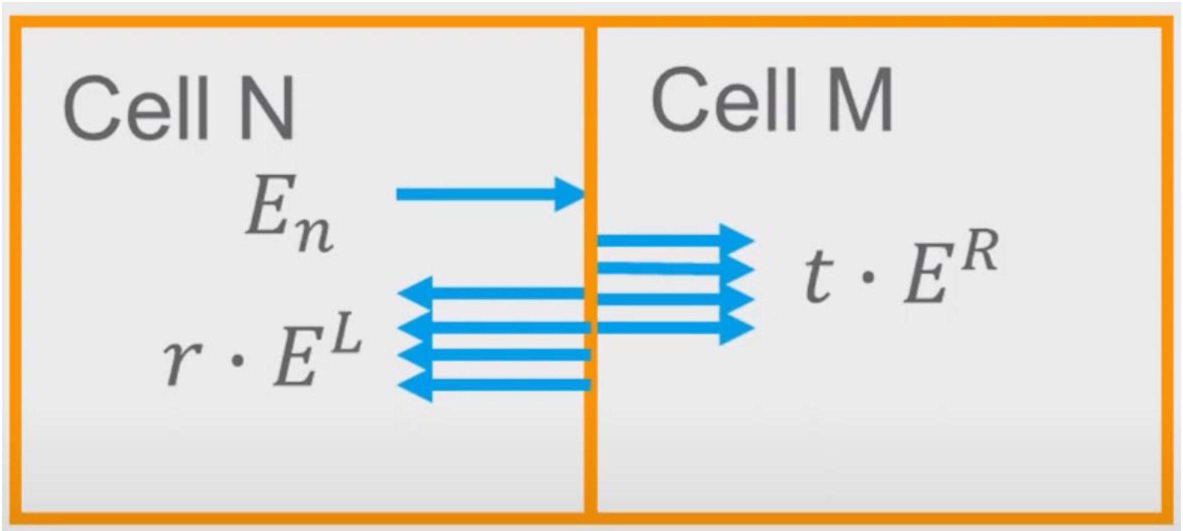


Normalization

$$- \int Re(\overrightarrow{E_i} \times \overrightarrow{H_i}) \cdot d\vec{S} = 1$$

Overlap integrals

$$- \int \overrightarrow{E_n^L} \times \overrightarrow{H_m^R} \cdot d\vec{S}$$

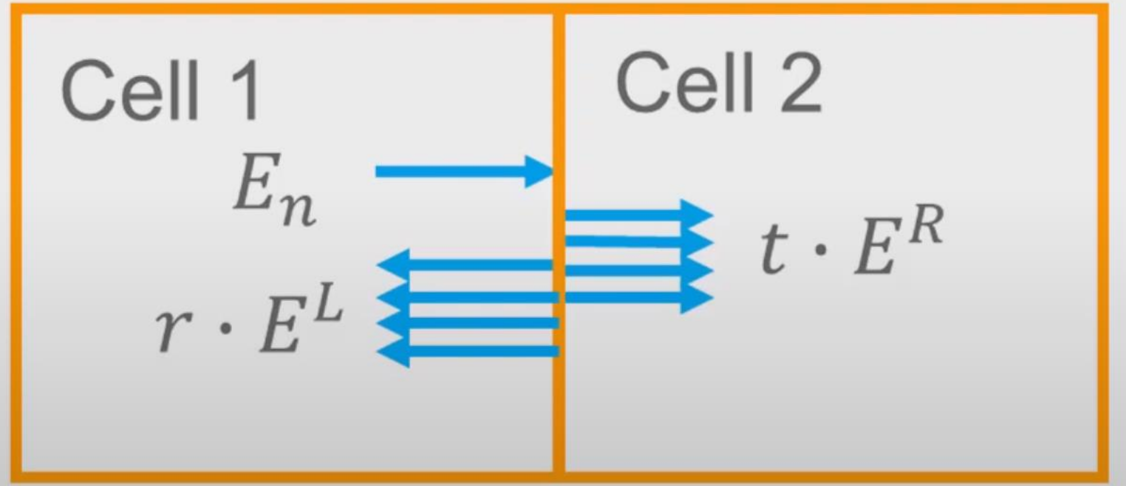
$$\vec{E}^{\parallel}_{incident} + \vec{E}^{\parallel}_{total\,R} = \vec{E}^{\parallel}_{total\,T}$$

