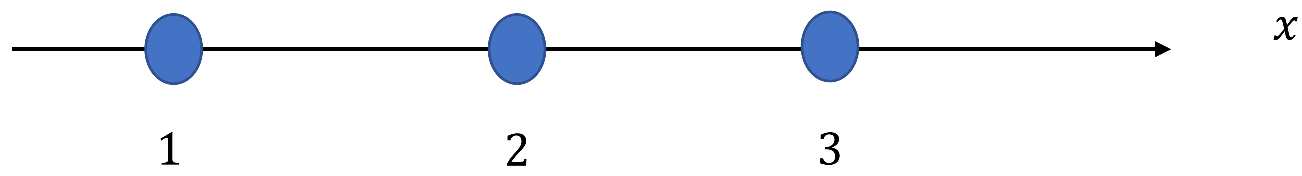
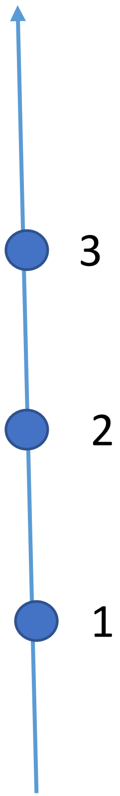


$$p\frac{\partial}{x}\left(\frac{1}{q}\frac{\partial}{x}(r\mathrm{m})\right)$$

$$p2\frac{1}{dx}\left[\frac{2}{q2+q3}\frac{r3m3-r2m2}{dx}-\frac{2}{q1+q2}\frac{r2m2-r1m1}{dx}\right]$$

$$\frac{2}{dx^2}\left[\frac{1}{q1+q2}p2r1m1-\left(\frac{1}{q1+q2}+\frac{1}{q2+q3}\right)p2r2m2+\frac{1}{q2+q3}p2r3m3\right]$$



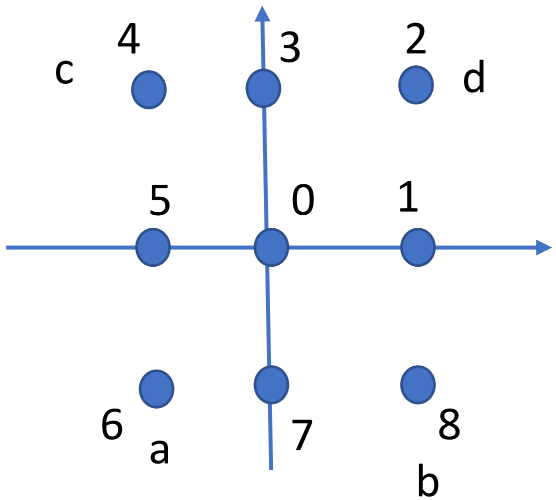
$$p\frac{\partial}{y}\left(\frac{1}{q}\frac{\partial}{y}\,(rm)\right)$$

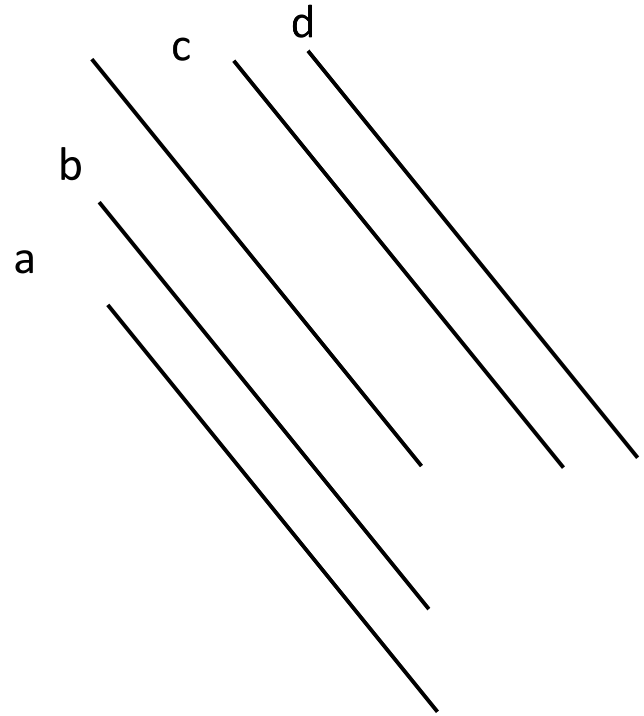
核心

$$\frac{\partial}{y}\frac{1}{q}\frac{\partial}{y}\Phi$$

$$\frac{p2}{dy}\left[\frac{2}{q2+q3}\frac{r3m3-r2m2}{dy}-\frac{2}{q1+q2}\frac{r2m2-r1m1}{dy}\right]$$

$$\frac{2}{dy^2}\left[\frac{1}{q1+q2}p2r1m1-\left(\frac{1}{q1+q2}+\frac{1}{q2+q3}\right)p2r2m2+\frac{1}{q2+q3}p2r3m\right]$$

