A CLUSTER ANALYTIC INVESTIGATION ON MATH ANXIETY

Sriram Chellappan, Raghavendra Kotikalapudi
Dept. of Computer Science
Missouri University of Science and Technology
Rolla, MO USA.
{chellaps, rkyvb}@mst.edu

Donald Wunsch
Dept. of Computer Engineering
Missouri University of Science and Technology
Rolla, MO USA.
dwunsch@mst.edu

Jacqueline Bichsel
Dept. of Psychological Sciences
Missouri University of Science and Technology
Rolla, MO USA
bichselj@mst.edu

Abstract

The purpose of this study is to understand math anxiety, what it entails, and what factors of cognitive abilities significantly contributes towards it. Statistical different groups are formed using cluster analysis. Each of these groups are further analyzed by finding inter correlations and ranking predictor variables. Data is collected along eight dimensions from 100 subjects, including visual and verbal working memory, state and trait anxiety, gender, applied and basic math performance, and math anxiety. Our analysis reveals interesting and previously unknown trends. Specifically, our analysis reveals that in males, verbal working memory has a greater importance towards of math anxiety. For females though, our analysis shows that it is the visual working memory that has the most importance to math anxiety.

Data Collection

Subjects

One hundred adults were recruited for this study. A total of 62 females and 38 males comprised the subject sample, ranging in age from 18 to 66. The sample consisted of 42 college students, 30 college graduates, 2 individuals with less than 2 years of college and no degree, and 26 individuals with no college education. This sample appears to be more representative than those used in previous studies of adult math performance, most of which have tested college students. Please see [1] for more details on the survey.

Procedure

Each subject was administered each of the five measures: math performance, math anxiety, state–trait anxiety, verbal working memory, and visual working memory. The five tasks were administered in a predetermined random order using a Latin square procedure.

Subjects were tested individually in sessions that lasted from 1 to 2 hours, depending on individual response times on each measure. Instructions and stimuli for the working memory measures were presented on a computer. The State–Trait Anxiety Inventory, the Math Anxiety Rating Scale, and the Woodcock–Johnson Tests of Achievement math subtests were administered by the examiner, as dictated by the instructions provided with each respective measure.

Analysis

The aim of this article is to understand math anxiety, how it is impacted, and (as part of future work) leverage from our conclusions to alleviate it. To understand these aspects, it is important to understand how predictor variables correlate with math anxiety. A prior study on math anxiety [1] concluded that it is associated with the visual subsystem of working memory than verbal working memory.

In this article, we expand on previous analysis in [1] by first determining the number of statistically relevant clusters in the data. Once these clusters are known, for each cluster, we perform correlation analysis and rank predictors as per their relevance to math anxiety. The idea is to fine tune analysis on different subgroups (for example, males and females) and study how math anxiety is affected within these groups. Based on this study, our subsequent motivation is to design effective strategies for alleviating math anxiety for each group.

Determining number of clusters

Hierarchical clustering was performed to get an initial idea on number of statistically relevant clusters. All variables were transformed into *z*-scores prior to analysis. Hierarchical clustering with squared Euclidean distance using wards method was performed. The number of statistically significant clusters was analyzed by finding a significant jump in agglomeration coefficient values [2].

The dendrogram (Figure 1) revealed three significant clusters. First, the data was partitioned into male and female clusters. Among females, two clusters were created, one for high math anxiety and another for low math anxiety. From Figure 2, it can be observed that there was a significant jump at step 97. Therefore, number of significant clusters are 100 – 97 = 3.

To eliminate the subjectivity involved in hierarchical clustering, we also ran v-fold cross validation algorithm using *k*-means clustering. The general idea of this method is to divide the overall sample into a number of v folds. The same type of analysis is then successively applied to the observations belonging to the v-1 folds (training sample), and the results of the analyses are applied to sample v to compute some index of predictive validity. The results for the v replications are aggregated (averaged) to yield a single measure of the stability of the respective model.

