Report: assigment 2

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March 1, 2017

1

1a)

```
def softmax(x):
    # ## YOUR CODE HERE
    all_constants = - tf.reduce_max(x, axis=1)
    x = x + tf.expand_dims(all_constants, 1)
    x = tf.exp(x)
    all_sums = tf.reduce_sum(x, 1)
    all_sums = tf.pow(all_sums, -1)
    out = x*tf.expand_dims(all_sums, 1)
    # ## END YOUR CODE
    return out
```

1b)

```
def cross_entropy_loss(y, yhat):
    # ## YOUR CODE HERE
    y = tf.cast(y, tf.float32)
    yhat = tf.log(yhat)
    out = - tf.reduce_sum(y*yhat)
    out = tf.reshape(out, (1,))
    # ## END YOUR CODE
    return out
```

1c) The placeholders variables are like their name suggest a placeholder for a tensor. We use it to form a computational graph before the training. In the training stage we use the dictionary feed_dict to 'load' the placeholders variables with real tensors.

```
    def add_placeholders(self):
        # ## YOUR CODE HERE
        self.input_placeholder = tf.placeholder(tf.float32,
                                                shape=[self.config.batch_size,
                                                       self.config.n_features],
                                                name="input_placeholder")
        self.labels_placeholder = tf.placeholder(tf.int32,
                                                 shape=[self.config.batch_size,
                                                        self.config.n_classes],
                                                 name="labels_placeholder")
        # ## END YOUR CODE
```

```python
def create_feed_dict(self, input_batch, label_batch):
    # ## YOUR CODE HERE
    feed_dict = {self.input_placeholder: input_batch,
                 self.labels_placeholder: label_batch}
    # ## END YOUR CODE
    return feed_dict
```

1d)

```python
def add_model(self, input_data):
    Wshape = [self.config.n_features, self.config.n_classes]
    bshape = [self.config.batch_size, self.config.n_classes]
    Winit = tf.zeros(Wshape)
    binit = tf.zeros(bshape)

    with tf.variable_scope("linear-model"):
        W = tf.get_variable("weights", dtype='float32', initializer=Winit)
        b = tf.get_variable("bias", dtype='float32', initializer=binit)
        out = softmax(tf.matmul(input_data, W) + b)
    # ## END YOUR CODE
    return out

def add_loss_op(self, pred):
    # ## YOUR CODE HERE
    loss = cross_entropy_loss(self.labels_placeholder,
                              pred)
    # ## END YOUR CODE
    return loss
```

1e)

```python
def add_training_op(self, loss):
    # ## YOUR CODE HERE
    optimizer = tf.train.GradientDescentOptimizer(self.config.lr)
    train_op = optimizer.minimize(loss)

    # ## END YOUR CODE
    return train_op
```

All the basic operations in TensorFlow have attached gradient operations. And so with the use of backpropagation TensorFlow computes the gradients for all variables in the computation graph.

2

We shall first understand **the Named Entity Recognition (NER) window model**. Suppose we have a corpus with a vocabulary $V = [\mathbb{w}_1, \ldots, \mathbb{w}_{|V|}]$ (we are assuming that every word $\mathbb{w}$ correspond to an index $i \in \{1, \ldots, |V|\}$), a number C of name entity categories (null-class,Person, Location, etc.) and a matrix $L \in \mathbb{R}^{|V|,d}$ where each row i correspond to the word embedding of size d of the word $\mathbb{w}_i$. Now we can choose the parameters m and H to be the size window and the size of the hidden layer, respectively. Let $n = (2m + 1)d$, $W \in \mathbb{R}^{n,H}$, $b_1 \in \mathbb{R}^H$, $U \in \mathbb{R}^{H,C}$ and $b^2 \in \mathbb{R}^C$. We assume that the training dataset is compose by training samples of the form $([\mathbb{w}_{t-m}, \ldots, \mathbb{w}_t, \ldots, \mathbb{w}_{t+m}], c)$ where $c \in \{1, \ldots, C\}$ (1 represent the null-class) – this sample tell us that the word $\mathbb{w}_t$ is a name

