MSCA 31000 - Introduction to Statistical Concepts

Chapter 10

Q1: When would the mean grade in a class on a final exam be considered a statistic? When would it be considered a parameter?

Answer: A parameter is a value calculated in a population. A statistic is a value computed in a sample to estimate a parameter.

In this case, a sample mean grade would be considered a statistic. The population mean grade would be considered a parameter.

Q5: When you construct a 95% confidence interval, what are you 95% confident about?

Answer: We are confident that 95% of the intervals would contain the population mean.

Q10: The effectiveness of a blood-pressure drug is being investigated. How might an experimenter demonstrate that, on average, the reduction in systolic blood pressure is 20 or more?

Answer: This can be done using the Difference between Means

In order to construct a confidence interval, we are going to make three assumptions: The two populations have the same variance. This assumption is called the assumption of homogeneity of variance. The populations are normally distributed. Each value is sampled independently from each other value.

1. Create samples S1 and S2 with sizes N1 and N2, where N1 = N2 = n
2. Calculate sample means (M_1 and M_2), Differences between Sample Means ($M_1 - M_2$) and Sample Variances (s_1^2 and s_2^2)
3. Calculate Mean Square Error (MSE) = $(s_1^2 + s_2^2) / 2$
4. Calculate sample standard dev ($s_{M1} - s_{M2}$) = sqrt(2 * MSE/ n)
5. Set confidence interval (say 99%)
6. Calculate $t_{CL.99}$ conversion factor using inverse t distribution for df = n - 2
7. Calculate Limits of confidence interval using
   Lower Limit = $(M_1 - M_2) - t_{CL.99} (s_{M1} - s_{M2})$
   Upper Limit = $(M_1 - M_2) + t_{CL.99} (s_{M1} - s_{M2})$

Q15: You take a sample of 22 from a population of test scores, and the mean of your sample is 60. (a) You know the standard deviation of the population is 10. What is the 99% confidence interval on the population mean. (b) Now assume that you do not know the population standard deviation, but the standard deviation in your sample is 10. What is the 99% confidence interval on the mean now?

Answer:

a. Std deviation of population is known

N = 22
sample mean (M) = 60
population std dev (σ) = 10
standard error (σ_M) = σ / sqrt(N) = 10/sqrt(22) = 2.132
$Z_{.99}$ = 2.58 (using normal distribution)

99% confidence interval on the population mean

Lower Limit = 60 - σ_M* $Z_{.99}$ = **54.5**
Upper Limit = 60 + σ_M* $Z_{.99}$ = **65.5**

Cross-checking by running qnorm(c(0.005,.995),mean=60,2.132), we get the interval limits as **54.5**, **65.5**

b. When Std Dev of population is not known, but std dev of the sample is given

N = 22
degrees of freedom = 21
sample mean (M) = 60
standard error (s_M) = 10 / sqrt(N) = 2.132
$t_{CL.99}$ = 2.831 (using inverse t distribution)

99% confidence interval on the mean

Lower Limit = 60 - s_M* $t_{CL.99}$ = **53.96**
Upper Limit = 60 + s_M* $t_{CL.99}$ = **66.04**

Q20: True/false: You have a sample of 9 men and a sample of 8 women. The degrees of freedom for the t value in your confidence interval on the difference between means is 16.

Answer: False. For difference of means, the degree of freedom is (n1 - 1) + (n2 - 1). So, here it should be (9 - 1) + (8 - 1) = 15.

Chapter 11

Q4: State the null hypothesis for:

a. An experiment testing whether echinacea decreases the length of colds.
b. A correlational study on the relationship between brain size and intelligence.
c. An investigation of whether a self-proclaimed psychic can predict the outcome of a coin flip.
d. A study comparing a drug with a placebo on the amount of pain relief. (A one-tailed test was used.)

Answer:

a. H0: Echinacea doesn't impact length of colds
b. H0: Brain size and intelligence are not correlated
c. H0: A self-proclaimed psychic can predict the outcome of a coin flip
d. H0: The drug provides less pain relief than the placebo.

Q8: A significance test is performed and p = .20. Why can't the experimenter claim that the probability that the null hypothesis is true is .20?

Answer: The probability value is the probability of a result as extreme or more extreme given that the null hypothesis is true. It (p-value = 0.2) is the probability of the data given the null hypothesis. It is not the probability that the null hypothesis is false or true. Hence, the experimenter cannot claim that the probability of the null hypothesis being true is 0.2.

Q14: Why is H0: M1 = M2 not a proper null hypothesis?

Answer: It is not a proper null hypothesis because the chances of getting two sample means the same is limited. This would be a proper null hypothesis if the stated hypothesis was at the population mean level.

Q18: You choose an alpha level of .01 and then analyze your data. (a) What is the probability that you will make a Type I error given that the null hypothesis is true? (b) What is the probability that you will make a Type I error given that the null hypothesis is false?

