

Normalization

$$- \int \text{Re}(\vec{E}_i \times \vec{H}_i) \cdot d\vec{S} = 1$$

Overlap integrals

$$- \int \vec{E}_n^L \times \vec{H}_m^R \cdot d\vec{S}$$

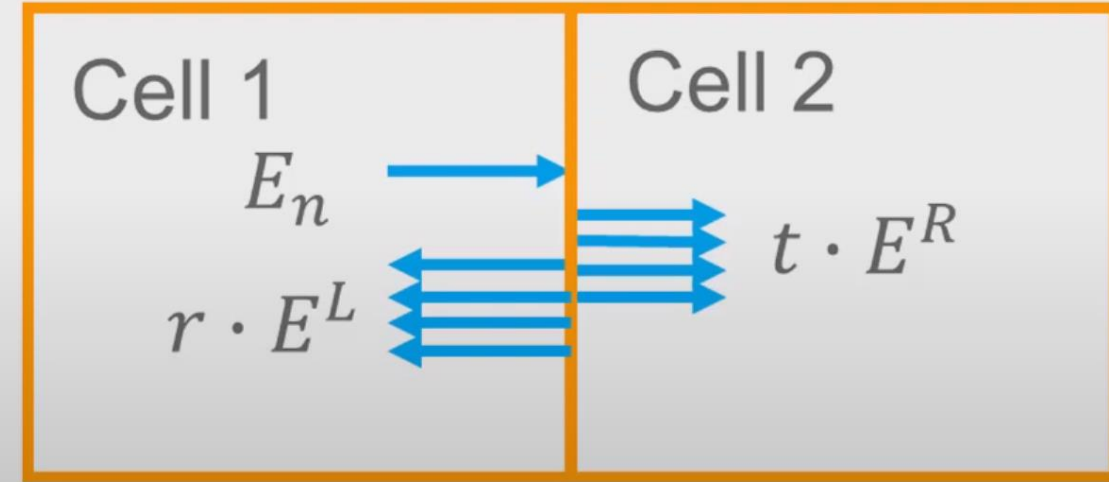
$$\vec{E}_{incident}^{\parallel} + \vec{E}_{total R}^{\parallel} = \vec{E}_{total T}^{\parallel}$$

$$\int \int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$

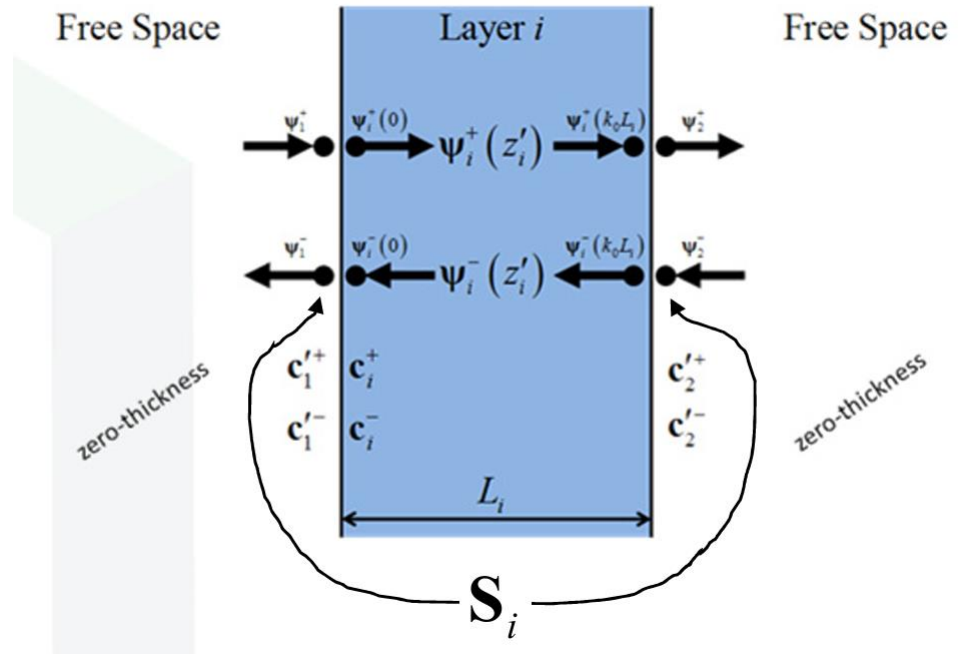
$$\vec{E}_n^{\parallel L} + \sum_{m=1}^N r_{mn} \vec{E}_m^{\parallel L} = \sum_{k=1}^M t_{kn} \vec{E}_k^{\parallel R}$$

