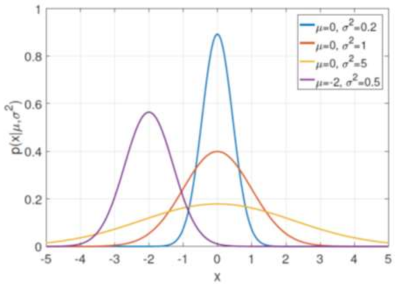
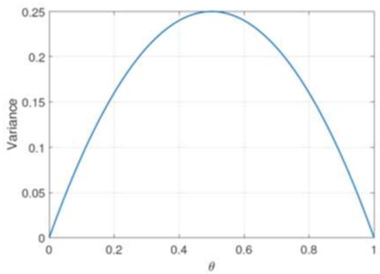
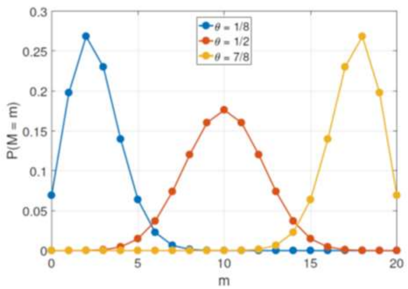
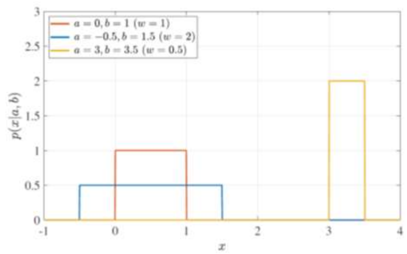
Probability Distribution Model

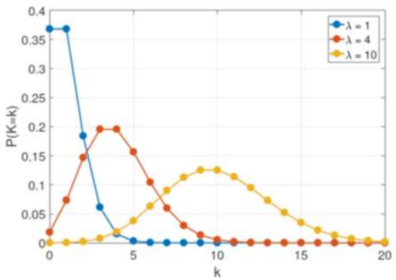
- Probability distribution model
  - Gaussian (normal) distribution
  - Bernoulli distribution
  - Binomial distribution
  - Uniform distribution
  - Poisson distribution











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Probability distribution model	pdf	parameters	X	mean	variance
Gaussian distribution $X \sim N(\mu, \sigma^2)$	$p(x \mid \mu, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{\frac{1}{2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	μ, σ^2	$X \in \mathbb{R},\ X \sim N(\mu, \sigma^2)$	$\mathbb{E}[X] = \mu,$	$\mathbb{V}[X] = \sigma^2$
Bernoulli distribution $X \sim \mathrm{Be}(\theta).$	$p(x \mid \theta) = \theta^x (1-\theta)^{(1-x)}.$	θ, *success*	$X \in \{0, 1\},\ X \sim \mathrm{Be}(\theta)$	$\mathbb{E}[X] = \theta$	$\mathbb{V}[X] = \theta(1-\theta)$
Binomial distribution $M \sim \mathrm{Bin}(\theta, n)$	$p(m \mid \theta) = \binom{n}{m} \prod_{i=1}^{n} p(x_i \mid \theta) = \binom{n}{m} \theta^m (1-\theta)^{(n-m)}$ where $\binom{n}{m} = \frac{n!}{(n-m)!m!}$	n, θ	$X \in \{0, 1, \ldots, n\},\ X \sim \mathrm{Bin}(\theta, n)$	$\mathbb{E}[X] = n\theta$	$\mathbb{V}[X] = n\theta(1-\theta)$
Discrete Uniform dist. $X \sim U(a, b)$	$\mathbb{P}(X = k \mid a, b) = \frac{1}{b-a+1}$	a, b $w = b - a$	$X \in \{a, \ldots, b\}$ with $b \geq a$	$\mathbb{E}[X] = \frac{a+b}{2}$	$\mathbb{V}[X] = \frac{(b-a+1)^2 - 1}{12}$
Continuous Uniform dist. $X \sim U(a, b)$	$p(x \mid a, b) = \begin{cases} 0 & \text{for } x < a \\ \frac{1}{b-a} & \text{for } a \leq x \leq b \\ 0 & \text{for } x > b \end{cases}$	a, b $w = b - a$	$X \in \{a, \ldots, b\}$ with $b \geq a$	$\mathbb{E}[X] = \frac{a+b}{2} = a + \frac{w}{2}$	$\mathbb{V}[X] = \frac{(b-a)^2}{12} = \frac{w^2}{12}$
Poisson distribution $X \sim \mathrm{Pois}(\lambda)$	$p(k \mid \lambda) = \frac{\lambda^k \exp(-\lambda)}{k!}$	λ, *called the rate*	$X \in \{0, 1, 2, \ldots\},\ X \sim \mathrm{Poi}(\lambda)$	$\mathbb{E}[X] = \lambda$	$\mathbb{V}[X] = \lambda$