_t) = (1 - 0.7325B)Z_t$$

here $boxcox()$ denotes the transformation performed on the original JJ_data.

trash content

start our fitting attempt with SARIMA(p,1,q)x(P,1,Q)[4].

Parameter analysis

Model fitting

Residual Time Plot   **Sample ACF**

Ljung–Box test P–values

From the plots above, we conclude that SARIMA(1,1,1)x(0,1,0)[4] is a fairly good fit due to following reasons:

    1) Time plot of the model residuals:

The time plot of the residuals looks similar to white noise, with mean zero, but the variance increases overtime.

    2) A plot of the sample ACF of the model residuals

For all lags > 0, the sample ACF are all close to zero except at lag $= 1$. This suggests that the residuals are almost independent(uncorrelated).

    3) A plot of the first ten P-values for the Ljung-Box test

Although the first p-value is fairly significant, all other p-values are greater than 0.05(non-significant), this suggests a fairly good model for JJ_data.

```
last_12 <- tail(JJ_data, 12)

# Apply a logarithmic transformation to the last 12 elements
transformed_last_12_log <- log(last_12)

# Replace the last 12 elements in the original time series with the transformed values
```

```r
# Calculate the starting index for the last 12 elements
start_index <- length(JJ_data) - length(transformed_last_12_log) + 1
end_index <- length(JJ_data)

JJ_transform <- JJ_data

# Replace the elements
JJ_transform[start_index:end_index] <- transformed_last_12_log
```

```r
subset <- window(JJ_transform, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_transform, start = c(1971+k,1), end = c(1971+k,4))

  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Variance",
     xlab="Sample Mean", ylab="Sample Variance", pch=19)
```
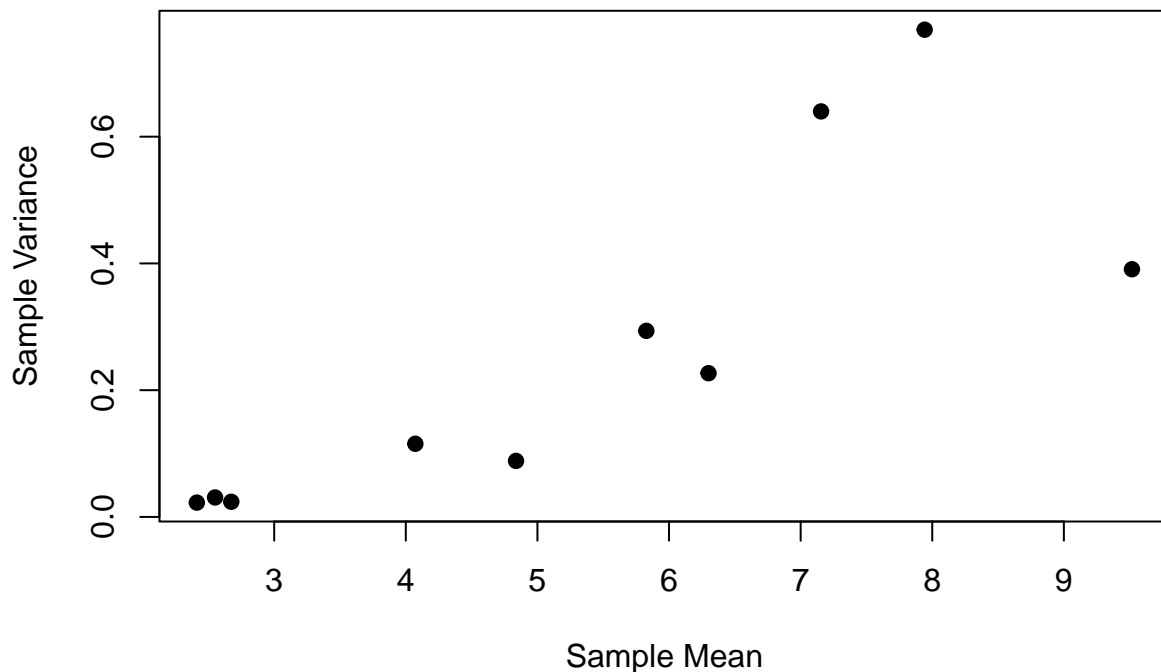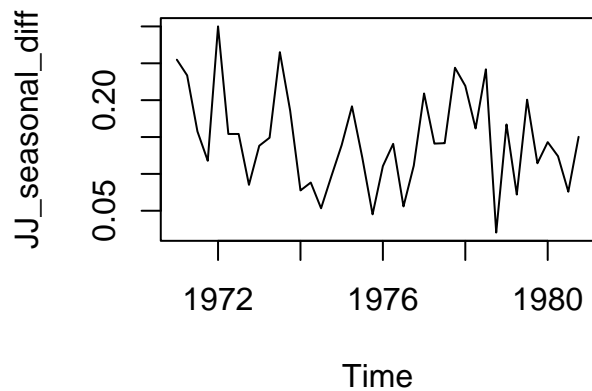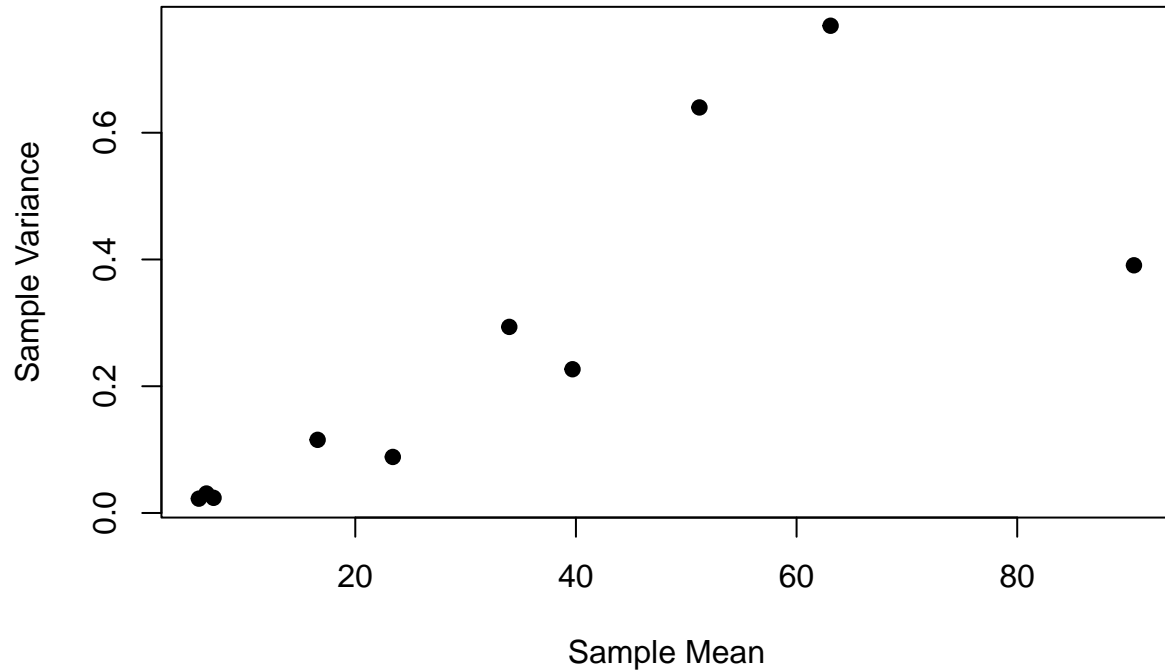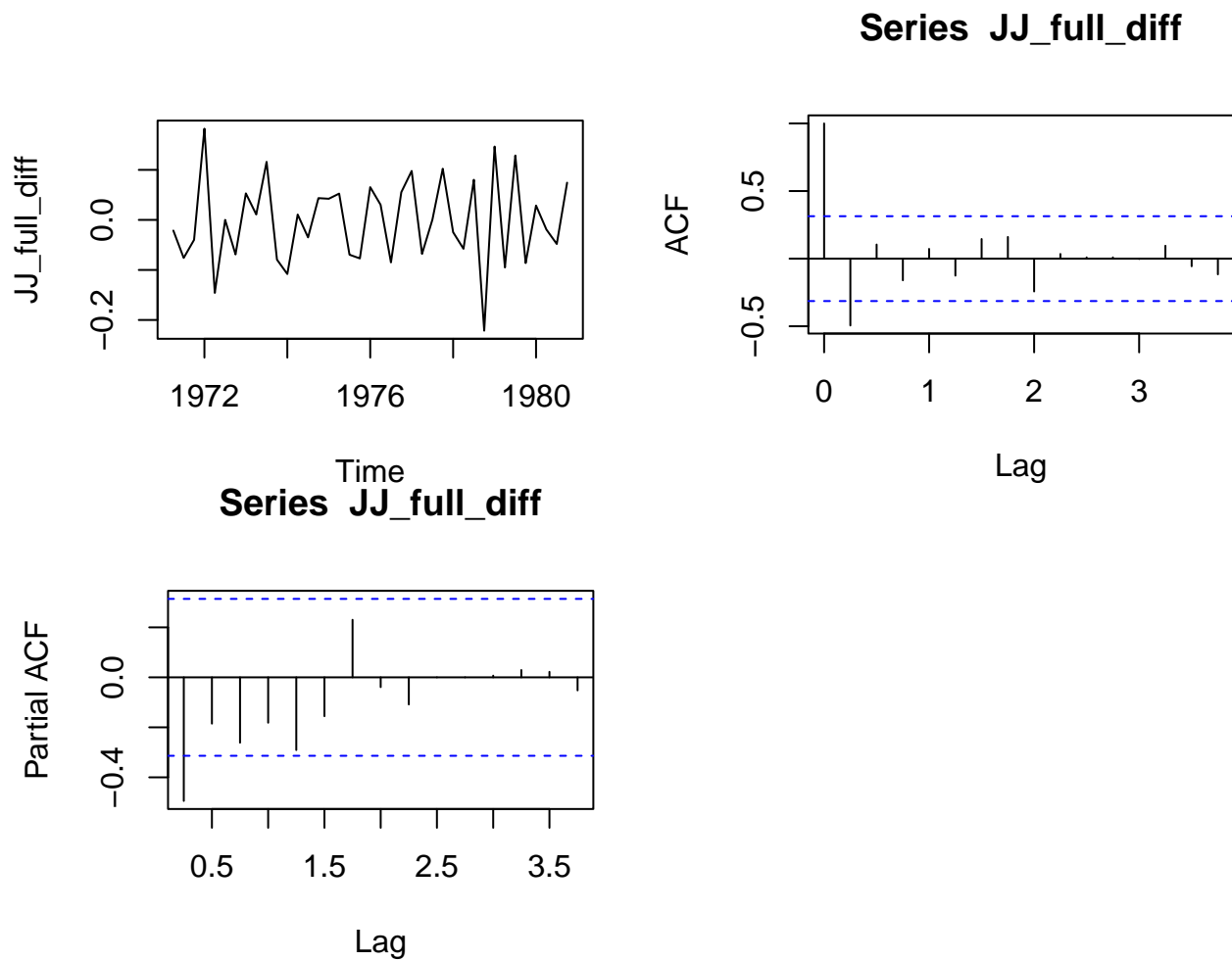
Scatter Plot of Mean vs Variance

```
plot(mean_vector^2, var_vector, main="Scatter Plot of Mean vs Variance",
     xlab="Sample Mean", ylab="Sample Variance", pch=19)
```

