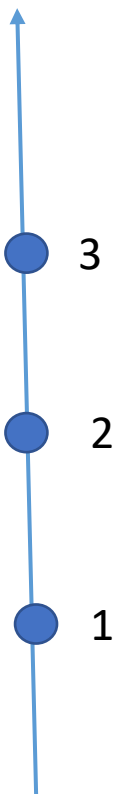


$$p \frac{\partial}{\partial x} \left(\frac{1}{q} \frac{\partial}{\partial x} (rm) \right)$$

$$p^2 \frac{1}{dx} \left[\frac{2}{q_2 + q_3} \frac{r_3 m_3 - r_2 m_2}{dx} - \frac{2}{q_1 + q_2} \frac{r_2 m_2 - r_1 m_1}{dx} \right]$$

$$\frac{2}{dx^2} \left[\frac{1}{q_1 + q_2} p_2 r_1 m_1 - \left(\frac{1}{q_1 + q_2} + \frac{1}{q_2 + q_3} \right) p_2 r_2 m_2 + \frac{1}{q_2 + q_3} p_2 r_3 m_3 \right]$$



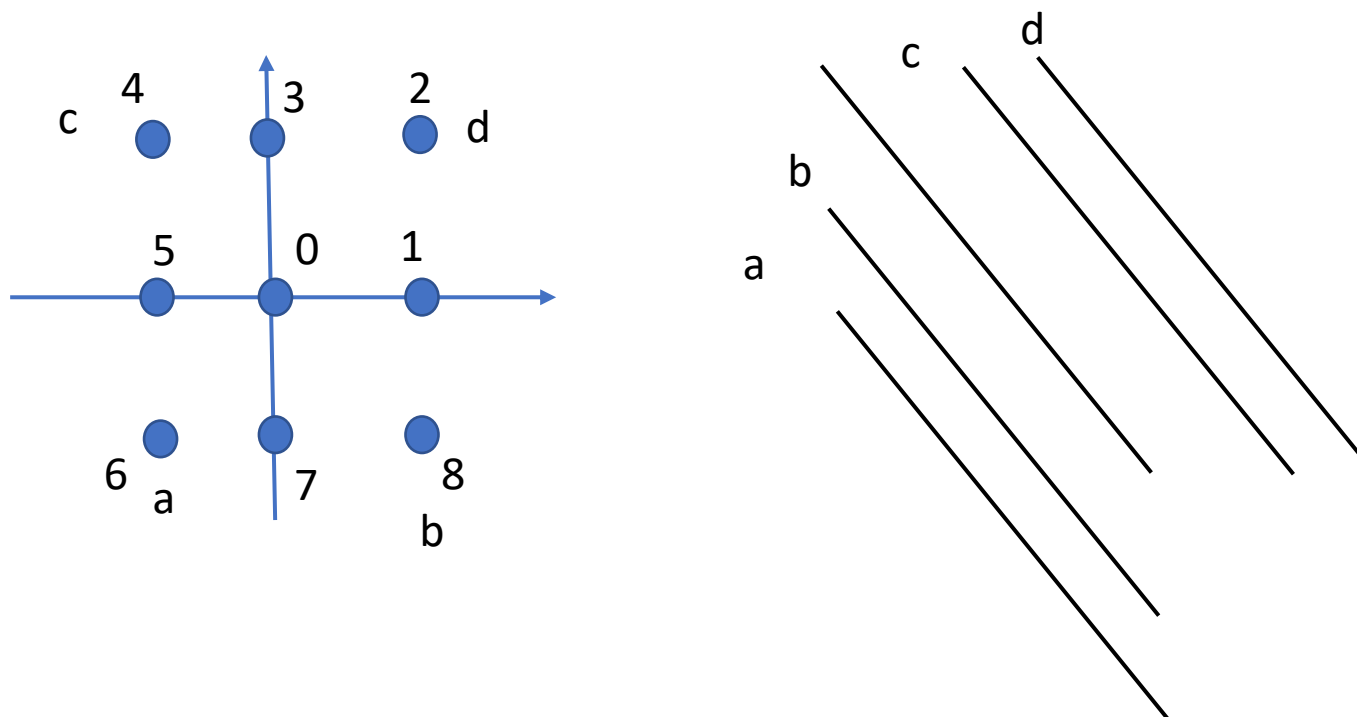
核心

$$p \frac{\partial}{\partial y} \left(\frac{1}{q} \frac{\partial}{\partial y} (rm) \right)$$

