

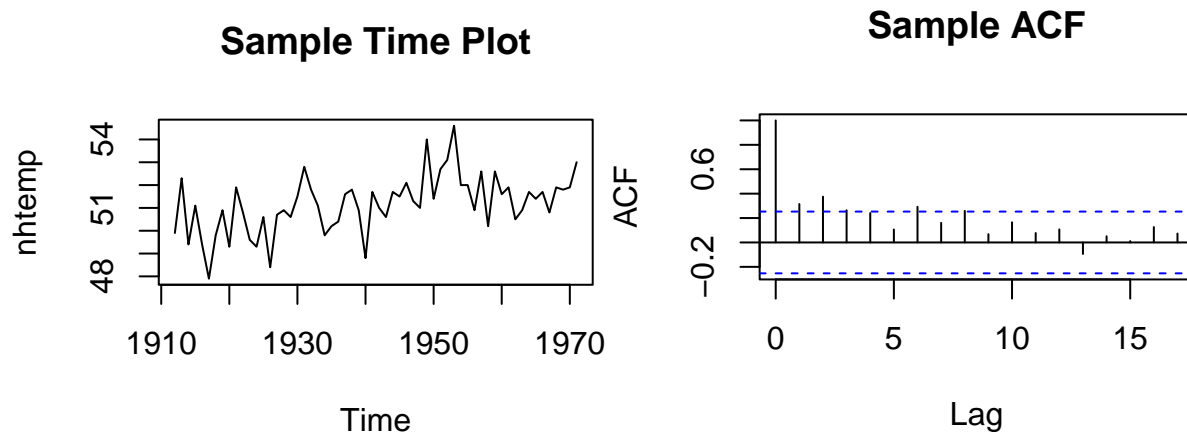
Time series CW2

Liangxiao LI, 2024-04-10

Q1: nhtemp

Part1: Check Stationarity and Seasonality

First we produce the time plot and ACF plot from the given data:

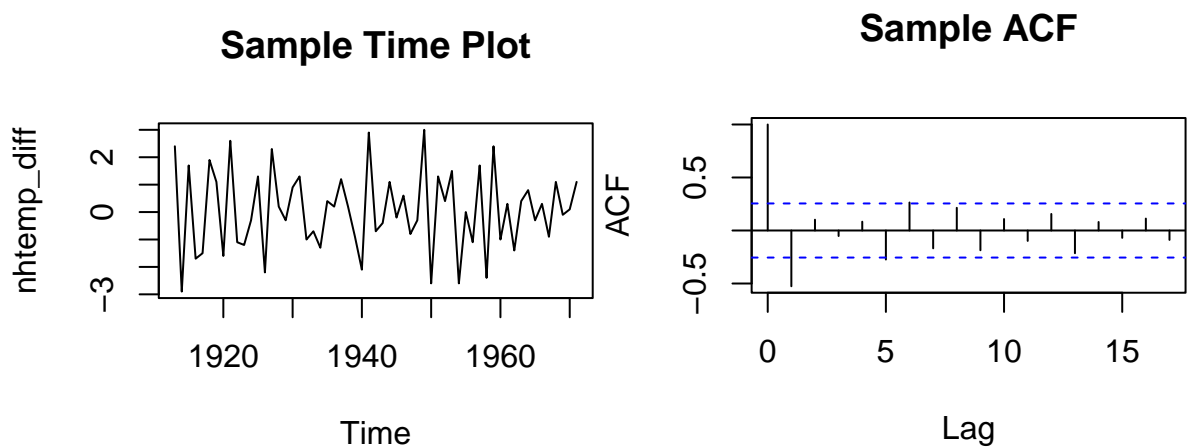


From the plots above, we conclude that the series is non-stationary and non-seasonal due to following reasons:

- 1) Time plot: the mean of the series appears higher between 1940-1970 to the period between 1910-1940.
- 2) Sample ACF plot: doesn't decline rapidly, therefore it's not stationary.

Part2: Remove non-stationarity through first difference

To remove non-stationarity, we take the first difference of the time series **nhtemp** as **nhtemp_diff**:



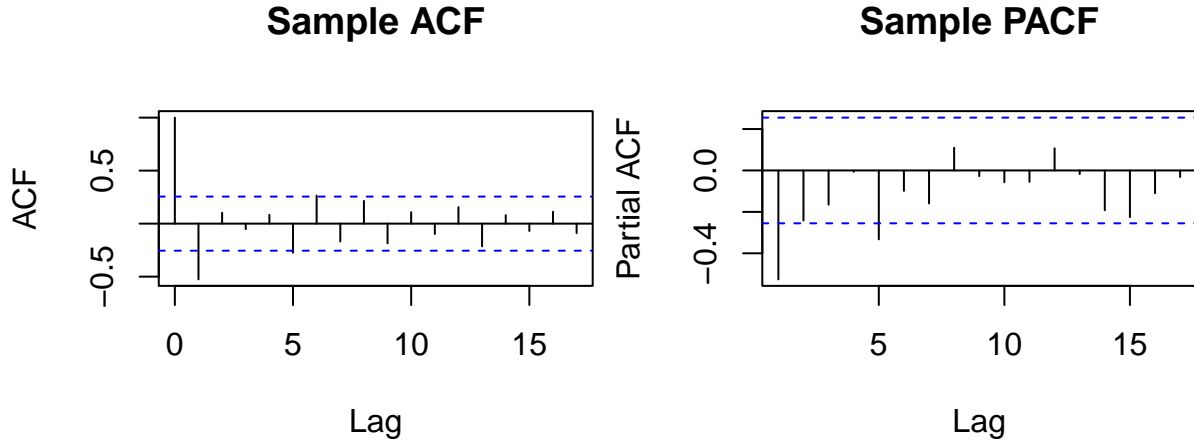
Therefore we conclude the series is (weakly) stationary without seasonality due to following reasons:

- 1) Time plot: has a mean equal to zero and shows constant variability over time.
- 2) Sample ACF plot: declines rapidly to zero as the lag increases, cut off after lag 1

In conclusion, we'll explore models with $d = 1$ in the following section.

Part3: Model fitting - Parameter analysis

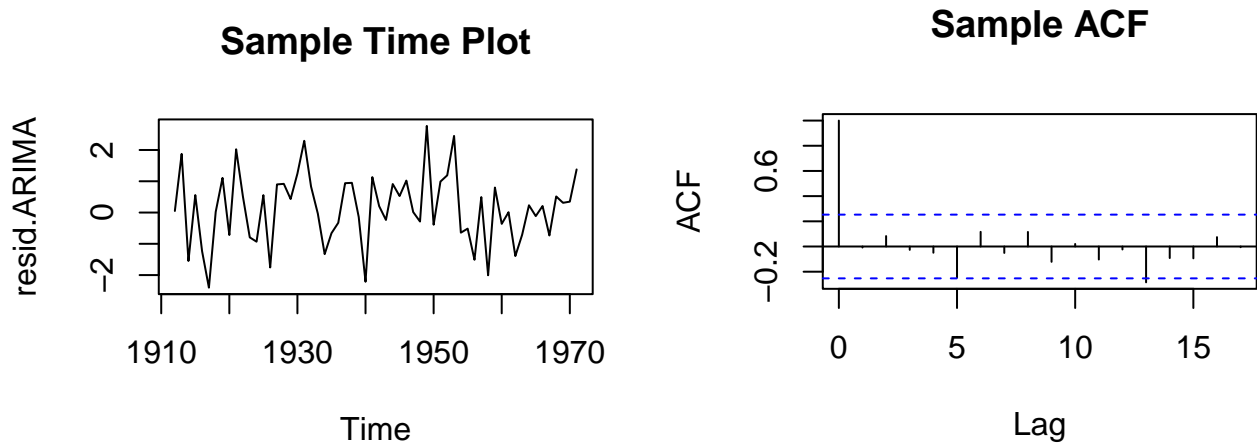
The analysis begins by analyzing the sample ACF and PACF plot for `nhtemp_diff`:



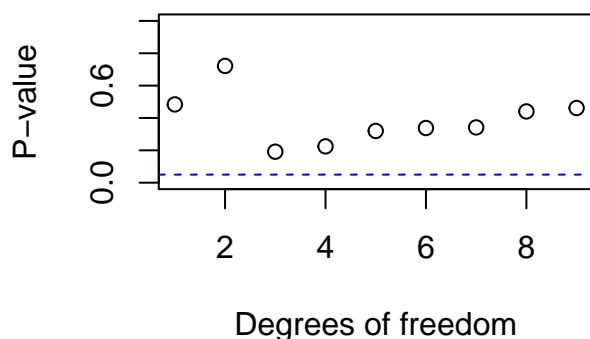
- 1) Since ACF cut off after lag 1, this suggest that we should begin by fitting an ARIMA(0,1,1) model
- 2) Since PACF doesn't cut off, this suggest the time series doesn't contain an AR component.

Part4: Model fitting - ARIMA(0,1,1)

We begins by fitting ARIMA(0,1,1), and we perform goodness of fit on the model based on following plots:



Ljung-Box test P-values



From the plots above, we conclude that ARIMA(0,1,1) is a good fit due to following reasons:

1) Time plot of the model residuals:

The time plot of the residuals looks similar to white noise, with mean zero and constant variance.

2) A plot of the sample ACF of the model residuals:

For all lags > 0 , the sample ACF are all close to zero. This suggests that the residuals are independent(uncorrelated).