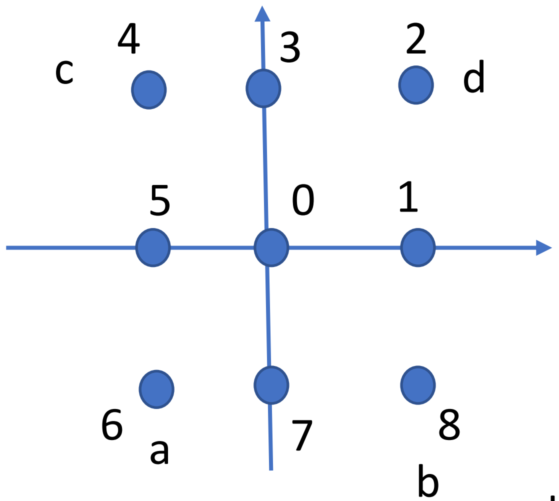


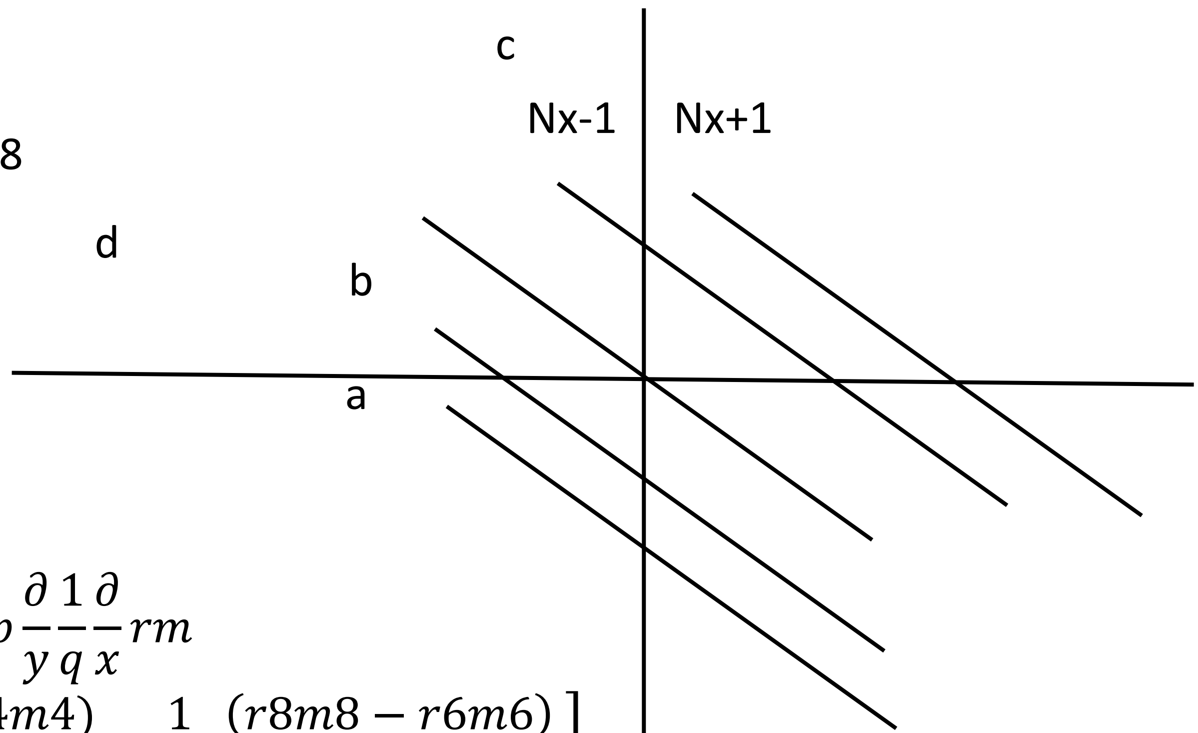


$$p\frac{\partial}{x}\left(\frac{1}{q}\frac{\partial}{y}rm\right)$$

$$\frac{p0}{2dx}\left[\frac{1}{q1}\frac{r2m2-r8m8}{2dy}-\frac{1}{q5}\frac{r4m4-r6m6}{2dy}\right]$$

