A Diversion on Binding Values to Symbol

How does R know which value to assign to which symbol? When I type

```
> lm <- function(x) { x * x }
> lm
function(x) { x * x }
```

how does R know what value to assign to the symbol `lm`? Why doesn't it give it the value of `lm` that is in the **stats** package?

A Diversion on Binding Values to Symbol

When R tries to bind a value to a symbol, it searches through a series of `environments` to find the appropriate value. When you are working on the command line and need to retrieve the value of an R object, the order is roughly

1. Search the global environment for a symbol name matching the one requested.
2. Search the namespaces of each of the packages on the search list

The search list can be found by using the `search` function.

```
> search()
[1] ".GlobalEnv"          "package:stats"       "package:graphics"
[4] "package:grDevices" "package:utils"       "package:datasets"
[7] "package:methods"    "Autoloads"           "package:base"
```

Binding Values to Symbol

- The *global environment* or the user's workspace is always the first element of the search list and the **base** package is always the last.
- The order of the packages on the search list matters!
- User's can configure which packages get loaded on startup so you cannot assume that there will be a set list of packages available.
- When a user loads a package with `library` the namespace of that package gets put in position 2 of the search list (by default) and everything else gets shifted down the list.
- Note that R has separate namespaces for functions and non-functions so it's possible to have an object named `c` and a function named `c`.

Scoping Rules

The scoping rules for R are the main feature that make it different from the original S language.

- The scoping rules determine how a value is associated with a free variable in a function
- R uses *lexical scoping* or *static scoping*. A common alternative is *dynamic scoping*.
- Related to the scoping rules is how R uses the *search list* to bind a value to a symbol
- Lexical scoping turns out to be particularly useful for simplifying statistical computations

Lexical Scoping

Consider the following function.

```
f <- function(x, y) {
        x^2 + y / z
}
```

This function has 2 formal arguments `x` and `y`. In the body of the function there is another symbol `z`. In this case `z` is called a *free variable*.
The scoping rules of a language determine how values are assigned to free variables. Free variables are not formal arguments and are not local variables (assigned insided the function body).

Lexical Scoping

Lexical scoping in R means that

> *the values of free variables are searched for in the environment in which the function was defined.*

What is an environment?

- An *environment* is a collection of (symbol, value) pairs, i.e. `x` is a symbol and `3.14` might be its value.
- Every environment has a parent environment; it is possible for an environment to have multiple "children"
- the only environment without a parent is the empty environment
- A function + an environment = a *closure* or *function closure*.

Lexical Scoping

Searching for the value for a free variable:

- If the value of a symbol is not found in the environment in which a function was defined, then the search is continued in the *parent environment*.
- The search continues down the sequence of parent environments until we hit the *top-level environment*; this usually the global environment (workspace) or the namespace of a package.
- After the top-level environment, the search continues down the search list until we hit the *empty environment*.
- If a value for a given symbol cannot be found once the empty environment is arrived at, then an error is thrown.

Why does all this matter?

- Typically, a function is defined in the global environment, so that the values of free variables are just found in the user's workspace
- This behavior is logical for most people and is usually the "right thing" to do
- However, in R you can have functions defined *inside other functions*
  - Languages like C don't let you do this
- Now things get interesting — In this case the environment in which a function is defined is the body of another function!

Lexical Scoping

```
make.power <- function(n) {
        pow <- function(x) {
                x^n
        }
        pow
}
```

This function returns another function as its value.

```
> cube <- make.power(3)
> square <- make.power(2)
> cube(3)
[1] 27
> square(3)
[1] 9
```

Exploring a Function Closure

What's in a function's environment?

```
> ls(environment(cube))
[1] "n"   "pow"
> get("n", environment(cube))
[1] 3

> ls(environment(square))
[1] "n"   "pow"
> get("n", environment(square))
[1] 2
```

Lexical vs. Dynamic Scoping

```
y <- 10

f <- function(x) {
        y <- 2
        y^2 + g(x)
}

g <- function(x) {
        x * y
}
```

What is the value of

```
f(3)
```

Lexical vs. Dynamic Scoping

- With lexical scoping the value of `y` in the function `g` is looked up in the environment in which the function was defined, in this case the global environment, so the value of `y` is 10.
- With dynamic scoping, the value of `y` is looked up in the environment from which the function was *called* (sometimes referred to as the *calling environment*).
    - In R the calling environment is known as the *parent frame*

So the value of `y` would be 2.

Lexical vs. Dynamic Scoping

When a function is *defined* in the global environment and is subsequently *called* from the global environment, then the defining environment and the calling environment are the same. This can sometimes give the appearance of dynamic scoping.

```
> g <- function(x) {
+          a <- 3
+          x + a + y
+ }
> g(2)
Error in g(2) : object "y" not found
> y <- 3
> g(2)
[1] 8
```

Other Languages

Other languages that support lexical scoping

- Scheme
- Perl
- Python
- Common Lisp (all languages converge to Lisp)

Consequences of Lexical Scoping

- In R, all objects must be stored in memory
- All functions must carry a pointer to their respective defining environments, which could be anywhere
- In S-PLUS, free variables are always looked up in the global workspace, so everything can be stored on the disk because the "defining environment" of all functions is the same.