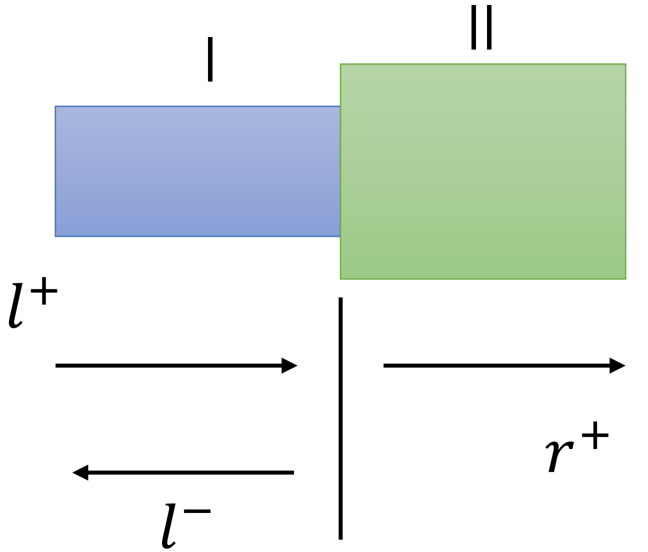
$$\vec{E}_n^{\parallel L} + \sum_{m=1}^{N} r_{mn} \vec{E}_m^{\parallel L} = \sum_{k=1}^{M} t_{kn} \vec{E}_k^{\parallel R}$$

$$\int \int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$

- If cell 1 has M modes, and cell 2 has P modes then S is NxN matrix where N=M+P

$$S = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix}$$





$$\int\int_S (\mathbf{E}_{m,t} \times \mathbf{H}_{n,t}) \cdot \mathbf{u}_z dS = 0$$

正向

$$\left[e_p^I + \sum_j e_j^I R_{jp} = \sum_j e_j^{II} T_{jp}\right] \times h_i^I$$

$$e_i^I \times \left[h_p^I - \sum_j h_j^I R_{jp} = \sum_j h_j^{II} T_{jp}\right]$$

$$\langle \mathbf{E}_m, \mathbf{H}_n \rangle \equiv \int\int_s (\mathbf{E}_m \times \mathbf{H}_n) \cdot \mathbf{u}_z dS$$

$$e_p^I \times h_i^I + \sum_j e_j^I \times h_i^I R_{jp} = \sum_j e_j^{II} \times h_i^I T_{jp}$$

$$e_i^I \times h_p^I - \sum_j e_i^I \times h_j^I R_{jp} = \sum_j e_i^I \times h_j^{II} T_{jp}$$

$$\delta_{ip} e_p^I \times h_p^I + e_i^I \times h_i^I R_{ip} = \sum_j e_j^{II} \times h_i^I T_{jp}$$

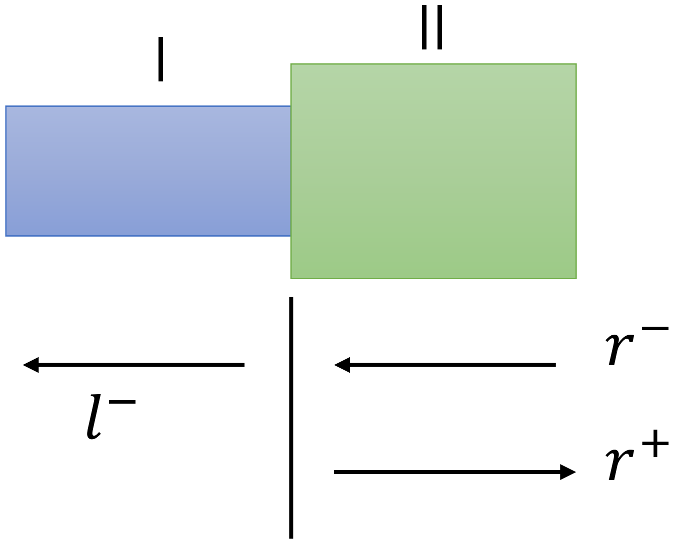
$$\delta_{ip} e_p^I \times h_p^I - e_i^I \times h_i^I R_{ip} = \sum_j e_i^I \times h_j^{II} T_{jp}$$