$$\frac{\partial}{\partial y} \frac{1}{q} \frac{\partial}{\partial y} \Phi$$

$$\frac{p_2}{dy} \left[\frac{2}{q_2 + q_3} \frac{r_3 m_3 - r_2 m_2}{dy} - \frac{2}{q_1 + q_2} \frac{r_2 m_2 - r_1 m_1}{dy} \right]$$

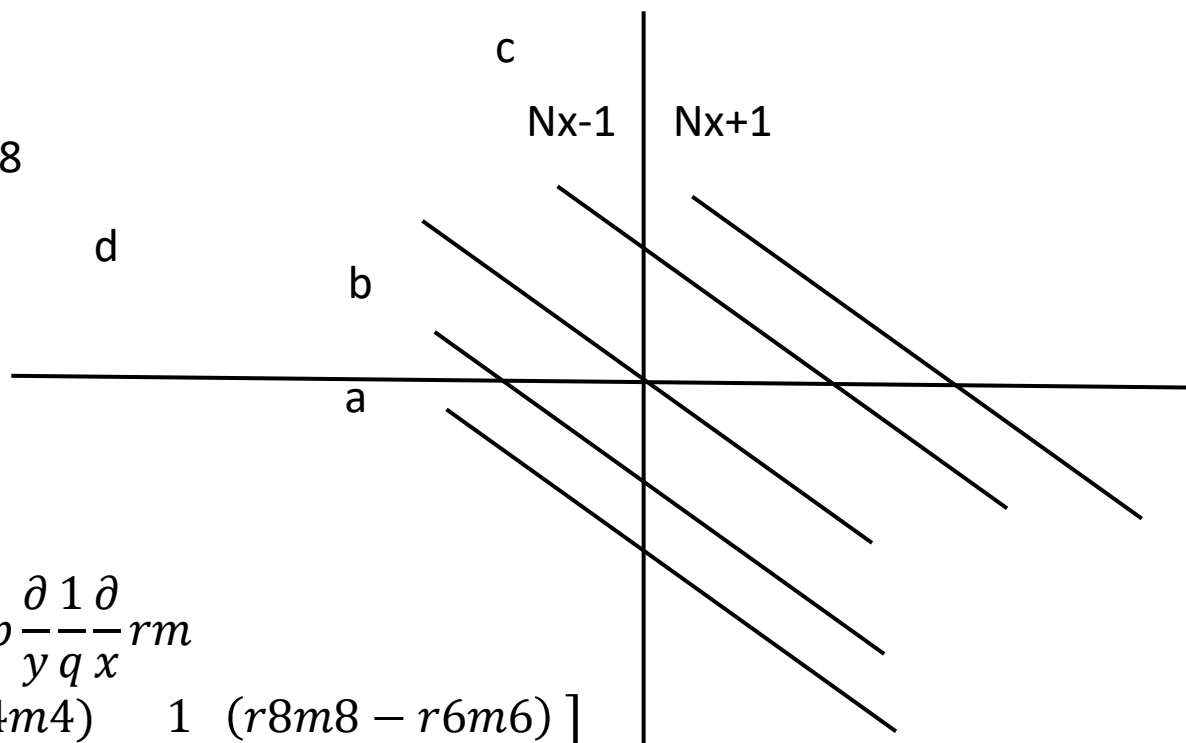
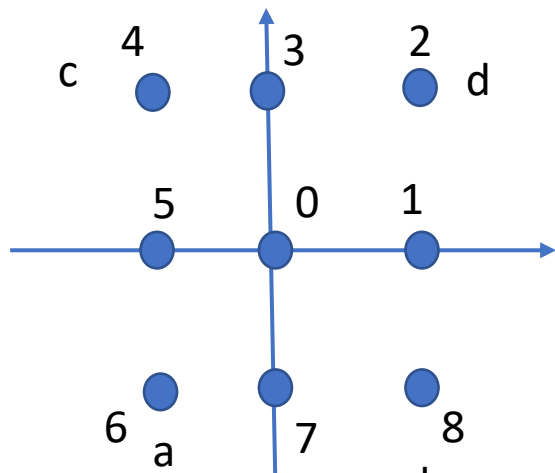
$$\frac{2}{dy^2} \left[\frac{1}{q_1 + q_2} p_2 r_1 m_1 - \left(\frac{1}{q_1 + q_2} + \frac{1}{q_2 + q_3} \right) p_2 r_2 m_2 + \frac{1}{q_2 + q_3} p_2 r_3 m_3 \right]$$



