Project for Operations Research 1

March 27, 2019

- Marks for project: 30%.

- Project due date: 23:55 Friday 12 April 2019 (Week 12).

1 Project Description

A file has been placed at http://jkcray.maths.ul.ie/ms4303/Projectdata.pdf containing a list of LP's, a different one for every student in the class.

The LP's are in Standard Form, so z is to be **minimised**.

2 Solution to Optimality

You should:

- Find and copy down "your" problem.

- Create a Matlab/Octave script m-file `Run.m` containing all the Matlab/Octave commands needed to solve the problem.

- Generate a Simplex tableau T_0 for your LP.

- Use the provided `Pivot.m` m-file with either Octave or Matlab to transform T_0 to a canonical form tableau T_B.

- Use the provided `colsortjk` m-file with either Octave or Matlab to transform T_B to a canonical form tableau T_C which has the basic columns in standard order on the right of the tableau.

  - ⋆ This corresponds to re-labelling the decision variables but does not change the optimal value of the LH–column and in particular z.

- Use the `Pivot.m` m-file with either Octave or Matlab to solve T_C to optimality.

- Name the optimal tableau T^*.

- What is the optimal solution vector $\mathbf{x}$ and the optimal z–value?

- **You don't need to — and shouldn't — "undo" the column sort.**

3 Sensitivity Analysis

For the remaining questions, you need to interpret your Standard Form LP (which is a **min** problem) as derived from a **max** problem with (of course) the objective coefficients reversed in sign.

The techniques needed for this part of the project will be explained in class from Week 8 on — they are covered in Ch. 5 of the Notes in the same sequence as the tasks in the following paragraphs.

(A) First deal with some changes in the Production Requirements.

(a) There has been a change in the Production Requirements.
- Pick the **last** **non-basic** variable in T^* (I'll refer to it as x_{nbl}) and increase it up to a value X — where X is either 1 unit or to **half** the maximum value of x_{nbl} (whichever is the greater).
- What is the new optimal solution $\mathbf{x}$ and optimal z–value?
- Check your result(s) by adding the constraint $x_{nbl} = X$ to T^* and pivot to optimality.

(b) There has been another change in the Production Requirements.
- $\star$ Find the last **basic** variable in T^* (I'll refer to it as x_{bl}) and increase it from its optimal value by 1 unit or by **half** its maximum increase (whichever is the greater).
    - $*$ Choose the "cheapest" non-basic variable to be increased from zero — i.e. the one has the least effect on the objective function value.
  - $\star$ If the last basic variable cannot be increased, choose the second-last, ... until you find a basic variable that can increased.
- What is the new optimal solution $\mathbf{x}$ and optimal z–value?
- Check your result(s) by adding the constraint $x_{bL} = X$ to T^* where X is the chosen larger value of x_{bL} and pivot to optimality.

(c) There has been yet another change in the Production Requirements.
- Find the **first** **non-basic** variable in T^* (I'll refer to it as x_{nbf}) and increase it by one unit **above** its maximum value, the minimum row ratio (**mrr**) for the variable. If x_{nbf} cannot be increased by one unit above its **mrr**, increase it by the maximum amount possible. (Use the two-phase method explained in Ch. 5 "When a Nonbasic Variable becomes Basic and Exceeds its Minimum Row Ratio" in the Notes.)
- What is the new optimal solution $\mathbf{x}$ and optimal z–value?
- Check your result(s) by adding the constraint $x_{nbf} = X$ to T^* where X is the chosen value of x_{nbF} and pivot to optimality.

(B) There has been a change in the availability of resources.

- **Use the tableau T_C as your STARTING tableau for this part of the project.**
- The availability of the resource corresponding to the last non-basic slack variable of your **optimal form tableau** T^* has changed.
- Work out (using the technique explained in Ch.5 of the Notes) the maximum amount by which the availability of this resource may be **reduced** and the maximum amount by which it can be **increased** while maintaining the optimality of your optimal tableau T^*.
- Use the technique explained in Ch. 4 of the Notes to **increase** the availability of this resource by **half** this maximum amount and find the new optimal solution **x** and optimal z–value.
- Check your result(s) by altering the availability of the resource in T_C and pivoting to optimality.

(C) A change in the price/cost (price if positive, cost if negative) of the first variable that is **basic** in your optimal tableau is required. (I'll call it x_{bf}.)

- **Use the tableau T_C as your STARTING tableau for this part of the project.**
- Use the technique explained in Ch. 5 of the Notes to find the range of price-changes q for that product in the canonical form tableau T_C that keeps the current set of basis variables basic in your optimal tableau.
- If possible, **Decrease** {if not then **Increase**} the price/cost in the "real" LP (equivalent to **Increasing** {otherwise **Decreasing**} the price/cost in the canonical form tableau T_C) for the selected basic variable by **half** the maximum amount possible that keeps the current set of basic variables basic in your optimal tableau.
- Find the new optimal z–value (the optimal **x**–vector will not change).
- Check your result(s) by altering the price in T_C and pivoting to optimality.

4 Report

- Write a report in PDF format using LaTeX explaining your steps and your interpretation of the results.
- Your report should include the succession of tableaux that you created when solving the problem, copy/pasted from the `diary` file.
- Use the LaTeX `verbatim` environment to enclose the successive tableaux.
- Use `\tiny` if necessary to allow your tableaux to fit on a page.

- You should submit a zip file containing your working folder, including your LaTeXfiles and your Matlab/Octave m-files.

- Email attachments (other than a single zip file) will not be accepted.

- Marks will be awarded for correct results and also for clear explanations of your steps.