A Novel Conditional Diagnostic Scheme for Hypercube-Based Multiprocessor Systems

Qi Wang, Jiafei Liu, Dajin Wang, Wenfei Liu, Jingli Wu, and Gaoshi Li

***Abstract*—With the scale of multiprocessor systems constantly increasing, the large number of interconnected processors (or nodes) makes faulty nodes inevitable. The fault diagnosis of multiprocessor systems therefore is a key technique for the system's robustness. In this paper, we first propose a novel diagnostic metric, the h-extra r-component diagnosability, denoted $ECD_r^h(G)$, which characterizes one special pattern of faults. We derive some theoretical results for the ECD of hypercube, denoted $ECD_r^h(Q_n)$, under the PMC model. Diagnostic algorithms is proposed and implemented to detect faulty nodes that will disconnect hypercube Q_n into r components each containing at least $h+1$ nodes. We also test the ECD-PMC algorithm to the hypercube network with different number of faulty processors satisfying the h-extra r-component condition. Extensive simulation results show that our proposed method achieves very good performance in terms of ACCR, TPR, FPR, and TNR.**

***Index Terms*—Interconnection networks, h-extra r-component diagnosability, hypercube, network reliability.**

I. Introduction

FAULT detection is an important issue in a wide range of research areas such as data center networks [9], social networks [11], supercomputer systems [2], and so on. A multiprocessor system, whose many processors are interconnected with a certain network, can be abstracted as a simple graph, where the nodes in the graph denote the processors and the edges denote the communication links between the processors. The hypercube is a topological structure that is widely used in the construction of network systems, such as IBM Quantum System Two: 7D hypercube for qubit connectivity, NASA's Pleiades: 8D hypercube interconnect. As the size of the interconnection network increases, node faults are inevitable. Therefore, rapid detection and isolation of faults are crucial to the robustness and reliability of the system. System-level diagnosis is one of the most important frameworks for fault diagnosis of multiprocessor systems.

Received 28 August 2024; revised 29 July 2025 and 16 September 2025; accepted 25 November 2025; approved by IEEE TRANSACTIONS ON NETWORKING Editor F. Ciucu. Date of publication 16 December 2025; date of current version 8 January 2026. This work was supported in part by the National Natural Science Foundation of China under Grant 62302107 and Grant 62366007, in part by Guangxi Natural Science Foundation under Grant 2025GXNSFBA069563 and Grant 2025GXNSFAA069507, in part by the Research Fund of Guangxi Key Laboratory of Multi-Source Information Mining and Security under Grant 24-A-03-01, in part by Guangxi Young Elite Scientist Sponsorship Program under Grant GXYESS2025010, and in part by the Science and Technology Project of Guangxi under Grant GuikeAD21220114. *(Corresponding author: Jiafei Liu.)*

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Digital Object Identifier 10.1109/TON.2025.3638453

The PMC diagnostic model, named after Preparata, Metze, and Chien, is a test-based diagnostic model. Under the PMC model, a network is said to be t-diagnosable if, given that the number of faulty nodes does not exceed t, all faulty nodes in the network can be identified [20]. Diagnosability is a traditional parameter for measuring the fault self-diagnosis capability of interconnection networks. However, from a practical perspective, the traditional diagnosability is often rather too restricted because it allows extreme, unlikely scenarios such as the simultaneous failing of all nodes around one particular node. In 2005, Lai et al. [6] proposed the notion of conditional diagnosability, so that certain conditions are assumed of the faulty nodes to accommodate to various practical situations. It turns out that conditional diagnosabilities are often larger than the traditional diagnosability.

The current diagnostic strategies consider only the number of components in the remaining network caused by the faulty nodes, without taking into account the number of nodes in each component. Similarly, the extra diagnosability considers only the size of each component, without taking into account the number of components in the remaining network. To evaluate the reliability and self-diagnosability of a network from a more comprehensive perspective, in this paper, we propose a new diagnostic strategy, named h*-extra* r*-component diagnosability*, considering both the number of components and the size of each component in the remaining network. The main works of this paper are as follows:

- We introduce a new diagnosis metric, h-extra r-component conditional diagnosability (denoted by $ECD_r^h(G)$) of interconnection networks in terms of the PMC model, which considers the network partitions and component size at the presence of faulty nodes, reflecting the self-diagnosis capability of a system.
- We characterize a theoretical framework for determining h-extra r-component conditional diagnosability for hypercube networks under the PMC model. Specifically, when $h = 1$ and $r = 2,3$, $ECD_2^1(Q_n) = 4n - 7, ECD_3^1(Q_n) = 6n - 17$
- We propose a h-extra r-component t-diagnosable algorithm to identify faulty nodes using the PMC model and DFS method. We apply the algorithm ECD-PMC to

the hypercube networks, with different number of faulty processors satisfying the h-extra r-component condition.

The rest of this paper is organized as follows. Section III introduces the terms, notations and definitions used in this paper. Section IV gives the theoretical value of $ECD_2^1(Q_n)\ \ (n \geq 7)$ and $ECD_3^1(Q_n)\ \ (n \geq 10)$. Section V presents a fast fault diagnostic algorithm in terms of ECD-PMC and the proof for its validity. Section VI performs simulation tests to validate the efficiency and accuracy of the algorithm, using the common criteria of ACCR, TPR, TNR, and FPR. Section VII summarizes the paper and discusses future research directions.

II. Related Work

The diagnosability under different conditions has been an active research area. In 2020, Chang and Hsieh [1] determined the conditional diagnosability of the n-dimensional alternating group network AN_n, while AN_n is isomorphic to $S_{n,n-2}$ when $k = n-2$, and the above result was extended to the $(n, n-2)$-star graph. Liu et al. [17] proposed a novel cyclic-based recursive network architecture, explored its fault diagnosis capabilities under the PMC and comparison diagnostic models, and developed a fast one-to-one (unicast) path construction algorithm. Wang et al. [25] presented a novel and highly scalable server-centric data center network topology, and designed efficient fault-free and fault-tolerant routing algorithms, significantly enhancing the network's performance and reliability. In 2016, Zhang and Yang [29] proposed the h-extra diagnosability, requiring that the each component in remaining network has at least $(h+1)$ nodes. Since then, numerous scholars have carried out findings in this direction. Zhu et al. [31] explored the relationship among classical diagnosability, strong diagnosability as well as conditional diagnosability of a class of strong networks, and applied theoretical results to BC networks, folded hypercube and augmented cube. Lin et al. [14] investigated the extra connectivity of alternating group graph, and derived the specific values of extra conditional diagnosability of this network under the PMC test model. In addition, they [13] further characterized the extra diagnosability of split-star networks under the comparison model. Liu et al. [15] determined the g-extra diagnosability of hypercubes and folded hypercube. Wang [26] determined specific values for the g-extra diagnosability of the möbius cube under the PMC and MM* models. Li et al. [8] depicted the extra connectivity, extra diagnosability, and t/k-diagnosability of data center network DCell under the PMC model, and provided a fast t/k-diagnosability algorithm, for identifying faulty nodes in the DCell network. Lv et al. [18] characterized the extra connectivity and used it as a bridge to determine extra diagnosability of regular interconnection networks, and designed two diagnostic algorithms with time complexity $O\left(n^2N\right)$. Recently, Tian et al. [22] investigated the extra conditional diagnosability of hypercube and proposed a new diagnostic model, the f-BPMC, which relaxes the restriction on the upper bound of the number of faulty processors as compared to the traditional PMC model. In the f-BPMC model, up to f faulty processors can be incorrectly evaluated as faulty free. Lin et al. [12] established a relationship between t/k-diagnosability and extra connectivity of large-scale interconnection networks and proposed a new diagnostic algorithm that obtains the t/k-diagnosability in terms of the h-extra connectivity. Wang et al. [24] introduced a new metric, the double-structure connectivity, applied it to hypercube network structures, which evaluates the reliability and fault tolerance of distributed computing networks. Zhang et al. [27] determined a generalization on the h-extra connectivity of the bubble sort network B_n and proved the extra diagnosability and component diagnosability of B_n under the PMC and MM* models.

