Analytical Jacobian – Coulomb Friction

$$T = \begin{cases} k_t(x_t - w) & \text{stick} \\ \xi\mu k_n x_n & \text{slip} \\ 0 & \text{gap} \end{cases} \qquad F_n = \begin{cases} k_n x_n & \text{contacting} \\ 0 & \text{gap} \end{cases} \qquad \xi = \pm 1$$

$$x_t = \frac{X_{tc}^0}{2} + \sum X_{tc}^k \cos(k\omega t) + X_{ts}^k \sin(k\omega t)$$

$$x_n = \frac{X_{nc}^0}{2} + \sum X_{nc}^k \cos(k\omega t) + X_{ns}^k \sin(k\omega t)$$

When there is no separation, the Jacobien in time domain can be written like following. t * is slip to stick transition time instant, if the system is pure stick, then the second term with t * is 0 .

$$\frac{\partial T(t)}{\partial X_{tc}^0} = \begin{cases} \frac{1}{2}k_t - \frac{1}{2}k_t|_{t=t^*} & \text{stick} \\ 0 & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial T(t)}{\partial X_{tc}^k} = \begin{cases} k_t \cdot \cos(kt) - k_t \cdot \cos(kt^*) & \text{stick} \\ 0 & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial T(t)}{\partial X_{ts}^k} = \begin{cases} k_t \cdot \sin(kt) - k_t \cdot \sin(kt^*) & \text{stick} \\ 0 & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial T(t)}{\partial X_{nc}^0} = \begin{cases} \frac{1}{2}\xi\mu k_n|_{t=t^*} & \text{stick} \\ \frac{1}{2}\xi\mu k_n & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial T(t)}{\partial X_{nc}^k} = \begin{cases} \xi\mu k_n \cdot \cos(kt^*) & \text{stick} \\ \xi\mu k_n \cdot \cos(kt) & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial T(t)}{\partial X_{ns}^k} = \begin{cases} \xi\mu k_n \cdot \sin(kt^*) & \text{stick} \\ \xi\mu k_n \cdot \sin(kt) & \text{slip} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial F_n(t)}{\partial X_{nc}^0} = \begin{cases} \frac{1}{2}k_n & \text{contact} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial F_n(t)}{\partial X_{nc}^k} = \begin{cases} k_n \cdot \cos(kt) & \text{contact} \\ 0 & \text{gap} \end{cases}$$

$$\frac{\partial F_n(t)}{\partial X_{ns}^k} = \begin{cases} k_n \cdot \sin(kt) & \text{contact} \\ 0 & \text{gap} \end{cases}$$