$$p \frac{\partial}{\partial x} \left(\frac{1}{q} \frac{\partial}{\partial y} r m \right)$$

$$\frac{p_0}{2dx} \left[\frac{1}{q_1} \frac{r_2 m_2 - r_8 m_8}{2dy} - \frac{1}{q_5} \frac{r_4 m_4 - r_6 m_6}{2dy} \right]$$

$$\frac{1}{4dxdy} \left[\frac{1}{q_5} p_0 r_6 m_6 - \frac{1}{q_1} p_0 r_8 m_8 - \frac{1}{q_5} p_0 r_4 m_4 + \frac{1}{q_1} p_0 r_2 m_2 \right]$$



$$p \frac{\partial}{\partial y} \frac{1}{q} \frac{\partial}{\partial x} r m$$

$$\frac{p_0}{2dy} \left[\frac{1}{q^3} \frac{(r^2 m^2 - r^4 m^4)}{2dx} - \frac{1}{q^7} \frac{(r^8 m^8 - r^6 m^6)}{2dx} \right]$$

$$\frac{1}{4dxdy} \left[\frac{1}{q^7} p_0 r^6 m^6 - \frac{1}{q^7} p_0 r^8 m^8 - \frac{1}{q^3} p_0 r^4 m^4 + \frac{1}{q^3} p_0 r^2 m^2 \right]$$

$$P_{xx}\Psi_x \equiv \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 \Psi_x) \right) + \frac{\partial^2 \Psi_x}{\partial y^2} + k_0^2 (n^2 - n_0^2) \Psi_x;$$

$$P_{yy}\Psi_y \equiv \frac{\partial^2 \Psi_y}{\partial x^2} + \frac{\partial}{\partial y} \left(\frac{1}{n^2} \frac{\partial}{\partial y} (n^2 \Psi_y) \right) + k_0^2 (n^2 - n_0^2) \Psi_y;$$

$$P_{xy}\Psi_y \equiv \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial y} (n^2 \Psi_y) \right) - \frac{\partial^2 \Psi_y}{\partial x \partial y};$$

$$P_{yx}\Psi_x \equiv \frac{\partial}{\partial y} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 \Psi_x) \right) - \frac{\partial^2 \Psi_x}{\partial y \partial x}.$$

$$2in_0k_0 \frac{\partial}{\partial z} \begin{bmatrix} \Psi_x \\ \Psi_y \end{bmatrix} = \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} \begin{bmatrix} \Psi_x \\ \Psi_y \end{bmatrix}. \quad \begin{bmatrix} P_{xx} & P_{xy} \\ P_{yx} & P_{yy} \end{bmatrix} = \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}.$$

$$A_x \Psi_x \equiv \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 \Psi_x) \right) + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_x;$$

$$A_y \Psi_x \equiv \frac{\partial^2 \Psi_x}{\partial v^2} + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_x.$$

$$B_x \Psi_y \equiv \frac{\partial^2 \Psi_y}{\partial x^2} + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_y;$$

$$B_y \Psi_y \equiv \frac{\partial}{\partial y} \left(\frac{1}{n^2} \frac{\partial}{\partial y} (n^2 \Psi_y) \right) + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_y.$$

$$C \Psi_y \equiv \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial y} (n^2 \Psi_y) \right) - \frac{\partial^2 \Psi_y}{\partial x \partial y};$$

$$D \Psi_x \equiv \frac{\partial}{\partial y} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 \Psi_x) \right) - \frac{\partial^2 \Psi_x}{\partial y \partial x}.$$