3) A plot of the first ten P-values for the Ljung-Box test:

All p-values are greater than 0.05(non-significant), this suggests the ARIMA(0,1,1) is a good fit to the data.

Part5: ARIMA(0,1,1) vs. ARIMA(1,1,1)

However, it's still worth checking if adding AR(p) component would be a better fit. Therefore we fit the model again with ARIMA(1,1,1)

```
##
## Call:
## arima(x = nhtemp, order = c(0, 1, 1), method = "ML")
##
## Coefficients:
##          ma1
##        -0.7983
## s.e.    0.0956
##
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 187.52
##
## Call:
## arima(x = nhtemp, order = c(1, 1, 1), method = "ML")
##
## Coefficients:
##          ar1          ma1
##        0.0073    -0.8019
## s.e.  0.1802    0.1285
##
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 189.52
```

From the summary above, we conclude that ARIMA(0,1,1) is better than ARIMA(1,1,1) due to following reasons:

- 1) AIC for ARIMA(0,1,1) is 187.12 is less than AIC for ARIMA(1,1,1), which is 189.52.
- 2) Perform hypothesis test: $H_0 : \phi_1 = 0$ vs. $H_1 : \phi_1 \neq 0$. The test statistic $= \frac{0.0073}{0.1802} < 2$, therefore we don't reject the null hypothesis and thus ARIMA(0,1,1) is better than ARIMA(1,1,1) model.
- 3) Overall we'd prefer a parsimonious model, thus ARIMA(0,1,1) is better than ARIMA(1,1,1)

Part6: ARIMA(0,1,1) vs. ARIMA(0,1,2)

We check further whether adding an additional MA(q) component would be a better fit. Therefore we fit the model again with ARIMA(0,1,2)

```
##
## Call:
## arima(x = nhtemp, order = c(0, 1, 2), method = "ML")
##
## Coefficients:
##           ma1           ma2
##      -0.7956    -0.0042
## s.e.    0.1224    0.1221
##
## sigma^2 estimated as 1.291:  log likelihood = -91.76,  aic = 189.52
```

From the summary above, we conclude ARIMA(0,1,1) is better than ARIMA(0,1,2) due to following reasons:

- 1) AIC for ARIMA(0,1,1) is 187.12 is less than AIC for ARIMA(0,1,2), which is 189.52.
- 2) Perform hypothesis test: $H_0 : \theta_1 = 0$ vs. $H_1 : \theta_1 \neq 0$. The test statistic $= \left| \frac{-0.0042}{0.1221} \right| < 2$, therefore we don't reject the null hypothesis and thus ARIMA(0,1,1) is better than ARIMA(0,1,2) model.
- 3) Overall we'd prefer a parsimonious model, thus ARIMA(0,1,1) is better than ARIMA(0,1,2)

Part7: Conclusion

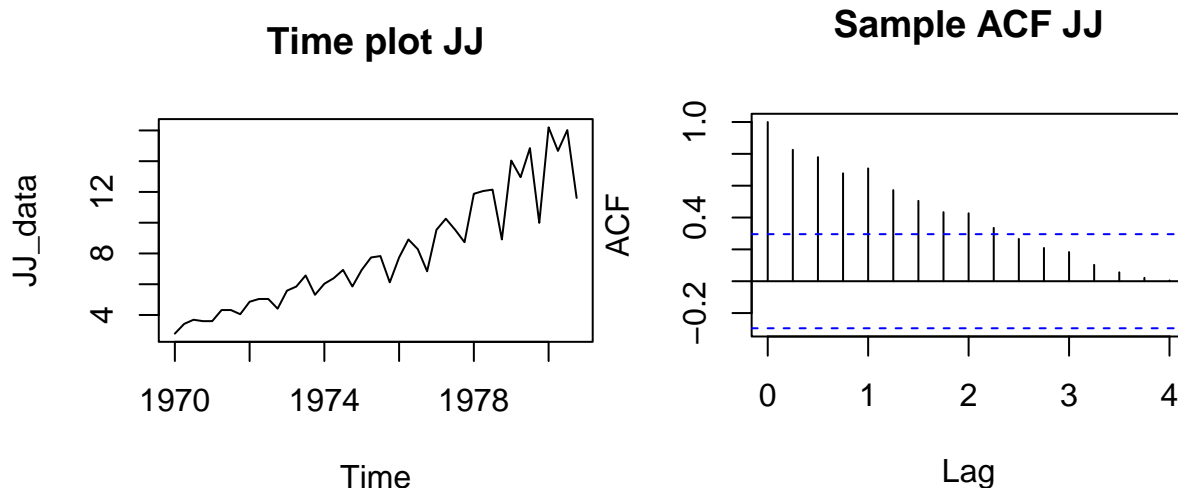
For question 1, the equation for the final fitted model is included below:

$$(1 - B)X_t = (1 - 0.7983B)Z_t$$

Q2: JJ_data

Part1: Check Stationarity and Seasonality

First we produce the time plot and ACF plot from the given data:



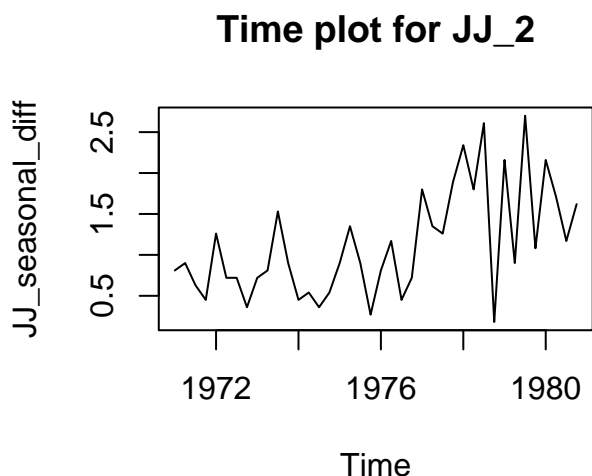
From the plots above, we conclude that the series is non-stationary and seasonal due to following reasons:

- 1) Time plot: both the mean and variance of the series appears to increase overtime, which indicate non-stationarity.
- 2) Sample ACF plot: doesn't decay rapidly, therefore it's not stationary.
- 3) Time plot: the data shows seasonality, as the earnings are higher in Qtr 2,3 and lower in Qtr 1,4

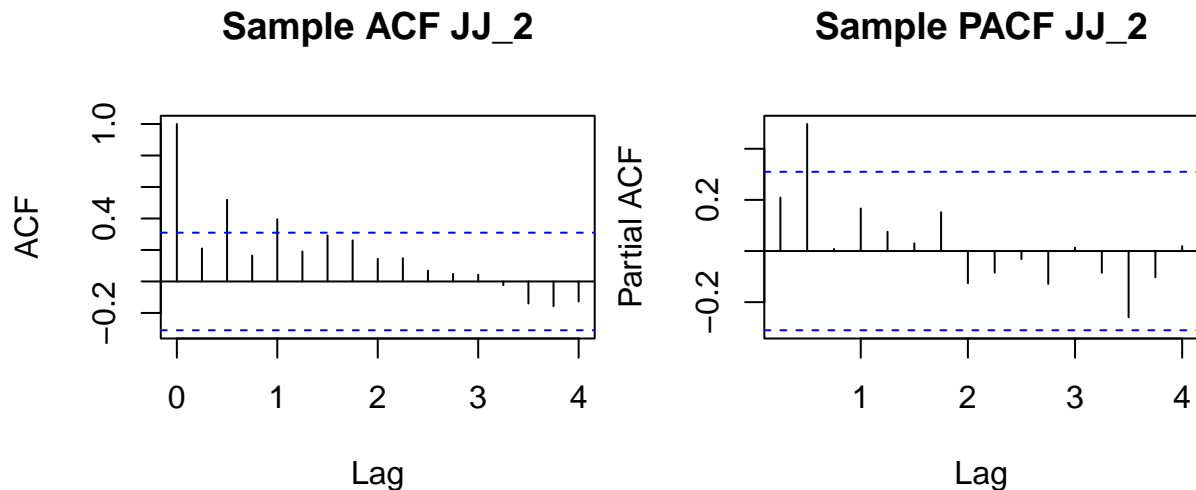
Therefore would need to apply a SARIMA model for JJ_data.

Part2: Apply Seasonal difference on JJ_1

According to the data description, JJ is a time series of the quarterly earnings between years, so the seasonal difference lag should be set to $h = 4$. There fore if JJ_1 denotes our original time series, we define the lag 4 difference time series JJ_2 as $JJ_2 = \nabla_4 JJ_1 = (1 - B^4)JJ_1$



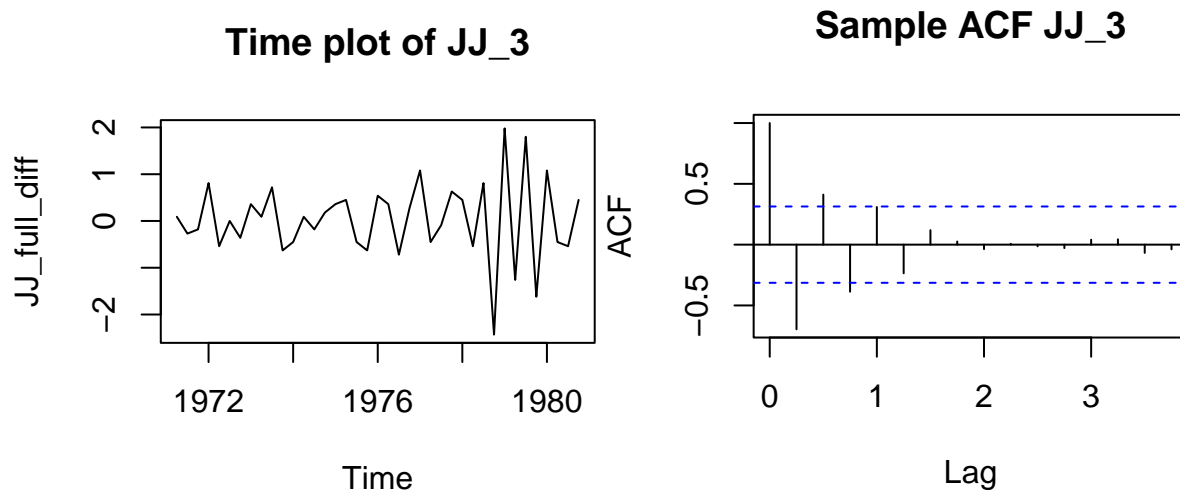
From the time plot, it seems that the seasonality has been removed in JJ_2 .