In a large-scale interconnected network, one set of faulty nodes may make the whole network more fragmented than another. To characterize this scenario, Zhang et al. [28] proposed the component diagnosability, which takes into account the number of components in the fragmented network caused by faulty nodes. Zhuang et al. [33] obtained the $(h+1)$-component diagnosability for several networks such as the hierarchical cube, the generalized exchange cube, the dual cube, under the PMC and MM* diagnostic models. Liu et al. [16] proposed a general scheme to establish the relationship between component connectivity and component diagnosability, and the theoretical results were applied to hypercube-based composite networks. Zhou et al. [30] studied the component connectivity and component diagnosability of DSC_n network. Recently, Huang et al. [5] proved the r-component diagnosability of the n-dimensional hierarchical cube network under the PMC and MM* models, and constructed the 0-PMC and 0-MM* subgraphs. Huang et al. [3] proposed the r-component diagnosability of alternating group network under the PMC model, compared this parameter with other classical networks and found that AG_n has a better self-diagnostic performance. Sun et al. [21] designed a component diagnostic algorithm with low time complexity and simulated it on several classical networks, and determined the component connectivity and component diagnosability of regular networks. Niu et al. [19] investigated the self-diagnosability performance of folded cross cube in terms of g-component diagnosability and g-good-neighbor diagnosability.

III. Preliminary

In this section, we introduces the terms, notations and diagnostic models throughout this paper. First, we provide some necessary terms from graph theory. Then, we put forward the concept of h-extra r-component conditional diagnosability and the descriptions of the PMC model. Furthermore, we present the definition and properties of an n-dimensional hypercube network.

A. Terminologies and Notations

Given a graph $G = (V, E)$, V denotes the set of nodes of G, and E denotes the set of edges of G. $|V(G)|$ denotes the number of nodes of G, and $|E(G)|$ denotes the number of edges of G. If $(u, v) \in E(G)$, then we call that u and v are adjacent. For any vertex $v \in$ G, $N_G(v)$denotes the set of all neighbors of v in the G. A graph is said to be n regular if and only if the degrees of each node is equal to n. For any

TABLE I
TEST RESULTS OF THE PMC MODEL

tester u	testee v	result $\sigma(u, v)$
Fault-Free	Fault-Free	0
Fault-Free	Fault	1
Fault	Fault-Free	0 or 1
Fault	Fault	0 or 1

TABLE II
TOPOLOGICAL PROPERTIES OF HYPERCUBES

hypercube	vertices	edges	diameter	regular
7	128	448	7	7
8	256	1024	8	8
9	512	2304	9	9
10	1024	5120	10	10
11	2048	11264	11	11
12	4096	24576	12	12
13	8192	53248	13	13
14	16384	114688	14	14

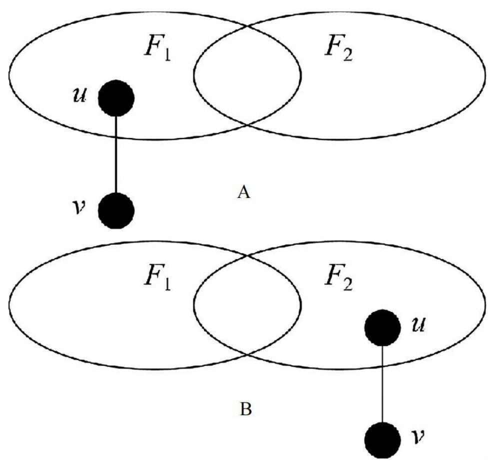

Fig. 1. An illustration on a pair of distinguishable pairs (F_1, F_2) under the PMC model.

two subsets F_1, F_2, we use $F_1 \Delta F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$ to denote the symmetric difference of F_1 and F_2. Given a set of vertices S, $G - S$ refers to the remaining part of G after removing a subgraph of size $|S|$ from it. The maximum connected subgraph in G is said to be a connected component of G.

The PMC model is a classical fault diagnosis model, which can be used to determine whether a vertex is faulty based on test results between two adjacent vertices in a directed graph $D(V, C)$, where V refers to a vertex set in $D(V, C)$, and C refers to an order edge set. Each $\text{arc}(u, v) \in C$ embodies a test from u to v, with test result denoted by $\sigma(u, v)$. It is widely accepted that, if the testing vertex is fault-free in $\sigma(u, v)$, the test result is considered to be reliable; while the testing vertex u is faulty, the test result is unreliable. In a test $\sigma(u, v)$, if both of u and v are fault-free vertices, then $\sigma(u, v) = 0$; if u is a fault-free vertex but v is a fault vertex, then $\sigma(u, v) = 1$. Table I gives the test symptoms of the PMC model.

In the following, we give a necessary and sufficient condition of two faulty sets being distinguishable. For any two distinct sets $F_1, F_2 \subset V$, (F_1, F_2) is a distinguishable pair if and only if there exists a vertex $u \in V(G) - (F_1 \cup F_2)$ and there exists a vertex $v \in F_1 \Delta F_2$ such that $(u, v) \in E(G)$. Conversely, if no such edge exists, F_1, F_2 are indistinguishable. Fig. 1 shows the constructed schemes of two distinguishable faulty sets under the PMC model. For example, in Fig. 1A, $u \in F_1 \Delta F_2$, $v \in G - F_1 - F_2$ and (u, v) is a testing link. Assuming that F_1 is the faulty set, according to the PMC model, it can be obtained that v test u as 1. Conversely, assuming that F_2 is the faulty set, according to the PMC model, it can be obtained that v test u as 0. Thus, two faulty sets are distinguishable.

By imposing a restriction on faulty set, Li et al. [7] proposed the definition of the h-extra r-component connectivity of G.

Definition 1: For any subset S in G, S is said to be an h-extra r-component cut of G if it satisfies that the remaining part of the graph after $G - S$ has at least r components and at least $h + 1$ vertices in each of the components. The minimum cardinality of all h-extra r-component vertex cuts is called the h-extra r-component connectivity of G, denoted by $ECC_r^h(G)$.

In terms of the conditional connectivity mentioned above, we put forward a novel fault diagnostic scheme, the h-extra r-component conditional diagnosability, which take the size of each component and the number of component in a faulty network into account. The characterization associated with the h-extra r-component diagnosability will be presented as follows.

