

# Forecasting Stock Index Futures Price Volatility: Linear vs. Nonlinear Models

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## Abstract

The study examines the relative ability of various models to forecast daily stock index futures volatility. The forecasting models that are employed range from naïve models to the relatively complex ARCH-class models. It is found that among linear models of stock index futures volatility, the autoregressive model ranks first using the RMSE and MAPE criteria. We also examine three nonlinear models. These models are GARCH-M, EGARCH, and ESTAR. We find that nonlinear GARCH models dominate linear models utilizing the RMSE and the MAPE error statistics and EGARCH appears to be the best model for forecasting stock index futures price volatility.

*Keywords:* stock index futures volatility, autoregressive, EGARCH, ESTAR

*JEL Classification:* G13

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## 1. Introduction

Recently, there has been an increasing interest in modeling the volatilities of stock returns. Understanding and modeling stock volatility is important since volatility forecasts have many practical applications. Investment decisions, as characterized by asset pricing models, depend heavily on the assessment of future returns and risk of various assets. The expected volatility of a security return also plays an important role in the option pricing theory.

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In the futures markets, stock index futures and options on stock index futures are important areas of research. These financial instruments have very high trading volume due to hedging, speculative trading, and arbitrage activities. Insight into the behavior of futures price volatility can have important implications for investors using stock index futures contracts for portfolio insurance purposes. One important implication is the delta decision a portfolio manager makes when implementing put replication portfolio insurance during period of high market volatility. For instance, should the hedge ratio be implemented using a hedge ratio based on the current high level of volatility or should forecast volatility be used to estimate a hedge ratio appropriate for the insurance horizon? Hill, Jain, and Wood (1988) show that unexpected changes in volatility are the most important risk factor in estimating the cost of portfolio insurance. Chu and Bubnys (1990) use a likelihood ratio test to compare the variance measure of price volatilities of stock market indexes and their corresponding futures contracts during the bull market of the 1980s. They find that spot market volatilities are significantly lower than their respective futures price volatilities.

In the empirical finance literature, many linear models are used to describe the stock return volatility. Poterba and Summers (1986) specify a stationary AR(1) process for the volatility of the S&P 500 Index. French, Schwert, and Stambaugh (1987) use a non-stationary ARIMA (0,1,3) model to describe the volatility of the S&P 500 Index. Schwert (1990) and Schwert and Seguin (1990) use a linear AR(12) as an approximation for monthly stock return volatility. The extensive use of such linear models is not surprising since they provide good first order approximation to many processes and the statistical theory is well developed for linear Gaussian models. However, certain features of a volatility series cannot be described by linear time series models. For example, empirical evidence shows that stock returns tend to exhibit clusters of outliers, implying that large variance tends to be followed by another large variance. Such limitations of linear models have motivated many researchers to consider nonlinear alternatives. The most commonly used nonlinear time-series models in the finance literature are the autoregressive conditional heteroscedastic (ARCH) model of Engle (1982), the generalized ARCH (GARCH) model of Bollerslev (1986) and the exponential GARCH (EGARCH) model of Nelson (1991). These ARCH-class models have been found to be useful in capturing certain nonlinear features of financial time-series such as heavy-tailed distributions and clusters of outliers.<sup>1</sup> Akgiray (1989) use a GARCH (1, 1) model to investigate the time series properties of the stock returns and finds GARCH to be the best of several models in describing and forecasting stock market volatility. Randolph and Najand (1991) compared out-of-sample forecasting power of the GARCH (1, 1) model and mean reversion models (MRM) for S&P 500 Index futures and conclude that MRM produces superior forecasts. Bera, Bubnys, and Park (1993) investigate

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<sup>1</sup> See Bollerslev, Chou, and Kroner (1992) for an extensive review of these models and their applications.

the validity of the conventional OLS model to estimate optimal hedge ratio using futures contracts. Comparing the OLS and ARCH hedge ratios, they find that the conventional hedge ratio estimates may cause investors to sell short too many or few futures contracts. Another complex class of nonlinear models is called exponential smooth transition autoregressive (ESTAR) models suggested by Teräsvirta and Anderson (1992). They apply ESTAR model to industrial production of 13 countries during different business cycles. They conclude that the STAR model family is a suitable tool for describing nonlinearity found in time series of industrial production.