However, according to the sample ACF and sample PACF for the seasonally differenced data, it suggests non-stationarity, because the ACF decays slowly.

Part3: Apply First difference on JJ_2

Therefore we'll take the first difference of JJ_2 and obtain $JJ_3 = \nabla^1 JJ_2 = (1 - B)JJ_2$



Now JJ_3 appears to be stationary without seasonality.

However, the Time plot for JJ_3 shows a trend of non-constant variance, as the final part of the time series has greater variance compared with the earlier part. Therefore we applied transformation to tackle this problem.

Part4: Apply Box-Cox transformation on JJ_1

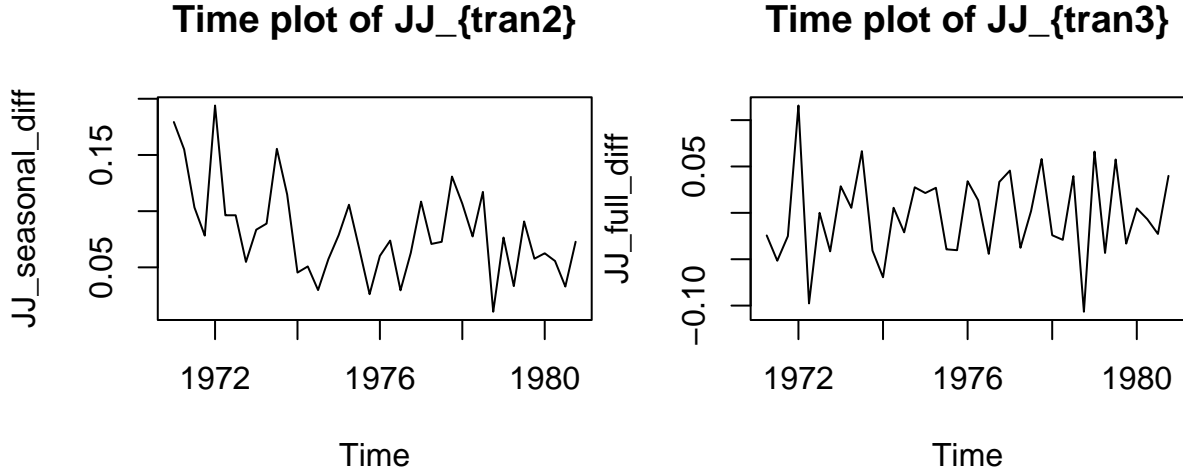
I've applied both log and sqrt transformation on JJ_3 but the final fitted model but the final model doesn't perform well compared with fitting a SARIMA(1,1,1)x(0,1,0)[4] on non-transformed data.

By Googling I found box-cox transformation is a nice way to solve the problem of non-constant variance, and I implemented it on JJ_1 , with optimal lambda set to be -0.305.

$$JJ_{tran1} = \text{boxcox}(JJ_1)$$

Part5: Remove non-stationarity and seasonality from JJ_tran1

Then we carry on the same process to remove the non-stationarity and seasonality. We first difference JJ_{tran1} with a seasonal difference lag $h = 4$ and gain $JJ_{tran2} = \nabla_4(JJ_{tran1}) = (1 - B^4)(JJ_{tran1})$, then we take the first difference on JJ_{tran2} and obtain $JJ_{tran3} = \nabla^1 JJ_{tran2} = (1 - B)JJ_{tran2}$. Below is the time plot for JJ_{tran2} and JJ_{tran3}

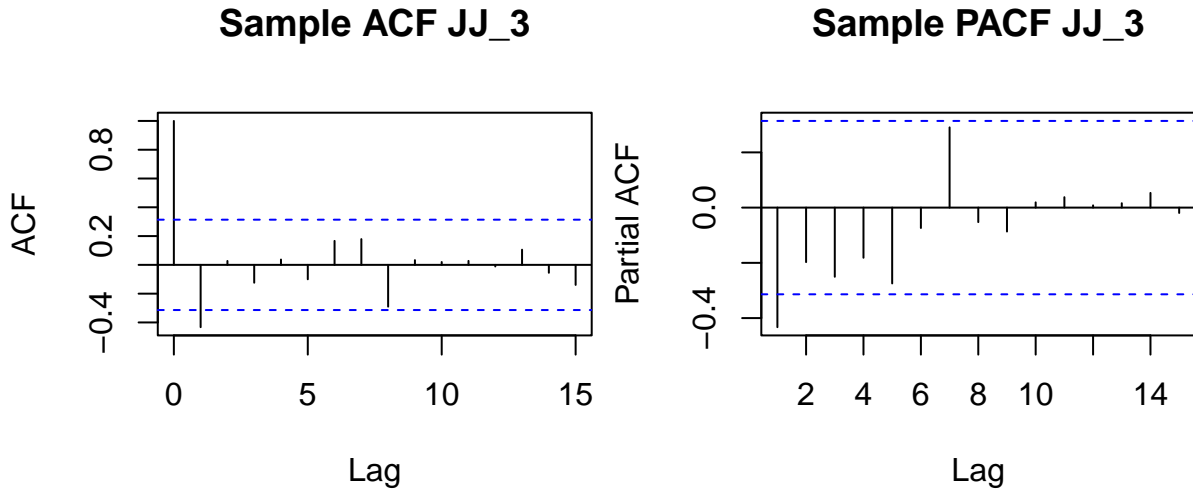


From the time plot, it seems that both non-stationarity and seasonality has been removed from JJ_{tran1} .

Part6: SARIMA Parameter analysis for JJ_3 and JJ_{tran3}

Now we start our fitting attempt with SARIMA(p,1,q)x(P,1,Q)[4].

Part 6.1: Non-transformed data JJ_3



The best model should be SARIMA(1,1,1) x (0,1,0)[4] due to following reasons:

Seasonal components: (P,Q)

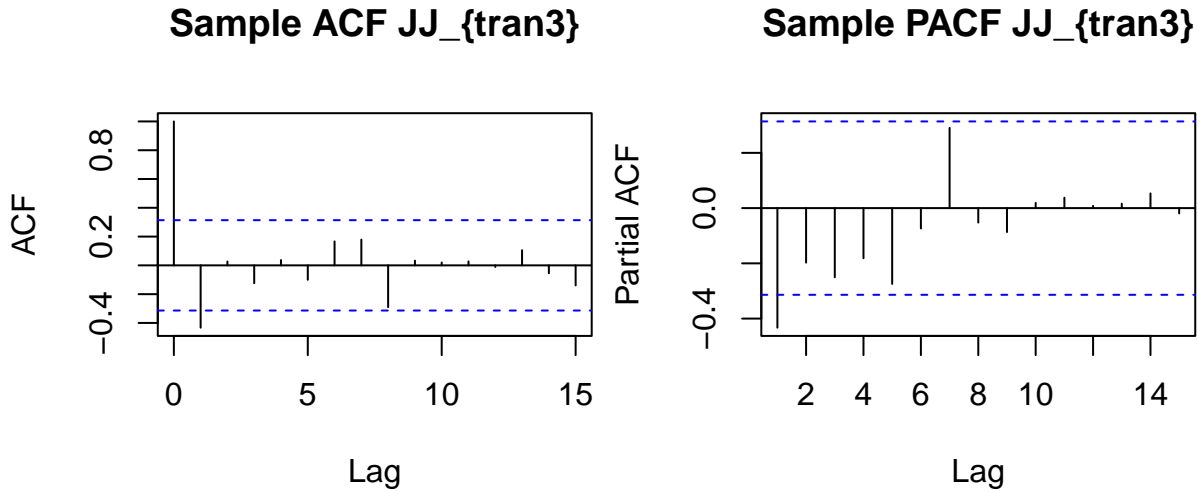
- 1) P: Check PACF at lag = 4,8,12 ... PACF cut off already at lag = 4, therefore we choose P = 0.
- 2) Q: Check ACF at lag = 4,8,12 ... ACF cut off already at lag = 4, therefore we choose Q = 0

Non Seasonal components : (p,q)

- 3) p: PACF cut off after lag = 1, therefore we choose p = 1

4) q: ACF cut off after lag = 1, therefore we choose $q = 1$.