Scatter Plot of Mean vs Variance

Series JJ_full_diff

Series JJ_full_diff


```
fit <- auto.arima(nhtemp)
summary(fit)
```
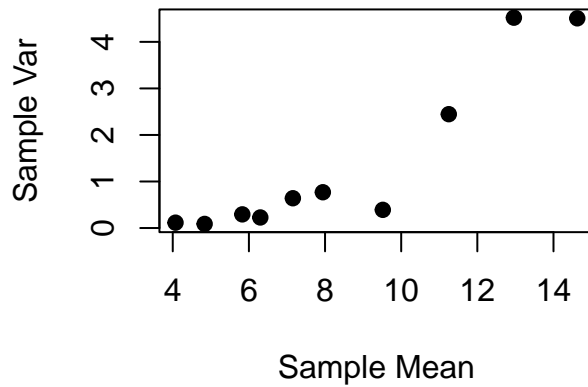
```
## Series: nhtemp
## ARIMA(0,1,1)
##
## Coefficients:
##           ma1
##        -0.7983
## s.e.   0.0956
##
## sigma^2 = 1.313:  log likelihood = -91.76
## AIC=187.52   AICc=187.73   BIC=191.67
##
## Training set error measures:
##                      ME      RMSE       MAE       MPE      MAPE      MASE
## Training set 0.1285716 1.126722 0.8952077 0.2080785 1.749865 0.7513123
##                     ACF1
## Training set -0.008258617
```

```
#fit <- auto.arima(JJ_data_transformed)
#summary(fit)
```
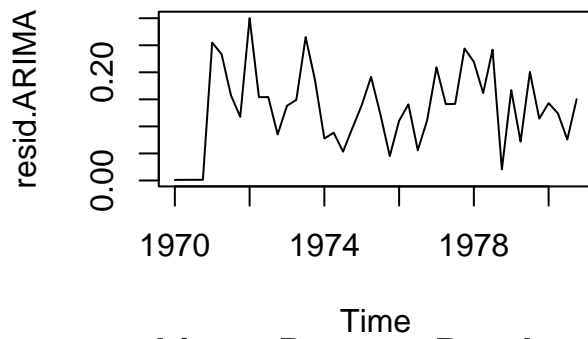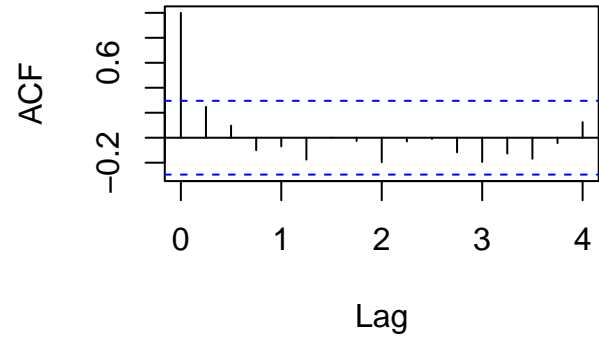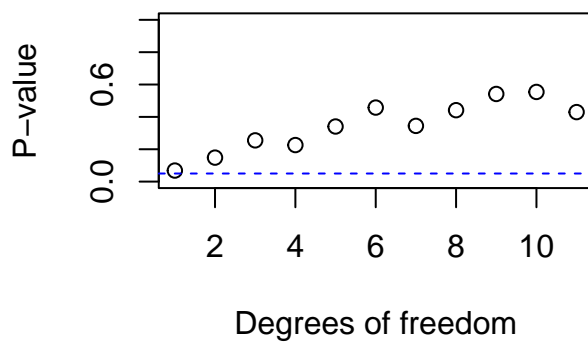
Scatter Plot of Mean vs Var

