A tool for systematically and directly identifying which of the components of the state vector x have significant participation in selected modes (*SM*) is first introduced: the *participation factor* p_{ki} of the k-th state variable in the i-th system mode (which corresponds to eigenvalue λ_i of A) is defined by

$$p_{ki} \triangleq u_{ki} y_{ki} (= \partial \lambda_i / \partial a_{kk}), \tag{2}$$

where y_{ki} (u_{ki}) is the k-th entry of the i-th right (left) eigenvector of A. Reasons for using the $\{p_{ki}\}$ rather than the $\{y_{ki}\}$ for measuring state-variable participation in a given mode (even though use of the latter has often been advocated in the literature) are presented, and illustrated by means of examples; the primary reason is that the former, unlike the latter, are dimensionless numbers, independent of state-variable scaling.

Now, given a selection of h (typically $<<N$) modes of interest, say those corresponding to $\lambda_1, \ldots, \lambda_h$, we may use the participation factors to help determine which state variables are relevant to these *SM*, and then (after possible reordering of state variables) rewrite (1) as

$$\begin{bmatrix} \dot{r} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} r \\ z \end{bmatrix}, \tag{3}$$

where r is an n-vector of *relevant states* ($N \geq h$ but typically $<<N$) to be retained in the reduced model. The desired reduced model, one that still contains the *SM*, is given by

$$\dot{\bar{r}} = (A_{11} + M)\bar{r} \tag{4}$$

where M is a constant matrix that reproduces the effect of the less relevant dynamics on the relevant dynamics when only the *SM* are excited. More precisely, M is any matrix that satisfies (cf. Fig. 1)

$$My_{ri} = H(\lambda_i) y_{ri},\ i = 1 \text{ to } h, \tag{5a}$$

$$H(s) \triangleq A_{12}(sI - A_{22})^{-1} A_{21}, \tag{5b}$$

where y_{ri} denotes the portion of y_i that corresponds to the $_r$ states. When $h = 1$, we can take $M = H\ H(\lambda(\lambda)$, which is equivalent to replacing z by λz in (3). Eigenstructure *sensitivities for the SM* can also be found from (4).

When the λ_i and y_{ri} are not known, which is usually the case, an iterative algorithm that follows naturally from (4) and (5) may be used. This prototype *SMA* algorithm involves eigenanalysis of a matrix of size $n \times n$ only at each stage, and requires just A_{11} and $H(s)$; eigenanalysis of the large matrix A is avoided. When $h = 1$, the algorithm has been proved to converge locally if, and only if, the sum of the participation factors for the r states in (3) exceed in absolute value the corresponding sum for the z states; this result strengthens our case for the use of the participation factors in picking relevant state variables.

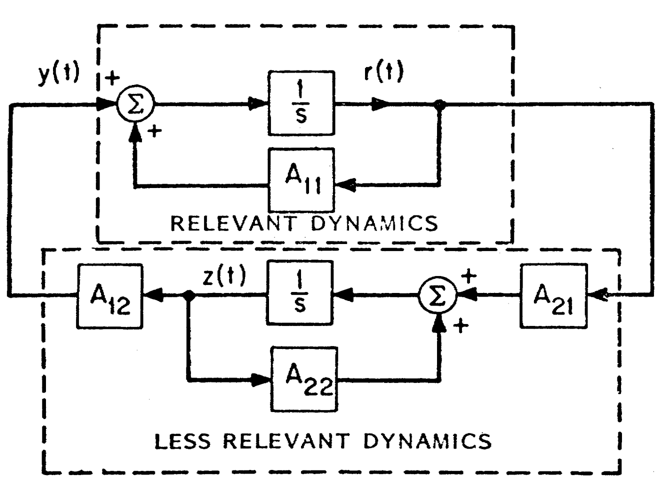

Fig. 1. The relevant and less relevant dynamics.