$$\frac{1}{4dxdy}\left[\frac{1}{q5}p0r6m6\ -\frac{1}{q1}p0r8m8-\frac{1}{q5}p0r4m4+\frac{1}{q1}p0r2m2\ \right]$$





$$p\frac{\partial}{y}\frac{1}{q}\frac{\partial}{x}rm$$

$$\frac{p0}{2dy}\left[\frac{1}{q3}\frac{(r2m2-r4m4)}{2dx}-\frac{1}{q7}\frac{(r8m8-r6m6)}{2dx}\right]$$

$$\frac{1}{4dxdy}\left[\frac{1}{q7}p0r6m6\ -\frac{1}{q7}p0r8m8-\frac{1}{q3}p0r4m4+\frac{1}{q3}p0r2m2\right]$$

$$P_{xx}\Psi_x \equiv \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\left(n^2\Psi_x\right)\right)+\frac{\partial^2\Psi_x}{\partial y^2}+k_0^2\left(n^2-n_0^2\right)\Psi_x;$$

$$P_{yy}\Psi_y \equiv \frac{\partial^2\Psi_y}{\partial x^2}+\frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\left(n^2\Psi_y\right)\right)+k_0^2\left(n^2-n_0^2\right)\Psi_y;$$

$$P_{xy}\Psi_y \equiv \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\left(n^2\Psi_y\right)\right)-\frac{\partial^2\Psi_y}{\partial x\partial y};$$

$$P_{yx}\Psi_x \equiv \frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\left(n^2\Psi_x\right)\right)-\frac{\partial^2\Psi_x}{\partial y\partial x}.$$

$$2in_0k_0\frac{\partial}{\partial z}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}=\begin{bmatrix}P_{xx} & P_{xy}\\ P_{yx} & P_{yy}\end{bmatrix}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}. \qquad \begin{bmatrix}P_{xx} & P_{xy}\\ P_{yx} & P_{yy}\end{bmatrix}=\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}.$$

$$A_x\Psi_x \equiv \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\left(n^2\Psi_x\right)\right) + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_x;$$

$$A_y\Psi_x \equiv \frac{\partial^2\Psi_x}{\partial y^2} + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_x.$$

$$B_x\Psi_y \equiv \frac{\partial^2\Psi_y}{\partial x^2} + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_y;$$

$$B_y\Psi_y \equiv \frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\left(n^2\Psi_y\right)\right) + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_y.$$

$$C\Psi_y \equiv \frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\left(n^2\Psi_y\right)\right) - \frac{\partial^2\Psi_y}{\partial x\partial y};$$

$$D\Psi_x \equiv \frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\left(n^2\Psi_x\right)\right) - \frac{\partial^2\Psi_x}{\partial y\partial x}.$$

三维			
半矢量	TM （Ex)		
	TE （Ey)		

三维半矢量 C=0,D=0

$$2in_0k_0\frac{\partial\Psi_x}{\partial z}=\left(A_x+A_y\right)\Psi_x; \longrightarrow \text{准TM}$$

$$2in_0k_0\frac{\partial\Psi_y}{\partial z}=\left(B_x+B_y\right)\Psi_y. \longrightarrow \text{准TE}$$

准TM半矢量(Ex)公式CN差分

$$2in_0k_0\frac{u^{m+1}-u^m}{dz}=(1-\alpha)\left(A_x+A_y\right)^m u^m+\alpha\left(A_x+A_y\right)^{m+1}u^{m+1}$$

$$\left(\frac{2in_0k_0}{dz}-\alpha\left(A_x+A_y\right)^{m+1}\right)u^{m+1}=\left(\frac{2in_0k_0}{dz}+(1-\alpha)\left(A_x+A_y\right)^m\right)u^m$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}\left(A_x+A_y\right)^{m+1}\right)u^{m+1}=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\left(A_x+A_y\right)^m\right)u^m$$

$$u^{m+1}=\frac{\left(1+\frac{(1-\alpha)dz}{2in_0k_0}(A_x)^m\right)\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\left(A_y\right)^m\right)}{\left(1-\frac{\alpha dz}{2in_0k_0}(A_x)^{m+1}\right)\left(1-\frac{\alpha dz}{2in_0k_0}\left(A_y\right)^{m+1}\right)}u^m$$

$$A_x\Psi_x\equiv\frac{\partial}{\partial x}\left(\frac{1}{n^2}\frac{\partial}{\partial x}\left(n^2\Psi_x\right)\right)+\frac{1}{2}k_0^2\left(n^2-n_0^2\right)\Psi_x;$$

$$A_y\Psi_x\equiv\frac{\partial^2\Psi_x}{\partial y^2}+\frac{1}{2}k_0^2\left(n^2-n_0^2\right)\Psi_x.$$

