Supplementary InGM-LIO: A Multiscale Gaussian Model-Based LiDAR-Inertial Odometry Using Invariant Kalman Filtering

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I. Dynamic Model

Given the discrete IMU inputs $\mathbf{u}$, the discrete state kinematic equations can be expressed as shown in Eq.(1), omitting the state variables $(\mathbf{b}_g, \mathbf{b}_a, \mathbf{g}^G)$ that remain unchanged during motion. The terms Γ_0, Γ_1, and Γ_2 account for the effects of angular velocity on the attitude, velocity, and position, respectively.

$$\begin{aligned}
\mathbf{R}_{k+1} &= \mathbf{R}_k \Gamma_0(\bar{\omega}_k \Delta t) = \mathbf{R}_k \operatorname{Exp}(\bar{\omega}_k \Delta t) \\
\mathbf{v}_{k+1} &= \mathbf{v}_k + \mathbf{R}_k \Gamma_1(\bar{\omega}_k \Delta t)\, \bar{\mathbf{a}}_k \Delta t + \mathbf{g}\Delta t \\
\mathbf{p}_{k+1} &= \mathbf{p}_k + \mathbf{v}_k \Delta t + \mathbf{R}_k \Gamma_2(\bar{\omega}_k \Delta t)\, \bar{\mathbf{a}}_k \Delta t^2 + \frac{1}{2}\mathbf{g}\Delta t^2
\end{aligned} \tag{1}$$

The specific expressions for Γ_0, Γ_1, and Γ_2 are shown in Eq. (2).

$$\begin{aligned}
\Gamma_0(\phi) &= I + \frac{\sin(\|\phi\|)}{\|\phi\|}(\phi^\wedge) + \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}(\phi^\wedge)^2 \\
\Gamma_1(\phi) &= I + \frac{1-\cos(\|\phi\|)}{\|\phi\|^2}(\phi^\wedge) + \frac{\|\phi\| - \sin(\|\phi\|)}{\|\phi\|^3}(\phi^\wedge)^2 \\
\Gamma_2(\phi) &= \frac{1}{2}I + \frac{\|\phi\| - \sin(\|\phi\|)}{\|\phi\|^3}(\phi^\wedge) + \frac{\|\phi\|^2 + 2\cos(\|\phi\|) - 2}{2\|\phi\|^4}(\phi^\wedge)^2 \\
\Gamma_m(\phi) &:= \left(\sum_{n=0}^{\infty} \frac{1}{(n+m)!}(\phi^\wedge)^n\right)
\end{aligned} \tag{2}$$

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