82 WM 071-9
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Selective Modal Analysis with Applications to Electric Power Systems, Part II: The Dynamic Stability Problem

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Selective Modal Analysis (or *SMA*) is a physically motivated framework for understanding, simplifying, and analyzing complicated linear time-invariant models of dynamic systems. SMA allows one to: focus on any prespecified dynamic modes of interest; identify those state variables that significantly participate in the selected modes; construct reduced-order physically-based models that retain only these relevant state variables and yet contain the selected modes; and efficiently and accurately compute these modes (and their sensitivities to parameter variations) when they are not known, using an iterative algorithm that has appealing interpretations. An introduction to the basic concepts of SMA, with several examples, is contained in the companion paper. The present paper concerns the application of SMA to the Dynamic Stability problem in electric power systems.

The term 'Dynamic Stability' is used to refer to the dynamic behavior of a power system under *small* perturbations about some operating condition, and more specifically to the phenomenon of slow and poorly damped, or sustained, or even diverging power oscillations in the 0.1 to 2 Hz range. The oscillations may involve extensive portions of the system, and many disparate system components; the number of troublesome oscillating modes (so-called 'swing modes') is, however, typically very small. Earlier work has established that Dynamic Stability is adequately modeled using a linear, time-invariant, state-space description of the form

$$\dot{x} = Ax \tag{1}$$

where the state vector x represents the perturbations of power system variables from their nominal values at the chosen operating condition. The dimension of x can easily be in the hundreds if large portions of the system have to be modeled. The few previous attempts at accurately and efficiently computing and analyzing just the modes of interest (the swing modes) in (1) have been deficient in several respects, with the result that Dynamic Stability studies have been off-line, and largely limited to highly simplified models. (These observations are elaborated in the paper.) SMA overcomes many of these deficiencies, and has the potential to deal with realistic models in a real-time setting.

A brief summary of the SMA framework described in Part I is presented in this paper: the definition of 'participation factors' and their use in identifying variables relevant to the selected modes are recalled; direct and iterative procedures for obtaining a reduced-order model (by collapsing the less relevant dynamics in such a way that the selected modes are preserved), and convergence conditions for the iterative algorithm (stated in terms of participation factors), are reviewed; formulas for determining the sensitivity of the selected eigenstructure to parameter changes, using only computations on the reduced model, are given. The ability of SMA to exploit composite structure in systems, by permitting and taking advantage of model reduction at subsystem level, is then described; this becomes especially important in the case of power systems, where separate subsystems (the power plants) are interconnected by static constraints (imposed by the network), thus naturally giving rise to a composite formulation; see Fig. 1.

The paper next brings together the discussions of Dynamic Stability and SMA: appropriate power plant models for Dynamic Stability are considered, see Fig. 2; the selection via SMA of relevant states (for the swing modes), and the subsequent reduction at subsystem level to lower order models that retain these relevant states and reproduce the selected modes are both treated in some detail; lessons obtained through computational experience with a 60-machine power system model (which was originally developed by Van Ness *et al.* to model an occurrence of dynamic instability in an actual power system, and which is discussed at some length in the companion paper) are mentioned; and guidelines for designing a comprehensie scheme for real-time analysis and control of Dynamic Stability, using information that can be made available via SMA, are suggested.

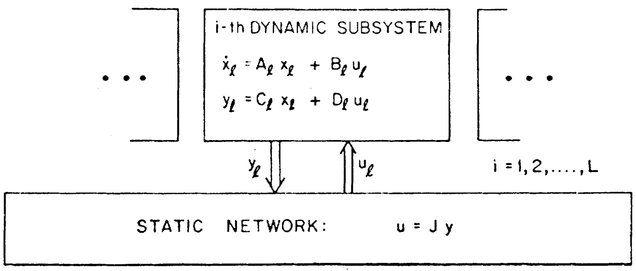

Fig. 1. Composite model.

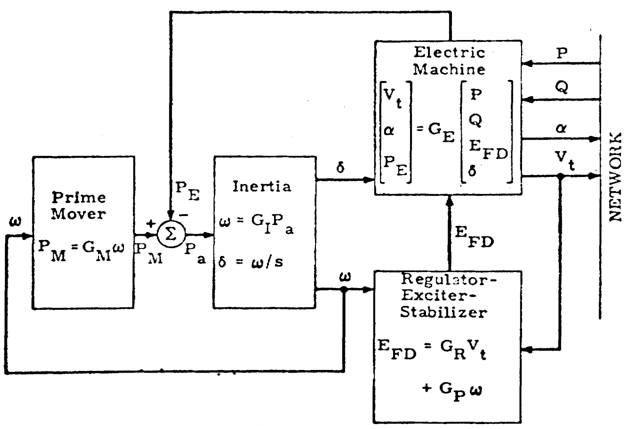

Fig. 2. Power plant model.