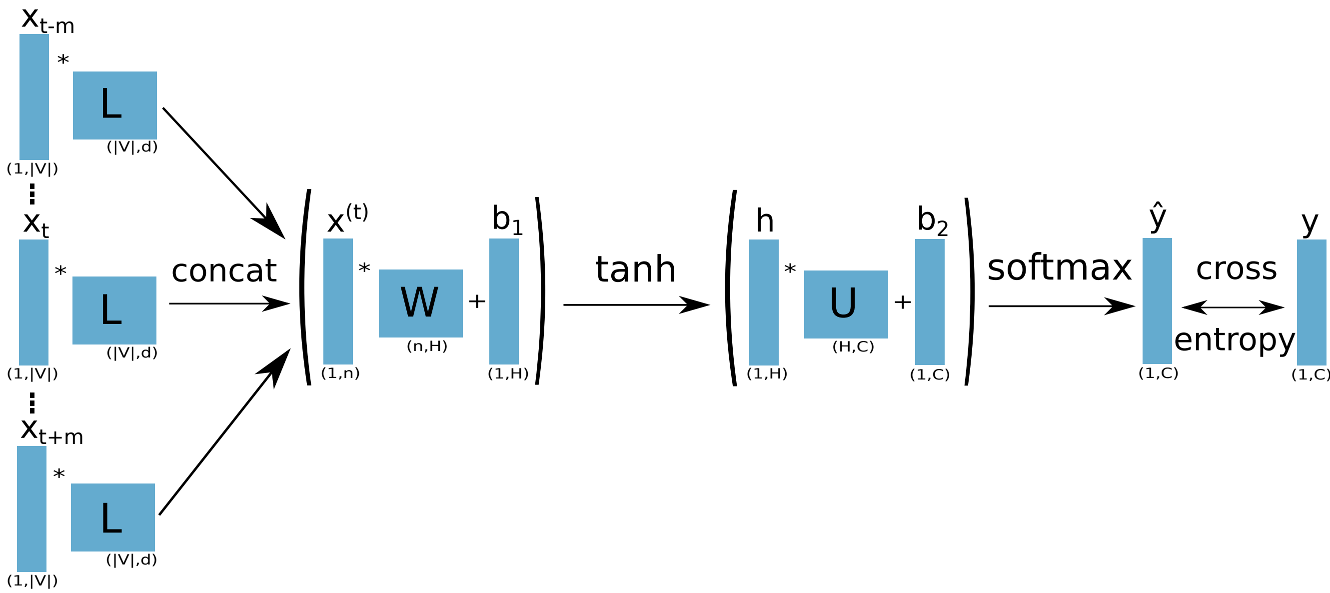


Figure 1: NER window model

entity of type c. Let y be the one-vector representation of c (i.e., $y \in \mathbb{R}^C$ such that $y_i = 1$ iff $i = c$), and let $x_{t-m}, \ldots, x_t, \ldots, x_{t+m} \in \mathbb{R}^{|V|}$ be the one-vector representation of $\mathbb{w}_{t-m}, \ldots, \mathbb{w}_t, \ldots, \mathbb{w}_{t+m}$, respectively. The model is composed by the following equations:

$$x^{(t)} = concat([x_{t-m}L, \ldots, x_tL, \ldots, x_{t+m}L]) \tag{1}$$

$$z = x^{(t)}W + b_1 \tag{2}$$

$$h = tanh(z) \tag{3}$$

$$\hat{y} = softmax(hU + b_2) \tag{4}$$

$$J(W, b_1, U, b_2) = CE(y, \hat{y}) = -\sum_{s=1}^{C} y_s \log(\hat{y}_s) \tag{5}$$

where *concat* is the operation of concatenate function and *tanh* is the hyperbolic tangent function ,i.e., $tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$, we can define this function using the sigmoid function, $tanh(z) = 2sigmoid(2z) - 1$. Figure 1 helps us to visualize the model.

For our particular implementation, let $C = 5$ $d = 50$, $m = 1$, $H = 100$ (hence $n = 150$) and let $J(\theta)$ be an abbreviation of $J(W, b_1, U, b_2)$. Thus, for every training sample the loss function is

$$J(\theta) = CE(y, \hat{y}) = -\sum_{s=1}^{5} y_s \log(\hat{y}_s) \tag{6}$$

2a) Since $tanh(z) = 2\sigma(2z) - 1$ we have that,

$$\begin{aligned}
tanh'(z) &= 2\sigma'(2z)2 \\
&= 4\sigma'(2z) \\
&= 4(\sigma(2z)(1 - \sigma(2z))) \\
&= 4\sigma(2z)(4 - 4\sigma(2z)) \\
&= 2(tanh(z) + 1)(4 - 2(tanh(z) + 1)) \\
&= 2((tanh(z) + 1)(2 - (tanh(z) + 1))) \ .
\end{aligned}$$

Let $\delta^{(3)} = \hat{y} - y \in \mathbb{R}^5$ be the outermost error vector, hence

$$\frac{\partial J}{\partial U} = h{\delta^{(3)}}^T \tag{7}$$

$$\frac{\partial J}{\partial b_2} = \delta^{(3)} \tag{8}$$

$$\delta^{(2)} = (\delta^{(3)} (U)^T) \circ tanh'(z) \tag{9}$$

$$\frac{\partial J}{\partial W} = x^{(t)}{\delta^{(2)}}^T \tag{10}$$

$$\frac{\partial J}{\partial b_1} = \delta^{(2)} \tag{11}$$

First let us define $\frac{\partial J}{\partial L_i}$ for the general case. Using the fact that $\frac{\partial J}{\partial x^{(t)}} = \delta^{(2)}(W)^T$ we will define the auxiliary vectors $v_1, \dots, v_{2m+1} \in \mathbb{R}^d$ such that for $j \in \{1, \dots, 2m+1\}$

$$v_j = \frac{\partial J}{\partial x^{(t)}}[(j-1)d + 1 : jd] \tag{12}$$

Let e be the enumeration function of the list $[t-m, \dots, t, \dots, t+m]$, so for $i \in \{t-m, \dots, t, \dots, t+m\}$

$$\frac{\partial J}{\partial L_i} = v_{e(i)} \tag{13}$$

And for $i \notin \{t-m, \dots, t, \dots, t+m\}$ $\frac{\partial J}{\partial L_i} = 0$. Now for the specific case where $m = 1$,

$$\frac{\partial J}{\partial L_{t-1}} = \frac{\partial J}{\partial x^{(t)}}[1 : d] \tag{14}$$

$$\frac{\partial J}{\partial L_t} = \frac{\partial J}{\partial x^{(t)}}[d + 1 : 2d] \tag{15}$$

$$\frac{\partial J}{\partial L_{t+1}} = \frac{\partial J}{\partial x^{(t)}}[2d + 1 : 3d] \tag{16}$$

And for $i \notin \{t-m, \dots, t, \dots, t+m\}$ $\frac{\partial J}{\partial L_i} = 0$.