Part 6.2: Transformed data JJ_{tran3}



The best model should be $SARIMA(0,1,1) \times (0,1,0)[4]$ due to following reasons:

Seasonal components: (P,Q)

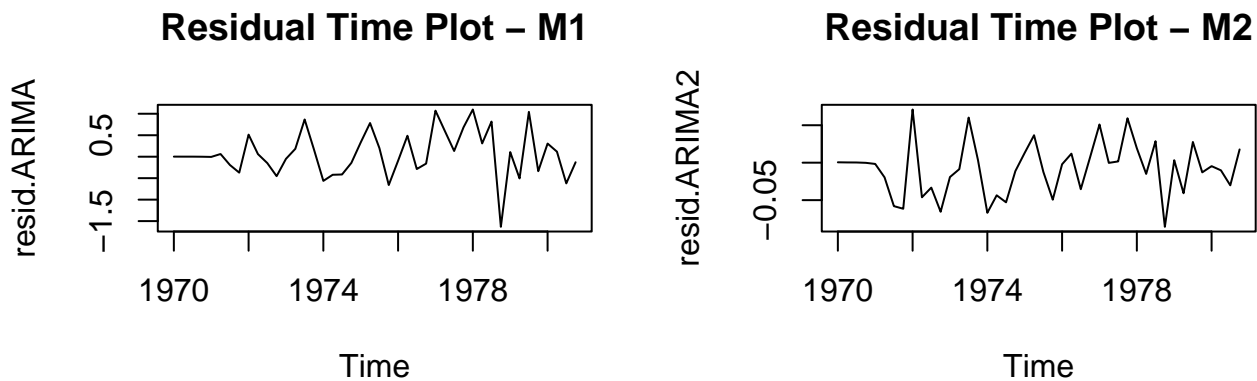
- 1) P: Check PACF at lag = 4,8,12 ... PACF cut off already at lag = 4, therefore we choose $P = 0$.
- 2) Q: Check ACF at lag = 4,8,12 ... ACF cut off already at lag = 4, therefore we choose $Q = 0$

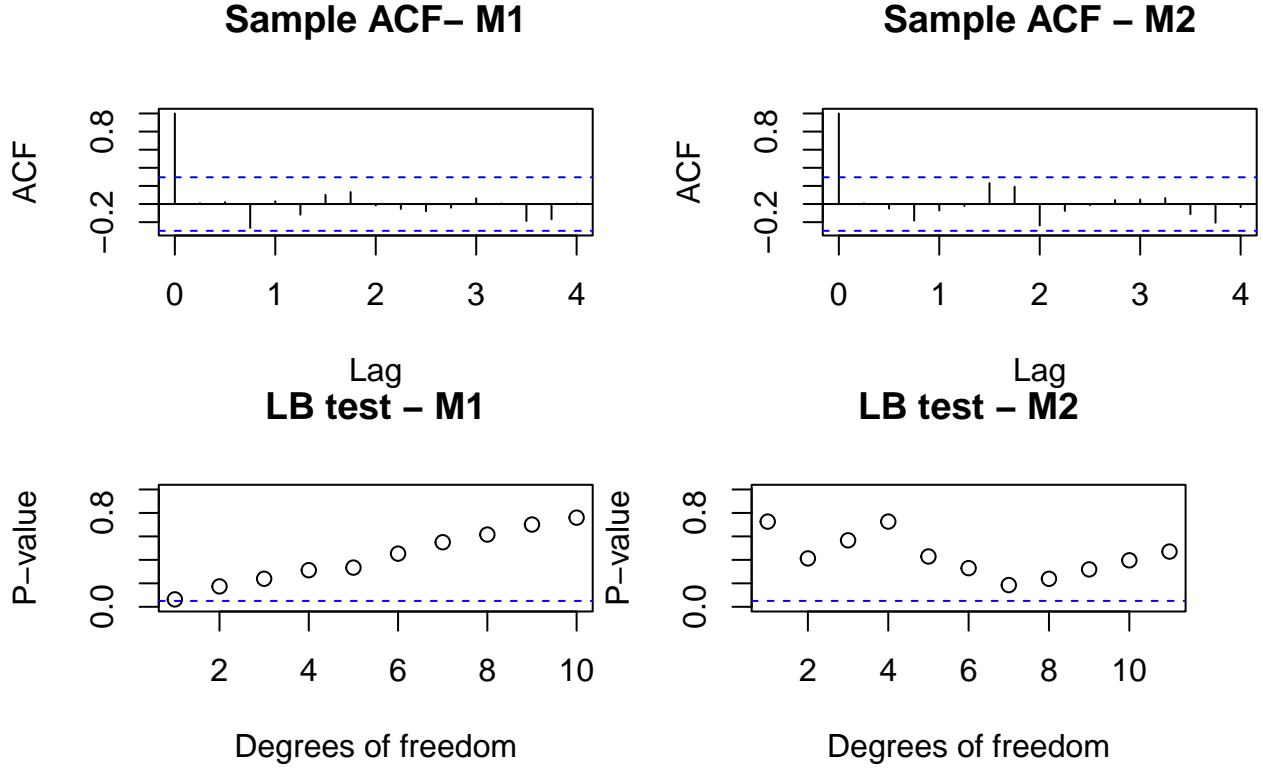
Non Seasonal components : (p,q)

- 3) p: PACF cut off after lag = 0, therefore we choose $p = 0$
- 4) q: ACF cut off after lag = 1, therefore we choose $q = 1$.

Part7: Model Diagnostic JJ_1 vs. JJ_{tran1}

To make expression concise, I'll use 'M1' to refer the $SARIMA(1,1,1) \times (0,1,0)[4]$ model for JJ_1 and 'M2' to refer the $SARIMA(0,1,1) \times (0,1,0)[4]$ model for JJ_{tran1}





From the plots above, we conclude that both M1 and M2 are good fits, while M2 is slightly better due to following reasons:

1) Time plot of the model residuals:

The time plot of the residuals for M2 looks to be white noise, with mean zero and constant variance. (However for M1 the variance isn't constant)

2) A plot of the sample ACF of the model residuals

For all lags > 0 , the sample ACF are all close to zero except at lag = 1. This suggests that the residuals are almost independent(uncorrelated).

3) A plot of the first ten P-values for the Ljung-Box test

All p-values for M2 are greater than 0.05(non-significant).

However for M1 the first p-value is significant

Part8: Conclusion

Let X_t denote the original JJ_data

For the non-transformed data(not included in the report due to page limit), the best model is SARIMA(1,1,1)x(0,1,0)[4], and the equation is:

$$(1 + 0.3465B)(1 - B)(1 - B^4)X_t = (1 - 0.6308B)Z_t$$

For the boxcox transformed data, the best model is SARIMA(0,1,1)x(0,1,0)[4], and the equation is:

$$(1 - B)(1 - B^4)boxcox(X_t) = (1 - 0.7325B)Z_t$$

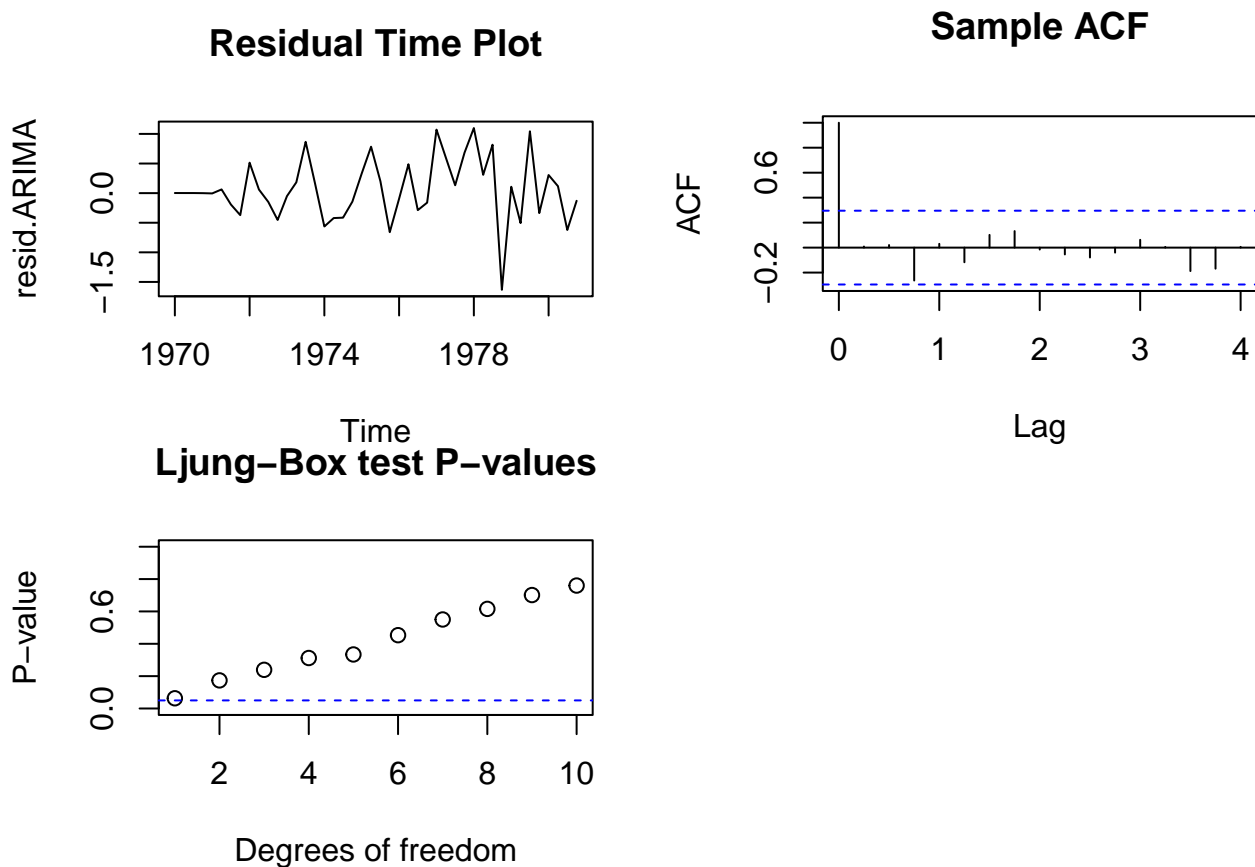
here $boxcox()$ denotes the transformation performed on the original JJ_data.