The existing literature contains conflicting evidence regarding the relative quality of stock market volatility forecasts. Evidence can be found supporting the superiority of complex (ARCH-class) and ESTAR models, while there is also evidence supporting of simple alternatives. These inconsistencies are of particular concern since volatility forecasts are key variables in asset/option pricing models.

The purpose of this paper is to examine the relative ability of various models to forecast daily stock index futures volatility. The forecasting models that are employed range from naïve models to the relatively complex ARCH-class and ESTAR models.

## **2. Data and methodology**

### *2.1. Data*

Daily closing prices data of S&P 500 futures index between January 1983 and December 1996 is obtained from Tick Data Inc. Since most trading activities take place in the near-month contract, only near-month contract data are examined. As a result, this controls the maturity effect on futures prices. A continuous sequence of 3561 observations of futures closing prices data are gathered over the fourteen-year period. The logarithm of price relatives multiplied by 100 is used to calculate price change. The use of logarithm price changes prevents nonstationarity of the price level of the data from affecting futures price volatility.

To assess the distributional properties of the daily stock index futures price change, various descriptive statistics are reported in Table 1 including: mean, variance, standard deviation, skewness, kurtosis, and the Kolmogorov-Smirnov (K-S) D statistics normality test. The null hypothesis of normality is rejected at the 1% level using K-S D statistics. Further evidence on the nature of deviation from normality may be gleaned from the sample skewness and kurtosis measures. While skewness is relatively small, kurtosis is very large.<sup>2</sup>

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<sup>2</sup> Kurtosis refers to excess kurtosis, so that a value of zero corresponds to normality.

Table 1

**Descriptive statistics on stock index futures price change series of near-month contracts between January 3, 1983, and December 30, 1996**

This table presents distributional properties of the daily stock index futures price change. The logarithm of price relatives multiplied by 100 is used to calculate price change. Various descriptive statistics, including mean, variance, standard deviation, skewness, kurtosis, and the K-S D statistics normality test, are reported here.

Statistics	
Sample size	3,560
Mean ( $\mu$ )	0.0463
Variance	1.3483
Standard deviation	1.1611
Skewness	−6.4232
Kurtosis	225.1621
D: Normal	0.0190*

\* Indicates statistical significance at the 0.10 level.

Following Schwert (1990) and Schwert and Seguin (1990), we estimate the volatility of daily futures stock returns by the following equation:<sup>3</sup>

$$\sigma_t = \sqrt{(\pi/2)}|R_t - \mu| \tag{1}$$

where  $R_t$  is return on S&P 500 futures contracts calculated as described above and  $\mu$  is mean of the series.<sup>4</sup> Summary statistics for volatility series  $\sigma_t$  are reported in Table 2. The results are very similar to the return series reported in Table 1. The null hypothesis of normality is rejected at the 1% level using K-S D statistics.

2.2. *Methodology*

The focus of this paper is on the forecasting accuracy of daily stock futures volatility from various statistical models. The basic methodology involves the estimation of the various models' parameters for an initial period and the application of these parameters to later data, thus forming out-of-sample forecasts. The linear models employed are: (1) a random walk model, (2) an autoregressive model, (3) a moving average model, (4) an exponential smoothing model, (5) and a double (Holt) exponential smoothing model. The nonlinear models utilized here are GARCH-M (1, 1), EGARCH (1, 1), and ESTAR models.

<sup>3</sup> Cao and Tsay (1992) also point out that  $\sigma_t = \sqrt{(\pi/2)}|R_t - \mu|$  is an unbiased estimator for the standard deviation.

<sup>4</sup> We also estimated volatility as:  $\sigma_t = \sqrt{(\pi/2)}|\varepsilon_t|$ , where  $\varepsilon_t$  is calculated as the residual of the AR(5) process for return series. The results are very similar to model (1) and are not reported here.

Table 2

**Descriptive statistics on stock index futures price volatility series of near-month contracts between January 3, 1983, and December 30, 1996**

This table presents distributional properties of the daily stock index futures price volatility. Volatility of daily futures stock returns are estimated by the following equation:

$$\sigma_t = \sqrt{(\pi/2)}|R_t - \mu|$$

Various descriptive statistics, including mean, variance, standard deviation, skewness, kurtosis, and the K-S D statistics normality test, are reported here.