2b) To add L2 regularization to our model, we can add the following function:

$$J_{reg}(\theta) = \frac{\lambda}{2}[\sum_{i=1}^{n}\sum_{j=1}^{H}(W_{i,j})^2) + \sum_{i'=1}^{H}\sum_{j'=1}^{C}(U_{i',j'})^2)] \tag{17}$$

where $\lambda \in \mathbb{R}$ is the regularization parameter. Hence,

$$J_{full}(\theta) = J(\theta) + J_{reg}(\theta) \tag{18}$$

The only grandients that change are in respect to U and W. Let $\delta^{(3)}$ and $\delta^{(2)}$ be as before; then,

$$\frac{\partial J_{full}}{\partial U} = h{\delta^{(3)}}^T + \lambda U \tag{19}$$

$$\frac{\partial J_{full}}{\partial b_2} = \frac{\partial J}{\partial b_2} \tag{20}$$

$$\frac{\partial J_{full}}{\partial W} = x^{(t)}{\delta^{(2)}}^T + \lambda W \tag{21}$$

$$\frac{\partial J_{full}}{\partial b_1} = \frac{\partial J}{\partial b_1} \tag{22}$$

And for $i \in \{1, \ldots, |V|\}$

$$\frac{\partial J_{full}}{\partial L_i} = \frac{\partial J}{\partial L_i} \tag{23}$$

2c)

```
def xavier_weight_init():

    def _xavier_initializer(shape, **kwargs):

        # ## YOUR CODE HERE
        epsilon = np.sqrt(6.)/np.sqrt(np.sum(shape))
        out = tf.random_uniform(shape,
                                minval=-epsilon,
                                maxval=epsilon,
                                dtype=tf.float32,
                                name='weights')
        # ## END YOUR CODE
        return out
    return _xavier_initializer
```

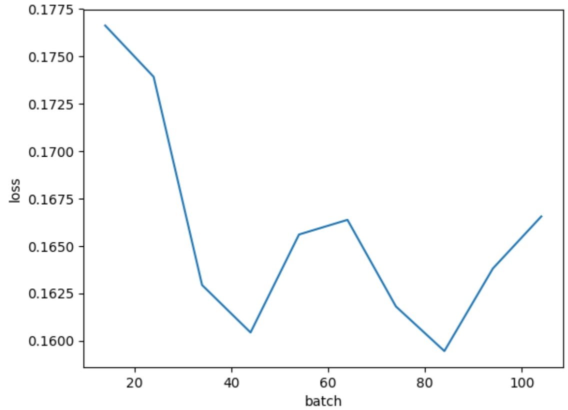
2d)

```
        def add_placeholders(self):
        # ## YOUR CODE HERE
        self.input_placeholder = tf.placeholder(tf.int32,
                                                shape=[None,
                                                       self.config.window_size],
                                                name="input_placeholder")
        self.labels_placeholder = tf.placeholder(tf.float32,
                                                 shape=[None,
                                                        self.config.label_size],
                                                 name="labels_placeholder")
        self.dropout_placeholder = tf.placeholder(tf.float32,
                                                  shape=[],
                                                  name="dropout_value")
        # ## END YOUR CODE

    def create_feed_dict(self, input_batch, dropout, label_batch=None):

        # ## YOUR CODE HERE
        if label_batch is None:
            feed_dict = {self.input_placeholder: input_batch,
                         self.dropout_placeholder: dropout}
        else:
            feed_dict = {self.input_placeholder: input_batch,
                         self.labels_placeholder: label_batch,
                         self.dropout_placeholder: dropout}
        # ## END YOUR CODE
        return feed_dict
```

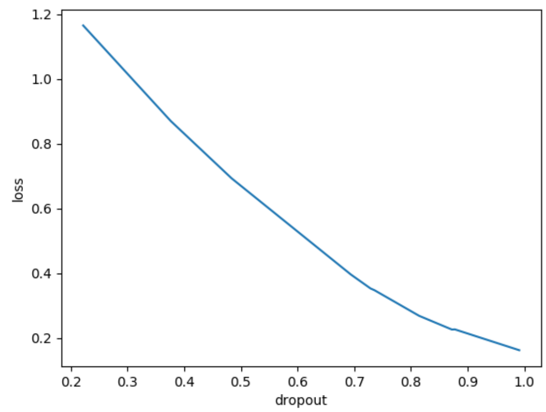
```python
    def add_embedding(self):
        with tf.device('/cpu:0'):
            # ## YOUR CODE HERE
            Linit = tf.constant_initializer(self.wv)
            L = tf.get_variable("L",
                                shape=[len(self.wv), self.config.embed_size],
                                dtype='float32',
                                initializer=Linit)
            window = tf.nn.embedding_lookup(L, self.input_placeholder)
            window = tf.reshape(window,
                                (-1,
                                 self.config.window_size*self.config.embed_size))
        # ## END YOUR CODE
        return window

    def add_model(self, window):

        # ## YOUR CODE HERE
        # shapes
        Wshape = (self.config.window_size*self.config.embed_size,
                  self.config.hidden_size)
        b1shape = (1, self.config.hidden_size)
        Ushape = (self.config.hidden_size, self.config.label_size)
        b2shape = (1, self.config.label_size)

        # initializers
        xavier_initializer = xavier_weight_init()
        Winit = xavier_initializer(Wshape)
        b1init = xavier_initializer(b1shape)
        Uinit = xavier_initializer(Ushape)
        b2init = xavier_initializer(b2shape)

        with tf.variable_scope("Layer"):
            self.W = tf.get_variable("weights",
                                     dtype='float32',
                                     initializer=Winit)
            self.b1 = tf.get_variable("bias",
                                      dtype='float32',
                                      initializer=b1init)
            linear_op = tf.matmul(window, self.W) + self.b1
            first_output = tf.nn.dropout(tf.tanh(linear_op),
                                         self.config.dropout,
                                         name="output")
            tf.add_to_collection("reg", tf.reduce_sum(tf.pow(self.W, 2)))
        with tf.variable_scope("Softmax"):
            self.U = tf.get_variable("weights",
                                     dtype='float32',
                                     initializer=Uinit)
            self.b2 = tf.get_variable("bias",
                                      dtype='float32',
                                      initializer=b2init)
            output = tf.nn.dropout(tf.matmul(first_output, self.U) + self.b2,
```

```
                                       self.config.dropout,
                                       name="output")
            tf.add_to_collection("reg", tf.reduce_sum(tf.pow(self.U, 2)))
        # END YOUR CODE
        return output

        def add_loss_op(self, y):
            # ## YOUR CODE HERE
        pred = self.labels_placeholder
        cross_entropy = tf.reduce_mean(tf.nn.softmax_cross_entropy_with_logits(y, pred))
        regularization = self.config.l2*0.5*sum(tf.get_collection("reg"))
        loss = cross_entropy + regularization
        # ## END YOUR CODE
        return loss

        def add_training_op(self, loss):

        # ## YOUR CODE HERE
        optimizer = tf.train.AdamOptimizer(self.config.lr)
        train_op = optimizer.minimize(loss)
        # ## END YOUR CODE
        return train_op
```