Definition 2: A graph G is h-extra r-component conditional t-diagnosable if and only if for any h-extra r-component faulty subset pair (F_1, F_2) with $F_1 \neq F_2$ and $|F_1| \leq t$, $|F_1| \leq t$, the pair (F_1, F_2) is distinguishable. The h-extra r-component diagnosability of G is the maximal value satisfying that G is h-extra r-component conditional t-diagnosable denoted by $ECD_r^h(G)$.

B. Definition and Properties of Hypercube

Hypercube is a classical network topology for fault tolerance analysis of networks, and its definition as well as properties are given below.

Definition 3: The topology of an n dimensional hypercube Q_n is defined as follows.

(1) The vertex set $V = \{x_1, x_2 \cdots x_n | x_i \in \{0, 1\}, i = 1, 2, \cdots n\}$;

(2) The edge set $E = \{(x, y) | x = x_1 x_2 \cdots x_n, y = y_1 y_2 \cdots y_n, \sum_{i=1}^{n} |x_i - y_i| = 1\}$.

The hypercube Q_n has 2^n vertices and $n2^{n-1}$ edges, while it is n regular, and $k(Q_n) = d(Q_n) = n$. Furthermore, hypercubes have excellent recursive properties, and each n dimensional hypercube consists of two $(n-1)$-dimensional hypercube.

IV. H-EXTRA R-COMPONENT DIAGNOSABILITY OF HYPERCUBE

In this section, we will determining the h-extra r-component diagnosability of Q_n, where $h = 1$, $r = 2, 3$. Before determine the theoretical results, we first introduce the following lemmas.

TABLE III
SUMMARY OF EVALUATION METRICS FOR 1-EXTRA 2-COMPONENT DIAGNOSTIC ALGORITHMS

Dimension	TPR Mean	TPR CI	FPR Mean	FPR CI	TNR Mean	TNR CI	ACCR Mean	ACCR CI
7	95.0%	[93.7%, 96.3%]	37.0%	[34.8%, 39.2%]	63.0%	[60.8%, 65.2%]	72.7%	[70.5%, 74.9%]
8	95.0%	[93.6%, 96.4%]	26.0%	[24.0%, 28.0%]	74.0%	[72.0%, 76.0%]	79.7%	[77.9%, 81.5%]
9	96.0%	[94.8%, 97.2%]	18.0%	[16.2%, 19.8%]	82.0%	[80.2%, 83.8%]	83.4%	[81.8%, 85.0%]
10	98.0%	[97.1%, 98.9%]	10.5%	[9.1%, 11.9%]	89.5%	[88.1%, 90.9%]	89.6%	[88.4%, 90.8%]
11	98.0%	[97.0%, 99.0%]	6.9%	[5.8%, 8.0%]	93.1%	[92.0%, 94.2%]	93.0%	[92.0%, 94.0%]
12	99.0%	[98.3%, 99.7%]	3.7%	[2.9%, 4.5%]	96.3%	[95.5%, 97.1%]	95.9%	[95.2%, 96.6%]
13	100.0%	[99.6%, 100%]	2.5%	[1.8%, 3.2%]	97.5%	[96.8%, 98.2%]	97.5%	[96.9%, 98.1%]
14	100.0%	[99.5%, 100%]	1.4%	[0.9%, 1.9%]	98.6%	[98.1%, 99.1%]	98.6%	[98.1%, 99.1%]

TABLE IV
SUMMARY OF EVALUATION METRICS FOR 1-EXTRA 3-COMPONENT DIAGNOSTIC ALGORITHMS

Dimension	TPR Mean	TPR CI	FPR Mean	FPR CI	TNR Mean	TNR CI	ACCR Mean	ACCR CI
10	97.0%	[95.8%, 98.2%]	15.7%	[13.8%, 17.6%]	84.3%	[82.4%, 86.2%]	90.9%	[89.5%, 92.3%]
11	97.0%	[95.7%, 98.3%]	10.1%	[8.6%, 11.6%]	89.9%	[88.4%, 91.4%]	94.9%	[93.9%, 95.9%]
12	98.0%	[97.0%, 99.0%]	6.2%	[5.1%, 7.3%]	93.8%	[92.7%, 94.9%]	97.0%	[96.3%, 97.7%]
13	99.0%	[98.2%, 99.8%]	3.8%	[2.9%, 4.7%]	96.2%	[95.3%, 97.1%]	98.1%	[97.6%, 98.6%]
14	100.0%	[99.5%, 100%]	2.2%	[1.5%, 2.9%]	97.8%	[97.1%, 98.5%]	98.9%	[98.5%, 99.3%]

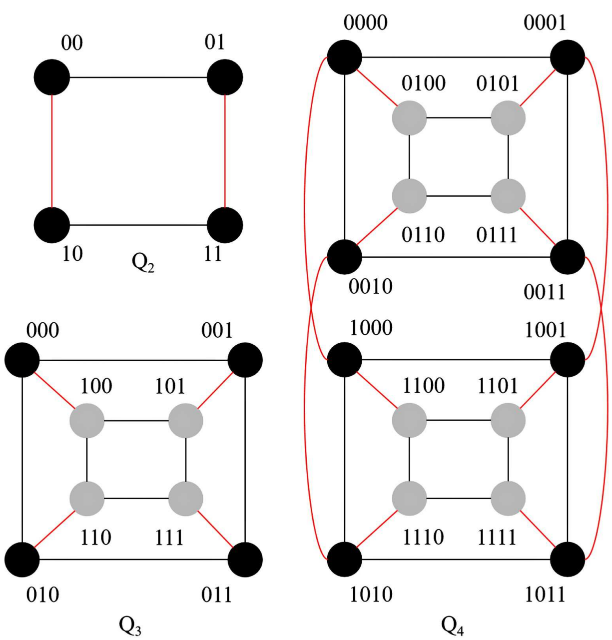

Fig. 2. Topology of Q_n $(n = 2, 3, 4)$.

lemma 1 [32]: For $n \geq 4$, $1 \leq h \leq n-1$, let $F \subset V(Q_n)$ and $|F| \leq hn - \frac{(h-1)(h+2)}{2} - 1$, then $Q_n - F$ is either connected or has a large component as well some small components with at most $h-1$ vertices.

lemma 2 [32]: $ECC_3^1(Q_n) = 4n - 8$, where $n \geq 7$.

lemma 3 [32]: $ECC_4^1(Q_n) = 6n - 18$, where $n \geq 9$.

lemma 4: For the hypercube $Q_n (n \geq 8)$, the upper bound of 1-extra 2-component diagnosability of Q_n under the PMC model is $ECD_2^1(Q_n) \leq 4n - 7$.

Proof: Let X $= \{u_1, u_2, u_3, u_4\}$ be a vertex set in $V(Q_n)$, where $(u_1, u_2), (u_3, u_4) \in E(Q_n)$ (An illustration of the construction refers to see Fig. 3). Assume that $F_1 = N_{Q_n}(X)$, $F_2 = F_1 \cup \{u_3, u_4\}$, then $F_1 \subseteq F_2$, and $F_1 \Delta F_2 = \{u_3, u_4\}$.