Scatter Plot of Mean^2 vs Var
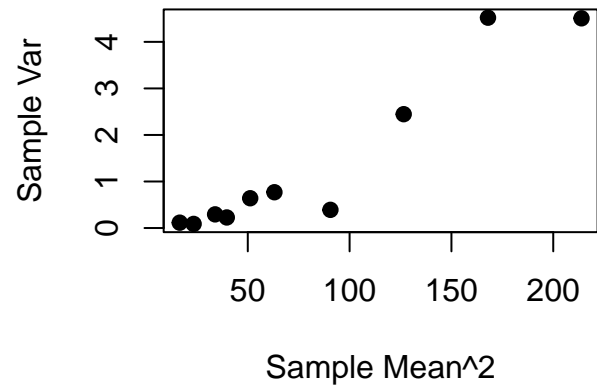
Residual Time Plot

Sample ACF

Ljung−Box test P−values

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Residual Time Plot
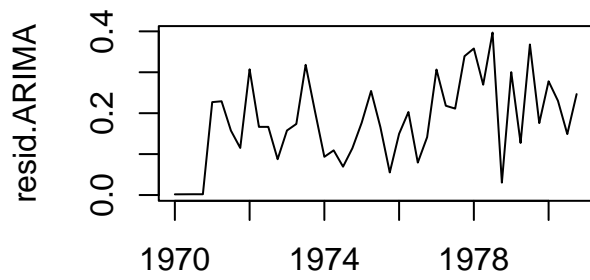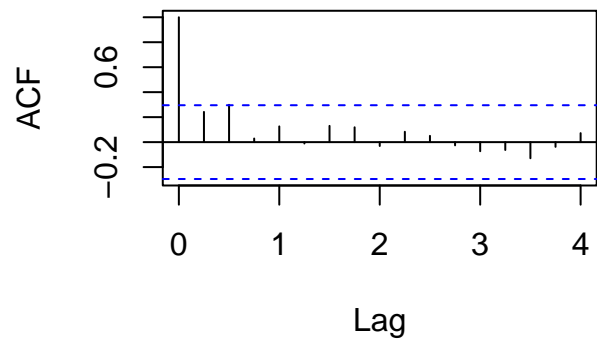
Sample ACF