Statistics	
Sample size	3,560
Mean ( $\mu$ )	0.8449
Variance	1.3548
Standard deviation	1.1639
Skewness	15.6722
Kurtosis	506.5896
D: Normal	0.2339*

\* Indicates significance at the 10% level.

### 2.2.1. Linear models

#### 2.2.1.1. Random walk model

Under a random walk model, the best forecast of today's volatility is yesterday's observed volatility.

$$\sigma_t(\text{RW}) = \sigma_{t-1} \quad (2)$$

where  $\sigma_t$  is the daily volatility measure defined in equation (1).

#### 2.2.1.2. Autoregressive AR (1) model

An AR (1) process is defined as:

$$(1 - \phi_1 B)\sigma_t(\text{AR}) = \mu + e_t \quad (3)$$

where  $\phi_1$  is the autoregressive parameter, B is backward shift operator,  $\mu$  is constant term, and  $e_t$  is the error term at time t.

#### 2.2.1.3. Moving average MA (1) model

The MA (1) process is defined as:

$$\sigma_t(\text{MA}) = \mu + (1 - \theta_1 B)e_t \quad (4)$$

where  $\theta_1$  is the moving average parameter, B is backward shift operator,  $\mu$  is constant term, and  $e_t$  is the error term at time t.

#### 2.2.1.4. Exponential smoothing model

Following Dimson and Marsh (1990), the following exponential smoothing model is used to forecast volatility:

$$\sigma_t(\text{ES}) = \phi\sigma_{t-1}(\text{ES}) + (1 - \phi)\sigma_{t-1} \quad (5)$$

In model (5), the forecast of volatility is assumed to be a function of the immediate past forecast and the immediate past observed volatility. The smoothing parameter ( $\phi$ ) is constrained to lie between zero and one. The optimal value of  $\phi$  must be determined empirically. If  $\phi$  is zero, then the exponential smoothing model collapses to the random walk model.

#### 2.2.1.5. Double (Holt) exponential smoothing model

Holt's linear exponential smoothing method is similar in principal to model (5) except that it smoothes the trend values separately.<sup>5</sup>

$$\sigma_t(\text{DES}) = \phi\sigma_{t-1}(\text{DES}) + (1 - \phi)(\sigma_{t-1} + b_{t-1}) \quad (6)$$

$$b_t = \gamma(\sigma_{t-1} - \sigma_{t-2}) + (1 - \gamma)b_{t-1} \quad (7)$$

Equation (6) adjusts  $\sigma_t$  directly for the trend of the previous period,  $b_{t-1}$ , by adding it to the last smoothed value,  $\sigma_{t-1}$ . This helps to eliminate the lag and brings  $\sigma_t$  to the approximate base of the current data value. Equation (7) then updates the trend, which is expressed as the difference between the last smoothed values. This is appropriate because if there is a trend in the data, new values should be higher or lower than the previous ones. Since there may be some randomness remaining, it is modified by smoothing with  $\gamma$ , the trend in the last period, and adding that to the previous estimate of the trend multiplied by  $(1 - \gamma)$ .

### 2.2.2. Nonlinear models

#### 2.2.2.1. GARCH-M (1, 1) model

The most commonly used nonlinear time-series models utilized in the finance literature are the autoregressive conditional heteroscedastic (ARCH) model of Engle (1982) and the generalized ARCH (GARCH) model of Bollerslev (1986). The GARCH model involves the joint estimation of a conditional mean and a conditional variance equation. The GARCH-in-Mean (GARCH-M) model has the added regressor that is the conditional standard deviation. The following GARCH-M (1, 1) is employed here:

$$\begin{aligned} R_t &= \beta_1 R_{t-1} + \delta \sqrt{h_t} + \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t) \\ h_t &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 h_{t-1} \end{aligned} \quad (8)$$

where  $R_t$  is the return on the stock index futures measured as the logarithm of relative price change and  $h_t$  is the conditional volatility. The key feature of the GARCH model is that the conditional variance of  $\varepsilon_t$  is predictable. Model (8) indicates that a large conditional variance tends to be followed by another large conditional

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<sup>5</sup> See Makridakis, Wheelwright, and McGee (1983).

variance. This feature implies that the GARCH model is capable of producing clusters of outliers. In financial time-series, clusters of outliers result in clusters of high volatilities.