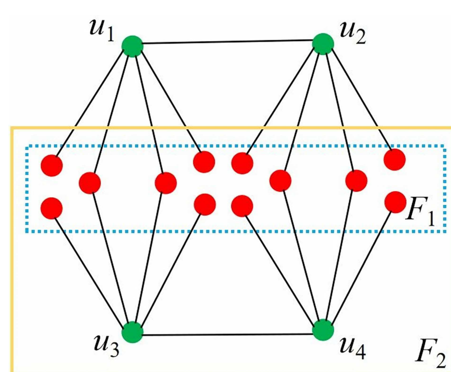

Fig. 3. Illustration on the upper bound of $ECD_2^1(Q_n)$.

Let

$$u_1 = 00\cdots000, \quad u_2 = 00\cdots001$$
$$u_3 = 11\cdots000, \quad u_4 = 11\cdots001$$

which means that $|N_{Q_n}(X)| = 4n - 8$. Thus we have $|F_1| \leq 4n - 8 \leq 4n - 6$, $|F_2| \leq 4n - 6$. Then

$$\begin{aligned} &|V(Q_n)| - |F_1 \cup F_2| - |X \setminus \{u_3, u_4\}| \\ &= |V(Q_n)| - |F_2| - |X \setminus \{u_3, u_4\}| \\ &\geq 2^n - (4n - 6) - 2 \\ &> 2 (n \geqslant 8). \end{aligned}$$

It is clear that there exist at least two components in $Q_n \setminus F_1$ or $Q_n \setminus F_2$, and each component has at least two vertices. Therefore, (F_1, F_2) is a pair of 1-extra 2-component faulty sets. Since $F_1 \subseteq F_2$ and $F_1 \Delta F_2 = \{u_3, u_4\}$, there is no edge between $V(Q_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$. Therefore under the PMC model, F_1, F_2 are indistinguishable. Thus $ECD_2^1(Q_n) \leq 4n - 7$.■

lemma 5: For the hypercube $Q_n (n \geq 8)$, the lower bound of 1-extra 2-component diagnosability of Q_n under the PMC model is $ECD_2^1(Q_n) \geq 4n - 7$.

Proof: Suppose, to the contrary, that $ECD_2^1(Q_n) \leq 4n - 8$. Let F_1, F_2 be two distinct 1-extra 2-component faulty vertex cuts in Q_n and $|F_1| \leq 4n-7, |F_2| \leq 4n-7$ such that F_1, F_2 are indistinguishable under the PMC model. Without loss of generality, assume that $F_2 - F_1 \neq \emptyset$. When $n \geq 8$, $4n-7 \leq 5n-15$. Applying Lemma 1, $Q_n - F_1$ has a large component of size larger than $|Q_n| - |F_1| - 4$. Let A denote the union of remaining small components in $Q_n - F_1$, and $B = Q_n - F_1 - V(A)$. Then $|V(A)| \leq 4$. So

$$\begin{aligned} |V(B)| &= |V(Q_n)| - |F_1| - |V(A)| \\ &\geq 2^n - (4n-7) - 4 \\ &= 2^n - 4n + 3. \end{aligned} \tag{1}$$

In terms of the inequality (1), when $n \geq 8$, the following inequality holds,

$$\begin{aligned} |V(B)| - |F_2| &\geq 2^n - 4n + 3 - (4n-7) \\ &= 2^n - 8n + 10 \\ &> 0. \end{aligned} \tag{2}$$

In terms of the inequality (2), we have $V(B) \backslash (F_2 \backslash F_1) \neq \emptyset$. Since F_1 and F_2 are indistinguishable, then $E[V(Q_n) - F_1 \cup F_2, F_2 \backslash F_1] = \emptyset$. Noting that B is a connected component, there exist no edge between B and $F_2 \backslash F_1$, and so $B \subseteq V(Q_n) - F_1 \cup F_2$. Therefore $V(B) \cap (F_2 \backslash F_1) = \emptyset$. In this case, $F_2 \backslash F_1 \subseteq V(A)$ and $|F_2 \backslash F_1| \leq |V(A)| \leq 4$. Similarly, because of the symmetry of F_1 and F_2, we have $|F_1 \backslash F_2| \leq 4$ and $E[F_1 \backslash F_2, V(B)] = \emptyset$. Thus, B is a connected component of $Q_n - F_1 \cap F_2$. By inequality (1), we have $|V(B)| - |V(A)| - |F_2 \backslash F_1| \geq (2^n - 4n + 3) - 4 - 4 > 0$, for $n \geq 8$. Therefore, B is the largest connected component in $Q_n - F_1 \cap F_2$. According to the relationship of F_1 and F_2, we consider the possible scenarios as below.

Case 1:$F_1 - F_2 \neq \emptyset$, $F_2 - F_1 \neq \emptyset$.

Since F_1 is a 1-extra 2-component vertex cut, we have $|F_2 - F_1| \geq 2$. Similarly, $|F_1 - F_2| \geq 2$. So $F_1 \cap F_2$ is a 1-extra 3-component vertex cut, and by Lemma 2, it follows that $|F_1 \cap F_2| \geq ECC_3^1(Q_n) = 4n-8$. Then $|F_1| = |F_1 - F_2| + |F_1 \cap F_2| \geq 4n-8+2 = 4n-6$, a contradiction with the hypothesis.

Case 2:$F_2 \subseteq F_1$(or $F_1 \subseteq F_2$)

Without loss generality, assume that $F_2 \subseteq F_1$. Since F_1 is a 1-extra 2-component vertex cut, $Q_n - F_1 - B$ contains at least one component. Noting that F_2 is a 1-extra 2-component cut, we have $|F_1 - F_2| \geq 2$. Therefore $Q_n - F_2 - B$ contains at least two components and each component has at least 2 vertices, and so $|F_2| \geq ECC_3^1(Q_n) = 4n-8$ by Lemma 2. Then $|F_1| = |F_1 - F_2| + |F_1 \cap F_2| \geq 4n-6$, contradicts the hypothesis.

In summary, we have $ECD_2^1(Q_n) \geq 4n-7$.■

Combining Lemma 4 and Lemma 5 yields the following result.

Theorem 1: The 1-extra 2-component diagnosability of the hypercube under the PMC model is $ECD_2^1(Q_n) = 4n-7$ for $n \geq 8$.

Next we prove the 1-extra 3-component diagnosability of the hypercube.

lemma 6: For the hypercube $Q_n (n \geq 11)$, the upper bound of 1-extra 3-component diagnosability of Q_n under the PMC model is $ECD_3^1(Q_n) \leq 6n-17$.

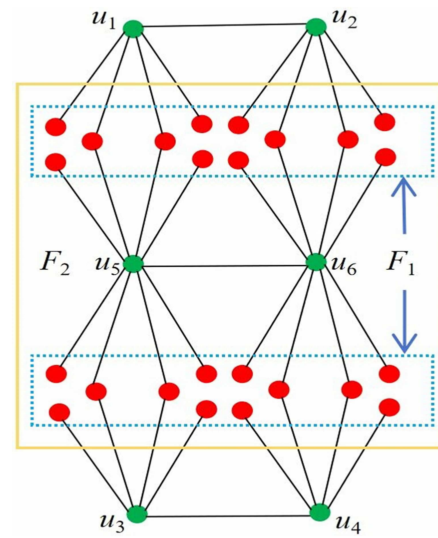

Fig. 4. Illustration on the upper bound of $ECD_3^1(Q_n)$.