Ljung–Box test P–values
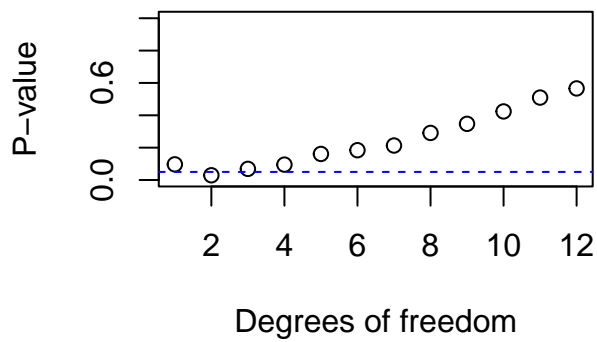
Test content

Time plot of JJ_{tran1}
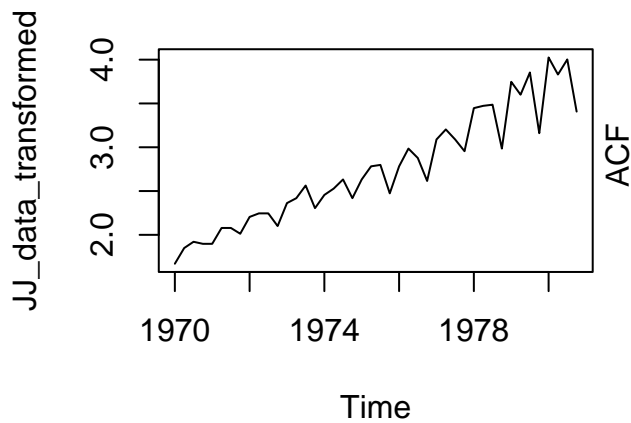
sample ACF of JJ_{tran1}
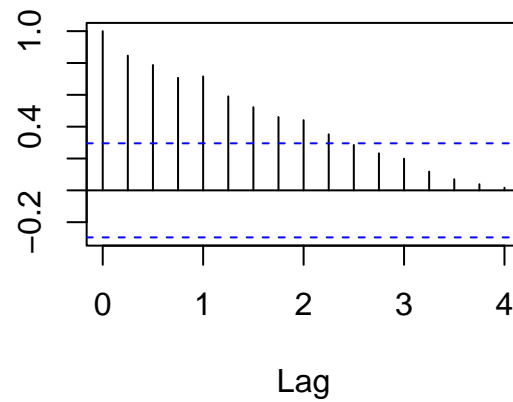

```r
JJ_seasonal_diff <- diff(sqrt(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff,main ="Time plot for JJ_2")
```

Time plot for JJ_2
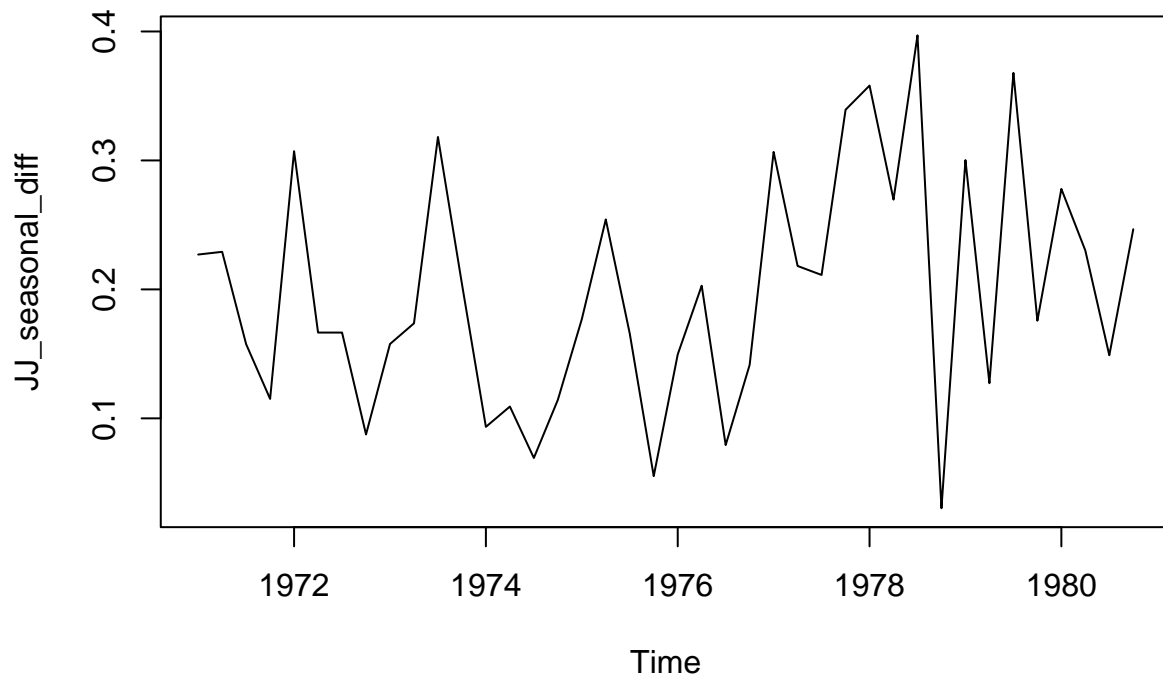

```
acf(JJ_seasonal_diff,main = "Sample ACF JJ_2")
```

