Problem Set 2

due data: 2023/5/28, PM 12:00

Pls send an attached email to jqian@ecust.edu.cn inlcuding both your `.ipynb` file and the corresponding `.html` or `.pdf` file. The homework topics below doesn't need to be inlcuded.

1. van der Pool oscillator

$$\frac{d^2x}{dt^2}+\mu(x^2-x_0^2)\frac{dx}{dt}+\omega_0^2x=0. \tag{1}$$

(a) Explain why Eq. (1) describes an oscillator with x-dependent damping.
(b) Plot the phase-space figure of the solution, that is, $x(t)$ versus $\dot{x}(t)$
(c) Verify that this equation produces a limit cycle in phase space, that is orbits internal to the limit cycle spiral out until they reach the limit cycle, and those external to it spiral in to it.

2. Duffing oscillator

Another example of a damped, driven nonlinear oscillator, which is given by

$$\frac{d^2x}{dt^2}+2\gamma\frac{dx}{dt}+\alpha x+\beta x^3=F\cos(\omega t). \tag{2}$$

(a) Modify your ODE solver to solve Eq. (2).
(b) First choose parameter values corresponding to a simple harmonic oscillator and verify that you obtain sinusoidal behavior for x(t) and a closed elliptical phase-space figure.
(c) Include a driving force, wait 100 cycles in order to eliminate transients, and then create a phase space plot. We used the parameters $\alpha=1.0, \beta=0.2, \gamma=0.2, \omega=1, F=4.0$ and the initial conditions $x(0)=0.009, \dot{x}(0)=0$.
(d) Search for a period-three solution, We used the parameters $\alpha=0.0$, $\beta=1.0$, $\gamma=0.04$, $\omega=1$, and $F=0.2$.

3. Lorenz Attractor

Lorenz attractor is described by the set of equations as follows

$$\begin{aligned}\frac{dx}{dt}&=\sigma(y-x),\\ \frac{dy}{dt}&=\rho x-y-xz,\\ \frac{dz}{dt}&=-\beta z+xy.\end{aligned} \tag{3}$$

where $x(t), y(t), z(t)$ are related to fluid velocity and the temperature distribution. σ, ρ, β are parameters and the terms xz, xy are nonlinear terms.
(a) Modify your ODE solver to solve Eq. (3) with $\sigma=10, \rho=28, \beta=8/3$.

(b) Plot independet figures of $x(t), y(t), z(t)$.
(c) Make a 3D plot of $x(t), y(t), z(t)$.
(d) Make a “phase-space” plot of $z(t)$ vs. $x(t)$ (the independent variable t does not appear in such a plot). The distorted, number eight-like figures you obtain are called Lorenz attractors, “attractors” because even chaotic solutions tend to be attracted to them.

4. Radiating Bar (Newton's cooling)

In the class we have discussed the temperature change of an iron bar that is in contact with the bath at $T_e = 100$ K. Now assuming the radiation of the bar, the modified heat equation is

$$\frac{\partial T(x,t)}{\partial t} = \frac{K}{C\rho}\frac{\partial^2 T}{\partial^2 x} - bT(x,t). \tag{4}$$

The parameters are $K = 237$ W/(mK), $C = 900$ J/(kg K), $\rho = 2700$ kg/m^3. You can choose the value of b by yourself.
(a) Modify your code to solve Eq. (4).
(b) Plot a 3D figure to show $T(x,t)$.
(c) Try to change the value of b and explain your results.