Proof 1: Let X $= \{u_1, u_2, u_3, u_4, u_5, u_6\}$ where $(u_1, u_2), (u_3, u_4), (u_5, u_6) \in E(Q_n)$ (see Fig. 4). Assuming that $F_1 = N_{Q_n}(X)$, $F_2 = F_1 \cup \{u_5, u_6\}$ then $F_1 \subseteq F_2$, and $F_1 \Delta F_2 = \{u_5, u_6\}$. Let

$$\begin{aligned} u_1 &= 000000 \cdots 0, \quad u_2 = 000000 \cdots 1 \\ u_3 &= 000110 \cdots 0, \quad u_4 = 000110 \cdots 1 \\ u_5 &= 001010 \cdots 0, \quad u_6 = 001010 \cdots 1 \end{aligned}$$

which means that $|N_{Q_n}(X)| = 6n-18$. Thus we have $|F_1| \leq 6n-18 \leq 6n-16, |F_2| \leq 6n-16$. Then

$$\begin{aligned} &|V(Q_n)| - |F_1 \cup F_2| - |X \setminus \{u_5, u_6\}| \\ &= |V(G)| - |F_2| - |X \setminus \{u_5, u_6\}| \\ &\geq 2^n - (6n-16) - 4 \\ &> 2 \quad (n \geqslant 10). \end{aligned}$$

it is clear that there exist at least three components in $Q_n \setminus F_1$ or $Q_n \setminus F_2$, and each component has at least two vertices. Therefore, (F_1, F_2) is a pair of 1-extra 3-component faulty sets. According to $F_1 \subseteq F_2$ and $F_1 \Delta F_2 = \{u_5, u_6\}$, there is no edge between $V(Q_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$, therefore under the PMC model F_1, F_2 is indistinguishable. Thus $ECD_3^1(Q_n) \leq 6n-17$.

lemma 7: For the hypercube $Q_n (n \geq 11)$, the lower bound of 1-extra 3-component diagnosability of Q_n under the PMC model is $ECD_3^1(Q_n) \geq 6n-17$.

Proof 2: Suppose, to the contrary, that $ECD_3^1(Q_n) \leq 6n-18$. Let F_1, F_2 be two distinct 1-extra 3-component faulty vertex cuts in Q_n and $|F_1| \leq 6n-17, |F_2| \leq 6n-17$ such that F_1, F_2 are indistinguishable under the PMC model. Without loss of generality, assume that $F_2 - F_1 \neq \emptyset$. When $n \geq 11$, $6n-17 \leq 7n-28$. From Lemma 1, $Q_n - F_1$ has a large component of size larger than $|Q_n| - |F_1| - 6$. The union of small components is represented in $Q_n - F_1$ by A, and $B = V(Q_n) - F_1 - V(A)$. Then $|V(A)| \leq 6$. So

$$|V(B)| = |Q_n| - |F_1| - |V(A)|$$

$$\begin{aligned} &\geq 2^n - (6n-17) - 6 \\ &= 2^n - 6n + 11. \end{aligned} \tag{3}$$

From the inequality (3), when $n \geq 11$, the following inequality holds

$$\begin{aligned} |V_B| - |F_1| &\geq 2^n - 6n + 11 - (6n-17) \\ &= 2^n - 12n + 28 \\ &> 0. \end{aligned} \tag{4}$$

In terms of the inequality (4), we have $V(B) \setminus (F_2 \setminus F_1) \neq \emptyset$. Since F_1 and F_2 are indistinguishable, then $E[V(Q_n) - F_1 \cup F_2, F_2 \setminus F_1] = \emptyset$. Noting that B is a connected component, there exist no edge between B and $F_2 \setminus F_1$, and so $B \subseteq V(Q_n) - F_1 \cup F_2$. Therefore $V(B) \cap (F_2 \setminus F_1) = \emptyset$. In this case, $F_2 \setminus F_1 \subseteq V(A)$ and $|F_2 \setminus F_1| \leq |V(A)| \leq 6$. Similarly, because of the symmetry of F_1 and F_2, we have $|F_1 \setminus F_2| \leq 6$ and $E[F_1 \setminus F_2, V(B)] = \emptyset$. Thus, B is a connected component of $Q_n - (F_1 \cap F_2)$. By inequality (3), we have

$$\begin{aligned} &|V(B)| - |V(A)| - |F_2 \setminus F_1| \\ &\geq (2^n - 6n + 11) - 6 - 6 \\ &> 0 \quad (n \geqslant 11). \end{aligned}$$

Therefore, B is the largest connected component in $Q_n - (F_1 \cap F_2)$. According to the relationship of F_1 and F_2, we consider the possible scenarios as below.

Case 1: $F_1 - F_2 \neq \emptyset$, $F_2 - F_1 \neq \emptyset$

Subcase 1.1: $A - (F_1 - F_2) \neq \emptyset$

Since F_1, F_2 are two 1-extra 3-component cuts, $Q_n - F_2$ contains at least three components, so $F_1 - F_2$ contains at least one component and has at least two nodes, i.e., $|F_1 - F_2| \geq 2$, similarly, $|F_2 - F_1| \geq 2$. Because $A - (F_2 - F_1) \neq \emptyset$, $F_1 \cap F_2$ is a 1-extra 4-component cut, it follows from Lemma 3 that $|F_1 \cap F_2| \geq 6n - 18$. Then $|F_1| = |F_1 - F_2| + |F_1 \cap F_2| \geq 6n - 16$, a contradiction with the hypothesis.

Subcase 1.2: $A - (F_1 - F_2) = \emptyset$

Since F_1, F_2 are two 1-extra 3-component cuts, $Q_n - F_2$ contains at least three components, so $F_1 - F_2$ contains at least two components and each component has at least two nodes, i.e., $|F_1 - F_2| \geq 4$, similarly, $|F_2 - F_1| \geq 4$. Because $A - (F_2 - F_1) = \emptyset$, $F_1 \cap F_2$ is a 1-extra 4-component cut, it follows from Lemma 3 that $|F_1 \cap F_2| \geq 6n - 18$. Then $|F_1| = |F_1 - F_2| + |F_1 \cap F_2| \geq 6n - 14$, a contradiction with the hypothesis.

Case 2: $F_2 \subseteq F_1$(or $F_1 \subseteq F_2$)

Without loss generality, assume that $F_2 \subseteq F_1$. Since F_1 is a 1-extra 3-component vertex cut, $Q_n - F_1 - B$ contains at least two components. Noting that F_2 is a 1-extra 3-component cut, we have $|F_1 - F_2| \geq 2$. Therefore $Q_n - F_2 - B$ contains at least three components and at least two vertices each component, i.e. F_2 is a 1-extra 4-component vertex cut, and so $|F_2| \geq ECC_4^1(Q_n) = 6n - 18$ by Lemma 3. Then $|F_1| = |F_1 - F_2| + |F_1 \cap F_2| \geq 6n - 16$, a contradiction with the hypothesis.

In summary, we have $ECD_3^1(Q_n) \geq 6n - 17$.

Combining Lemma 7 and Lemma 8 yields the result as follows.