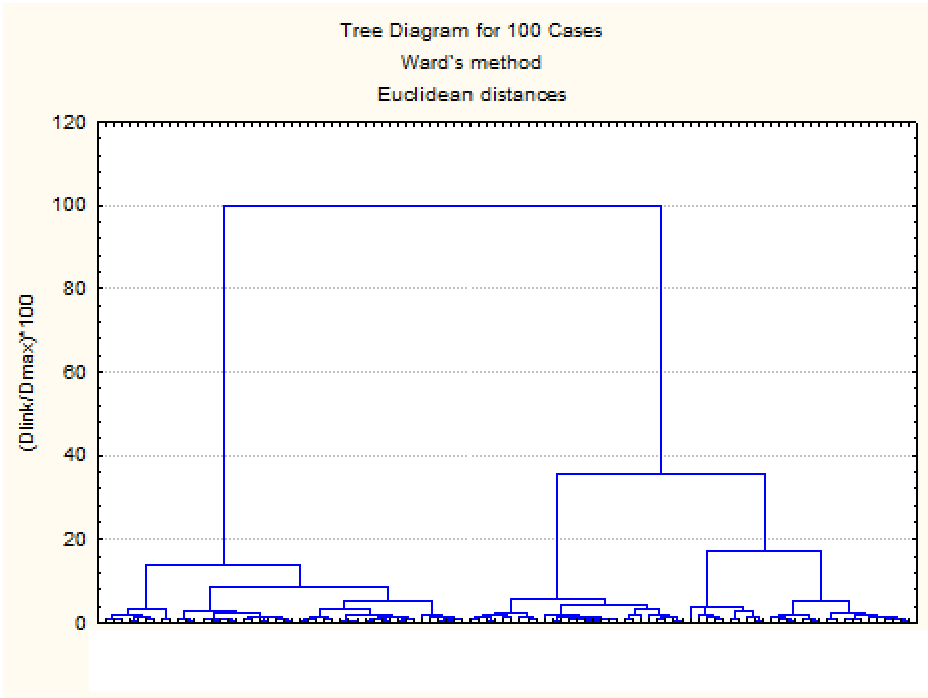

Figure 1: Dendrogram produced by hierarchical clustering using wards method

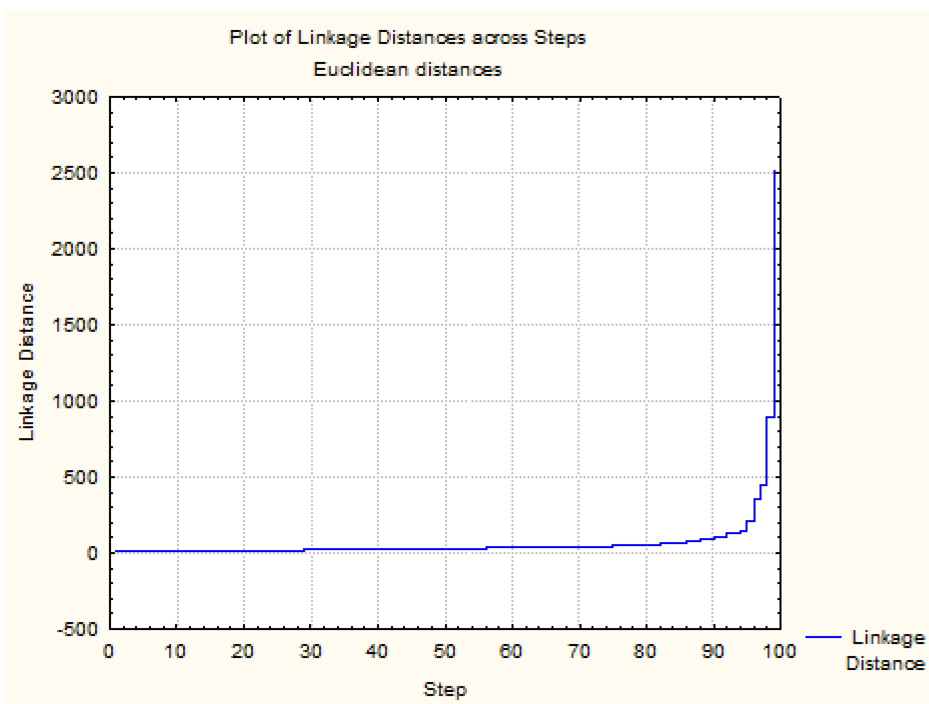

Figure 2: Plot of linkage distances across steps

In our analysis, *v* fold cross validation with three folds (*v* value), *k*-means clustering with Euclidean distance measure yielded three clusters verifying the number of clusters claimed by hierarchical clustering. Figure 3 shows the amount of reduction in cluster cost on addition of a new cluster. Addition of a 4^{th} cluster didn't yield significant decrease in cluster cost. Thus, a three group solution reasonably accounts for the variance present in eight variables chosen in the sample.