Answer:

a) .01
b) 0

Chapter 12

Q8: Participants threw darts at a target. In one condition, they used their preferred hand; in the other condition, they used their other hand. All subjects performed in both conditions (the order of conditions was counterbalanced). Their scores are shown below.

Preferred	Non-preferred
12	7
7	9
11	8
13	10
10	9

a. Which kind of t-test should be used?
b. Calculate the two-tailed t and p values using this t test.
c. Calculate the one-tailed t and p values using this t test.

Answer:

a. Correlated t-test, since there is only one group of subjects in both cases

- mean of differences = 2
- std dev of difference = 2.65
- # of samples (N) = 5
- degrees of freedom = 4
- S_M = 2.65/sqrt(5) = 1.185

b. In R: t.test(c(ch$preferred), c(ch$nonpreferred), paired=T)
t = 1.69, p-value = 0.1662

c. In R: t.test(c(ch$preferred), c(ch$nonpreferred), alternative = c("less"), paired=T)
t = 1.69, p-value = 0.9169

Q9: Assume the data in the previous problem were collected using two different groups of subjects: One group used their preferred hand and the other group used their non-preferred hand. Analyze the data and compare the results to those for the previous problem

Answer:

	n	mean	variance
preferred	5	10.6	5.3
nonpreferred	5	8.6	1.3

N = 5
M1 - M2 = 2
MSE = 3.3
$S_{M1 - M2}$ = sqrt(2*MSE/N) = 1.15

1. Two-tailed test (t.test(c(ch$preferred), c(ch$nonpreferred)))
   t = 1.7408
   p-value = 0.1336

2. One-tailed test (t.test(c(ch$preferred), c(ch$nonpreferred), alternative=c("l")))
   t = 1.7408
   p-value = 0.9332

Comparing to previous problem Q8,
- the t values in Q9 are higher
- p-value for Q8, two-tail is higher than corresponding for Q9
- p-value for Q9, one-tail is higher than corresponding for Q8

Q11: In an experiment, participants were divided into 4 groups. There were 20 participants in each group, so the degrees of freedom (error) for this study was 80 - 4 = 76. Tukey's HSD test was performed on the data. (a) Calculate the p value for each pair based on the Q value given below. (b) Which differences are significant at the .05 level?

Comparison of Groups	Q
A - B	3.4
A - C	3.8
A - D	4.3
B - C	1.7
B - D	3.9
C - D	3.7

Answer: next page

Comparison of Groups	Q	p-value	significant at 0.05 level
A - B	3.4	0.0849	no
A - C	3.8	0.043	yes
A - D	4.3	0.0167	yes
B - C	1.7	0.6274	no
B - D	3.9	0.0359	yes
C - D	3.7	0.0513	no

Chapter 13

Q1: Define power in your own words.

Answer: Power is the probability of correctly rejecting the null hypothesis when the null hypothesis is false. Equivalently, it is the probability of accepting the alternate hypothesis when the alternate hypothesis is true.

Adapted from Wikipedia.

Q2: List 3 measures one can take to increase the power of an experiment. Explain why your measures result in greater power.

Answer:

1. Sample size: A larger sample size has a greater potential of resulting in an accurate test. A larger sample size reduces the standard error of a sample, thereby resulting in increased power.
2. Standard deviation: Smaller value of standard deviation for the same sample size results in increased power. This is because the variability of the sample is decreased and the probability of the population mean falling within the range of the sample mean is increased.
3. One- versus Two-Tailed Tests: With the assumption that the hypothesized direction is correct, a one-tailed test results in a greater confidence interval in that direction. This is conducive for the population mean to fall within the range of the mean and std dev in that single direction since more area is covered.

Q3: Population 1 mean = 36

Population 2 mean = 45

Both population standard deviations are 10.

Sample size (per group) 16.

What is the probability that a t test will find a significant difference between means at the 0.05 level? Give results for both one- and two-tailed tests. Hint: the power of a one-tailed test at 0.05 level is the power of a two-tailed test at 0.10.

Answer:

$\mu1 = 36$, $\mu2 = 45$
$\sigma1 = 10$, $\sigma2 = 10$
$N1 = 16$, $N2 = 16$
$df = 30$

$M2 - M1 = 9$
$\sigma_{M2 - M1} = 3.54$

$S_1 = S_2 = 10/\text{sqrt}(16) = 2.5$
MSE = 6.25
$S_{M2 - M1} = 0.875$

$t = (M2 - M1) / \sigma_{M2 - M1} = 9 / 3.54 = 2.54$

From this link: http://onlinestatbook.com/2/calculators/power_calc.html

Power(one-tailed test at alpha = 0.05) = **.8**
Power(two-tailed test at alpha = 0.05) = **.693**