#### 2.2.2.2. EGARCH (1, 1) model

A limitation of the GARCH model described above is that the conditional variance responds to positive and negative residuals  $\varepsilon_{t-1}$  in the same manner. However, empirical evidence in financial time-series shows that there is a negative correlation between the current returns and future return volatility. The GARCH model imposes the nonnegative constraints on the parameters,  $\alpha_1$ , and  $\gamma_1$ , while there are no restrictions on these parameters in the exponential GARCH model proposed by Nelson (1991). In the EGARCH (1, 1) model, the conditional variance,  $h_t$ , is an asymmetric function of lagged residuals,  $\varepsilon_{t-1}$ :

$$\begin{aligned} R_t &= \beta_1 R_{t-1} + \varepsilon_t \\ \ln(h_t) &= \omega + \alpha_1 g(z_{t-1}) + \gamma_1 \ln(h_{t-1}) \end{aligned} \quad (9)$$

where  $g(z_t) = \theta z_t + \gamma [|z_t| - E|z_t|]$  and  $z_t = \varepsilon_t / \sqrt{h_t}$  ( $z_t$  is i.i.d. with zero mean and unit variance). Consider the  $g(z_t)$  function above. If  $z_t$  is positive then  $g(z_t)$  is linear function of the slope changes,  $z_t$ , with slope  $(\theta + \gamma)$ . If  $z_t$  is negative then the slope changes to  $(\theta - \gamma)$ . Consequently, the conditional variance  $h_t$  responds asymmetrically to the sign of innovation  $z_{t-1}$ .

#### 2.2.2.3. Smooth transition autoregressive (STAR) models

Teräsvirta and Anderson (1992) investigate the issue of nonlinearity of business cycles using STAR models. They find that in many cases the STAR models is a suitable tool for describing the nonlinearity found in the industrial output of different countries. Consider the following STAR model of order  $p$  and  $d$ :

$$Y_t = c_1 + c_2 W_t + (c_3 + c_4 W_t) [1 - \exp(-c_5 (Y_{t-d} - c_6)^2)] + \mu_t \quad c_5 > 0 \quad (10)$$

where  $\mu_t \sim (0, \sigma^2)$ ,  $W_t = (Y_{t-1}, \dots, Y_{t-p})$  and  $[1 - \exp(-c_5 (Y_{t-d} - c_6)^2)]$  is a function which by convention is bounded by zero and one. In the above model,  $d$  is the delay parameter and lag  $p$  is usually unknown and has to be determined from the data. For example, in the case of  $d = 1$  and  $p = 3$ , it takes one period for the volatility to respond to a shock and takes three periods to complete the responses to a shock. Model (10) is called an exponential STAR (ESTAR) Model in which the parameters change symmetrically about  $c_6$  with  $Y_{t-d}$ . The ESTAR model implies that high volatility and low volatility have similar dynamic structures.

## 3. Results

In Tables 3 and 4 estimates for GARCH (model (8)) and EGARCH (model (9)) are reported. We estimate the models using 3500 and 3380 observations and saving

Table 3

**Estimates of GARCH-M(1, 1) model**

The table presents parameter estimates for the following GARCH-M(1, 1) model:

$$\begin{aligned} R_t &= \beta_1 R_{t-1} + \delta \sqrt{h_t} + \varepsilon_t \\ \varepsilon_t &\sim N(0, h_t) \\ h_t &= \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma_1 h_{t-1} \end{aligned}$$

where:  $R_t$  is the return on the stock index futures measured as the logarithm of relative price change, and  $h_t$  is the conditional volatility. Standard errors are in parentheses. The diagnostics are the AIC, the LnL, Q, and the LM. Q(10) and LM(10) denote the tests for the significance of residuals correlations up to lag 10 in the estimated standardized residuals  $\varepsilon_t / \sqrt{h_t}$ .

*Panel A: Parameters*

	Coefficient
Intercept	−0.05529 (0.0561)
$\beta_1$	0.0337* (0.0207)
$\delta$	0.1436** (0.0663)
$\omega$	0.0469*** (0.0024)
$\alpha_1$	0.1414*** (0.0047)
$\gamma_1$	0.8258*** (0.0090)

*Panel B: Diagnostics*

AIC	9193.63
LnL	−4590.82
Q(10)	3.237
LM(10)	3.226

\*\*\* Indicates statistical significance at the 0.01 level.

