$$\boxed{\begin{aligned}J &= \frac{-1}{N} * \sum_{i=1}^{N}\left(y_i * \log(\hat{y}_i) + (1-y_i) * \log(1-\hat{y}_i)\right)\\ \hat{y}_i &= \sigma\left(w_1 \cdot x_i^{(1)} + w_2 \cdot x_i^{(2)} + w_0\right)\end{aligned}}$$

$$\frac{\partial \hat{y}}{\partial w} = \frac{\partial \hat{y}}{\partial z} * \frac{\partial z}{\partial w}$$

$$\frac{\partial \hat{y}}{\partial z} = \frac{\partial}{\partial z} \frac{1}{1+e^{-z}} = \frac{\partial}{\partial z} (1+e^{-z})^{-1} = -1 \cdot (1+e^{-z})^{-2} \cdot (1+e^{-z})$$

$$= \frac{-1}{(1+e^{-z})^2} \cdot (1+e^{-z}) = \frac{1}{(1+e^{-z})}$$

$$\frac{\partial z}{\partial w} = \text{?} \dots$$

$$\frac{\partial J}{\partial w_1} = \frac{-1}{N} \sum_{i=1}^{N} y \cdot \frac{\partial}{\partial w_1}(\log \hat{y}) + (1-y)\frac{\partial (1-\hat{y})}{d w_1}$$

$$\frac{\partial}{\partial w_1} \log \hat{y} = \frac{1}{\hat{y}} \frac{\partial}{\partial w_1} \hat{y}$$

$$\frac{\partial}{\partial w_1} \log(1-\hat{y}) = \frac{1}{1-\hat{y}} \frac{\partial}{\partial w_1}(1-\hat{y}) = \frac{-1}{1-\hat{y}} \frac{\partial}{\partial w_1} \hat{y}$$

$$\Rightarrow \frac{\partial}{\partial w} J = \frac{-1}{N} \sum_{i=1}^{N} y \cdot \frac{1}{\hat{y}} \frac{\partial}{\partial w} \hat{y} + (1-y)\left(\frac{-1}{1-\hat{y}}\right) \frac{\partial}{\partial w} \hat{y}$$

$$= \frac{-1}{N} \sum_{i=1}^{N} \left[\frac{y}{\hat{y}} + \frac{-1+y}{1-\hat{y}}\right] * \frac{\partial}{\partial w} \hat{y}$$

$$\rightarrow \sigma(z) = \frac{1}{1+e^{-z}}$$

$$\frac{\partial J}{\partial w_1} = -\frac{1}{N} \sum_{i=1}^{N} \left(\frac{y_i}{\hat{y}_i} - \frac{1-y_i}{1-\hat{y}_i} \right) \cdot \sigma(\hat{y}_i) \cdot (1-\sigma(\hat{y}_i)) \cdot x_i^{(1)}$$

$$\frac{\partial J}{\partial w_2} = -\frac{1}{N} \sum_{i=1}^{N} \left(\frac{y_i}{\hat{y}_i} - \frac{1-y_i}{1-\hat{y}_i} \right) \left((1-\sigma\hat{y}_i) \cdot x_i \right)$$

$$\frac{\partial J}{\partial w_0} = -\frac{1}{N} \sum_{i=1}^{N} \left(\frac{y_i}{\hat{y}_i} - \frac{1-y_i}{1-\hat{y}_i} \right) (1-\sigma\hat{y}_i)$$