Theorem 2: The 1-extra 3-component diagnosability of the hypercube under the PMC model is $ECD_3^1(Q_n) = 6n - 17$ for $n \geq 11$.

V. Extra Component Diagnosis Algorithm for Hypercubes

In this section, we propose a new h-extra r-component t-diagnosable algorithm to diagnose faulty processors under PMC model. Under this model, test results can be obtained by testing two adjacent vertices of the system. The test results of the fault-free vertex are reliable, therefore, we exploit them to determine whether the other vertices are faulty or not.

Algorithm 1 Generating Components by DFS

Require: the network G, StartNode $node$, SymptomMatrix sm, AccessToArrays $visited$
Ensure: Connected components ccs
$visited \leftarrow$ [False] $\times G$ {Set all nodes of the hypercube as unvisited}
$visited[node] \leftarrow$ TRUE {Change the access array of visited nodes to TRUE}
$c \leftarrow \{node\}$ {Adding nodes to the component collection}
for each node u in G **do**
  for each node x in $N_G(u)$ **do**
    if $visited[x] ==$ False **then**
      if $sm(node, x) == 0$ **then**
        $DFS(G, x, sm, visited)$
      end if
    end if
  end for
  $ccs \leftarrow ccs \cup \{c\}$
end for
return ccs

In Algorithm 1, we generate connected components by the depth-first search (DFS) method and test symptom matrix of hypercube under the PMC model. The algorithm requires a network G, the start node $node$ and the symptom matrix sm corresponding to the network. Construct two vertex sets X and $\overline{\mathrm{X}}$, where X denotes a visited vertex $\overline{\mathrm{X}}$ denotes an unvisited vertex, and a list CC denoting connected component, and a set CCS for holding all connected components. In the first step, a vertex v is selected as the starting vertex and added to X and CC. In the second step, determine all the neighbors v_i of v. If v_i has not been visited and the test result of v with v_i is 0 then add v_i to CC, then repeat the above steps with one of the neighbors as a new starting vertex. If the test result of this vertex with all its neighbors is 1 then return to the parent of this vertex, select another vertex that satisfies the condition and repeat the above steps. Until all the neighbors of vertices which in CC have been visited or the test result with them is 1 then end. Output a connected component CC. In the third step, hold a connected component CC into the set CCS. Then return to the first step and find a new starting vertex to find the next connected component. Until all nodes have been visited, output all connected components CCS.

In Algorithm 2, we design an extra component fault diagnosis algorithm(ECD-PMC) to detect faulty nodes and fault-free

Algorithm 2 Extra Component Fault Diagnosis Algorithm for networks(ECD-PMC)

Require: a network G, Symptom Matrix sm, Connected components ccs
Ensure: Fault-free set FF, faulty set F
1: $FF \leftarrow \emptyset$, $F \leftarrow \emptyset$ {Initialize the sets of fault-free and fault to the empty set}
2: **for** j in $ccs[0]$ **do**
3:     FF.add(j) {Consider the node in the largest connected component as fault-free}
4: **end for**
5: $i \leftarrow 1$
6: **while** $|ccs[i]| > ECD_r^h(G)$ **do**
7:     **for** j in $ccs[i]$ **do**
8:         FF.add(j) {If the number of nodes of $ccs[i]$ is greater than the theoretical value, it is considered as fault-free}
9:     **end for**
10:     $i \leftarrow i+1$
11: **end while**
12: **for** each node u in FF **do**
13:     **if** $N_G(u) \cap (V(G) - FF - F) \neq \emptyset$ **then**
14:         **for** each node v in $N_G(u) \cap (V(G) - FF - F)$ **do**
15:             **if** $sm(u, v) == 0$ **then**
16:                 FF.add(v)
17:             **else**
18:                 F.add(v)
19:             **end if**
20:         **end for**
21:     **end if**
22: **end for**
23: **if** $|F| == ECD_r^h(G)$ **then**
24:     **for** each node v in $V(G) - FF - F$ **do**
25:         FF.add(v)
26:     **end for**
27: **end if**
28: **return** (FF, F)

nodes of a networks under the PMC model. The algorithm requires the network G, the symptom matrix sm and connected components ccs. Initially, we set F to be the faulty node set and FF to be the fault-free node set. First, on the basis of Algorithm 1, sort all connected components obtained from largest to smallest in terms of the number of vertices, denoted by $C_1, C_2, \cdots, C_m$. Then we select the largest connected component C_1. If the size of C_1 is more than $ECD_2^1(G)$, then take all vertices of C_1 as fault-free vertices and add them to the set FF. Next, we determine C_i where $i = 2, 3, 4, \ldots, m$. If the number of vertices in C_i is greater than the theoretical results, all vertices of C_i are considered fault-free simultaneously. Then we traverse all the vertices in S and determine the test result of S with its neighborhood. If the test result is 0, then we take the neighbor as fault-free, otherwise, we take it as faulty. The faulty set is then judged and if its size is greater than or equal to the theoretical value, then all the remaining vertices without judgement are considered as fault-free. Otherwise, considered as faulty. After checking all the nodes, output the faulty set F and the fault-free set FF.

The aim of Algorithm 3 is to generate a high-dimensional hypercube using a computer in the following steps.

1) Input the dimension of the hypercube as a parameter and generate all vertices in the hypercube by an ordered binary string.
2) Add an edge between two nodes if the binary string between them differs from only one bit.

Theorem 3: Let G be a network with its node number being N. Then the time complexity of the algorithm ECD-PMC is $O(Nd)$ where d represents the maximum degree of the network.

Proof 3: The number of nodes of G is N and the time complexity of the algorithm is specified as follows.

Step 1: $O(1)$, initialize the set of fault-free nodes and the set of faulty nodes.

Steps 2-4: $O(|ccs[0]|)$, adding all the nodes in $ccs[0]$ to the fault-free set requires traversing every node in it, so the time complexity is $O(|ccs[0]|)$.

Steps 5-11: $O(m|C_i|)$, in this process, traverse all the ccs that satisfies the condition. Assuming that we traverse m components and the size of each component is on average $|C_i|$, then the time complexity of these steps is $O(m|C_i|)$.

Steps 12-22: $O(Nd)$, assuming that the average number of neighbors of u is d, then time complexity of outer for-loop is $O(N)$, inner for-loop is $O(d)$, $i \leftarrow i+1$: $O(1)$. In the worst case, inner loops and conditional checks are executed for each node $O(d)$ operations, so the overall complexity is $O(Nd)$.

Steps 23-27: $O(N)$, this progress iterates $N - |FF| - |F|$, therefore its time complexity is $O(N)$.

Algorithm 3 Generate Hypercube

Require: Dimension of the hypercube $dimensions$
Ensure: node sets v and edge sets e of hypercubes
$v \leftarrow$ list(itertools.product([0,1], repeat=dimensions)) {Generate the node set of a hypercube using an ordered $(0,1)$ permutation based on the dimension}
$e \leftarrow$ Empty list {Initialize the set of edges}
$v\_i \leftarrow$ {vertex : indexfor index, vertex inenumerate(v)} {Create a dictionary that maps vertices to their indexes}
$n\_v \leftarrow$ Length of $v$list {Number of nodes}
for i**in**0**to**$n\_v - 1$ **do**
    for j**in**$i+1$**to**$n\_v - 1$ **do**
        if the Hamming distance between $v[i]$ and $v[j]$ is 1 **then**
            add the edge $(v\_i[v[i]], v\_i[v[j]])$ to e {Determine the Hamming distance between two nodes; if it is 1, add an edge between the two nodes}
        end if
    end for
end for
return v, e

Therefore, the final time complexity is $O(Nd)$, where N is the number of nodes.