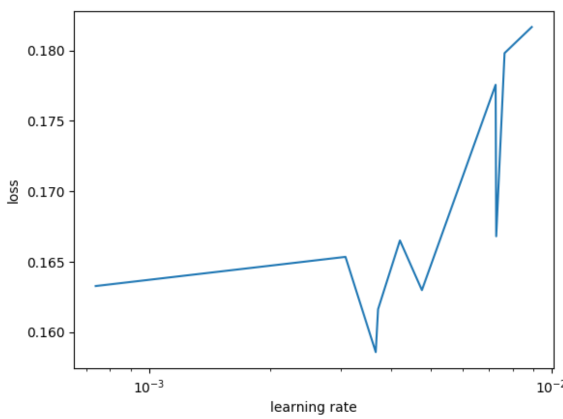
After some experiments (some related plots can be seen in Figure 2) we choose the following hyper parameters: $batch\_size = 84, dropout = 0.991323729933, lr = 0.00365884577219, l2 = 1.7095245617e-05$. This choice yields val_loss = 0.156871527433.



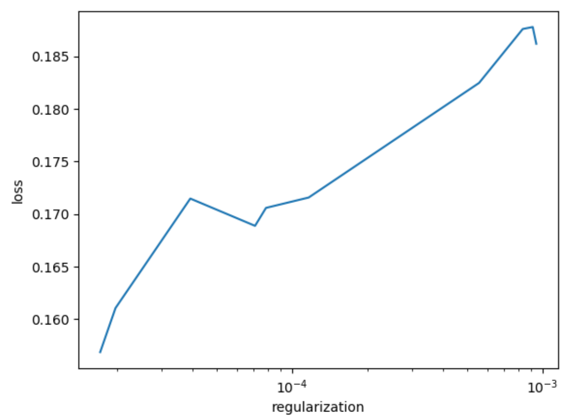
(a) Validation loss vs batch size



(b) Validation loss vs dropout



(c) Validation loss vs learning rate



(d) Validation loss vs regularization

Figure 2: Experiments with the NER window model

3

We will use a **recurrent neural network (RNN)** to build a language model. Given words $x_1, \ldots, x_{n-1}$ a language model predicts the following word x_n by modeling:

$$P(x_n = v_j | x_1, \ldots, x_{n-1})$$

where v_j is a word in the vocabulary. The model can be described as $n-1$ feed-forward neural networks, say $NN^{(1)}, \ldots, NN^{(n-1)}$ such that each $NN^{(t)}$ uses an vector from $NN^{(t-1)}$ to perform the computation on its hidden layer. Since the model is the same, we can describe each $NN^{(t)}$ as a time step. Figure 3 introduces the main idea:

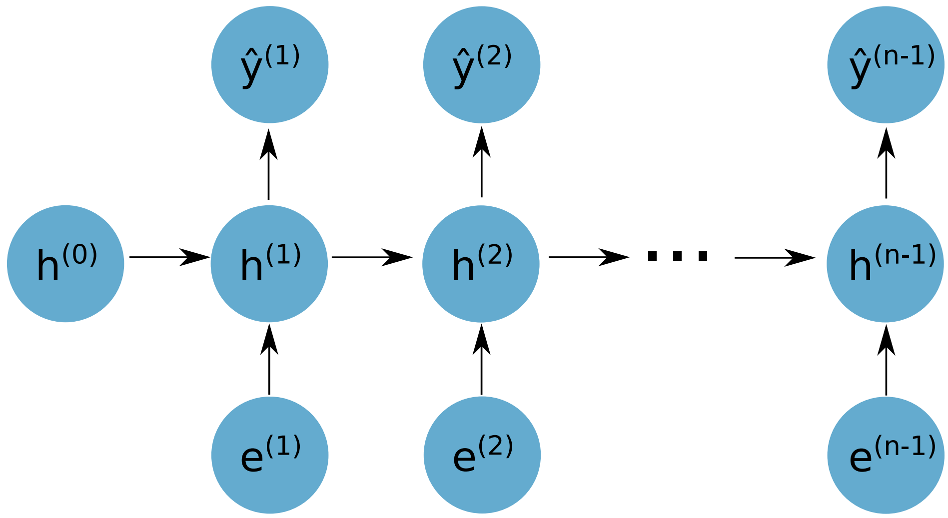


Figure 3: A RNN representation

Since $h^{(n-1)}$ is a function of $h^{(0)}, h^{(1)}, \ldots, h^{(n-2)}$ we can interpret that at the time step $n-1$ the model $NN^{(n-1)}$ has the "history" of the words $x_1, x_2, \ldots, x_{(n-2)}$.