\*\* Indicates statistical significance at the 0.05 level.

\* Indicates statistical significance at the 0.10 level.

the last 60 and 180 observations, respectively, for out-of-sample forecasting comparisons between models. In Table 3, we report relevant parameter estimates as well as diagnostics for GARCH-M (1, 1) model. In panel A, GARCH parameter estimates are reported. The coefficients for  $\alpha_1$  and  $\gamma_1$  are significant at the 1% level indicating that the constant variance model can be rejected. Also the coefficient for the conditional standard deviation ( $\delta$ ) in the conditional mean equation is significant at the 5% level. This indicates that an increase in the conditional variance will be associated with an increase in the conditional mean (tradeoff between the risk and the expected return). In panel B, we report the diagnostics for the GARCH-M (1, 1) model. The Akaike Information Criterion (AIC) and the Log Likelihood (LnL) are used to compare GARCH-M and EGARCH (reported in Table IV) models. For nonlinear time series models, the portmanteau Q-test statistics (Q) based on standardized residuals ( $\varepsilon_t / \sqrt{h_t}$ ) is used to test for nonlinear effects. The Q (10) statistic cannot reject the null hypothesis of no nonlinear effects for up to lag 10 for our GARCH-M (1, 1) model. Thus, it appears that the nonlinearity in the volatility series has been successfully removed by our GARCH model specifications. We also report the Lagrange multiplier



Table 4

**Estimates of EGARCH (1, 1) model**

This table presents parameter estimates for the following GARCH-M (1, 1) model:

$$R_t = \beta_1 R_{t-1} + \varepsilon_t$$

$$\ln(h_t) = \omega + \alpha_1 g(z_{t-1}) + \gamma_1 \ln(h_{t-1})$$

where:  $R_t$  is the return on the stock index futures measured as the logarithm of relative price change, and  $h_t$  is the conditional volatility,  $g(z_t) = \theta z_t + \gamma[|z_t| - E|z_t|]$  and  $z_t = \varepsilon_t/\sqrt{h_t}$ . Standard errors are in parentheses. The diagnostics are the AIC, the LnL, Q, and the LM. Q(10) and LM(10) denote the tests for the significance of residuals correlations up to lag 10 in the estimated standardized residuals  $\varepsilon_t/\sqrt{h_t}$ .

*Panel A: Parameters*

	Coefficient
Intercept	0.0319** (0.0137)
$\beta_1$	0.0295** (0.0125)
$\omega$	0.0115*** (0.0035)
$\alpha_1$	0.2069*** (0.0217)
$\gamma_1$	0.9607*** (0.0079)
$\theta$	-0.4944*** (0.0679)

*Panel B: Diagnostics*

AIC	9118.86
LnL	-4553.43
Q(10)	3.54
LM(10)	3.53

\*\*\* Indicates statistical significance at the 0.01 level.

\*\* Indicates statistical significance at the 0.05 level.

\* Indicates statistical significance at the 0.10 level.

test (LM) for ARCH disturbances proposed by Engle (1982) in Panel B. The null hypothesis that the disturbances lack ARCH effects is not rejected.

We report the parameter estimates of the EGARCH (1, 1) model in Table 3. It is interesting to note that parameters  $\theta$  and  $\gamma$  are statistically significant, suggesting that the conditional variance of futures stock returns indeed responds differently to 'positive' and 'negative' innovations. The possible superiority of EGARCH (1, 1) over GARCH-M (1, 1) is confirmed by AIC and LnL values. Again, both the Q and LM statistics indicate that our EGARCH model is well specified.

Finally, we estimate the ESTAR (1) model for volatility of stock index futures contracts. The specified and estimated model is:

$$\sigma_t = 0.646 + 0.235\sigma_{t-1} + (.468 + 0.353\sigma_{t-1}[1 - \exp(-0.002(\sigma_{t-1} - 0.001)^2)] + \mu_t$$

(0.018)   (0.042)   (.168)   (.013)                      (0.001)                      (0.004)

$$s = 1.136$$

where  $s$  is the residual standard deviation and the figures in parentheses are the estimated standard errors.