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Reduced Order Models for Double-Cage Induction Motors

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The need to include transient models in certain power system studies is generally recognized. In those cases worth considering, however, the induction motors are usually of the large size and equipped with a double-cage or deep-bar rotor both of which having essentially the same transient properties.

By modeling each polyphase winding in the two-axis machine theory as a second-order *d,q* equation, the transient model of a double-cage induction machine is of the seventh order whereas the complete representation of an induction motor with a single rotor winding is of the fifth order. Using this fifth-order model or its derivatives to represent large induction machines would mean ignoring the presence of the second rotor winding.

In devéloping the reduced order models, the seventh order machine model is first presented in state-variable form with the winding flux-linkages rather than currents used as the state variables since in the digital simulation of the model these variables provide more stability with the same integration time-step; or a larger time step can be used to obtain the same accuracy thus requiring less computer time. This is particularly true if fast transient components due to an electromagnetically coupled winding are neglected in which case computational instability can even occur as has been observed in the simulation studies.

Due to the objective of the starting (outer) cage, its leakage reactance and resistance are designed to have respectively lower and much higher values than the corresponding parameters of the running (inner) cage. This results in transient components associated with the starting cage which are much faster than those associated with the running cage. These statements are confirmed by an eigenvalue analysis performed for several induction motors at different slip values. It shows that for all slip values, the damping constant of the transients associated with the starting cage is indeed at least one order of magnitude larger than that associated with the running cage.

An order reduction of the machine model is then obtained by neglecting the fast transients due to the starting cage. This amounts to setting the derivative of the starting cage flux-linkage in the differential equations equal to zero so that this flux linkage can be algebraically expressed in terms of the other flux linkages and substituted in the remaining differential equations hereby reducing the system order to five.

Although the simplified model and the conventional complete model for a single-cage motor are of the same order, their structures are quite different so that no parameter values for the conventional fifth-order model can be found which will provide the same transient behavior as that of the double-cage motor. It is also noted that the order reduction described above can be rigorously established by the singular perturbation theory.

The reduced order model can be further simplified if the transients associated with the stator winding are neglected as is customary in stabilty studies. This amounts to ignoring the dc offset in stator currents caused by magnitude changes in the stator voltages. Dc offsets in stator currents cause decaying oscillations of almost the line frequency in the transient active and reactive power flows at the machine terminals, and the torque developed. Since the time constant of the transient component associated with the stator winding is usually much larger than its oscillation period thus corresponding to a small damping ratio; the the time constant associated with machine speed changes is relatively large in general, these decaying alternating components in the power flows and torque present in the full-order model have approximately zero mean and little effect on the speed values of all machines in the power system including the induction motor itself. Consequently, the reduced order model which results from neglecting stator transients is an accurate indicator of the average power flows and machine speed.

The order reduction by neglecting stator transients is achieved by setting the derivative of the stator flux-linkage equal to zero in the equations referred to a synchronously rotating reference frame. However, it is noted that the order reduction, although established in a quite similar way as when neglecting the fast transient associated with the starting cage, is based on a completely different consideration. While the time constant of the transients associated with the starting cage is at least an order of magnitude smaller than that associated with the running cage, the time constants of the transients associated with the stator winding and the running cage are generally of the same order of magnitude.

The different reduced order models are validated by comparing their simulated responses with that of the full-order model for a full-voltage start-up as well as for the case when the motor is exposed to a temporary symmetrical fault at its terminals. The simulation results show that the fifth-order machine model is an excellent replica of the complete seventh-order model and thus can substitute for the seventh-order model in any simulation study. The third-order machine model is appropriate for use in cases where oscillations of almost the line frequency in the active and reactive powers are not of interest. Linearized versions fo the reduced order models can also be employed for small signal analysis such as in dynamic stability studies where only sufficiently small perturbations from the operating point are considered.