To be more precise, suppose we have a corpus with a vocabulary V of size $|V|$ and a matrix $L \in \mathbb{R}^{|V|,d}$ where each row i correspond to the word embedding of size d of the word x_i. Let D_h be the size of the hidden layer. We use $x^{(i)} \in \mathbb{R}^{|V|}$ to be the one hot representation of x_i. We also define the following parameters: $H \in \mathbb{R}^{D_h,D_h}$ is the hidden transformation matrix, $I \in \mathbb{R}^{d,D_h}$ is the input word representation matrix, $U \in \mathbb{R}^{D_h,|V|}$ is the output word representation matrix, $h^{(0)} \in \mathbb{R}^{D_h}$ is the initialization vector for the hidden layer, and $b_1 \in \mathbb{R}^{D_h}$ and $b_2 \in \mathbb{R}^{|V|}$ are the biases. Let $x_1, \ldots, x_{n-1}$ be a sequence of words, the target word (the one that we are trying to predict) is x_n, so for $t = 1, \ldots, n-1$ the model is defined by the following equations:

$$e^{(t)} = x^{(t)} L \tag{24}$$

$$h^{(t)} = \sigma(h^{(t-1)} H + e^{(t)} I + b_1) \tag{25}$$

$$\hat{y}^{(t)} = softmax(h^{(t)} U + b_2) \tag{26}$$

The output vector $\hat{y}^{(t)} \in \mathbb{R}^{|V|}$ is a probability over the vocabulary,

$$\hat{y}_j^{(t)} = P(x_{t+1} = v_j | x_t, \ldots, x_1) \tag{27}$$

Let $y^{(t)} \in \mathbb{R}^{|V|}$ be the one hot representation of x_{t+1}, then the point-wise loss is

$$J(\theta)^{(t)} = CE(y^{(t)}, \hat{y}^{(t)}) \tag{28}$$

In order to evaluate the model performance we need to compute the loss for the whole dataset. Suppose we have a dataset as a collection of N words sequences, say $(x_{1,1}, \ldots, x_{1,n_1}), \ldots, (x_{N,1}, \ldots, x_{1,n_N})$ (and let $m = \sum_{i=1}^{N} n_i$). For each $i = 1, \ldots, n_i$ we calculate:

$$J^{(i)} = \sum_{k=1}^{(n_i-1)} CE(y^{(i,k)}, \hat{y}^{((i,k))}) \tag{29}$$

where $y^{(i,s)}$ is the one hot representation of $x_{i,s+1}$. It should be noted that for each i we compute $h_0, h_1, \ldots, h_{n_i}$. Since it does not make sense to use this for the next dataset entry, I think that we set

h_0 with random numbers again and delete $h_1, \dots, h_{n_i}$. Hence the cross-entropy error over the dataset is:

$$J = -\frac{1}{m} \sum_{i=1}^{N} J^{(i)} \tag{30}$$

The dataset can be a corpus with T words, say $x_1, x_2, \dots, x_T$. As before for $t = 1, \dots, T-1$ $y^{(t)}$ is the one hot representation of x_{t+1}. So the cross-entropy error over a corpus of size T is

$$J = -\frac{1}{T} \sum_{t=1}^{T} CE(y^{(t)}, \hat{y}^{((t))}) \tag{31}$$

3a) First, let $j^* \in \{1, \dots, |V|\}$ be the hot index from $y^{(t)}$, now consider:

$$\begin{aligned}
J^{(t)} &= CE(y^{(t)}, \hat{y}^{((t))}) \\
&= -\sum_{j=1}^{|V|} y_j \log(\hat{y}_j) \\
&= -\log(\hat{y}_{j^*}) \\
&= \log(1) - \log(\hat{y}_{j^*}) \\
&= \log(\frac{1}{\hat{y}_{j^*}}) \\
&= \log(\frac{1}{\sum_{j=1}^{|V|} y_j \hat{y}_j}) \\
&= \log(\frac{1}{P(x_{t+1}^{pred} = x_{t+1} | x_t, \dots, x_1)}) \\
&= \log(PP^{(t)}(y^{(t)}, \hat{y}^{((t))})) \; .
\end{aligned}$$

Therefore,

$$PP^{(t)}(y^{(t)}, \hat{y}^{((t))}) = e^{J^{(t)}} \tag{32}$$

With (32) we got the following equality:

$$\frac{1}{T} \sum_{t=1}^{T} J^{(t)} = \log((\prod_{t=1}^{T} PP^{(t)})^{\frac{1}{T}}) \tag{33}$$

And so when we minimize $\frac{1}{T} \sum_{t=1}^{T} J^{(t)}$ we also minimize $(\prod_{t=1}^{T} PP^{(t)})^{\frac{1}{T}}$.

Now, suppose that our model is completely random. Then for each sequence $x_1, \dots, x_t$ $P(x_{t+1}^{pred} = x_{t+1} | x_t, \dots, x_1) = \frac{1}{|V|}$. Thus,

$$PP^{(t)} = e^{-\log(\frac{1}{|V|})}$$

In a similar way, the general cross-entropy loss J becomes a function of $|V|$:

$$J(|V|) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)} = \frac{1}{T} \sum_{t=1}^{T} -\log(\frac{1}{|V|}) = -\log(\frac{1}{|V|})$$

Hence, for $|V| = 2000$, $J = 7.6009024595420822$ and $PP = e^J = 1999.9999999999998$. Similarly, for $|V| = 10000$, $J = 9.2103403719761818$ and $PP = e^J = 9999.9999999999909$.