$$O_{I,II} = e_i^{I} \times h_j^{II}$$
$$O_{II,I} = e_i^{II} \times h_j^{I}$$

$$I + R_{I,II} = O_{II,I}^{T} T_{I,II}$$
$$I - R_{I,II} = O_{I,II} T_{I,II}$$
$$T_{I,II} = 2I\left(O_{I,II} + O_{II,I}^{T}\right)^{-1}$$
$$R_{I,II} = \frac{1}{2}\left(O_{II,I}^{T} - O_{I,II}\right)T_{I,II}$$

验证X轴传播

$$\int\int \left(e_i \times h_j\right)_x dS = \delta_{ij}$$
$$= \int\int e_y\, h_z - e_z h_y dS$$



反向

$$O_{I,II}^T T_{II,I} = I + R_{II,I}$$
$$-O_{II,I} T_{II,I} = -I + R_{II,I}$$

$$T_{II,I} = 2I\left(O_{I,II}^T + O_{II,I}\right)^{-1}$$
$$R_{II,I} = \frac{1}{2}\left(O_{I,II}^T - O_{II,I}\right)T_{II,I}$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} R_{I,II} & T_{II,I} \\ T_{I,II} & R_{II,I} \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

$$\left[\sum_j e_j^I T_{jp} = e_p^{II} + \sum_j e_j^{II} R_{jp}\right] \times h_i^{II}$$
$$e_i^{II} \times \left[-\sum_j h_j^I\, T_{jp} = -h_p^{II} + \sum_j h_j^{II}\, R_{jp}\right]$$

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} & \mathbf{S}_{12}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} & \mathbf{S}_{22}^{(\text{global})} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} \mathbf{c}_{\text{ref}} &= \mathbf{S}_{11}^{(\text{global})} \mathbf{c}_{\text{inc}} \\ \mathbf{c}_{\text{trn}} &= \mathbf{S}_{21}^{(\text{global})} \mathbf{c}_{\text{inc}} \end{aligned}$$

$$\begin{bmatrix} \mathbf{c}_{\text{ref}} \\ \mathbf{c}_{\text{trn}} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{(\text{global})} & \mathbf{S}_{12}^{(\text{global})} \\ \mathbf{S}_{21}^{(\text{global})} & \mathbf{S}_{22}^{(\text{global})} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{\text{inc}} \\ \mathbf{0} \end{bmatrix} \quad \Rightarrow \quad \begin{aligned} \mathbf{c}_{\text{ref}} &= \mathbf{S}_{11}^{(\text{global})} \mathbf{c}_{\text{inc}} \\ \mathbf{c}_{\text{trn}} &= \mathbf{S}_{21}^{(\text{global})} \mathbf{c}_{\text{inc}} \end{aligned}$$

$$r^+ = e^{-j[\beta]z} l^+$$
$$l^- = e^{j[\beta]z} r^-$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} R_{I,II} & T_{II,I} \\ T_{I,II} & R_{II,I} \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$

$$\begin{bmatrix} l^- \\ r^+ \end{bmatrix} = \begin{bmatrix} 0 & e^{j[\beta]z} \\ e^{-j[\beta]z} & 0 \end{bmatrix} \begin{bmatrix} l^+ \\ r^- \end{bmatrix}$$