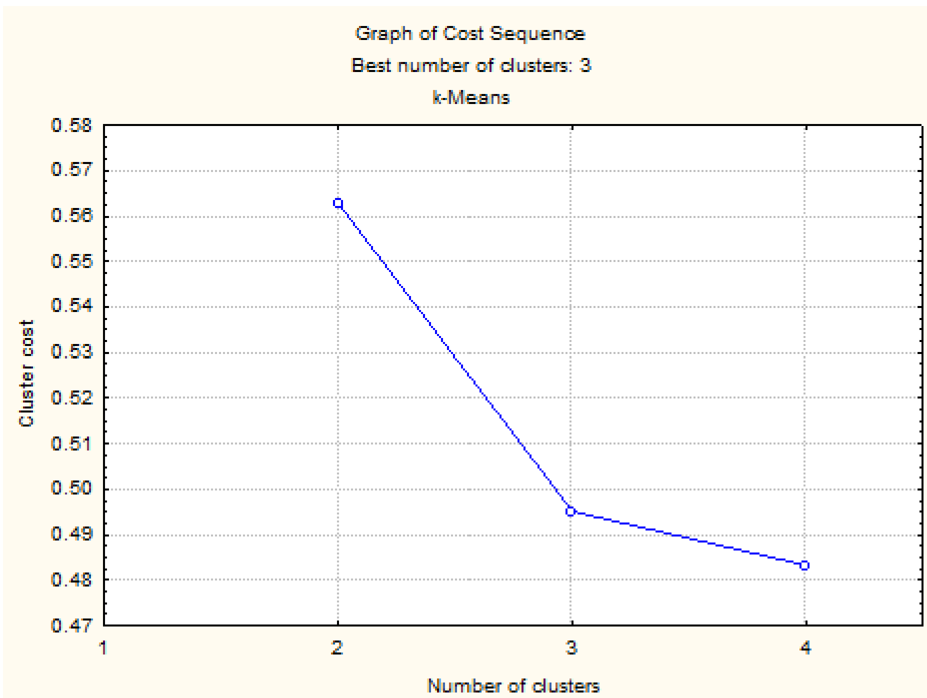

Figure 3: Graph of cost sequence from v fold cross validation using k-means.

Cluster Analysis

Hierarchical clustering demonstrated a clear partition of male and female subjects. Females were further subdivided into groups corresponding to high and low math. Cluster membership formed by *k*-means clustering (*k*=3) as shown in Figure 4.

From Cluster 1, it may be inferred that low applied math in females cause insignificant increase in math anxiety. Cluster 3 shows that low state and trait anxiety, followed by high basic math performance may be the key to low math anxiety in females. Hence, it may be inferred that basic math is more significant predictor of math anxiety as compared to applied math. From Cluster 2 and Cluster 3, it can be observed that low state and trait anxiety, followed by high basic math performance is a characteristic of subjects with low math anxiety.

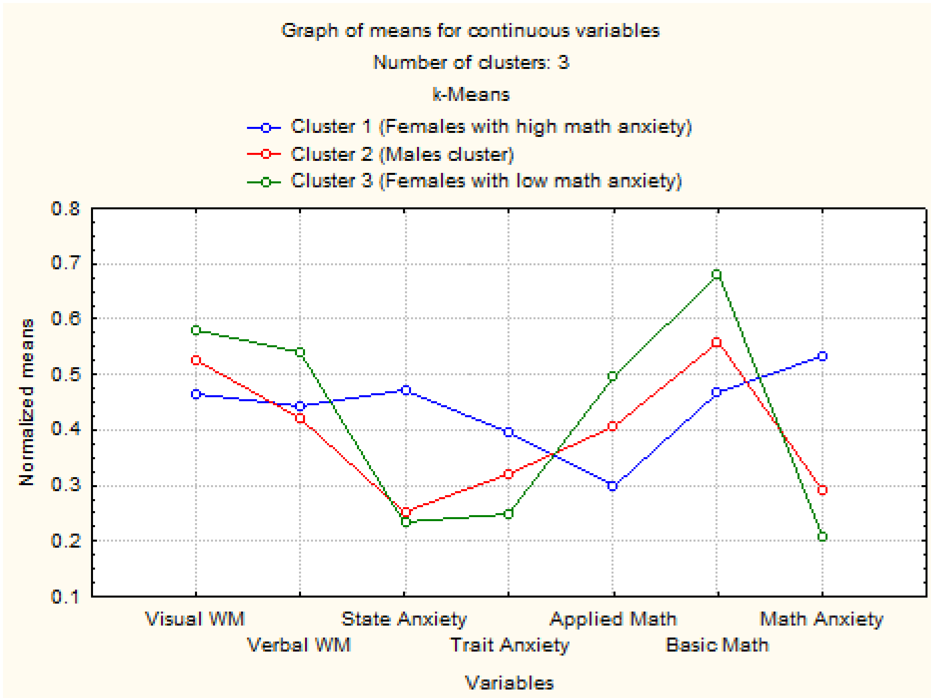

Figure 4: Cluster membership for overall data using k-means with k=3

Analysis of male and female clusters

To further elaborate on the analysis, we perform inter correlations between all measures for male only cluster (Cluster 2) and female only cluster (Cluster 1 + Cluster 3). Inter correlations are tabulated in Table 1 and Table 2.

For male only cluster (Table 1), visual and verbal working memory had significant positive correlation with basic math performance. Basic math performance had a significant inverse correlation with math anxiety. Furthermore, visual and verbal working memory correlated inversely with math anxiety, with verbal working memory being slightly more than visual working memory. As visual and verbal working memory significantly correlate with basic math, it indirectly follows that they are important predictors of math anxiety.