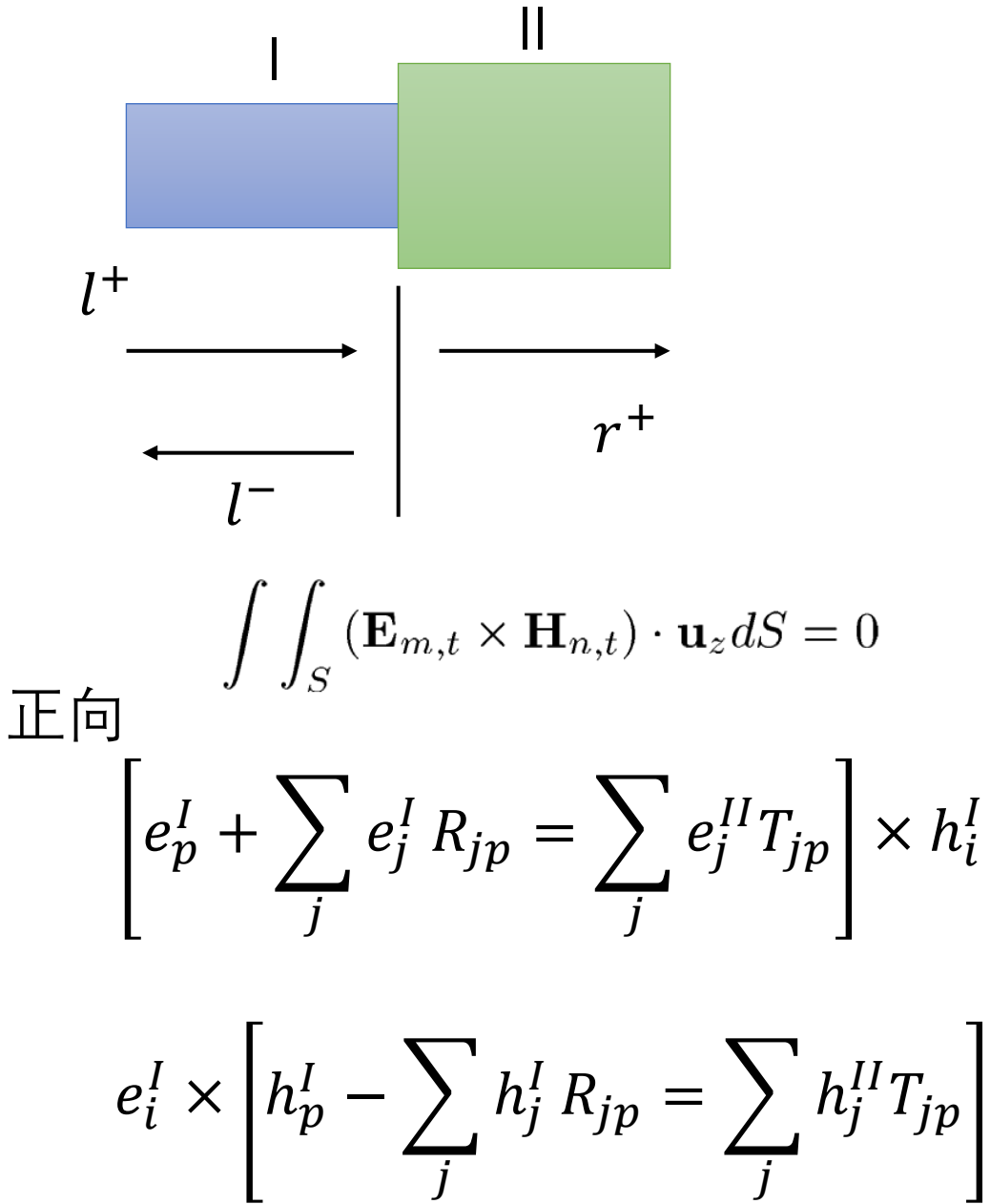
- If cell 1 has M modes, and cell 2 has P modes then S is NxN matrix where $N=M+P$

$$S = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix}$$



$$\begin{bmatrix} \mathbf{c}'_1{}^- \\ \mathbf{c}'_2{}^+ \end{bmatrix} = \mathbf{S}^{(i)} \begin{bmatrix} \mathbf{c}'_1{}^+ \\ \mathbf{c}'_2{}^- \end{bmatrix}$$