三维			
半矢量	TM (E_x)		
	TE (E_y)		

三维半矢量 $C=0, D=0$

$$2in_0k_0\frac{\partial\Psi_x}{\partial z} = (A_x + A_y)\Psi_x; \quad \longrightarrow \quad \text{准TM}$$

$$2in_0k_0\frac{\partial\Psi_y}{\partial z} = (B_x + B_y)\Psi_y. \quad \longrightarrow \quad \text{准TE}$$

准TM半矢量(Ex)公式CN差分

$$\begin{aligned} 2in_0k_0 \frac{u^{m+1} - u^m}{dz} &= (1 - \alpha)(A_x + A_y)^m u^m + \alpha(A_x + A_y)^{m+1} u^{m+1} \\ \left(\frac{2in_0k_0}{dz} - \alpha(A_x + A_y)^{m+1} \right) u^{m+1} &= \left(\frac{2in_0k_0}{dz} + (1 - \alpha)(A_x + A_y)^m \right) u^m \\ \left(1 - \frac{\alpha dz}{2in_0k_0} (A_x + A_y)^{m+1} \right) u^{m+1} &= \left(1 + \frac{(1 - \alpha) dz}{2in_0k_0} (A_x + A_y)^m \right) u^m \\ u^{m+1} &= \frac{\left(1 + \frac{(1 - \alpha) dz}{2in_0k_0} (A_x)^m \right) \left(1 + \frac{(1 - \alpha) dz}{2in_0k_0} (A_y)^m \right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} (A_x)^{m+1} \right) \left(1 - \frac{\alpha dz}{2in_0k_0} (A_y)^{m+1} \right)} u^m \end{aligned}$$

$$A_x \Psi_x \equiv \frac{\partial}{\partial x} \left(\frac{1}{n^2} \frac{\partial}{\partial x} (n^2 \Psi_x) \right) + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_x;$$

$$A_y \Psi_x \equiv \frac{\partial^2 \Psi_x}{\partial y^2} + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_x.$$

准TE半矢量(Ey)公式CN差分

$$v^{m+1} = \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0} (B_x)^m\right) \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} (B_y)^m\right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} (B_x)^{m+1}\right) \left(1 - \frac{\alpha dz}{2in_0k_0} (B_y)^{m+1}\right)} v^m$$

$$B_x \Psi_y \equiv \frac{\partial^2 \Psi_y}{\partial x^2} + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_y;$$

$$B_y \Psi_y \equiv \frac{\partial}{\partial y} \left(\frac{1}{n^2} \frac{\partial}{\partial y} (n^2 \Psi_y) \right) + \frac{1}{2} k_0^2 (n^2 - n_0^2) \Psi_y.$$

全矢量公式C!=0,D!=0

$$\begin{aligned}
 \frac{2in_0k_0}{dz} \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} - \begin{bmatrix} u \\ v \end{bmatrix}^m \right) &= (1-\alpha) \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^m \begin{bmatrix} u \\ v \end{bmatrix}^m + \alpha \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^{m+1} \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} \\
 \left(\frac{2in_0k_0}{dz} - \alpha \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^{m+1} \right) \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} &= \left(\frac{2in_0k_0}{dz} + (1-\alpha) \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^m \right) \begin{bmatrix} u \\ v \end{bmatrix}^m \\
 \left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^{m+1} \right) \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} &= \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x + A_y & C \\ D & B_x + B_y \end{bmatrix}^m \right) \begin{bmatrix} u \\ v \end{bmatrix}^m \\
 \left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1} \right) \left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1} \right) \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} \\
 &= \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m \right) \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m \right) \begin{bmatrix} u \\ v \end{bmatrix}^m \\
 \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} &= \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m \right) \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m \right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1} \right) \left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1} \right)} \begin{bmatrix} u \\ v \end{bmatrix}^m
 \end{aligned}$$