准TE半矢量(Ey)公式CN差分

$$v^{m+1} = \frac{\left(1 + \frac{(1-\alpha)dz}{2in_0k_0}(B_x)^m\right)\left(1 + \frac{(1-\alpha)dz}{2in_0k_0}\left(B_y\right)^m\right)}{\left(1 - \frac{\alpha dz}{2in_0k_0}(B_x)^{m+1}\right)\left(1 - \frac{\alpha dz}{2in_0k_0}\left(B_y\right)^{m+1}\right)} v^m$$

$$B_x\Psi_y \equiv \frac{\partial^2 \Psi_y}{\partial x^2} + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_y;$$

$$B_y\Psi_y \equiv \frac{\partial}{\partial y}\left(\frac{1}{n^2}\frac{\partial}{\partial y}\left(n^2\Psi_y\right)\right) + \frac{1}{2}k_0^2\left(n^2 - n_0^2\right)\Psi_y.$$

全矢量公式C!=0,D!=0

$$\frac{2in_0k_0}{dz}\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+1}-\begin{bmatrix}u\\v\end{bmatrix}^{m}\right)=(1-\alpha)\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m}\begin{bmatrix}u\\v\end{bmatrix}^{m}+\alpha\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m+1}\begin{bmatrix}u\\v\end{bmatrix}^{m+1}$$

$$\left(\frac{2in_0k_0}{dz}-\alpha\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m+1}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m+1}=\left(\frac{2in_0k_0}{dz}+(1-\alpha)\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m}$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m+1}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m+1}=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_x+A_y & C\\ D & B_x+B_y\end{bmatrix}^{m}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m}$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_x & C\\ 0 & B_x\end{bmatrix}^{m+1}\right)\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\ D & B_y\end{bmatrix}^{m+1}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m+1}$$

$$=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_x & C\\ 0 & B_x\end{bmatrix}^{m}\right)\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\ D & B_y\end{bmatrix}^{m}\right)\begin{bmatrix}u\\v\end{bmatrix}^{m}$$

$$\begin{bmatrix}u\\v\end{bmatrix}^{m+1}=\frac{\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_x & C\\ 0 & B_x\end{bmatrix}^{m}\right)\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\ D & B_y\end{bmatrix}^{m}\right)}{\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_x & C\\ 0 & B_x\end{bmatrix}^{m+1}\right)\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\ D & B_y\end{bmatrix}^{m+1}\right)}\begin{bmatrix}u\\v\end{bmatrix}^{m}$$

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m}\right)\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m}\right)}{\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m}$$

第一步 $$\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} = \frac{\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m}\right)}{\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m}$$

第二步 $$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m}\right)}{\left(1-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}}$$

第一步走

$$\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\D & B_y\end{bmatrix}^{m+1}\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}\right)=\left(\begin{bmatrix}u\\v\end{bmatrix}^{m}+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_y & 0\\D & B_y\end{bmatrix}^{m}\begin{bmatrix}u\\v\end{bmatrix}^{m}\right)$$

$$\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_y^{m+1}u^{m+\frac{1}{2}}\\D^{m+1}u^{m+\frac{1}{2}}+B_y^{m+1}v^{m+\frac{1}{2}}\end{bmatrix}\right)=\left(\begin{bmatrix}u\\v\end{bmatrix}^{m}+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_y^{m}u^{m}\\D^{m}u^{m}+B_y^{m}v^{m}\end{bmatrix}\right)$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}\left[A_y^{m+1}\right]\right)[u]^{m+\frac{1}{2}}=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}\left[A_y^{m}\right]\right)[u]^{m}$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}B_y^{m+1}\right)\ [v]^{m+\frac{1}{2}}-\frac{\alpha dz}{2in_0k_0}\left[D^{m+1}u^{m+\frac{1}{2}}\right]=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}B_y^{m}\right)[v]^{m}+\frac{(1-\alpha)dz}{2in_0k_0}[D^{m}u^{m}]$$

