

# Project for Operations Research 1

March 20, 2019

- Marks for project: 30%.
- Project due date: 23:55 Friday 12 April 2019 (Week 12).

## 1 Project Description

A file has been placed at <http://jkcray.maths.ul.ie/ms4303/Projectdata.pdf> containing a list of LP's, a different one for every student in the class.

The LP's are in Standard Form, so  $z$  is to be **minimised**.

## 2 Solution to Optimality

You should:

- Find and copy down “your” problem.
- Create a Matlab/Octave script m-file `Run.m` containing all the Matlab/Octave commands needed to solve the problem.
- Generate a Simplex tableau  $T_0$  for your LP.
- Use the provided `Pivot.m` m-file with either Octave or Matlab to transform  $T_0$  to a canonical form tableau  $T_B$ .
- Use the provided `colsort.jk` m-file with either Octave or Matlab to transform  $T_B$  to a canonical form tableau  $T_C$  which has the basic columns in standard order on the right of the tableau.
  - ★ This corresponds to re-labelling the decision variables but does not change the optimal value of the LH-column and in particular  $z$ .
- Use the `Pivot.m` m-file with either Octave or Matlab to solve  $T_C$  to optimality.
- Name the optimal tableau  $T^*$ .
- What is the optimal solution vector  $\mathbf{x}$  and the optimal  $z$ -value?
- **You don't need to — and shouldn't — “undo” the column sort.**

### 3 Sensitivity Analysis

For the remaining questions, you need to interpret your Standard Form LP (which is a **min** problem) as derived from a **max** problem with (of course) the objective coefficients reversed in sign.

**The techniques needed for this part of the project will be explained in class from Week 8 on — they are covered in Ch. 5 of the Notes in the same sequence as the tasks in the following paragraphs.**

- (A) First deal with some changes in the Production Requirements.
- (a) There has been a change in the Production Requirements.
    - Pick the **last non-basic** variable in  $T^*$  (I'll refer to it as  $x_{nbl}$ ) and increase it up to a value  $X$  — where  $X$  is either 1 unit or to **half** the maximum value of  $x_{nbl}$  (whichever is the greater).
    - What is the new optimal solution  $\mathbf{x}$  and optimal  $z$ -value?
    - Check your result(s) by adding the constraint  $x_{nbl} = X$  to  $T^*$  and pivot to optimality.
  - (b) There has been another change in the Production Requirements.
    - ★ Find the last **basic** variable in  $T^*$  (I'll refer to it as  $x_{bl}$ ) and increase it from its optimal value by 1 unit or by **half** its maximum increase (whichever is the greater).
      - \* Choose the “cheapest” non-basic variable to be increased from zero — i.e. the one has the least effect on the objective function value.
      - \* If the last basic variable cannot be increased, choose the second-last, ... until you find a basic variable that can be increased.
    - What is the new optimal solution  $\mathbf{x}$  and optimal  $z$ -value?
    - Check your result(s) by adding the constraint  $x_{bl} = X$  to  $T^*$  where  $X$  is the chosen larger value of  $x_{bl}$  and pivot to optimality.
  - (c) There has been yet another change in the Production Requirements.
    - Find the **first non-basic** variable in  $T^*$  (I'll refer to it as  $x_{nbf}$ ) and increase it by one unit **above** its maximum value, the minimum row ratio (**mrr**) for the variable. If  $x_{nbf}$  cannot be increased by one unit above its **mrr**, increase it by the maximum amount possible. (Use the two-phase method explained in Ch. 5 “When a Nonbasic Variable becomes Basic and Exceeds its Minimum Row Ratio” in the Notes.)
    - What is the new optimal solution  $\mathbf{x}$  and optimal  $z$ -value?
    - Check your result(s) by adding the constraint  $x_{nbf} = X$  to  $T^*$  where  $X$  is the chosen value of  $x_{nbf}$  and pivot to optimality.

- (B) There has been a change in the availability of resources.
- **Use the tableau  $T_C$  as your STARTING tableau for this part of the project.**
  - The availability of the resource corresponding to the last non-basic slack variable of your **optimal form tableau  $T^*$**  has changed.
  - Work out (using the technique explained in Ch.5 of the Notes) the maximum amount by which the availability of this resource may be **reduced** and the maximum amount by which it can be **increased** while maintaining the optimality of your optimal tableau  $T^*$ .
  - Use the technique explained in Ch. 4 of the Notes to **increase** the availability of this resource by **half** this maximum amount and find the new optimal solution  $\mathbf{x}$  and optimal  $z$ -value.
  - Check your result(s) by altering the availability of the resource in  $T_C$  and pivoting to optimality.
- (C) A change in the price/cost (price if positive, cost if negative) of the first variable that is **basic** in your optimal tableau is required. (I'll call it  $x_{bf}$ .)
- (D) **Use the tableau  $T_C$  as your STARTING tableau for this part of the project.**
- Use the technique explained in Ch. 5 of the Notes to find the range of price-changes  $q$  for that product in the canonical form tableau  $T_C$  that keeps the current set of basis variables basic in your optimal tableau.
  - If possible, **Decrease** {if not then **Increase**} the price/cost in the “real” LP (equivalent to **Increasing** {otherwise **Decreasing**} the price/cost in the canonical form tableau  $T_C$ ) for the selected basic variable by **half** the maximum amount possible that keeps the current set of basic variables basic in your optimal tableau.
  - Find the new optimal  $z$ -value (the optimal  $\mathbf{x}$ -vector will not change).
  - Check your result(s) by altering the price in  $T_C$  and pivoting to optimality.

## 4 Report

- Write a report in PDF format using  $\text{\LaTeX}$  explaining your steps and your interpretation of the results.
- Your report should include the succession of tableaux that you created when solving the problem, copy/pasted from the **diary** file.
- Use the  $\text{\LaTeX}$  **verbatim** environment to enclose the successive tableaux.
- Use `\tiny` if necessary to allow your tableaux to fit on a page.

- You should submit a zip file containing your working folder, including your L<sup>A</sup>T<sub>E</sub>X files and your Matlab/Octave m-files.
- Email attachments (other than a single zip file) will not be accepted.
- Marks will be awarded for correct results and also for clear explanations of your steps.