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start our fitting attempt with SARIMA(p,1,q)x(P,1,Q)[4].

Parameter analysis

Model fitting



From the plots above, we conclude that SARIMA(1,1,1)x(0,1,0)[4] is a fairly good fit due to following reasons:

- 1) Time plot of the model residuals:

The time plot of the residuals looks similar to white noise, with mean zero, but the variance increases overtime.

- 2) A plot of the sample ACF of the model residuals

For all lags > 0 , the sample ACF are all close to zero except at lag = 1. This suggests that the residuals are almost independent(uncorrelated).

- 3) A plot of the first ten P-values for the Ljung-Box test

Although the first p-value is fairly significant, all other p-values are greater than 0.05(non-significant), this suggests a fairly good model for JJ_data.

```
last_12 <- tail(JJ_data, 12)

# Apply a logarithmic transformation to the last 12 elements
transformed_last_12_log <- log(last_12)

# Replace the last 12 elements in the original time series with the transformed values
```

```

# Calculate the starting index for the last 12 elements
start_index <- length(JJ_data) - length(transformed_last_12_log) + 1
end_index <- length(JJ_data)

JJ_transform <- JJ_data

# Replace the elements
JJ_transform[start_index:end_index] <- transformed_last_12_log

subset <- window(JJ_transform, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_transform, start = c(1971+k,1), end = c(1971+k,4))

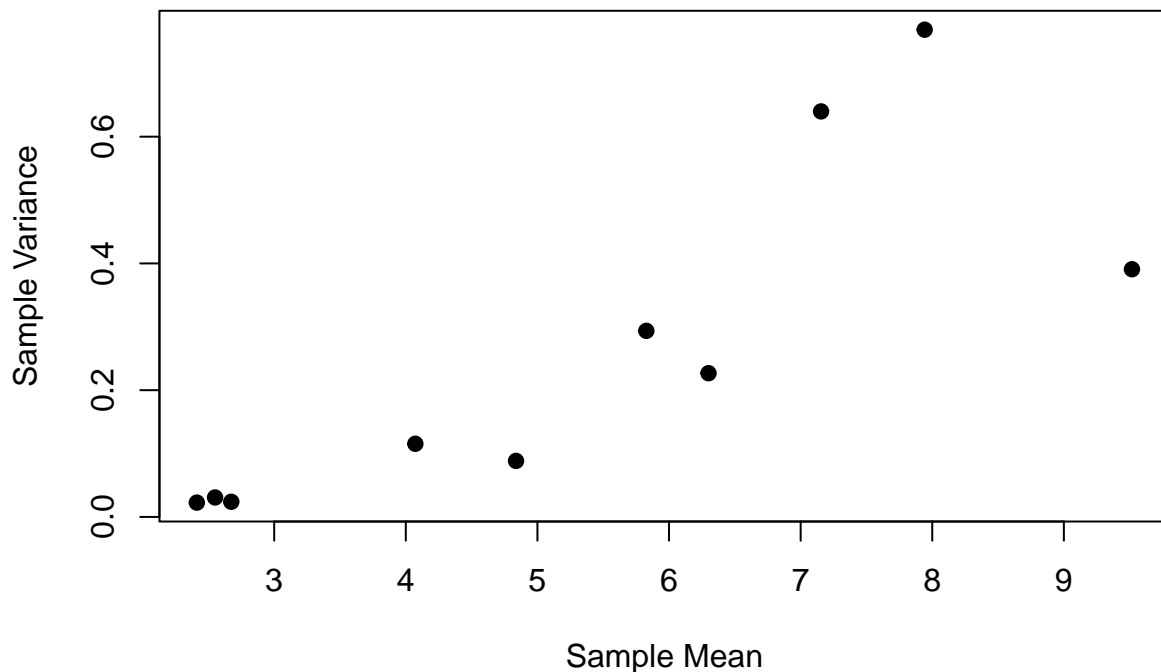
  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Variance",
      xlab="Sample Mean", ylab="Sample Variance", pch=19)

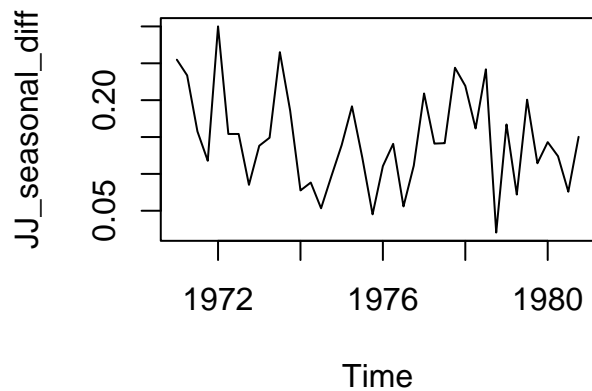
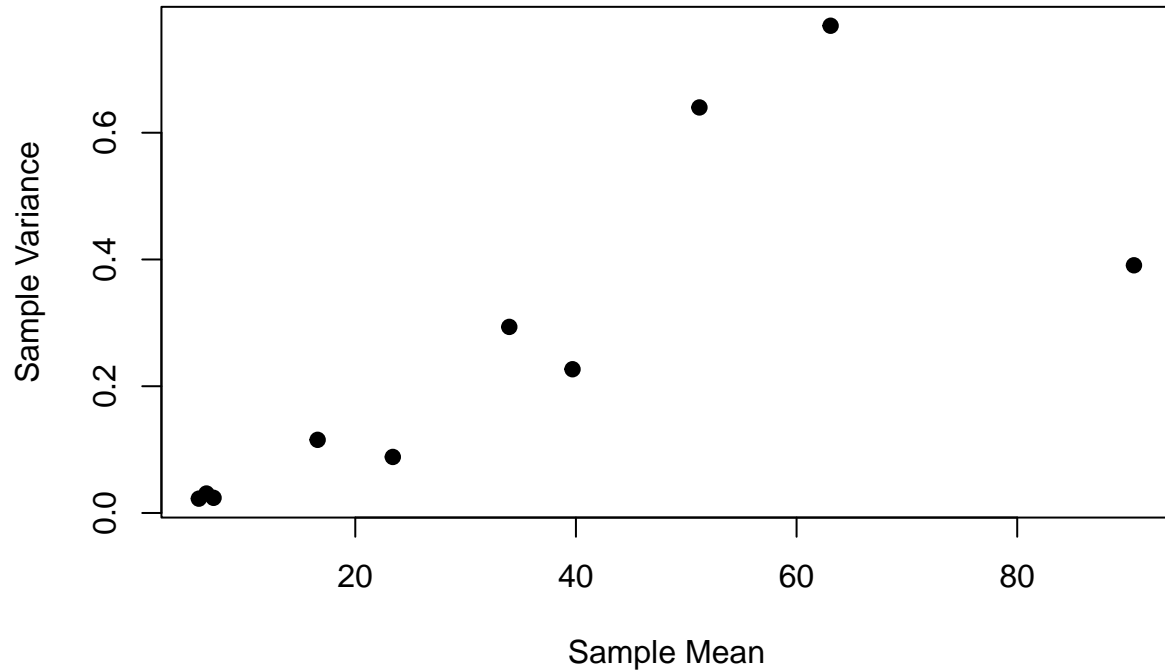
```