Next, we proceed to perform a comparative analysis regarding the time complexity of the ECD-PMC scheme in relation to other fault diagnosis algorithms. Define the following parameters: the mean node degree of a system as d, the total number of nodes in the system as N, and the maximum number of faulty nodes in the system as t. Under these definitions, the time complexity of the ECD-PMC scheme is Nd. Lin et al. [10] established a local fault diagnosis strategy, the time complexity of which is $O(t^2N)$. In 2016, Ziwich et al. [34] presented a compared model-based diagnosis scheme and its time complexity is $O(t^2N)$. More recently, Huang et al. [4] put forward a fast t/k-diagnosis scheme of complexity for general networks under the MM* model, whose time complexity is $O(Nd^2)$. Wan et al. [23] proposed the FCFDSn algorithm, which also exhibits a time complexity of $O(N^2)$. Upon comparing the time complexities of the aforementioned algorithms, we observe the following order: $O(Nd) < O(t^2N) < O(t^2N) < O(Nd^2) < O(N^2)$. Based on this comparative analysis of the algorithms, it is evident that the ECD-PMC algorithm demonstrates a lower execution time, thereby highlighting its efficiency in fault diagnosis scenarios.

VI. Simulation and Performance

To measure the effectiveness of the proposed scheme and compare its superiority with the existing schemes, we implement experiments in terms of the performance of the algorithm in different dimensions based on hypercube. The analysis of the simulation experiment is first carry out in the hypercube, regarding the hypercube from 7 to 14 dimensions the dataset is shown in Table II. Hypercube dimensions 7-14 are selected based on theoretical requirements: $n \geq 8$ is required for ECD_2^1 (Theorem 1) $n \geq 11$ is required for ECD_3^1 (Theorem 2) Lower dimensions (7-11) demonstrate boundary cases, while higher dimensions (12-14) validate scalability. In addition, the performance of the algorithm was measured by Accuracy(ACCR), Recall(TPR), False Positive Rate(FPR), and True Negative Rate(TNR). The definitions of each metric are as follows:

• $Accuracy(ACCR)$: It represents the ratio of correctly diagnosed faulty and fault-free nodes to all nodes. The specific formula is as follows,

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

• $Recall(TPR)$: It represents the proportion of correctly diagnosed faulty nodes to all faulty nodes. The specific formula is as follows,

$$Recall/TPR = \frac{TP}{TP + FN}$$

• $FalsePositiveRate(FPR)$: It represents the proportion of fault-free nodes misdiagnosed as faulty to all fault-free nodes. The specific formula is as follows,

$$FPR = \frac{FP}{FP + TN}$$

• $TrueNegativeRate(TNR)$: It represents the proportion of correctly diagnosed fault-free nodes to all fault-free nodes. The specific formula is as follows,

$$TNR = \frac{TN}{FP + TN}$$

where TP refers to the number of correctly diagnosed faulty nodes; TN refers to the number of correctly diagnosed fault-free nodes; FP refers to the number of fault-free nodes incorrectly diagnosed as faulty; and FN refers to the number of faulty nodes incorrectly diagnosed as fault-free.

First, we programme the diagnostic algorithm for 1-extra 2-component diagnosis strategy, with hypercube dimensions ranging from 7 to 14, and perform the diagnosis algorithm 1000 times respectively, and finally calculate the average of the above metrics. All experimental results are obtained by averaging outcomes from 1000 independent trials. Since our algorithm is deterministic (given fixed input fault sets), the observed variation stems solely from random fault injections. Therefore, we report standard deviations to quantify variability, though confidence intervals are not applicable here as the mean values converge to deterministic outcomes under repeated sampling.

As depicted in Fig. 7, one can observe that, with the hypercube dimension increasing, TPR gradually increases, from 95% even to 100%, which indicates that the ability of correctly diagnosed faulty nodes. In addition, FPR is characterized by network scale and network dimension, and we set the dimension of hypercube being from 7 to 14. As the dimension increases, FPR decreases from 35% to 1.4%, eventually converge to 0, which implies that the possibility of fault-free nodes misdiagnosed as faulty is very little. At the same time, TNR increases from 63% to 98.6%, which is consistent with the downward trend in FPR, demonstrating that the algorithm also performs excellently in accurately identifying fault-free nodes. It is clearly noted from Fig. 7 that, ACCR gradually increases from 72.7% to 98.6%.

As shown in Fig. 8, we consider the case of 1-extra 3-component diagnosability. Compared with 1-extra 2-component diagnosability, the fault detection rate of 1-extra 3-component diagnosability is more stable. In Fig. 5, The TPR remains at a high level and shows a continuous upward trend, increasing from 97% to 100%. This indicates that the algorithm's ability to correctly identify faulty nodes improves with the increase in dimensions. The FPR shows a significant downward trend, decreasing from 15.7% to 2.3%. This suggests that with the increase in dimensions, the likelihood of the algorithm incorrectly classifying fault-free nodes as faulty ones significantly reduces. Corresponding to the decline in FPR, the TNR shows a continuous increase from 84.3% to 97.8%. This further confirms that the accuracy of the algorithm in identifying fault-free nodes significantly improves with the increase in dimensions. The overall accuracy of the algorithm (ACCR) gradually improves from 90.9% to 98.9%. Although the improvement in ACCR is not as pronounced as in TPR and TNR, the steady increase indicates that the overall performance of the algorithm benefits from higher dimensions, leading to better classification of both faulty nodes and fault-

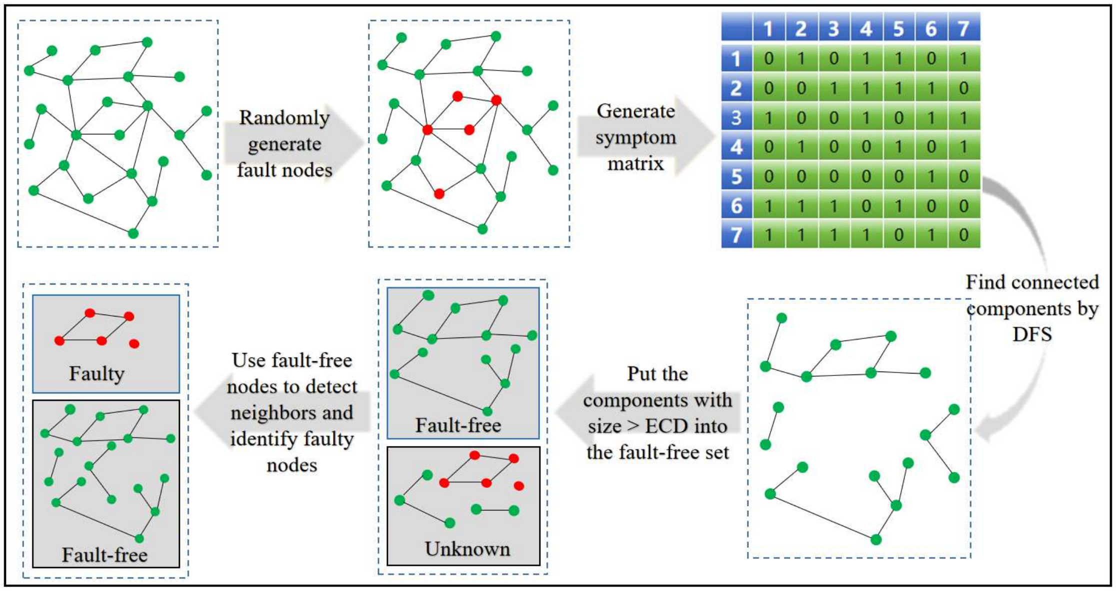

Fig. 5. Illustration of the ECD-PMC algorithm.