Sample ACF JJ_2
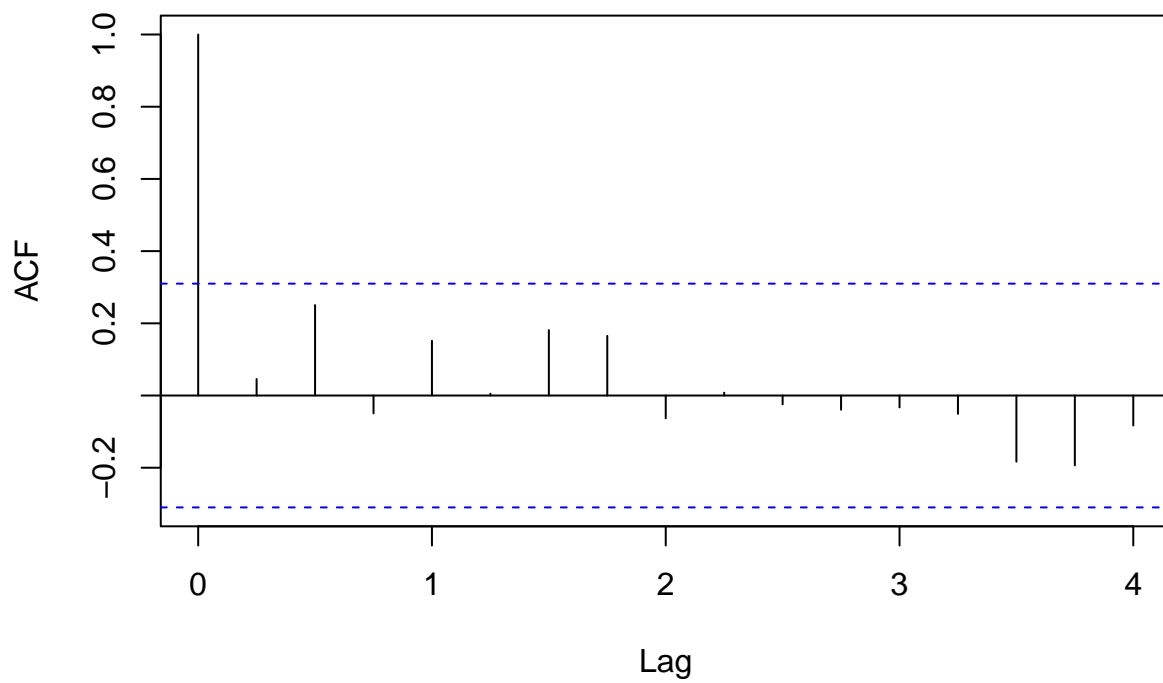

```
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
```