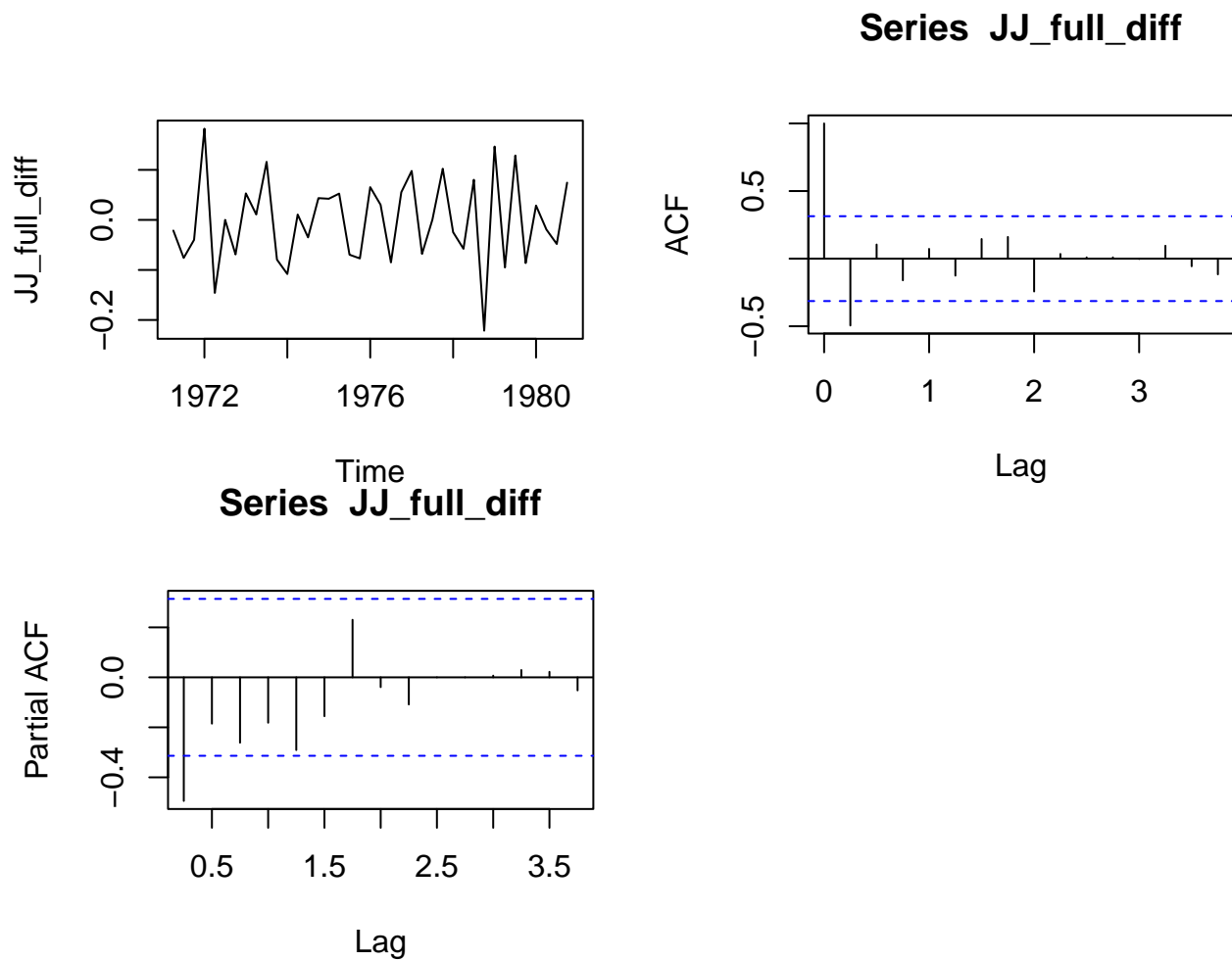
Scatter Plot of Mean vs Variance



```
plot(mean_vector^2, var_vector, main="Scatter Plot of Mean vs Variance",  
      xlab="Sample Mean", ylab="Sample Variance", pch=19)
```

Scatter Plot of Mean vs Variance



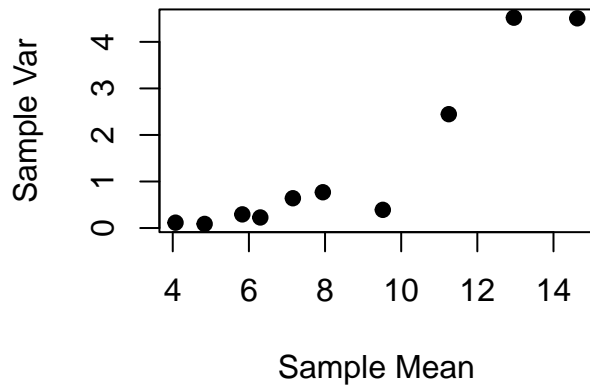


```
fit <- auto.arima(log(JJ_data))
summary(fit)

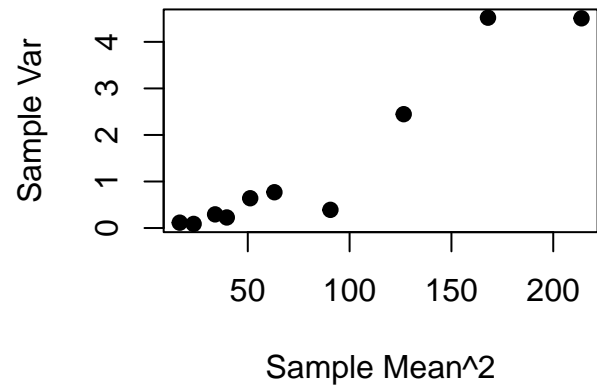
## Series: log(JJ_data)
## ARIMA(0,0,0)(0,1,0)[4] with drift
##
## Coefficients:
##      drift
##      0.0366
## s.e.  0.0026
##
## sigma^2 = 0.004255: log likelihood = 52.94
## AIC=-101.88   AICc=-101.56   BIC=-98.51
##
## Training set error measures:
##              ME      RMSE      MAE      MPE      MAPE      MASE
## Training set 0.0001017352 0.06141291 0.04580905 0.1018794 2.394819 0.3129954
##              ACF1
## Training set 0.1119376

#fit <- auto.arima(JJ_data_transformed)
#summary(fit)
```