I II

l^+

l^-

r^+

$$\int \int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$

正向

$$\left[e_p^I + \sum_j e_j^I R_{jp} = \sum_j e_j^{II} T_{jp} \right] \times h_i^I$$

$$e_i^I \times \left[h_p^I - \sum_j h_j^I R_{jp} = \sum_j h_j^{II} T_{jp} \right]$$

$$\langle \mathbf{E}_m, \mathbf{H}_n \rangle \equiv \int \int_s (\mathbf{E}_m \times \mathbf{H}_n) \cdot \mathbf{u}_z dS$$

$$e_p^I \times h_i^I + \sum_j e_j^I \times h_i^I R_{jp} = \sum_j e_j^{II} \times h_i^I T_{jp}$$

$$e_i^I \times h_p^I - \sum_j e_i^I \times h_j^I R_{jp} = \sum_j e_i^I \times h_j^{II} T_{jp}$$

$$\delta_{ip} e_p^I \times h_p^I + e_i^I \times h_i^I R_{ip} = \sum_j e_j^{II} \times h_i^I T_{jp}$$

$$\delta_{ip} e_p^I \times h_p^I - e_i^I \times h_i^I R_{ip} = \sum_j e_i^I \times h_j^{II} T_{jp}$$

$$O_{I,II} = e_i^I \times h_j^{II}$$

$$O_{II,I} = e_i^{II} \times h_j^I$$

$$I + R_{I,II} = O_{II,I}^T T_{I,II}$$

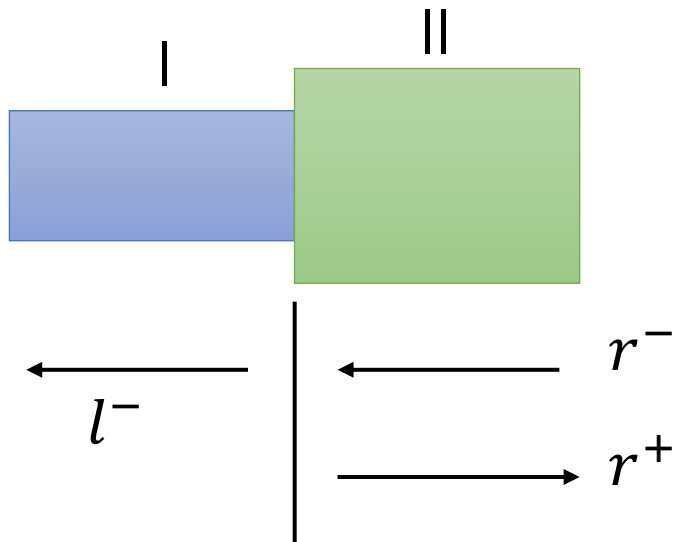
$$I - R_{I,II} = O_{I,II} T_{I,II}$$

$$T_{I,II} = 2I(O_{II,I}^T + O_{I,II})^{-1}$$

$$R_{I,II} = \frac{1}{2}(O_{II,I}^T - O_{I,II})T_{I,II}$$

验证x轴传播

$$\begin{aligned} \iint (e_i \times h_j)_x dS &= \delta_{ij} \\ &= \iint e_y h_z - e_z h_y dS \end{aligned}$$



反向

$$\begin{aligned} O_{I,II}^T T_{II,I} &= I + R_{II,I} \\ -O_{II,I} T_{II,I} &= -I + R_{II,I} \end{aligned}$$

$$\begin{aligned} T_{II,I} &= 2I(O_{I,II}^T + O_{II,I})^{-1} \\ R_{II,I} &= \frac{1}{2}(O_{I,II}^T - O_{II,I})T_{II,I} \end{aligned}$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} R_{I,II} & T_{II,I} \\ T_{I,II} & R_{II,I} \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

$$\begin{aligned} &\left[\sum_j e_j^I T_{jp} = e_p^{II} + \sum_j e_j^{II} R_{jp} \right] \times h_i^{II} \\ e_i^{II} \times &\left[-\sum_j h_j^I T_{jp} = -h_p^{II} + \sum_j h_j^{II} R_{jp} \right] \end{aligned}$$