For female only cluster (Table 2), visual working memory has a significant inverse correlation with math anxiety when compared to verbal working memory. They also have significant positive correlation with basic and applied math performance.

From these observations, it may be inferred that in females, visual working memory is more significant towards math anxiety when compared to verbal working memory. This trend is nearly opposite in males. To reinforce these observations, we rank predictors of math anxiety using feature selection algorithm for both groups. Results of ranking are shown in Figures 5 and 6.

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	0.22	0.22	0.04	0.22	0.39	-0.17
Verbal WM	0.22	1.00	-0.35	-0.06	0.24	0.37	-0.21
State Anxiety	0.22	-0.35	1.00	0.22	-0.12	-0.22	0.29
Trait Anxiety	0.04	-0.06	0.22	1.00	0.03	0.02	0.13
Applied Math	0.22	0.24	-0.12	0.03	1.00	0.77	-0.25
Basic Math	0.39	0.37	-0.22	0.02	0.77	1.00	-0.53
Math Anxiety	-0.17	-0.21	0.29	0.13	-0.25	-0.53	1.00

(Marked correlations are significant at p < .05, N = 38)

Table 1: Inter correlations between all measures for male only cluster (cluster 2)

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	0.18	0.03	0.07	0.30	0.27	-0.29
Verbal WM	0.18	1.00	0.03	-0.03	0.25	0.29	-0.13
State Anxiety	0.03	0.03	1.00	0.46	-0.17	-0.14	0.46
Trait Anxiety	0.07	-0.03	0.46	1.00	-0.11	-0.17	0.35
Applied Math	0.30	0.25	-0.17	-0.11	1.00	0.70	-0.34
Basic Math	0.27	0.29	-0.14	-0.17	0.70	1.00	-0.37
Math Anxiety	-0.29	-0.13	0.46	0.35	-0.34	-0.37	1.00

(Marked correlations are significant at p < .05, N = 62)

Table 2: Inter correlations between all measures for female only cluster (cluster 1 + cluster 3)

Figure 5 shows predictor importance for male only data (Cluster 2). As seen by cluster membership, basic math performance has a lot to do with math anxiety. Another interesting observation is that verbal working memory is more than twice as important as visual working memory. Figure 6 shows predictor importance for female only data. Again as expected, basic math performance has a lot to do with math anxiety. However, it may be noted that visual working memory is almost twice as important when compared to verbal working memory. This leads us to an interesting observation. In males, verbal working memory relates more to math anxiety, whereas in females, its visual working memory. As visual and verbal working memory are inversely correlated with math anxiety, it can be concluded that higher verbal working memory is more important that higher visual working memory for having low math anxiety in males and vice-versa for females.

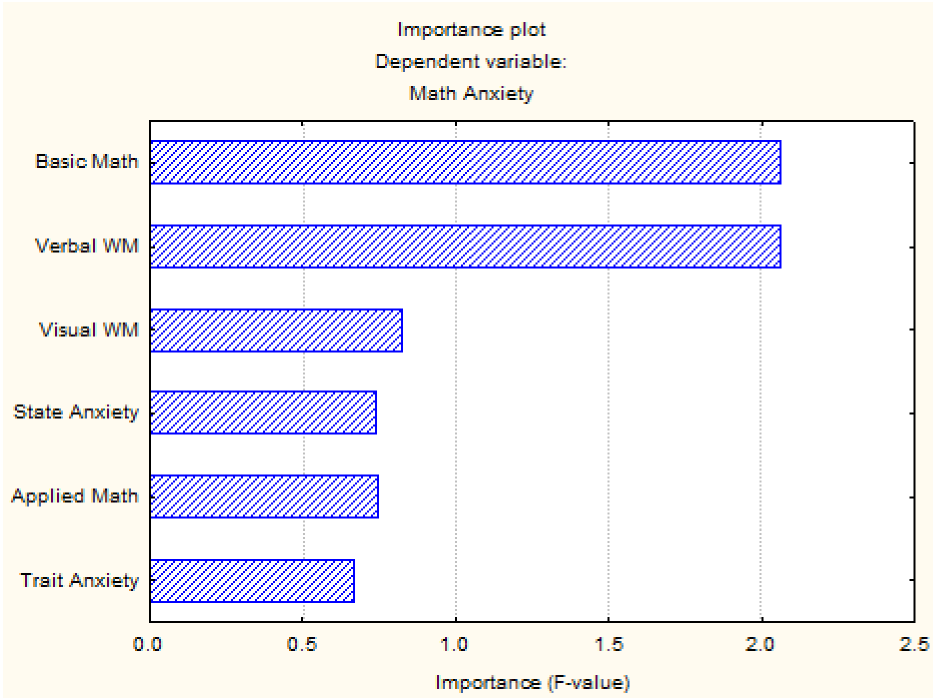

Figure 5: Predictor importance for male only data (cluster 2)