第二步走

$$\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+1}-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_x & C\\0 & B_x\end{bmatrix}^{m+1}\begin{bmatrix}u\\v\end{bmatrix}^{m+1}\right)=\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_x & C\\0 & B_x\end{bmatrix}^{m}\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}\right)$$

$$\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+1}-\frac{\alpha dz}{2in_0k_0}\begin{bmatrix}A_x^{m+1}u^{m+1}+C^{m+1}v^{m+1}\\B_x^{m+1}v^{m+1}\end{bmatrix}\right)=\left(\begin{bmatrix}u\\v\end{bmatrix}^{m+\frac{1}{2}}+\frac{(1-\alpha)dz}{2in_0k_0}\begin{bmatrix}A_x^{m}u^{m+\frac{1}{2}}+C^{m}v^{m+\frac{1}{2}}\\B_x^{m}v^{m+\frac{1}{2}}\end{bmatrix}\right)$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}A_x^{m+1}\right)u^{m+1}-\frac{\alpha dz}{2in_0k_0}C^{m+1}v^{m+1}=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}A_x^{m}\right)u^{m+\frac{1}{2}}+\frac{(1-\alpha)dz}{2in_0k_0}C^{m}v^{m+\frac{1}{2}}$$

$$\left(1-\frac{\alpha dz}{2in_0k_0}B_x^{m+1}\right)v^{m+1}=\left(1+\frac{(1-\alpha)dz}{2in_0k_0}B_x^{m}\right)v^{m+\frac{1}{2}}$$

广角BPM

$$2in_0k_0\frac{\partial}{\partial z}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}=\begin{bmatrix}P_{xx} & P_{xy}\\ P_{yx} & P_{yy}\end{bmatrix}\begin{bmatrix}\Psi_x\\ \Psi_y\end{bmatrix}.$$

旁轴近似

$$\frac{\partial^2}{\partial z^2}\Psi-2in_0k_0\frac{\partial}{\partial z}\Psi=-P\Psi$$

非缓慢包络近似

$$D^2\Psi-2in_0k_0D\Psi+P\Psi=0$$

$$D=\frac{\left(2in_0k_0\pm\sqrt{(-4n_0^2k_0^2-4P}\right)}{2}=in_0k_0\pm i\sqrt{P+n_0^2k_0^2}$$

$$\mathcal{P} = \frac{P}{n_0^2 k_0^2}$$

$$\mathcal{K} = n_0 k_0$$

$$D = -i\mathcal{K}\left(\pm\sqrt{\mathcal{P}+1} - 1\right)$$

$$D = -i\mathcal{K}\left(\sqrt{\mathcal{P}+1} - 1\right)$$

$\longrightarrow$ 单向波

$$\sqrt{1+P_s} - 1 \approx \frac{M_m(\mathcal{P})}{N_n(\mathcal{P})}$$

$$\left.\frac{\partial}{\partial z}\right|_n \Psi = -i\mathcal{K}\frac{M_m(\mathcal{P})}{N_n(\mathcal{P})}\Psi$$

Pade近似

$$\frac{\partial}{\partial z}\Big|_n = -i\mathcal{K}\frac{\frac{\mathcal{P}}{2}}{1+\frac{i}{2\mathcal{K}}\frac{\partial}{\partial_z}|_{n-1}}$$

基于该式
得到多项
式的系数

基于CN差分的Pade公式

$$\frac{\Psi^{m+1}-\Psi^{m}}{dz} = -i\mathcal{K}\alpha\left[\frac{M}{N}\right]^{m+1}\Psi^{m+1} - i\mathcal{K}(1-\alpha)\left[\frac{M}{N}\right]^{m}\Psi^{m}$$

$$\Psi^{m+1} = \left(\frac{N-i\mathcal{K}dz(1-\alpha)M}{N+i\mathcal{K}dz\alpha M}\right)\Psi^{m}$$

书本

$$\Psi^{m+1} \approx \left(\frac{N^{m}-i\mathcal{K}dz(1-\alpha)M^{m}}{N^{m+1}+i\mathcal{K}dz\alpha M^{m+1}}\right)\Psi^{m}$$

本方法交叉

$$\Psi^{m+1} \approx \frac{\Pi(1+a_n\mathcal{P}^{m})}{\Pi(1+b_n\mathcal{P}^{m+1})}\Psi^{m}$$