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} & \mathbf{S}_{12}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} & \mathbf{S}_{22}^{(\text{global})} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \rightarrow \begin{aligned} \mathbf{c}_{\text{ref}} &= \mathbf{S}_{11}^{(\text{global})} \mathbf{c}_{\text{inc}} \\ \mathbf{c}_{\text{trn}} &= \mathbf{S}_{21}^{(\text{global})} \mathbf{c}_{\text{inc}} \end{aligned}$$

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} & \mathbf{S}_{12}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} & \mathbf{S}_{22}^{(\text{global})} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} \end{bmatrix} \mathbf{c}_{\text{inc}}$$



$$r^+ = e^{-j[\beta]z} l^+$$

$$l^- = e^{j[\beta]z} r^-$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} R_{I,II} & T_{II,I} \\ T_{I,II} & R_{II,I} \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

$$\begin{bmatrix} r^+ \\ r^- \end{bmatrix} = \begin{bmatrix} e^{-j[\beta]z} & 0 \\ 0 & e^{+j[\beta]z} \end{bmatrix} \begin{bmatrix} l^+ \\ l^- \end{bmatrix}$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} 0 & e^{j[\beta]z} \\ e^{-j[\beta]z} & 0 \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

传输矩阵

$$\begin{bmatrix} r^+ \\ r^- \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} l^+ \\ l^- \end{bmatrix}$$
$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

$$\begin{aligned} S_{11} &= -T_{22}^{-1}T_{21} \\ S_{12} &= T_{22}^{-1} \\ S_{21} &= T_{11} - T_{12}T_{22}^{-1}T_{21} \\ S_{22} &= T_{12}T_{22}^{-1} \end{aligned}$$

$$\underline{\mathbf{S}^{(\text{global})} = \mathbf{S}^{(\text{global})} \otimes \mathbf{S}^{(i)}}$$

$$\mathbf{D} = \mathbf{S}_{12}^{(\text{global})} \left[\mathbf{I} - \mathbf{S}_{11}^{(i)} \mathbf{S}_{22}^{(\text{global})} \right]^{-1}$$

$$\mathbf{F} = \mathbf{S}_{21}^{(i)} \left[\mathbf{I} - \mathbf{S}_{22}^{(\text{global})} \mathbf{S}_{11}^{(i)} \right]^{-1}$$

$$\mathbf{S}_{11}^{(\text{global})} = \mathbf{S}_{11}^{(\text{global})} + \mathbf{D} \mathbf{S}_{11}^{(i)} \mathbf{S}_{21}^{(\text{global})}$$

$$\mathbf{S}_{12}^{(\text{global})} = \mathbf{D} \mathbf{S}_{12}^{(i)}$$

$$\mathbf{S}_{21}^{(\text{global})} = \mathbf{F} \mathbf{S}_{21}^{(\text{global})}$$

$$\mathbf{S}_{22}^{(\text{global})} = \mathbf{S}_{22}^{(i)} + \mathbf{F} \mathbf{S}_{22}^{(\text{global})} \mathbf{S}_{12}^{(i)}$$

standard order
↓

$$\underline{\mathbf{S}^{(\text{global})} = \mathbf{S}^{(i)} \otimes \mathbf{S}^{(\text{global})}}$$

$$\mathbf{D} = \mathbf{S}_{12}^{(i)} \left[\mathbf{I} - \mathbf{S}_{11}^{(\text{global})} \mathbf{S}_{22}^{(i)} \right]^{-1}$$

$$\mathbf{F} = \mathbf{S}_{21}^{(\text{global})} \left[\mathbf{I} - \mathbf{S}_{22}^{(i)} \mathbf{S}_{11}^{(\text{global})} \right]^{-1}$$