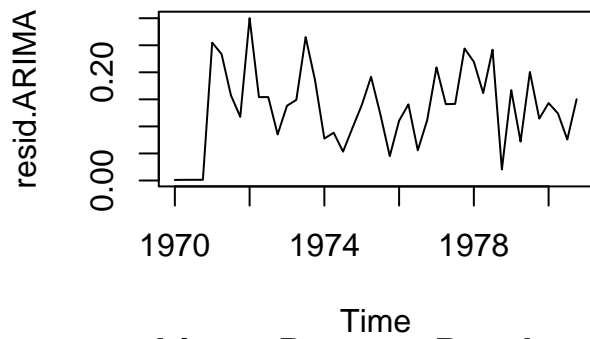
Scatter Plot of Mean vs Var



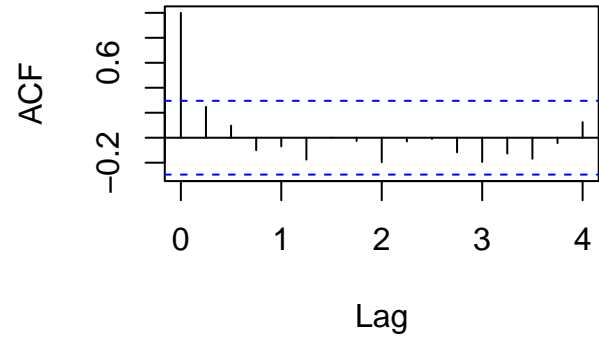
Scatter Plot of Mean^2 vs Var



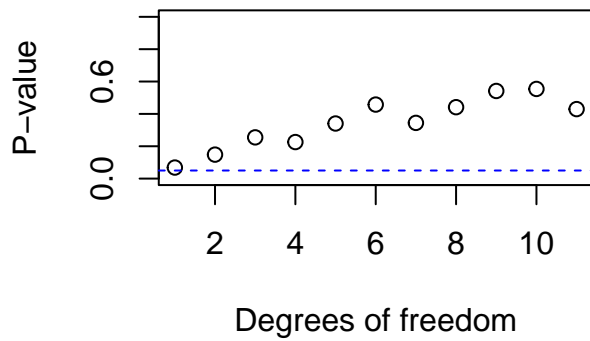
Residual Time Plot



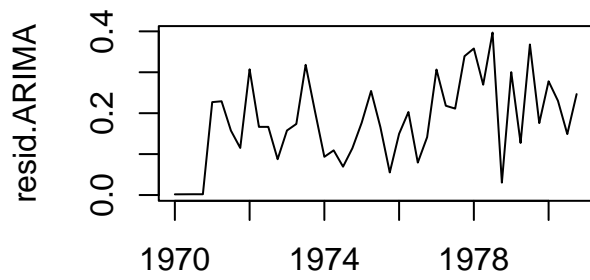
Sample ACF



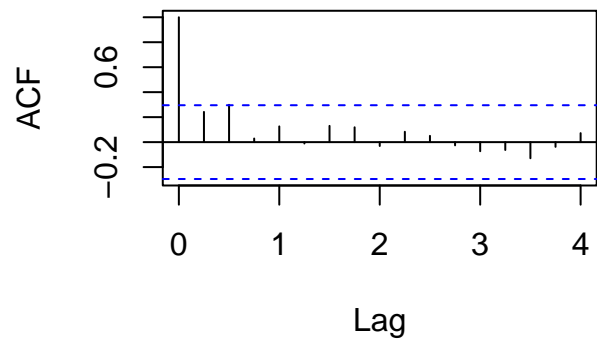
Ljung-Box test P-values



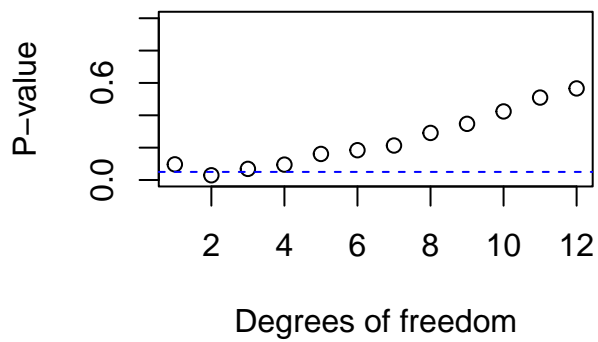
Residual Time Plot



Sample ACF

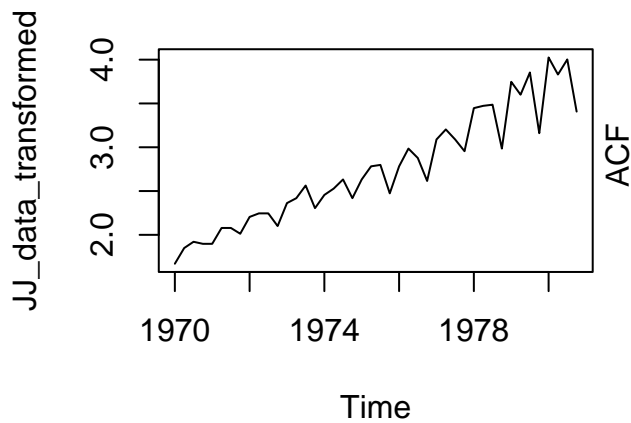


Ljung-Box test P-values

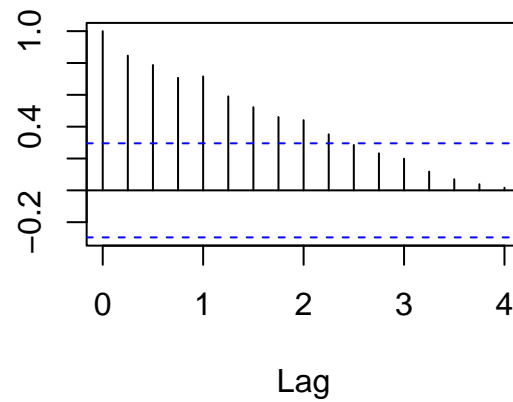


Test content

Time plot of JJ_{tran1}

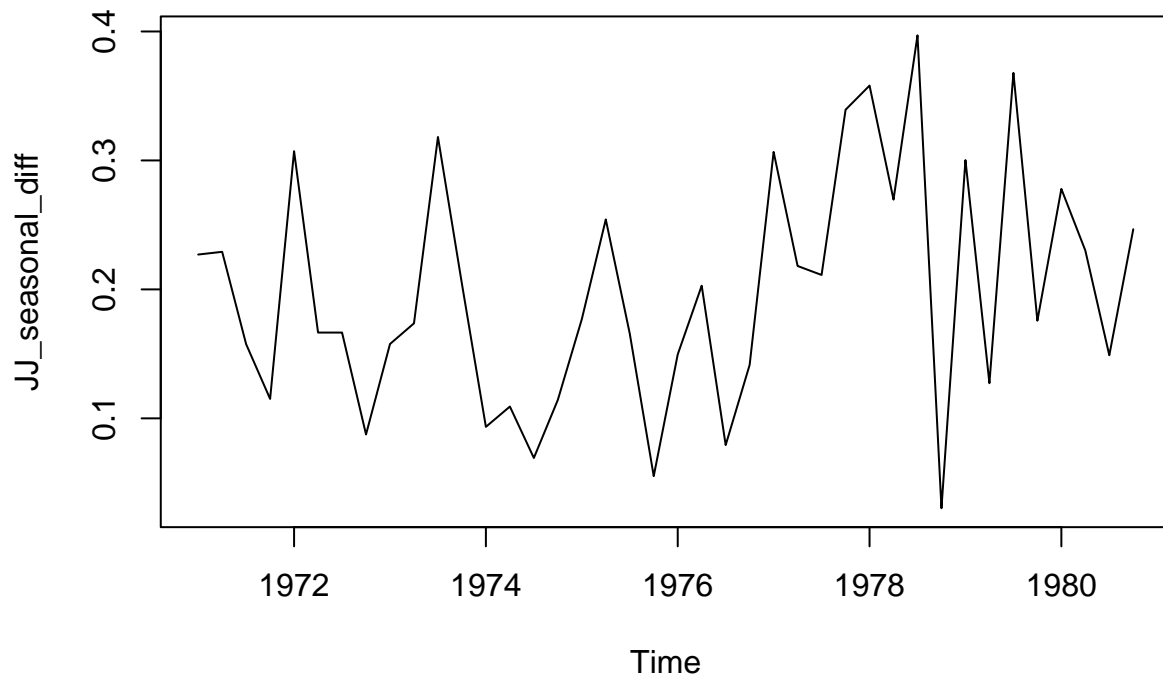


sample ACF of JJ_{tran1}



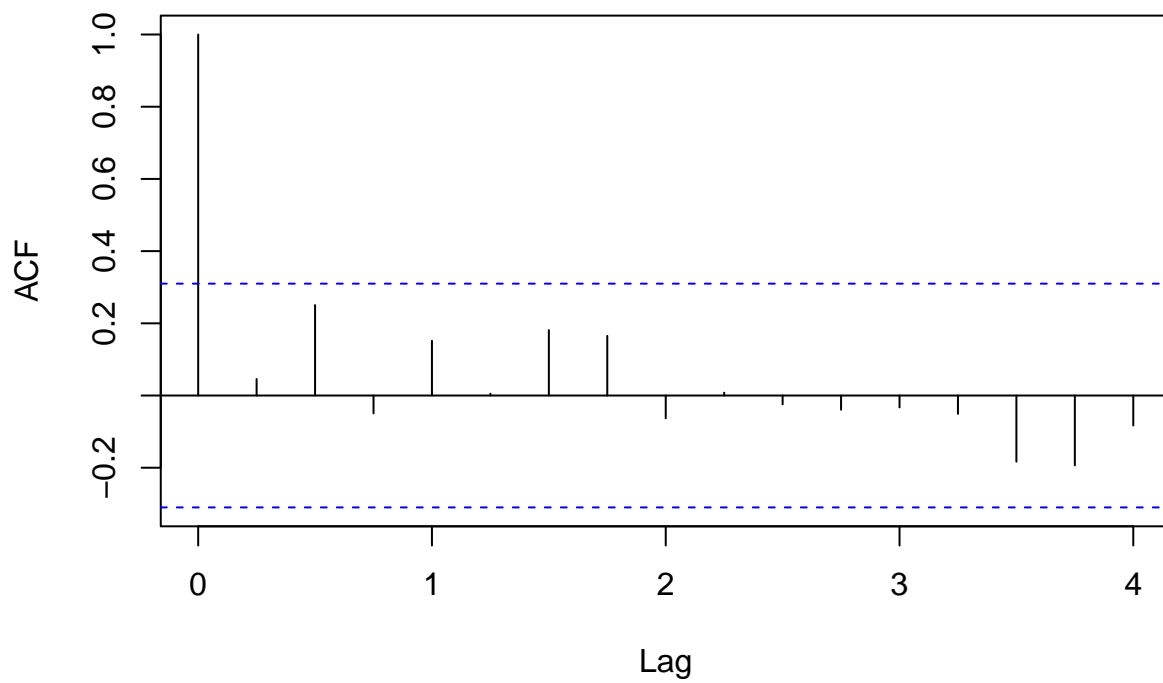
```
JJ_seasonal_diff <- diff(sqrt(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff,main = "Time plot for JJ_2")
```

Time plot for JJ_2



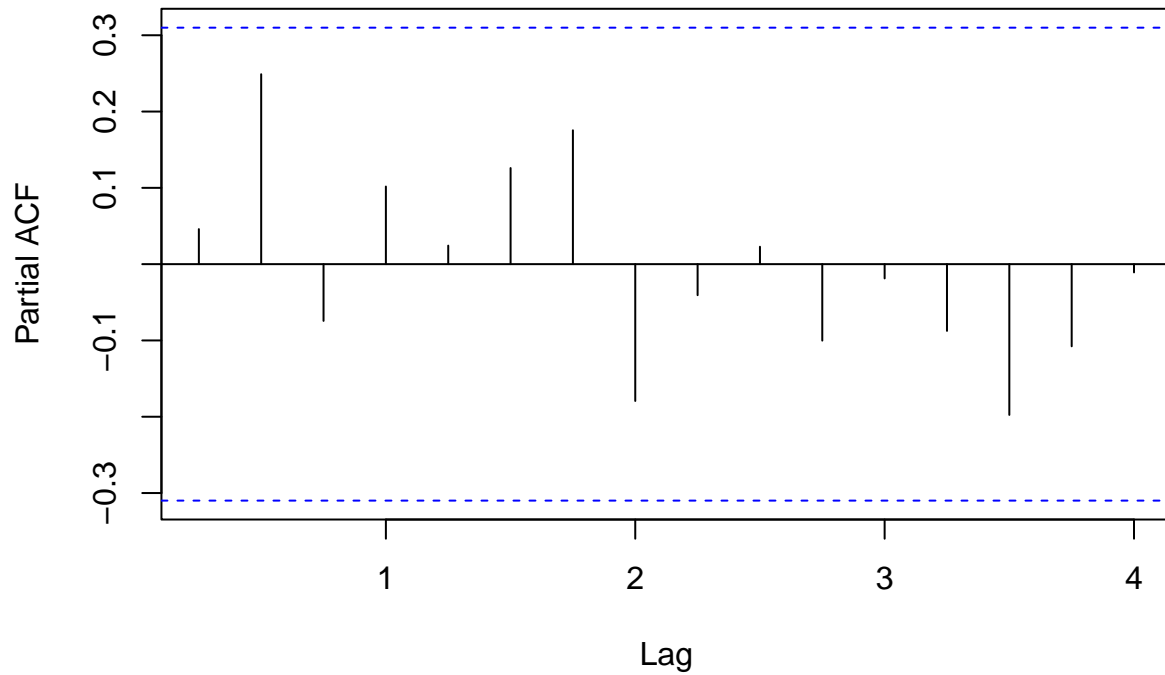
```
acf(JJ_seasonal_diff, main = "Sample ACF JJ_2")
```