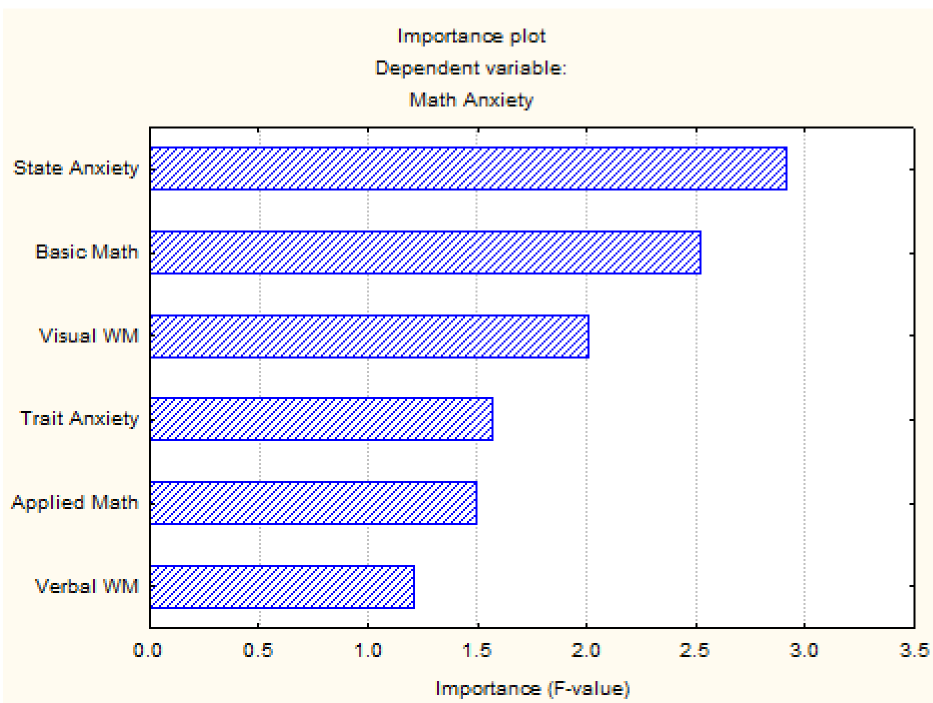

Figure 6: Predictor importance for female only data (cluster 1 + cluster 3)

Indicators of basic math performance for either gender

In both cases, it is observed that basic math is a stronger indicator of math anxiety when compared to applied math performance. This leads us to another interesting question. In males, is verbal working memory responsible for high basic math performance? And, could there be an inverse trend observed for females?

To explore these possibilities, we consider basic math performance as dependent and all the other variables as predictors and rank them using feature selection algorithm. Ranking results are shown in Figures 7 and 8. However, contrary to our hypothesis, in both cases, it turns out that verbal working memory is more strongly associated with basic math performance.