N步走

Pade order	M	$\boldsymbol{N}$
1,0	$\frac{\mathcal{P}}{2}$	1
1,1	$\frac{\mathcal{P}}{2}$	$1+\frac{\mathcal{P}}{4}$
2,2	$\frac{\mathcal{P}}{2}+\frac{\mathcal{P}^2}{4}$	$1+\frac{3\mathcal{P}}{4}+\frac{\mathcal{P}^2}{16}$
3,3	$\frac{\mathcal{P}}{2}+\frac{\mathcal{P}^2}{2}+\frac{3\mathcal{P}^3}{32}$	$1+\frac{5\mathcal{P}}{4}+\frac{3\mathcal{P}^2}{8}+\frac{\mathcal{P}^3}{64}$
4,4	$\frac{\mathcal{P}}{2}+\frac{3\mathcal{P}^2}{4}+\frac{5\mathcal{P}^3}{16}+\frac{\mathcal{P}^4}{32}$	$1+\frac{7\mathcal{P}}{4}+\frac{15\mathcal{P}^2}{16}+\frac{5\mathcal{P}^3}{4}+\frac{\mathcal{P}^4}{256}$

Pade每一步详细

$$\Psi^{n+1} = \frac{1 + \frac{a_n}{n_0^2 k_0^2} P^m}{1 + \frac{b_n}{n_0^2 k_0^2} P^{m+1}} \Psi^n$$

CN差分

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right)\left(1 + \frac{a_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)\left(1 + \frac{b_n}{n_0 k_0} \begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$$

CN差分第一步

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} = \frac{\left(1+\frac{a_n}{n_0 k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^m\right)}{\left(1+\frac{b_n}{n_0 k_0}\begin{bmatrix} A_y & 0 \\ D & B_y \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^m$$

$$\left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{b_n}{n_0 k_0}\begin{bmatrix} A_y^{m+1} u^{m+\frac{1}{2}} \\ D^{m+1} u^{m+\frac{1}{2}} + B_y^{m+1} v^{m+\frac{1}{2}} \end{bmatrix}\right) = \left(\begin{bmatrix} u \\ v \end{bmatrix}^m + \frac{a_n}{n_0 k_0}\begin{bmatrix} A_y^m u^m \\ D^m u^m + B_y^m v^m \end{bmatrix}\right)$$

$$\left(1+\frac{b_n}{n_0 k_0} A_y^{m+1}\right) u^{m+\frac{1}{2}} = \left(1+\frac{a_n}{n_0 k_0} A_y^m\right) u^m$$

$$\left(1+\frac{b_n}{n_0 k_0} B_y^{m+1}\right) v^{m+\frac{1}{2}} + \frac{b_n}{n_0 k_0} D^{m+1} u^{m+\frac{1}{2}} = \left(1+\frac{a_n}{n_0 k_0} B_y^m\right) v^m + \frac{a_n}{n_0 k_0} D^m u^m$$

CN差分第二步

$$\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} = \frac{\left(1 + \frac{a_n}{n_0 k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^m\right)}{\left(1 + \frac{b_n}{n_0 k_0}\begin{bmatrix} A_x & C \\ 0 & B_x \end{bmatrix}^{m+1}\right)} \begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}}$$

$$\left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+1} + \frac{b_n}{n_0 k_0}\begin{bmatrix} A_x^{m+1} u^{m+1} + C^{m+1} v^{m+1} \\ B_x^{m+1} v^{m+1} \end{bmatrix}\right) = \left(\begin{bmatrix} u \\ v \end{bmatrix}^{m+\frac{1}{2}} + \frac{a_n}{n_0 k_0}\begin{bmatrix} A_x^m u^{m+\frac{1}{2}} + C^m v^{m+\frac{1}{2}} \\ B_x^m v^{m+\frac{1}{2}} \end{bmatrix}\right)$$

$$\left(1 + \frac{b_n}{n_0 k_0} A_x^{m+1}\right) u^{m+1} + \frac{b_n}{n_0 k_0} C^{m+1} v^{m+1} = \left(1 + \frac{a_n}{n_0 k_0} A_x^m\right) u^{m+\frac{1}{2}} + \frac{a_n}{n_0 k_0} C^m v^{m+\frac{1}{2}}$$

$$\left(1 + \frac{b_n}{n_0 k_0} B_x^m\right) v^{m+1} = \left(1 + \frac{a_n}{n_0 k_0} B_x^m\right) v^{m+\frac{1}{2}}$$