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right) \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right) \left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$$

第一步 $\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} = \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$

第二步 $\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right)}{\left(1 - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}}$

第一步走

$$\left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1} \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} \right) = \left(\begin{bmatrix} u \\ v \end{bmatrix}^m + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m \begin{bmatrix} u \\ v \end{bmatrix}^m \right)$$

$$\left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_y^{m+1} u^{m+\frac{1}{2}} \\ D^{m+1} u^{m+\frac{1}{2}} + B_y^{m+1} v^{m+\frac{1}{2}} \end{bmatrix} \right) = \left(\begin{bmatrix} u \\ v \end{bmatrix}^m + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_y^m u^m \\ D^m u^m + B_y^m v^m \end{bmatrix} \right)$$

$$\left(1 - \frac{\alpha dz}{2in_0k_0} [A_y^{m+1}] \right) [u]^{m+\frac{1}{2}} = \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} [A_y^m] \right) [u]^m$$

$$\left(1 - \frac{\alpha dz}{2in_0k_0} B_y^{m+1} \right) [v]^{m+\frac{1}{2}} - \frac{\alpha dz}{2in_0k_0} [D^{m+1} u^{m+\frac{1}{2}}] = \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} B_y^m \right) [v]^m + \frac{(1-\alpha)dz}{2in_0k_0} [D^m u^m]$$

第二步走

$$\begin{aligned} \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1} \begin{bmatrix} u \\ v \end{bmatrix}^{m+1} \right) &= \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} \right) \\ \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} - \frac{\alpha dz}{2in_0k_0} \begin{bmatrix} A_x^{m+1}u^{m+1} + C^{m+1}v^{m+1} \\ B_x^{m+1}v^{m+1} \end{bmatrix} \right) &= \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{(1-\alpha)dz}{2in_0k_0} \begin{bmatrix} A_x^m u^{m+\frac{1}{2}} + C^m v^{m+\frac{1}{2}} \\ B_x^m v^{m+\frac{1}{2}} \end{bmatrix} \right) \\ \left(1 - \frac{\alpha dz}{2in_0k_0} A_x^{m+1} \right) u^{m+1} - \frac{\alpha dz}{2in_0k_0} C^{m+1} v^{m+1} &= \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} A_x^m \right) u^{m+\frac{1}{2}} + \frac{(1-\alpha)dz}{2in_0k_0} C^m v^{m+\frac{1}{2}} \\ \left(1 - \frac{\alpha dz}{2in_0k_0} B_x^{m+1} \right) v^{m+1} &= \left(1 + \frac{(1-\alpha)dz}{2in_0k_0} B_x^m \right) v^{m+\frac{1}{2}} \end{aligned}$$

广角BPM

$$2in_0k_0\frac{\partial}{\partial z}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}=\begin{bmatrix}P_{xx}&P_{xy}\\ P_{yx}&P_{yy}\end{bmatrix}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}.$$

旁轴近似

$$\frac{\partial^2}{\partial z^2}\Psi-2in_0k_0\frac{\partial}{\partial z}\Psi=-P\Psi$$

非缓慢包络
近似

$$D^2\Psi-2in_0k_0D\Psi+P\Psi=0$$

$$D=\frac{\left(2in_0k_0\pm\sqrt{(-4n_0^2k_0^2-4P)}\right)}{2}=in_0k_0\pm i\sqrt{P+n_0^2k_0^2}$$

$$\begin{aligned}
\mathcal{P} &= \frac{P}{n_0^2 k_0^2} \\
\mathcal{K} &= n_0 k_0 \\
D &= -i\mathcal{K}(\pm\sqrt{\mathcal{P}+1}-1) \\
D &= -i\mathcal{K}(\sqrt{\mathcal{P}+1}-1)
\end{aligned}
\longrightarrow \text{单向波}$$

$$\begin{aligned}
\sqrt{1+P_s}-1 &\approx \frac{M_m(\mathcal{P})}{N_n(\mathcal{P})} \\
\left.\frac{\partial}{\partial z}\right|_n \Psi &= -i\mathcal{K} \frac{M_m(\mathcal{P})}{N_n(\mathcal{P})} \Psi
\end{aligned}$$