Application: Optimization

Why is any of this information useful?

- Optimization routines in R like `optim`, `nlm`, and `optimize` require you to pass a function whose argument is a vector of parameters (e.g. a log-likelihood)
- However, an object function might depend on a host of other things besides its parameters (like *data*)
- When writing software which does optimization, it may be desirable to allow the user to hold certain parameters fixed

Maximizing a Normal Likelihood

Write a "constructor" function

```
make.NegLogLik <- function(data, fixed=c(FALSE,FALSE)) {
        params <- fixed
        function(p) {
                params[!fixed] <- p
                mu <- params[1]
                sigma <- params[2]
                a <- -0.5*length(data)*log(2*pi*sigma^2)
                b <- -0.5*sum((data-mu)^2) / (sigma^2)
                -(a + b)
        }
}
```

Note: Optimization functions in R *minimize* functions, so you need to use the negative log-likelihood.

Maximizing a Normal Likelihood

```
> set.seed(1); normals <- rnorm(100, 1, 2)
> nLL <- make.NegLogLik(normals)
> nLL
function(p) {
                params[!fixed] <- p
                mu <- params[1]
                sigma <- params[2]
                a <- -0.5*length(data)*log(2*pi*sigma^2)
                b <- -0.5*sum((data-mu)^2) / (sigma^2)
                -(a + b)
        }
<environment: 0x165b1a4>
> ls(environment(nLL))
[1] "data"   "fixed"  "params"
```

```
> optim(c(mu = 0, sigma = 1), nLL)$par
      mu    sigma
1.218239 1.787343
```

Fixing $\sigma = 2$

```
> nLL <- make.NegLogLik(normals, c(FALSE, 2))
> optimize(nLL, c(-1, 3))$minimum
[1] 1.217775
```

Fixing $\mu = 1$

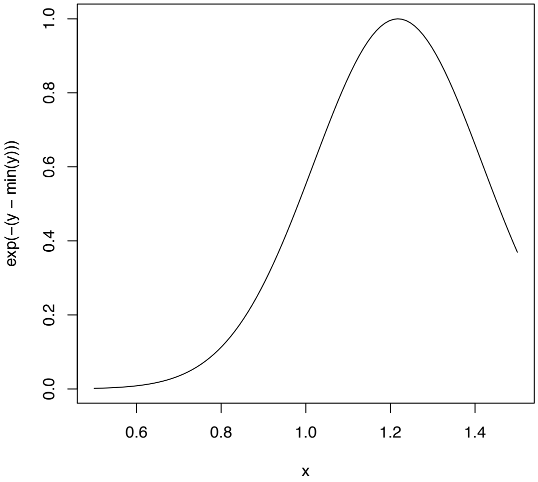
```
> nLL <- make.NegLogLik(normals, c(1, FALSE))
> optimize(nLL, c(1e-6, 10))$minimum
[1] 1.800596
```

Plotting the Likelihood

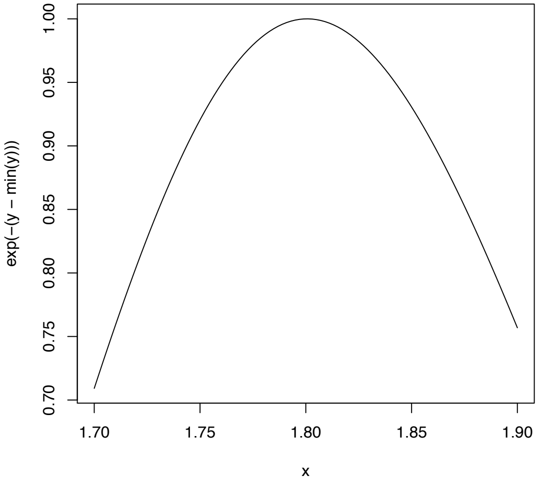
```
nLL <- make.NegLogLik(normals, c(1, FALSE))
x <- seq(1.7, 1.9, len = 100)
y <- sapply(x, nLL)
plot(x, exp(-(y - min(y))), type = "l")

nLL <- make.NegLogLik(normals, c(FALSE, 2))
x <- seq(0.5, 1.5, len = 100)
y <- sapply(x, nLL)
plot(x, exp(-(y - min(y))), type = "l")
```

Plotting the Likelihood



Plotting the Likelihood



Lexical Scoping Summary

- Objective functions can be “built” which contain all of the necessary data for evaluating the function
- No need to carry around long argument lists — useful for interactive and exploratory work.
- Code can be simplified and cleand up
- Reference: Robert Gentleman and Ross Ihaka (2000). “Lexical Scope and Statistical Computing,” *JCGS*, 9, 491–508.