Sample PACF JJ_2
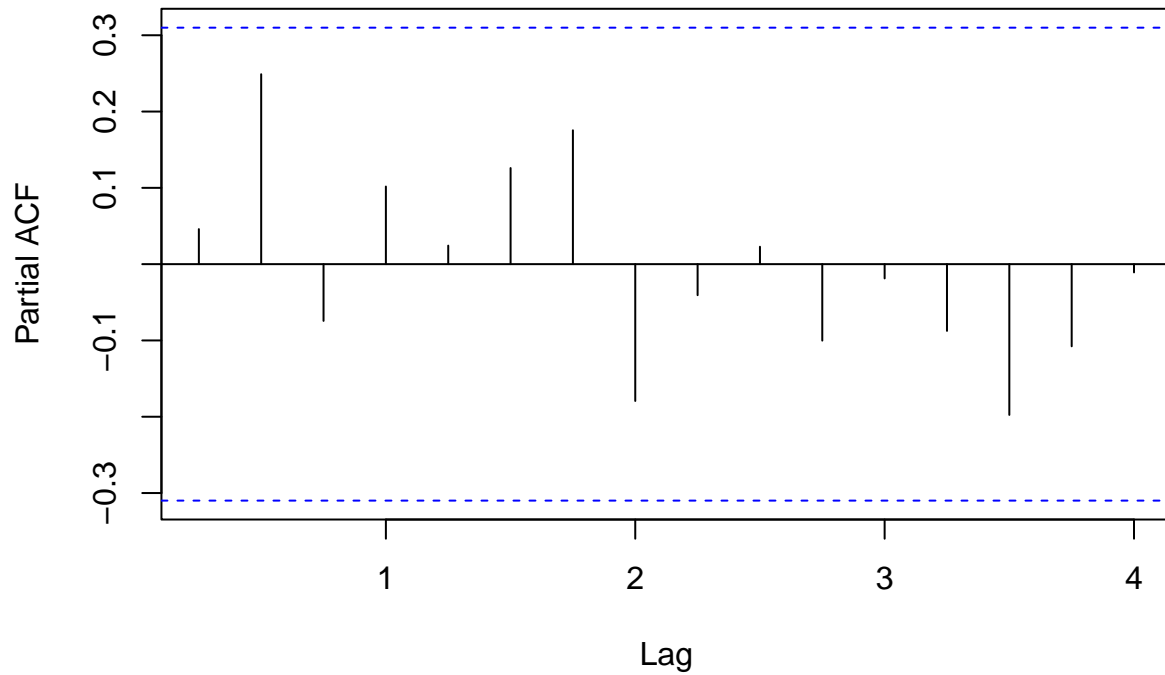
Appendix

```
knitr::opts_chunk$set(echo = TRUE)

library(forecast)
LB_test<-function(resid,max.k,p,q){
  lb_result<-list()
  df<-list()
  p_value<-list()
  for(i in (p+q+1):max.k){
    lb_result[[i]]<-Box.test(resid,lag=i,type=c("Ljung-Box"),fitdf=(p+q))
    df[[i]]<-lb_result[[i]]$parameter
    p_value[[i]]<-lb_result[[i]]$p.value
  }
  df<-as.vector(unlist(df))
  p_value<-as.vector(unlist(p_value))
  test_output<-data.frame(df,p_value)
  names(test_output)<-c("deg_freedom","LB_p_value")
  return(test_output)
}
load("nhtemp.rda")
# Time Series Plot
ts.plot(nhtemp, main="Sample Time Plot")

# ACF Plot
acf(nhtemp, main="Sample ACF")
```

```r
# PACF Plot
#pacf(nhtemp, main="Sample PACF")
nhtemp_diff<-diff(nhtemp)
ts.plot(nhtemp_diff, main="Sample Time Plot")
acf(nhtemp_diff, main="Sample ACF")
#pacf(nhtemp_diff)
acf(nhtemp_diff, main="Sample ACF")
pacf(nhtemp_diff, main = "Sample PACF")
ARIMA<-arima(nhtemp,order=c(0,1,1),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Sample Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test(resid.ARIMA,max.k=11,p=0,q=2)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box
abline(h=0.05,col="blue",lty=2)
ARIMA
ARIMA<-arima(nhtemp,order=c(1,1,1),method="ML")
ARIMA
ARIMA<-arima(nhtemp,order=c(0,1,2),method="ML")
ARIMA
load("JJ_data.rda")
#JJ_data_ts <- ts(JJ_data, start=c(1970, 1))
LB_test_SARIMA<-function(resid,max.k,p,q,P,Q){
 lb_result<-list()
 df<-list()
 p_value<-list()
  for(i in (p+q+P+Q+1):max.k){
   lb_result[[i]]<-Box.test(resid,lag=i,type=c("Ljung-Box"),fitdf=(p+q+P+Q))
   df[[i]]<-lb_result[[i]]$parameter
   p_value[[i]]<-lb_result[[i]]$p.value
  }
 df<-as.vector(unlist(df))
 p_value<-as.vector(unlist(p_value))
 test_output<-data.frame(df,p_value)
 names(test_output)<-c("deg_freedom","LB_p_value")
 return(test_output)
 }
ts.plot(JJ_data, main = "Time plot JJ")
acf(JJ_data,main = "Sample ACF JJ")
#pacf(JJ_data)
JJ_seasonal_diff <- diff(JJ_data,lag=4)
ts.plot(JJ_seasonal_diff,main ="Time plot for JJ_2")
acf(JJ_seasonal_diff,main = "Sample ACF JJ_2")
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
JJ_full_diff <- diff(JJ_seasonal_diff)
ts.plot(JJ_full_diff,main = "Time plot of JJ_3")
acf(JJ_full_diff, main = "Sample ACF JJ_3")
#pacf(JJ_full_diff, main = "Sample PACF JJ_3")
# Estimate the optimal lambda for the Box-Cox transformation
lambda <- BoxCox.lambda(JJ_data)
```

```r
# Apply the Box-Cox transformation
JJ_data_transformed <- BoxCox(JJ_data, lambda)
ts.plot(JJ_data_transformed, main ="Time plot of JJ_{tran1}")
acf(JJ_data_transformed, main = "sample ACF of JJ_{tran1}")
#pacf(JJ_data)
JJ_seasonal_diff <- diff(JJ_data_transformed,lag=4)
ts.plot(JJ_seasonal_diff,main ="Time plot of JJ_{tran2}")
JJ_full_diff <- diff(JJ_seasonal_diff)
ts.plot(JJ_full_diff,main = "Time plot of JJ_{tran3}")
JJ_data_ts <- ts(JJ_full_diff, start=c(1970, 1))
acf(JJ_data_ts, main = "Sample ACF JJ_3")
pacf(JJ_data_ts, main = "Sample PACF JJ_3")
JJ_data_ts <- ts(JJ_full_diff, start=c(1970, 1))
acf(JJ_data_ts, main = "Sample ACF JJ_{tran3}")
pacf(JJ_data_ts, main = "Sample PACF JJ_{tran3}")
fit <- auto.arima(JJ_data_transformed)
summary(fit)
ARIMA<-arima(JJ_data,order=c(1,1,1),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA2<-arima(JJ_data_transformed,order=c(0,1,1),seasonal=list(order=c(0,1,0),period=4),method="ML")

