3.

$$p(N,\theta|x) = \frac{p(x|N,\theta)\times p(N,\theta)}{p(x)}$$

$$= \frac{p(x|N,\theta)\times \mathrm{Beta}(\theta|a,b)}{\int p(x,\theta)\,d\theta}$$

$$= \frac{\binom{N}{m}\theta^{m}(1-\theta)^{N-m}\times\theta^{a-1}(1-\theta)^{b-1}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}{\int_0^1\binom{N}{m}\theta^{m}(1-\theta)^{N-m}\times\theta^{a-1}(1-\theta)^{b-1}\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\,d\theta}$$

$$= \frac{\cancel{\binom{N}{m}}\theta^{m+a-1}(1-\theta)^{N-m+b-1}\cancel{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}}{\cancel{\binom{N}{m}}\cancel{\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}}\int_0^1\theta^{m+a-1}(1-\theta)^{N-m+b-1}\,d\theta}$$

$$\int_0^1\beta(\theta|m+a,N-m+b)\,d\theta=1$$

$$\Rightarrow\frac{\Gamma(N+a+b)}{\Gamma(m+a)\Gamma(N-m+b)}\int_0^1\theta^{m+a-1}(1-\theta)^{N-m+b-1}\,d\theta=1$$

$$\Rightarrow\int_0^1\theta^{m+a-1}(1-\theta)^{N-m+b-1}\,d\theta=\frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(N+a+b)}$$

$$= \frac{\theta^{m+a-1}(1-\theta)^{N-m+b-1}}{\frac{\Gamma(m+a)\Gamma(N-m+b)}{\Gamma(N+a+b)}}$$

$$= \theta^{m+a-1}(1-\theta)^{N-m+b-1}\frac{\Gamma(N+a+b)}{\Gamma(m+a)\Gamma(N-m+b)}$$

$$= \mathrm{Beta}(\theta|m+a,N-m+b)$$