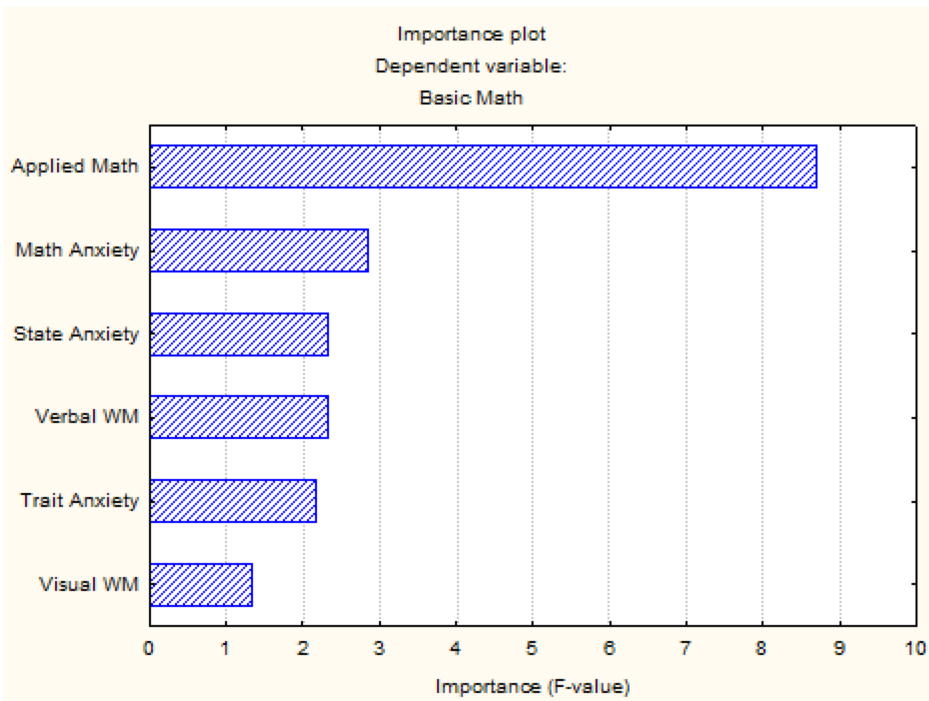

Figure 7: Predictor importance for basic math performance in males

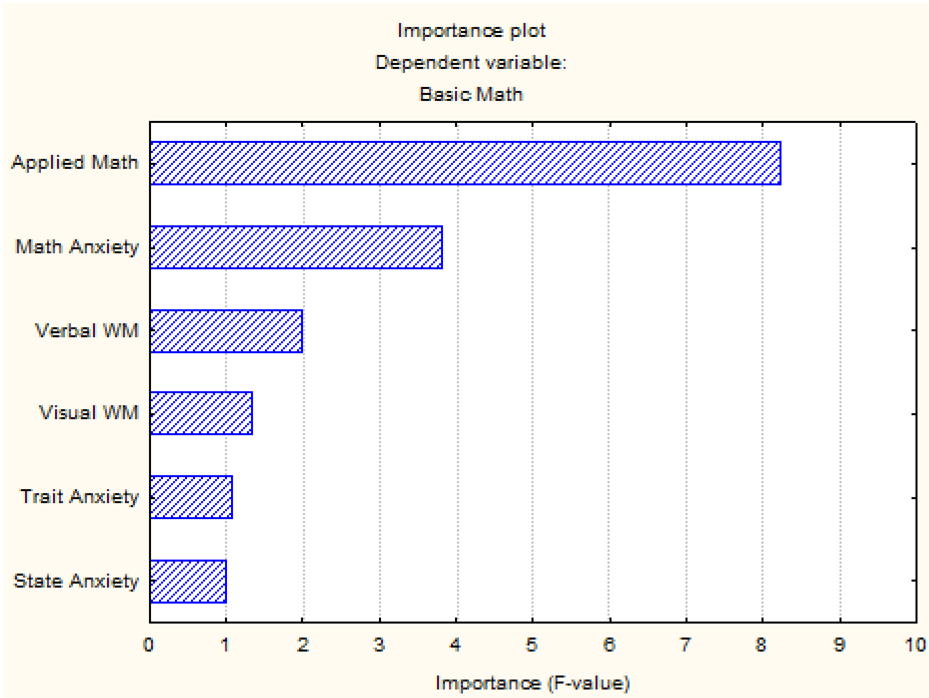

Figure 8: Predictor importance for basic math performance in females

Analysis for females with high and low math anxiety

Inter correlations between all measures for females with high math anxiety (Cluster 1) and females with low math anxiety (Cluster 3) are tabulated in Tables 3 and 4.

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	0.24	0.41	0.37	0.02	0.12	-0.10
Verbal WM	0.24	1.00	0.23	0.10	0.28	0.23	0.08
State Anxiety	0.41	0.23	1.00	0.33	0.15	0.15	0.17
Trait Anxiety	0.37	0.10	0.33	1.00	0.13	-0.03	0.06
Applied Math	0.02	0.28	0.15	0.13	1.00	0.67	0.15
Basic Math	0.12	0.23	0.15	-0.03	0.67	1.00	0.05
Math Anxiety	-0.10	0.08	0.17	0.06	0.15	0.05	1.00

(Marked correlations are significant at p < .05, N = 38)

Table 3: Inter correlations between all measures for females with high math anxiety (cluster 1)

In Cluster 1, it should be noted that while visual working has an inverse correlation with math anxiety, verbal working memory is positively correlated with it.

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	-0.06	-0.45	-0.30	0.40	0.21	-0.19
Verbal WM	-0.06	1.00	-0.05	-0.08	0.00	0.19	-0.07
State Anxiety	-0.45	-0.05	1.00	0.46	0.04	0.06	0.49
Trait Anxiety	-0.30	-0.08	0.46	1.00	0.09	0.16	0.56
Applied Math	0.40	0.00	0.04	0.09	1.00	0.48	0.01
Basic Math	0.21	0.19	0.06	0.16	0.48	1.00	-0.17
Math Anxiety	-0.19	-0.07	0.49	0.56	0.01	-0.17	1.00

(Marked correlations are significant at $p < .05$, $N = 24$)

Table 3: Inter correlations between all measures for females with low math anxiety (cluster 3)

Results from feature selection and ranking algorithm for Cluster 1 and Cluster 3 are shown in Figures 9 and 10.