Sample ACF JJ_2



```
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
```


Sample PACF JJ_2



Appendix

```
knitr::opts_chunk$set(echo = TRUE)

library(forecast)
LB_test<-function(resid,max.k,p,q){
  lb_result<-list()
  df<-list()
  p_value<-list()
  for(i in (p+q+1):max.k){
    lb_result[[i]]<-Box.test(resid,lag=i,type=c("Ljung-Box"),fitdf=(p+q))
    df[[i]]<-lb_result[[i]]$parameter
    p_value[[i]]<-lb_result[[i]]$p.value
  }
  df<-as.vector(unlist(df))
  p_value<-as.vector(unlist(p_value))
  test_output<-data.frame(df,p_value)
  names(test_output)<-c("deg_freedom", "LB_p_value")
  return(test_output)
}

load("nhtemp.rda")
# Time Series Plot
ts.plot(nhtemp, main="Sample Time Plot")

# ACF Plot
acf(nhtemp, main="Sample ACF")
```

```

# PACF Plot
#pacf(nhtemp, main="Sample PACF")
nhtemp_diff<-diff(nhtemp)
ts.plot(nhtemp_diff, main="Sample Time Plot")
acf(nhtemp_diff, main="Sample ACF")
#pacf(nhtemp_diff)
acf(nhtemp_diff, main="Sample ACF")
pacf(nhtemp_diff, main = "Sample PACF")
ARIMA<-arima(nhtemp,order=c(0,1,1),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Sample Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test(resid.ARIMA,max.k=11,p=0,q=2)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box")
abline(h=0.05,col="blue",lty=2)
ARIMA
ARIMA<-arima(nhtemp,order=c(1,1,1),method="ML")
ARIMA
ARIMA<-arima(nhtemp,order=c(0,1,2),method="ML")
ARIMA
load("JJ_data.rda")
#JJ_data_ts <- ts(JJ_data, start=c(1970, 1))
LB_test_SARIMA<-function(resid,max.k,p,q,P,Q){
  lb_result<-list()
  df<-list()
  p_value<-list()
  for(i in (p+q+P+Q+1):max.k){
    lb_result[[i]]<-Box.test(resid,lag=i,type=c("Ljung-Box"),fitdf=(p+q+P+Q))
    df[[i]]<-lb_result[[i]]$parameter
    p_value[[i]]<-lb_result[[i]]$p.value
  }
  df<-as.vector(unlist(df))
  p_value<-as.vector(unlist(p_value))
  test_output<-data.frame(df,p_value)
  names(test_output)<-c("deg_freedom","LB_p_value")
  return(test_output)
}
ts.plot(JJ_data, main = "Time plot JJ")
acf(JJ_data,main = "Sample ACF JJ")
#pacf(JJ_data)
JJ_seasonal_diff <- diff(JJ_data,lag=4)
ts.plot(JJ_seasonal_diff,main = "Time plot for JJ_2")
acf(JJ_seasonal_diff,main = "Sample ACF JJ_2")
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
JJ_full_diff <- diff(JJ_seasonal_diff)
ts.plot(JJ_full_diff,main = "Time plot of JJ_3")
acf(JJ_full_diff, main = "Sample ACF JJ_3")
#pacf(JJ_full_diff, main = "Sample PACF JJ_3")
# Estimate the optimal lambda for the Box-Cox transformation
lambda <- BoxCox.lambda(JJ_data)

```

```

# Apply the Box-Cox transformation
JJ_data_transformed <- BoxCox(JJ_data, lambda)
ts.plot(JJ_data_transformed, main = "Time plot of JJ_{tran1}")
acf(JJ_data_transformed, main = "sample ACF of JJ_{tran1}")
#pacf(JJ_data)
JJ_seasonal_diff <- diff(JJ_data_transformed, lag=4)
ts.plot(JJ_seasonal_diff, main = "Time plot of JJ_{tran2}")
JJ_full_diff <- diff(JJ_seasonal_diff)
ts.plot(JJ_full_diff, main = "Time plot of JJ_{tran3}")
JJ_data_ts <- ts(JJ_full_diff, start=c(1970, 1))
acf(JJ_data_ts, main = "Sample ACF JJ_3")
pacf(JJ_data_ts, main = "Sample PACF JJ_3")
JJ_data_ts <- ts(JJ_full_diff, start=c(1970, 1))
acf(JJ_data_ts, main = "Sample ACF JJ_{tran3}")
pacf(JJ_data_ts, main = "Sample PACF JJ_{tran3}")
fit <- auto.arima(JJ_data_transformed)
summary(fit)
ARIMA<-arima(JJ_data, order=c(1,1,1), seasonal=list(order=c(0,1,0), period=4), method="ML")
ARIMA2<-arima(JJ_data_transformed, order=c(0,1,1), seasonal=list(order=c(0,1,0), period=4), method="ML")

resid.ARIMA<-residuals(ARIMA)
resid.ARIMA2<-residuals(ARIMA2)

ts.plot(resid.ARIMA, main = "Residual Time Plot - M1")
ts.plot(resid.ARIMA2, main = "Residual Time Plot - M2")

acf(resid.ARIMA, main = "Sample ACF- M1")
acf(resid.ARIMA2, main = "Sample ACF - M2")

ARIMA.LB<-LB_test_SARIMA(resid.ARIMA, max.k=12, p=1, q=1, P=0, Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom, ARIMA.LB$LB_p_value, xlab="Degrees of freedom", ylab="P-value", main="LB test - M1")
abline(h=0.05, col="blue", lty=2)

ARIMA.LB2<-LB_test_SARIMA(resid.ARIMA2, max.k=12, p=0, q=1, P=0, Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB2$deg_freedom, ARIMA.LB2$LB_p_value, xlab="Degrees of freedom", ylab="P-value", main="LB test - M2")
abline(h=0.05, col="blue", lty=2)

ARIMA<-arima(JJ_data, order=c(1,1,1), seasonal=list(order=c(0,1,0), period=4), method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")

```

```

ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=1,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box")
abline(h=0.05,col="blue",lty=2)
library(forecast)
fit <- auto.arima(JJ_data)
summary(fit)

fit <- auto.arima(sqrt(JJ_data))
summary(fit)

last_12 <- tail(JJ_data, 12)

# Apply a logarithmic transformation to the last 12 elements
transformed_last_12_log <- log(last_12)

# Replace the last 12 elements in the original time series with the transformed values
# Calculate the starting index for the last 12 elements
start_index <- length(JJ_data) - length(transformed_last_12_log) + 1
end_index <- length(JJ_data)

JJ_transform <- JJ_data

# Replace the elements
JJ_transform[start_index:end_index] <- transformed_last_12_log
subset <- window(JJ_transform, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_transform, start = c(1971+k,1), end = c(1971+k,4))

  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Variance",
      xlab="Sample Mean", ylab="Sample Variance", pch=19)

plot(mean_vector^2, var_vector, main="Scatter Plot of Mean vs Variance",
      xlab="Sample Mean", ylab="Sample Variance", pch=19)

JJ_seasonal_diff <- diff(log(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff)
JJ_full_diff <- diff(JJ_seasonal_diff)

```

```

ts.plot(JJ_full_diff)
acf(JJ_full_diff)
pacf(JJ_full_diff)
fit <- auto.arima(log(JJ_data))
summary(fit)
#fit <- auto.arima(JJ_data_transformed)
#summary(fit)
subset <- window(JJ_data, start = c(1971,1), end = c(1971,4))
mean_subset = list()
var_subset = list()
mean_subset <- mean(subset)
var_subset <- var(subset)

for (k in 1:9){
  subset <- window(JJ_data, start = c(1971+k,1), end = c(1971+k,4))

  mean_subset[[k+1]] <- mean(subset)
  var_subset[[k+1]] <- var(subset)
}

mean_vector <- unlist(mean_subset)
var_vector <- unlist(var_subset)

plot(mean_vector, var_vector, main="Scatter Plot of Mean vs Var",
      xlab="Sample Mean", ylab="Sample Var", pch=19)

plot(mean_vector^2, var_vector, main="Scatter Plot of Mean^2 vs Var",
      xlab="Sample Mean^2", ylab="Sample Var", pch=19)

ARIMA<-arima(log(JJ_data),order=c(0,0,0),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=0,q=1,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box",
      abline(h=0.05,col="blue",lty=2)
ARIMA<-arima(sqrt(JJ_data),order=c(0,0,0),seasonal=list(order=c(0,1,0),period=4),method="ML")
ARIMA
resid.ARIMA<-residuals(ARIMA)
ts.plot(resid.ARIMA, main = "Residual Time Plot")
acf(resid.ARIMA, main = "Sample ACF")
ARIMA.LB<-LB_test_SARIMA(resid.ARIMA,max.k=12,p=0,q=0,P=0,Q=0)
#To produce a plot of the P-values against the degrees of freedom and
#add a blue dashed line at 0.05, we run the commands
plot(ARIMA.LB$deg_freedom,ARIMA.LB$LB_p_value,xlab="Degrees of freedom",ylab="P-value",main="Ljung-Box",
      abline(h=0.05,col="blue",lty=2)
JJ_data_transformed <- sqrt(JJ_data)
ts.plot(JJ_data_transformed, main = "Time plot of JJ_{tran1}")
acf(JJ_data_transformed, main = "sample ACF of JJ_{tran1}")
#pacf(JJ_data)

```

```
JJ_seasonal_diff <- diff(sqrt(JJ_data),lag=4)
ts.plot(JJ_seasonal_diff,main = "Time plot for JJ_2")
acf(JJ_seasonal_diff,main = "Sample ACF JJ_2")
pacf(JJ_seasonal_diff, main = "Sample PACF JJ_2")
```