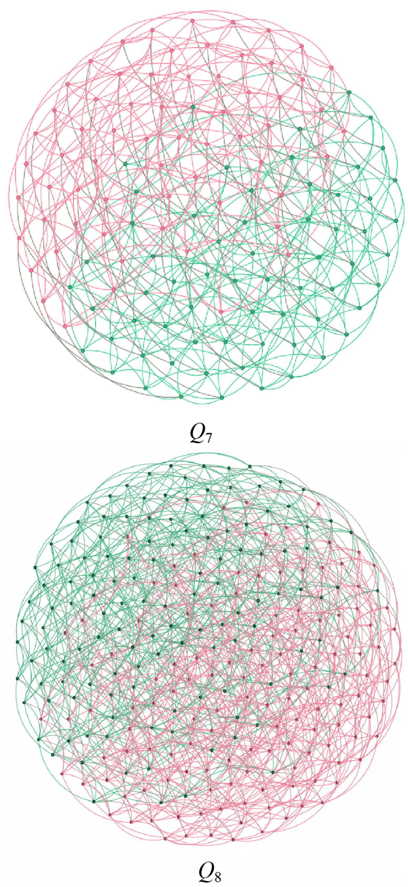

Fig. 6. Visualization of high-dimensional hypercubes.

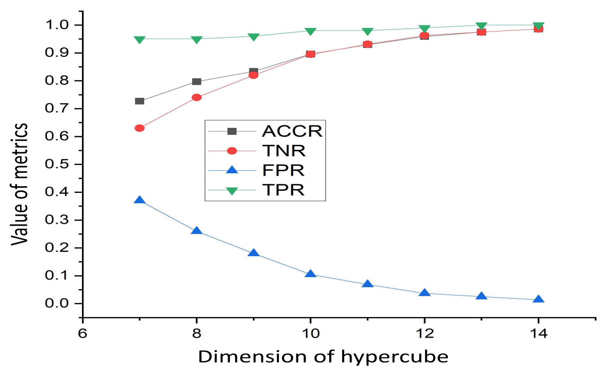

Fig. 7. Simulation of 1-Extra 2-Component Diagnostic Algorithm in Hypercube Simulation.

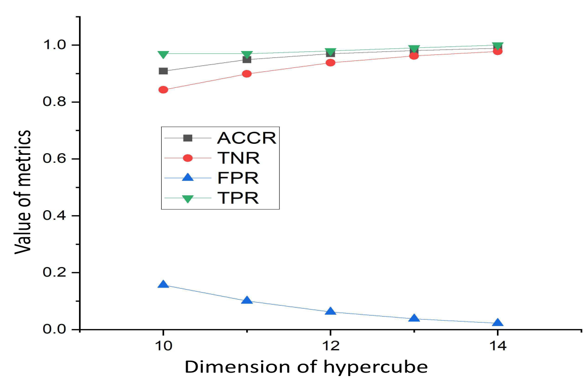

Fig. 8. Simulation of 1-Extra 3-Component Diagnostic Algorithm in Hypercube.

free nodes. Overall, we conclude that, a positive correlation is presented among TPR, TNR as well as ACCR, while ACCR and FPR form a negative correlation.

In Fig. 10, the bar chart shows the comparisons among five distinct diagnostics strategies in hypercube-based networks under different dimensions. Specifically, the blue represents 1-extra 2-component diagnosability, the orange represents

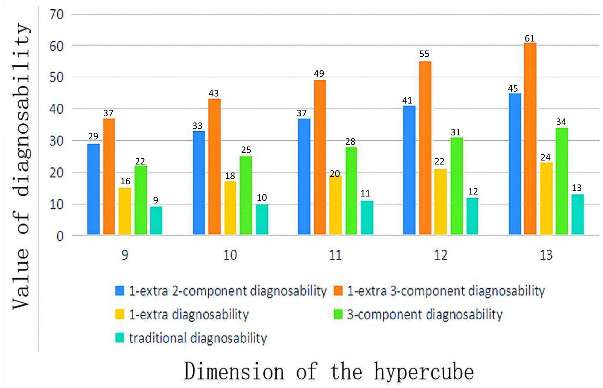

Fig. 9. Comparison results of different diagnosability of hypercubes.

1-extra 3-component diagnosability, the green represents 3-component diagnosability, the yellow represents 1-extra diagnosability and the cyan represents traditional diagnosability. It can be seen that blue and orange are significantly higher than the other three, indicating that 1-extra 2-component diagnosability and 1-extra 3-component diagnosability perform better than the other three diagnosabilities on all dimensions of the graph. These two methods improve the efficiency and accuracy of diagnosis by considering the number of components in the network and the size of each component. Therefore, the ECD-PMC scheme can be used as an effective strategy to improve the reliability of hypercube networks.

VII. Conclusion

In this work, we proposed a new diagnostic scheme, named h-extra r-component diagnosability, which assumes conditions of the faulty nodes, so that they disconnect the network into r component, and each component contains at least $h+1$ nodes. We determined the 1-extra 2-component diagnosability of the hypercube network, denoted $ECD_2^1(Q_n)$ to be $4n-7$, and the 1-extra 3-component diagnosability is $ECD_3^1(Q_n) = 6n-17$, under the PMC model. In addition, we developed the hypercube dimension generation algorithm (HDG) and the fault diagnosis algorithm (ECD-PMC), which are applied to the hypercube networks. We conducted simulation experiments, in terms of ACCR, TPR, FPR, and TNR, for ECD-PMC, demonstrating its effectiveness in detecting faulty nodes in hypercubes networks.

Looking forward, we hope to establish h-extra r-component diagnosability for larger ranges of h and r, and also for more interconnection networks, as well as data center networks. In addition, we will explore the possibility of extending the framework to other networking realms, such as Internet of Things and industrial control systems.

Finally, while the ECD-PMC algorithm demonstrates high diagnostic accuracy under standard PMC assumptions, it relies on fault-free testers (healthy nodes) providing perfect test results. Future work will explore robust extensions under adversarial or noisy testing conditions (e.g., f-BPMC variants [22]) where this assumption may be relaxed.

Declaration of Competing Interest

The authors declare that there is no conflict of interest regarding the publication of this article. Neither the entire article nor any part of its content has been published or has been accepted else where. It not submitted to any other journal.

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