From Figure 9 and Figure 4, it can be seen that high math anxiety is primarily attributed to low visual working memory, followed by low basic and applied math performance. However, it is interesting to note (from Figure 10 and Figure 4) that for females with low math anxiety, high applied math performance seems to outweigh basic math performance by a factor of three. It should also be noted that low state anxiety is almost as equally ranked as high basic math performance. From these observations, it may be the case that confidence in good applied math performance alleviates unnecessary state anxiety, thus exhibiting reduced math anxiety for this group.

In both cases, visual working memory is more important than verbal working memory supporting previously concluded claim that in females, visual working memory is far more important than verbal working memory.

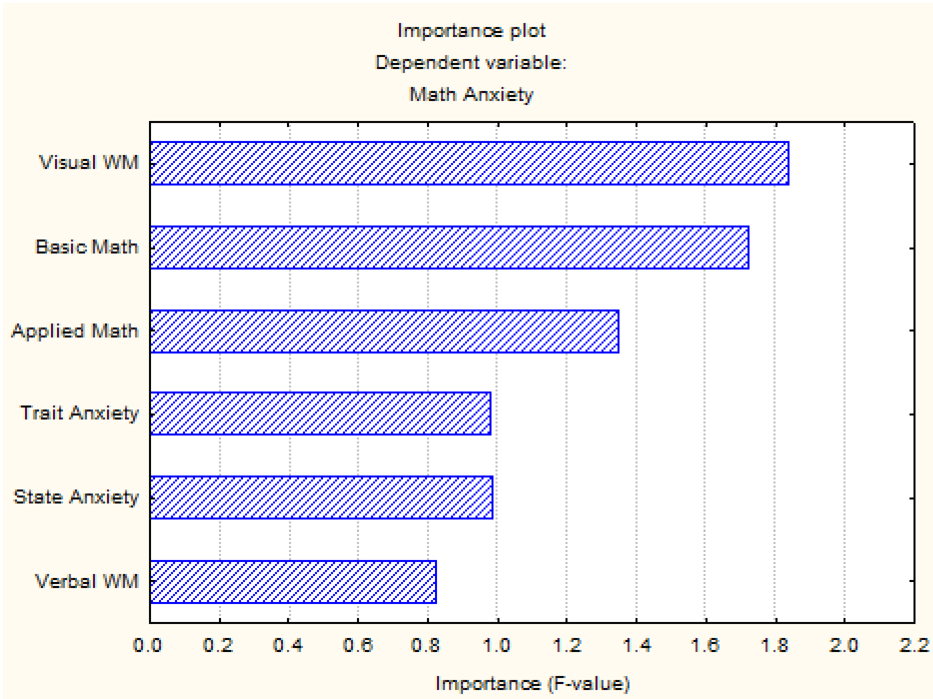

Figure 9: Predictor importance for females with high math anxiety (Cluster 1)

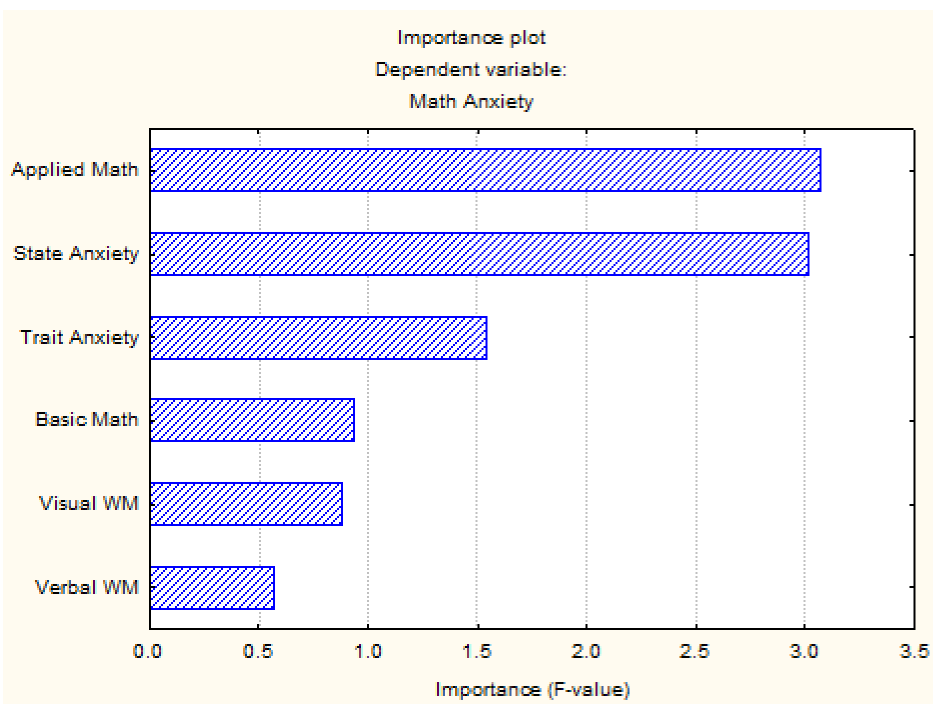

Figure 10: Predictor importance for females with low math anxiety (Cluster 3)

Analysis for males with high and low math anxiety

As we have analyzed high and low math anxiety groups for females, it is imperative that we look for interesting patterns in male subjects also.

k-means with k=2, for male only data yielded two clusters, one with high math anxiety and another with low math anxiety. Cluster memberships for these two clusters are illustrated in Figure 11.