resid.ARIMA<-residuals(ARIMA)
resid.ARIMA2<-residuals(ARIMA2)

ts.plot(resid.ARIMA, main = "Residual Time Plot - M1")
ts.plot(resid.ARIMA2, main = "Residual Time Plot - M2")


acf(resid.ARIMA, main = "Sample ACF- M1")
acf(resid.ARIMA2, main = "Sample ACF - M2")


ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=1,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="LB test - 
abline(h=0.05,col="blue",lty=2)


ARIMA.LB2<-LB_test_SARIMA(resid.ARIMA2,max.k=12,p=0,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB2$deg_freedom,ARIMA.LB2$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="LB test 
abline(h=0.05,col="blue",lty=2)



ARIMA<-arima(JJ_data,order=c(1,1,1),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
```

```
ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=1,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box
abline(h=0.05,col="blue",lty=2)
library(forecast)
fit <- auto.arima(JJ_data)
summary(fit)

fit <- auto.arima(sqrt(JJ_data))
summary(fit)




last_12 <- tail(JJ_data, 12)

# Apply a logarithmic transformation to the last 12 elements
transformed_last_12_log <- log(last_12)

# Replace the last 12 elements in the original time series with the transformed values
# Calculate the starting index for the last 12 elements
start_index <- length(JJ_data) - length(transformed_last_12_log) + 1
end_index <- length(JJ_data)

JJ_transform <- JJ_data

# Replace the elements
JJ_transform[start_index:end_index] <- transformed_last_12_log
subset <- window(JJ_transform, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_transform, start = c(1971+k,1), end = c(1971+k,4))

  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Variance",
     xlab="Sample Mean", ylab="Sample Variance", pch=19)

plot(mean_vector^2, var_vector, main="Scatter Plot of Mean vs Variance",
     xlab="Sample Mean", ylab="Sample Variance", pch=19)

JJ_seasonal_diff <- diff(log(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff)
JJ_full_diff <- diff(JJ_seasonal_diff)
```

```r
ts.plot(JJ_full_diff)
acf(JJ_full_diff)
pacf(JJ_full_diff)
fit <- auto.arima(nhtemp)
summary(fit)
#fit <- auto.arima(JJ_data_transformed)
#summary(fit)
subset <- window(JJ_data, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_data, start = c(1971+k,1), end = c(1971+k,4))

  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Var",
     xlab="Sample Mean", ylab="Sample Var", pch=19)

plot(mean_vector^2, var_vector, main="Scatter Plot of Mean^2 vs Var",
     xlab="Sample Mean^2", ylab="Sample Var", pch=19)

ARIMA<-arima(log(JJ_data),order=c(0,0,0),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=0,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box
abline(h=0.05,col="blue",lty=2)
ARIMA<-arima(sqrt(JJ_data),order=c(0,0,0),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=0,q=0,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box
abline(h=0.05,col="blue",lty=2)
JJ_data_transformed <- sqrt(JJ_data)
ts.plot(JJ_data_transformed, main ="Time plot of JJ_{tran1}")
acf(JJ_data_transformed, main = "sample ACF of JJ_{tran1}")
#pacf(JJ_data)
```

```r
JJ_seasonal_diff <- diff(sqrt(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff,main ="Time plot for JJ_2")
acf(JJ_seasonal_diff,main = "Sample ACF JJ_2")
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
```