Table 5

**Out-of-sample forecast comparison of linear and nonlinear models for the volatility of stock index futures**

This table presents the actual error statistics for each linear and nonlinear model across two error measures and for two forecasting periods. The forecast performance is judged by using RMSE and MAPE. The forecasts are for 60 and 180 periods ahead.

Model	Equation Number	RMSE		MAPE	
		N = 60	N = 180	N = 60	N = 180
Linear Models					
Random Walk	(2)	1.2090	1.9156	1.0209	1.2299
AR (1)	(3)	0.9457	1.5022	0.8311	0.9545
MA (1)	(4)	0.9658	1.5248	0.8787	0.9665
Exponential Smoothing	(5)	0.9946	1.5801	0.8986	0.9884
Double Exponential Smoothing	(6) and (7)	1.052	1.9098	0.9032	1.1935
Nonlinear Models					
GARCH-M (1, 1)	(8)	0.9426	1.0958	0.8106	0.8770
EGARCH (1, 1)	(9)	0.9401	0.7901	0.6698	0.7066
ESTAR (1)	(10)	0.9428	0.8634	0.8148	0.8023

To evaluate the performance of the linear and nonlinear models in describing stock index futures volatility, we compare their out-of-sample forecasts with our benchmark model (1). The post-sample forecast comparisons are carried out as follows. First, we reserve the last 60 and 180 observations for forecast comparison. Secondly, all the models used in forecasting are estimated using the first 3500 and 3380 observations. Such a scheme provides 60 and 180 one-step ahead forecasts. The objective is to evaluate forecasting capability of different models during the low (stable prices) and high (volatile prices) volatility periods. We summarize the forecast performance by considering the root mean squared error (RMSE) and the mean absolute percentage error (MAPE) which are defined as follows:

$$RMSE = 1/n \sqrt{\sum_{t=1}^n (\sigma_t - \hat{\sigma}_t)^2} \quad (11)$$

$$MAPE = 1/n \sum_{t=1}^n |(\sigma_t - \hat{\sigma}_t)|/\sigma_t \quad (12)$$

Table 5 presents the actual error statistics for each linear and nonlinear model across the two error measures and for two forecasting periods. An examination of Table 5 reveals that within the linear models the AR(1) model dominates all of the models using RMSE criterion. The MA(1) model is a close second and the exponential smoothing model ranks last. It is interesting to note that the AR(1) model outperforms the random walk model based on RMSE for both periods. The MAPE statistic

indicates the same ranking and AR(1) outperforms all the linear models followed by MA(1).

Within nonlinear models, the EGARCH (1, 1) model clearly dominates the GARCH-M (1, 1) model using either measure of performance and the ESTAR (1) model ranks last for 60 periods ahead. It is interesting to note that ESTAR (1) model ranks second to EGARCH (1, 1) model and dominates GARCH-M (1, 1) model when the forecasting period is extended to 180 periods ahead.

In comparison of all the models, linear and nonlinear, the EGARCH model is superior to all the models followed closely by the GARCH-M model and the ESTAR model during the low volatility period (60 periods ahead). During the high volatility period (180 periods ahead), EGARCH model again dominates all the models followed closely by the ESTAR and GARCH-M models. Thus, both the RMSE and the MAPE statistics clearly identify the nonlinear GARCH class models and ESTAR model as superior to linear models.

#### 4. Conclusions

The purpose of this paper is to examine the relative ability of various models to forecast daily stock index futures volatility. Understanding and modeling stock volatility is important since volatility forecasts have many practical applications. Investment decisions and asset pricing models depend heavily on the assessment of future returns and risk of various assets. The expected volatility of a security return also plays an important role in the option pricing theory. The forecasting models that are employed range from naïve models to the relatively complex ARCH-class models. The five linear models considered here are: (1) random walk, (2) autoregressive (AR(1)), (3) moving average (MA(1)), (4) exponential smoothing, and (5) double exponential smoothing. It is found that among linear models of stock index futures volatility, the autoregressive model ranks first using the RMSE and the MAPE. We also examine three nonlinear models. These models are GARCH-M, EGARCH, and ESTAR. We find that nonlinear GARCH models and ESTAR model dominate linear models utilizing the RMSE and the MAPE error statistics and EGARCH appears to be the best model for forecasting stock index futures price volatility followed closely by GARCH-M and ESTAR models.

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