3b) First, to make things simple, we will add a new variable. So, for at time t we have the model:

$$e^{(t)} = x^{(t)}L \tag{34}$$
$$z^{(t)} = h^{(t-1)}H + e^{(t)}I + b_1 \tag{35}$$
$$h^{(t)} = \sigma(z^{(t)}) \tag{36}$$
$$\hat{y}^{(t)} = softmax(h^{(t)}U + b_2) \tag{37}$$

We will assume that $h^{(t-1)}$ if fixed, i.e., this vector does not depend on H, I and b_1 (note that is only true for $t = 1$). This assumption change the gradients with respect to H, I and b_1. To make this assumption explicit we use the notation $\frac{\partial J^{(t)}}{\partial H}|_{(t)}$, $\frac{\partial J^{(t)}}{\partial I}|_{(t)}$ and $\frac{\partial J^{(t)}}{\partial b_1}|_{(t)}$. Now, let $\gamma^{(t,3)} = \hat{y}^{(t)} - y^{(t)}$ be the outermost error vector, hence

$$\frac{\partial J^{(t)}}{\partial U} = h^{(t)}\gamma^{(t,3)^T} \tag{38}$$

$$\frac{\partial J^{(t)}}{\partial b_2} = \gamma^{(t,3)} \tag{39}$$

$$\gamma^{(t,2)} = (\gamma^{(t,3)}(U)^T) \circ \sigma'(z^{(t)}) \tag{40}$$

$$\frac{\partial J^{(t)}}{\partial I}|_{(t)} = e^{(t)}\gamma^{(t,2)^T} \tag{41}$$

$$\frac{\partial J^{(t)}}{\partial b_1}|_{(t)} = \gamma^{(t,2)} \tag{42}$$

$$\frac{\partial J^{(t)}}{\partial H}|_{(t)} = h^{(t-1)}\gamma^{(t,2)^T} \tag{43}$$

$$\frac{\partial J^{(t)}}{\partial L_{x^{(t)}}} = \frac{\partial J^{(t)}}{\partial e^{(t)}} = \gamma^{(t,2)}(I)^T \tag{44}$$

$$\frac{\partial J^{(t)}}{\partial h^{(t-1)}} = \gamma^{(t,2)}(H)^T \tag{45}$$

3c) Figure 4 shows a detailed version of a RNN for 3 time steps.

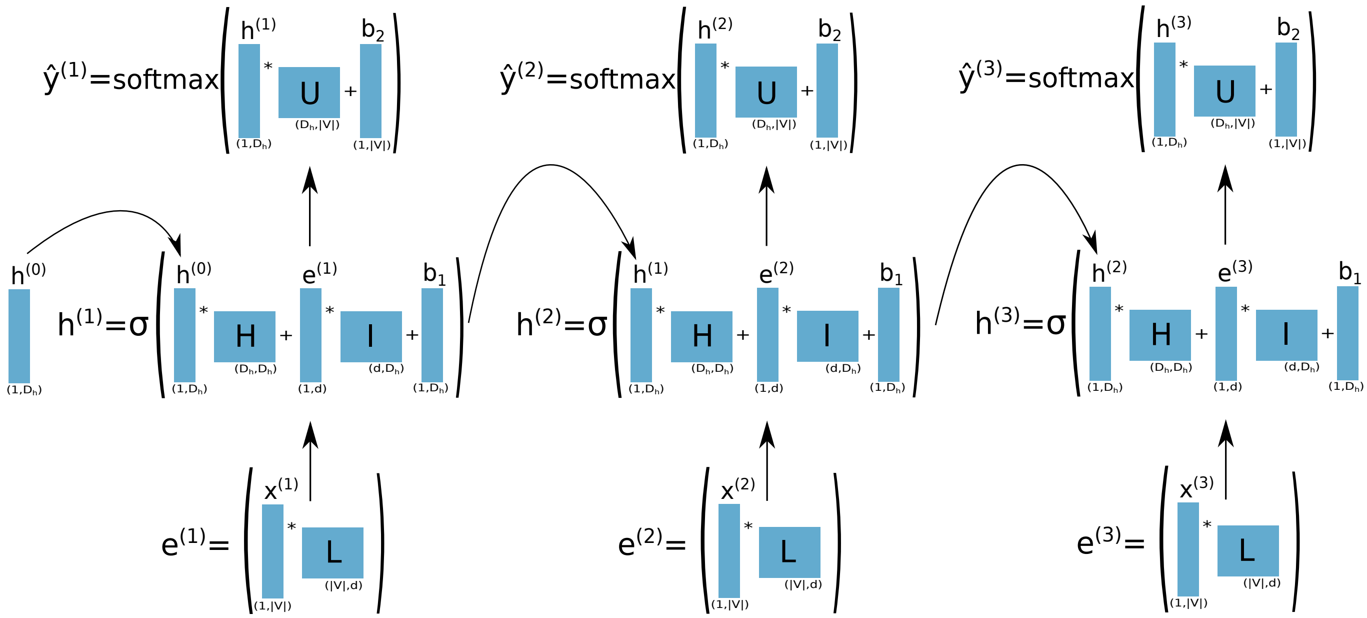


Figure 4: A detailed representation of RNN for 3 time steps

This is a non-vectorized formulation.
For $j \in \{1, \ldots, d\}$:

$$\frac{\partial J^{(t)}}{\partial L_{x^{(t-1)}}{}_j} = \frac{\partial J^{(t)}}{\partial e_j^{(t-1)}} = \sum_{i=1}^{D_h} \frac{\partial J^{(t)}}{\partial h_i^{(t-1)}} \frac{\partial h_i^{(t-1)}}{\partial e_j^{(t-1)}} = \sum_{i=1}^{D_h} \delta_i^{(t-1)} I_{i,j} \tag{46}$$