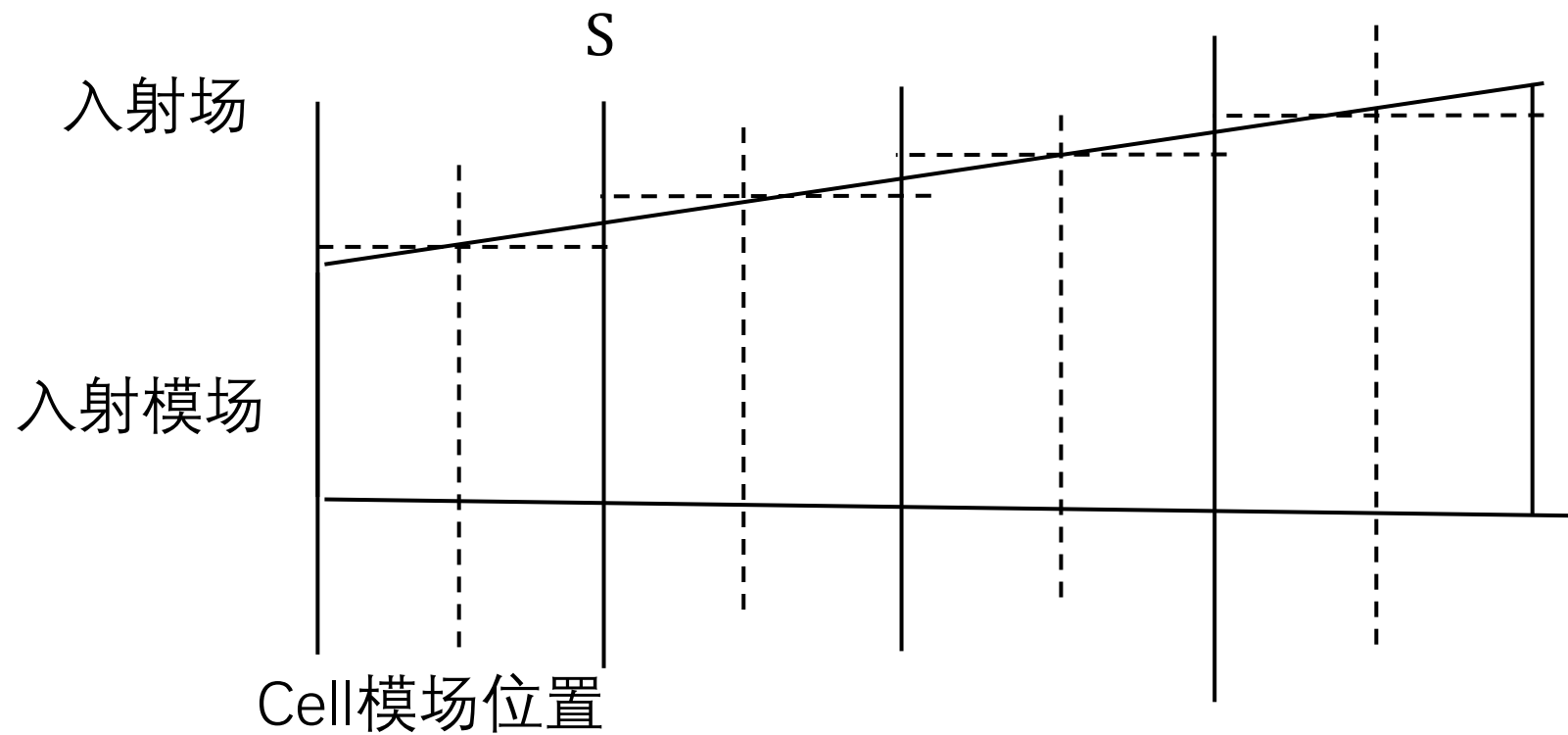
$$\mathbf{S}_{22}^{(\text{global})} = \mathbf{S}_{22}^{(\text{global})} + \mathbf{F} \mathbf{S}_{22}^{(i)} \mathbf{S}_{12}^{(\text{global})}$$

$$\mathbf{S}_{21}^{(\text{global})} = \mathbf{F} \mathbf{S}_{21}^{(i)}$$

$$\mathbf{S}_{12}^{(\text{global})} = \mathbf{D} \mathbf{S}_{12}^{(\text{global})}$$

$$\mathbf{S}_{11}^{(\text{global})} = \mathbf{S}_{11}^{(i)} + \mathbf{D} \mathbf{S}_{11}^{(\text{global})} \mathbf{S}_{21}^{(i)}$$

reverse order
↑



入射S矩
阵