$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix}.$$

$$\frac{\partial E_z}{\partial z} = -\frac{1}{\varepsilon_{zz}} \nabla_t \cdot (\varepsilon_{tt} E_t)$$

$$\begin{aligned} &\nabla_t^2 E_t + \left(k_0^2 \varepsilon_{tt} - \beta^2\right) E_t = \nabla_t \left[ \nabla_t \cdot E_t - \frac{1}{\varepsilon_{zz}} \nabla_t \cdot (\varepsilon_{tt} E_t) \right] \\ &\nabla_t^2 E_t + k_0^2 \varepsilon_{tt} E_t - \nabla_t \left[ \nabla_t \cdot E_t - \frac{1}{\varepsilon_{zz}} \nabla_t \cdot (\varepsilon_{tt} E_t) \right] = \beta^2 E_t \end{aligned}$$

$$\begin{split} \nabla_t^2 E_t + k_0^2 \varepsilon_{tt} E_t - \nabla_t \left[ \nabla_t \cdot E_t - \frac{1}{\varepsilon_{zz}} \nabla_t \cdot (\varepsilon_{tt} E_t) \right] &= \beta^2 E_t \\ \nabla_t^2 \begin{pmatrix} E_x \\ E_y \end{pmatrix} + k_0^2 \begin{pmatrix} \varepsilon_{xx} E_x + \varepsilon_{xy} E_y \\ \varepsilon_{yx} E_x + \varepsilon_{yy} E_y \end{pmatrix} - \nabla_t \left[ \nabla_t \cdot \begin{pmatrix} E_x \\ E_y \end{pmatrix} - \frac{1}{\varepsilon_{zz}} \nabla_t \cdot \begin{pmatrix} \varepsilon_{xx} E_x + \varepsilon_{xy} E_y \\ \varepsilon_{yx} E_x + \varepsilon_{yy} E_y \end{pmatrix} \right] &= \beta^2 \begin{pmatrix} E_x \\ E_y \end{pmatrix} \end{split}$$

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + k_0^2 (\epsilon_{xx} E_x + \epsilon_{xy} E_y) - \frac{\partial}{\partial x} \left[ \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} \right) - \frac{1}{\epsilon_{zz}} \left( \frac{\partial (\epsilon_{xx} E_x + \epsilon_{xy} E_y)}{\partial x} + \frac{\partial (\epsilon_{yx} E_x + \epsilon_{yy} E_y)}{\partial y} \right) \right] = \beta^2 E_x$$

$$\frac{\partial^{2} E_{y}}{\partial x^{2}} + \frac{\partial^{2} E_{y}}{\partial y^{2}} + k_{0}^{2} (\epsilon_{yx} E_{x} + \epsilon_{yy} E_{y}) - \frac{\partial}{\partial y} \left[ \left( \frac{\partial E_{x}}{\partial x} + \frac{\partial E_{y}}{\partial y} \right) - \frac{1}{\epsilon_{zz}} \left( \frac{\partial (\epsilon_{xx} E_{x} + \epsilon_{xy} E_{y})}{\partial x} + \frac{\partial (\epsilon_{yx} E_{x} + \epsilon_{yy} E_{y})}{\partial y} \right) \right] = \beta^{2} E_{y}$$

$$P_{xx} = \frac{\partial}{\partial x} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{xx} E_x)}{\partial x} + \frac{\partial^2 E_x}{\partial y^2} + k_0^2 (\epsilon_{xx} E_x) + \frac{\partial}{\partial x} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{yx} E_x)}{\partial y}$$

$$P_{xy} = \frac{\partial}{\partial x} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{yy} E_y)}{\partial y} - \frac{\partial}{\partial x} \left( \frac{\partial E_y}{\partial y} \right) + k_0^2 (\epsilon_{xy} E_y) + \frac{\partial}{\partial x} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{xy} E_y)}{\partial x}$$

$$P_{yx} = \frac{\partial}{\partial y} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{xx} E_x)}{\partial x} - \frac{\partial}{\partial y} \left( \frac{\partial E_x}{\partial x} \right) + k_0^2 (\epsilon_{yx} E_x) + \frac{\partial}{\partial y} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{yx} E_x)}{\partial y}$$

$$P_{yy} = \frac{\partial}{\partial y} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{yy} E_y)}{\partial y} + \frac{\partial^2 E_y}{\partial x^2} + k_0^2 (\epsilon_{yy} E_y) + \frac{\partial}{\partial y} \frac{1}{\epsilon_{zz}} \frac{\partial (\epsilon_{xy} E_y)}{\partial x}$$

$$\begin{bmatrix} P_{\chi\chi} & P_{\chi y} \\ P_{y\chi} & P_{yy} \end{bmatrix} \begin{bmatrix} E_{\chi} \\ E_{y} \end{bmatrix} = \beta^{2} \begin{bmatrix} E_{\chi} \\ E_{y} \end{bmatrix}$$

$$\frac{\partial E_z}{\partial z} = -j\beta E_z = -\frac{1}{\epsilon_{zz}} \left( \frac{\partial (\epsilon_{xx} E_x + \epsilon_{xy} E_y)}{\partial x} + \frac{\partial (\epsilon_{yx} E_x + \epsilon_{yy} E_y)}{\partial y} \right)$$

$$j\omega\mu_0 H_x = \frac{\partial E_z}{\partial y} + j\beta E_y$$
$$j\omega\mu_0 H_y = -j\beta E_x - \frac{\partial E_z}{\partial x}$$
$$j\omega\mu_0 H_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$