Pade近似

$$\left. \frac{\partial}{\partial z} \right|_n = -i\mathcal{K} \frac{\frac{\mathcal{P}}{2}}{1 + \frac{i}{2\mathcal{K}} \left. \frac{\partial}{\partial z} \right|_{n-1}}$$

基于该式
得到多项
式的系数

基于CN差分的Pade公式

$$\frac{\psi^{m+1} - \psi^m}{dz} = -i\mathcal{K}\alpha \left[\frac{M}{N} \right]^{m+1} \psi^{m+1} - i\mathcal{K}(1 - \alpha) \left[\frac{M}{N} \right]^m \psi^m$$

$$\psi^{m+1} = \left(\frac{N - i\mathcal{K}dz(1 - \alpha)M}{N + i\mathcal{K}dz\alpha M} \right) \psi^m$$

$$\psi^{m+1} \approx \left(\frac{N^m - i\mathcal{K}dz(1 - \alpha)M^m}{N^{m+1} + i\mathcal{K}dz\alpha M^{m+1}} \right) \psi^m$$

$$\psi^{m+1} \approx \frac{\Pi(1 + a_n \mathcal{P}^m)}{\Pi(1 + b_n \mathcal{P}^{m+1})} \psi^m$$

书本

本方法交叉

N步走

Pade每一步详细

$$\Psi^{n+1} = \frac{1 + \frac{a_n}{n_0 k_0} p^m}{1 + \frac{b_n}{n_0 k_0} p^{m+1}} \Psi^n$$

CN差分

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right) \left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right) \left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$$

CN差分第一步

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} = \frac{\left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$$

$$\left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_y^{m+1} u^{m+\frac{1}{2}} \\ D^{m+1} u^{m+\frac{1}{2}} + B_y^{m+1} v^{m+\frac{1}{2}} \end{bmatrix} \right) = \left(\begin{bmatrix} u \\ v \end{bmatrix}^m + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_y^m u^m \\ D^m u^m + B_y^m v^m \end{bmatrix} \right)$$

$$\left(1 + \frac{b_n}{n_0 k_0} A_y^{m+1}\right) u^{m+\frac{1}{2}} = \left(1 + \frac{a_n}{n_0 k_0} A_y^m\right) u^m$$

$$\left(1 + \frac{b_n}{n_0 k_0} B_y^{m+1}\right) v^{m+\frac{1}{2}} + \frac{b_n}{n_0 k_0} D^{m+1} u^{m+\frac{1}{2}} = \left(1 + \frac{a_n}{n_0 k_0} B_y^m\right) v^m + \frac{a_n}{n_0 k_0} D^m u^m$$

CN差分第二步

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right)}{\left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}}$$

$$\begin{aligned} \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_x^{m+1} u^{m+1} + C^{m+1} v^{m+1} \\ B_x^{m+1} v^{m+1} \end{bmatrix} \right) &= \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_x^m u^{m+\frac{1}{2}} + C^m v^{m+\frac{1}{2}} \\ B_x^m v^{m+\frac{1}{2}} \end{bmatrix} \right) \\ \left(1 + \frac{b_n}{n_0 k_0} A_x^{m+1}\right) u^{m+1} + \frac{b_n}{n_0 k_0} C^{m+1} v^{m+1} &= \left(1 + \frac{a_n}{n_0 k_0} A_x^m\right) u^{m+\frac{1}{2}} + \frac{a_n}{n_0 k_0} C^m v^{m+\frac{1}{2}} \\ \left(1 + \frac{b_n}{n_0 k_0} B_x^m\right) v^{m+1} &= \left(1 + \frac{a_n}{n_0 k_0} B_x^m\right) v^{m+\frac{1}{2}} \end{aligned}$$