For $i, j \in \{1, \ldots, D_h\}$:

$$\frac{\partial J^{(t)}}{\partial H_{i,j}}|_{(t-1)} = \frac{\partial J^{(t)}}{\partial z_j^{(t)}} \frac{\partial z_j^{(t)}}{\partial H_{i,j}}|_{(t-1)} \tag{47}$$

hence

$$\frac{\partial J^{(t)}}{\partial H_{i,j}}|_{(t-1)} = \gamma_j^{(t,2)}(h_i^{(t-1)} + H_{j,j}\sigma'(z_j^{(t-1)})h_i^{(t-2)}) \tag{48}$$

For $i \in \{1, \ldots, d\}$ and $j \in \{1, \ldots, D_h\}$:

$$\frac{\partial J^{(t)}}{\partial I_{i,j}}|_{(t-1)} = \frac{\partial J^{(t)}}{\partial z_j^{(t)}} \frac{\partial z_j^{(t)}}{\partial I_{i,j}}|_{(t-1)} \tag{49}$$

thus

$$\frac{\partial J^{(t)}}{\partial I_{i,j}}|_{(t-1)} = \gamma_j^{(t,2)}(e_i^{(t)} + H_{j,j}\sigma'(z_j^{(t-1)})e_i^{(t-1)}) \tag{50}$$

And for $j \in \{1, \ldots, D_h\}$:

$$\frac{\partial J^{(t)}}{\partial b_{1j}}|_{(t-1)} = \frac{\partial J^{(t)}}{\partial z_j^{(t)}} \frac{\partial z_j^{(t)}}{\partial b_{1j}}|_{(t-1)} \tag{51}$$

thus

$$\frac{\partial J^{(t)}}{\partial b_{1j}}|_{(t-1)} = \gamma_j^{(t,2)}(1 + H_{j,j}\sigma'(z_j^{(t-1)})) \tag{52}$$

3d) Give $h^{(t-1)}$, in one step of forward propagation we perform 4 matrix multiplication, 3 vector sums, one application of the sigmoid function in a vector of size D_h and one application of the softmax function in a vector of size $|V|$. Let $f(D_h)$ and $g(|V|)$ be the cost of sigmoid function and the softmax function in that vectors, respectively. We can then express the cost with the equation:

$$O(|V|d + D_h D_h + dD_h + 2D_h + f(D_h) + D_h|V| + |V| + g(|V|)) \tag{53}$$

Regarding backpropagation, assuming that $h^{(t-1)}$ if fixed, for each single step we perform 4 outer products, one Hadamard product, 2 matrix multiplication and one application of σ' in a vector of size D_h (let $f^*(D_h)$ be the cost of this application). Let α be the following sum:

$$\alpha = 2D_h|V| + D_h + f^*(D_h) + 2dD_h + 2D_h D_h \tag{54}$$

hence, for a backpropagation for a single step we have:

$$O(\alpha) \tag{55}$$

and for τ steps

$$O(\tau\alpha) \tag{56}$$

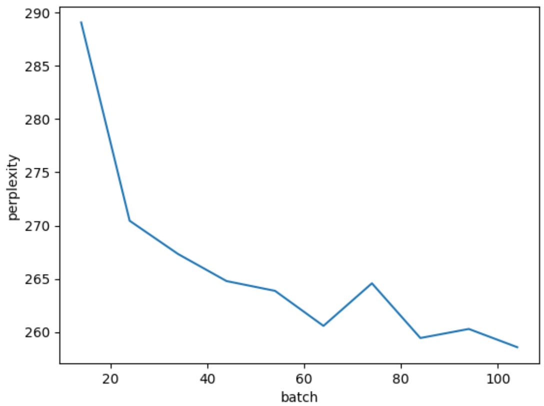
3e)

```python
def add_placeholders(self):
        # ## YOUR CODE HERE
        self.input_placeholder = tf.placeholder(tf.int32,
                                                shape=[None,
                                                       self.config.num_steps],
                                                name="input_placeholder")
        self.labels_placeholder = tf.placeholder(tf.int64,
                                                 shape=[None,
                                                        self.config.num_steps],
                                                 name="labels_placeholder")
        self.dropout_placeholder = tf.placeholder(tf.float32,
                                                  shape=[],
                                                  name="dropout_value")

        # ## END YOUR CODE
def add_embedding(self):
        with tf.device('/cpu:0'):
            # ## YOUR CODE HERE
            Lshape = (len(self.vocab), self.config.embed_size)
            L = tf.get_variable("L", shape=Lshape)
            look = tf.nn.embedding_lookup(L, self.input_placeholder)
            split = tf.split(1, self.config.num_steps, look)
            inputs = [tf.squeeze(tensor, squeeze_dims=[1]) for tensor in split]
            # ## END YOUR CODE
            return inputs

def add_projection(self, rnn_outputs):

        # ## YOUR CODE HERE
```

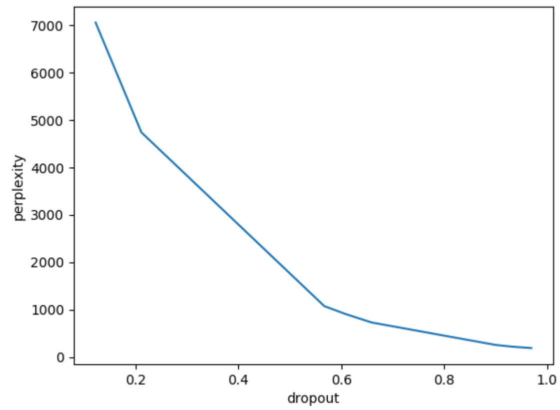
```python
        # shapes
        Ushape = (self.config.hidden_size, len(self.vocab))
        b2shape = (1, len(self.vocab))

        with tf.variable_scope("Projection_layer"):
            self.U = tf.get_variable("weights", shape=Ushape)
            self.b2 = tf.get_variable("bias", shape=b2shape)
            outputs = [tf.matmul(tensor, self.U) + self.b2
                       for tensor in rnn_outputs]

        # ## END YOUR CODE
        return outputs

def add_loss_op(self, output):
        # ## YOUR CODE HERE
        loss = sequence_loss([output],
                             [tf.reshape(self.labels_placeholder,
                              [self.config.batch_size * self.config.num_steps,
                               -1])],
                             [tf.constant(1.0)])
        # ## END YOUR CODE
        return loss

def add_training_op(self, loss):
        # ## YOUR CODE HERE
        optimizer = tf.train.AdamOptimizer(self.config.lr)
        train_op = optimizer.minimize(loss)
        # ## END YOUR CODE
        return train_op

def add_model(self, inputs):
        # ## YOUR CODE HERE

        rnn_outputs = []

        # shapes
        initialshape = (self.config.batch_size, self.config.hidden_size)
        Hshape = (self.config.hidden_size, self.config.hidden_size)
        Ishape = (self.config.embed_size, self.config.hidden_size)
        b1shape = (1, self.config.hidden_size)

        # initializers
        self.initial_state = tf.zeros(initialshape)

        with tf.variable_scope("RNN"):
            self.H = tf.get_variable("hidden_weights", shape=Hshape)
            self.I = tf.get_variable("input_weights", shape=Ishape)
            self.b1 = tf.get_variable("bias", shape=b1shape)

        previous_h = self.initial_state
```

```python
for i, tensor in enumerate(inputs):
    with tf.variable_scope("RNN", reuse=True):
        drop_tensor = tf.nn.dropout(tensor, self.config.dropout)
        h = tf.sigmoid(tf.matmul(previous_h, self.H) +
                       (tf.matmul(drop_tensor, self.I) + self.b1))
        h = tf.nn.dropout(h, self.config.dropout)
        rnn_outputs.append(h)
        previous_h = h
        if i == (len(inputs)-1):
            self.final_state = h
# ## END YOUR CODE
return rnn_outputs
```

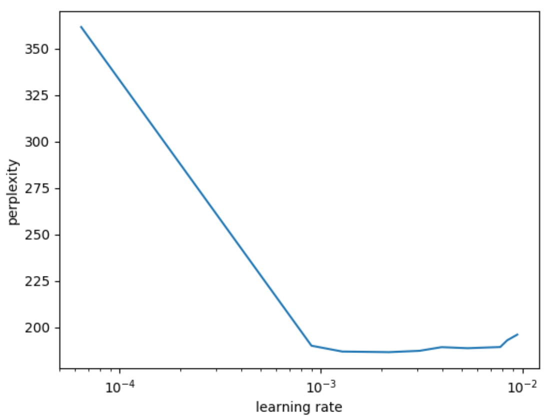
After some experiments we choose the following hyper parameters: $batch\_size = 104, dropout = 0.991323729933, lr = 0.00217346380124, num\_steps = 14$. This choice yields validation perplexity $= 163.170974731$. As can be seen in the plots from Figure 5 we could continue searching better hyper parameters, but due the lack of time (we use CPU only) we decide to stop the search at those parameters.



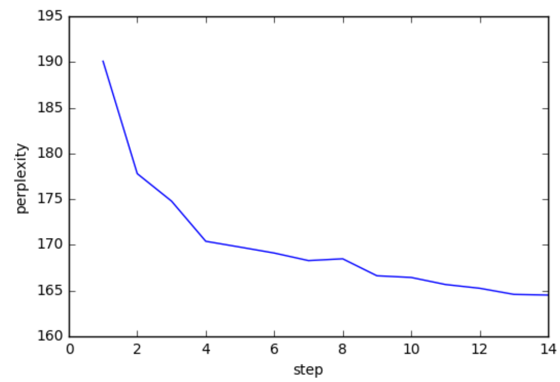
(a) Perplexity vs batch size



(b) Perplexity vs dropout



(c) Perplexity vs learning rate



(d) Perplexity vs step size

Figure 5: Experiments with the RNN language model

3f)

```
def generate_text(session, model, config, starting_text='<eos>',
                  stop_length=100, stop_tokens=None, temp=1.0):
    state = model.initial_state.eval()
    tokens = [model.vocab.encode(word) for word in starting_text.split()]
    for i in xrange(stop_length):
        # ## YOUR CODE HERE
        feed = {model.input_placeholder: [[tokens[-1]]],
                model.initial_state: state,
                model.dropout_placeholder: 1.0}
        state, y_pred = session.run([model.final_state, model.predictions[-1]],
                                    feed_dict=feed)
        # ## END YOUR CODE
```

Examples of generated sentence:

- > **sex is:** sex is chairman of two-year operation <eos>
- > **violence is:** violence is expected well but by the german social concept of public conditions around what if the sec <unk> with fujis at greenville odd a <unk> formerly won away with editor of the federal court term the news spokeswoman says mr. <unk> said the <unk> 's is one location of the development of a new active position he says dick green louisville ky. representatives of l.j. hooker maker increased N N to N at N <eos>
- > **this is a great:** this is a great japanese she had the president maker was the ceo for a time when i do n't any shared with potential but going to be hampered by the noriega is that last winter <eos>