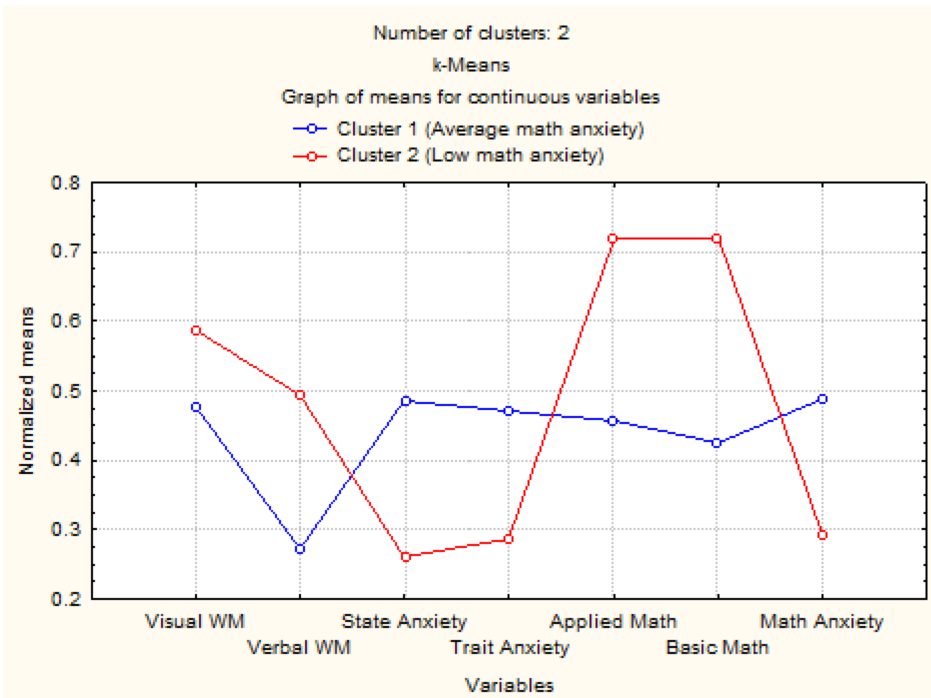

Figure 11: Cluster membership for male only data using k-means with k=2

Inter correlations among all measures for Cluster 1 and Cluster 2 are tabulated in Tables 4 and 5.

To further analyze these clusters, we rank predictor variables for these clusters. Results of feature selection and ranking are shown in Figure 12, 13.

In both the clusters it may be noted that verbal working memory is almost twice as important when compared to visual working memory.

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	0.23	0.30	0.01	-0.10	0.14	-0.02
Verbal WM	0.23	1.00	-0.08	0.16	0.31	0.35	0.07
State Anxiety	0.30	-0.08	1.00	-0.01	-0.05	-0.18	0.16
Trait Anxiety	0.01	0.16	-0.01	1.00	0.52	0.36	0.04
Applied Math	-0.10	0.31	-0.05	0.52	1.00	0.76	-0.21
Basic Math	0.14	0.35	-0.18	0.36	0.76	1.00	-0.43
Math Anxiety	-0.02	0.07	0.16	0.04	-0.21	-0.43	1.00

(Marked correlations are significant at $p < .05$, $N = 21$)

Table 4: Inter correlations between all measures for males with average math anxiety (Cluster 1)

	Visual WM	Verbal WM	State Anxiety	Trait Anxiety	Applied Math	Basic Math	Math Anxiety
Visual WM	1.00	0.01	0.50	0.33	0.30	0.51	-0.13
Verbal WM	0.01	1.00	-0.23	0.15	-0.43	-0.19	-0.12
State Anxiety	0.50	-0.23	1.00	0.15	0.44	0.43	0.08
Trait Anxiety	0.33	0.15	0.15	1.00	-0.07	0.24	-0.12
Applied Math	0.30	-0.43	0.44	-0.07	1.00	0.50	0.23
Basic Math	0.51	-0.19	0.43	0.24	0.50	1.00	-0.33
Math Anxiety	-0.13	-0.12	0.08	-0.12	0.23	-0.33	1.00

(Marked correlations are significant at $p < .05$, $N = 17$)

Table 5: Inter correlations between all measures for males with low math anxiety (cluster 2)

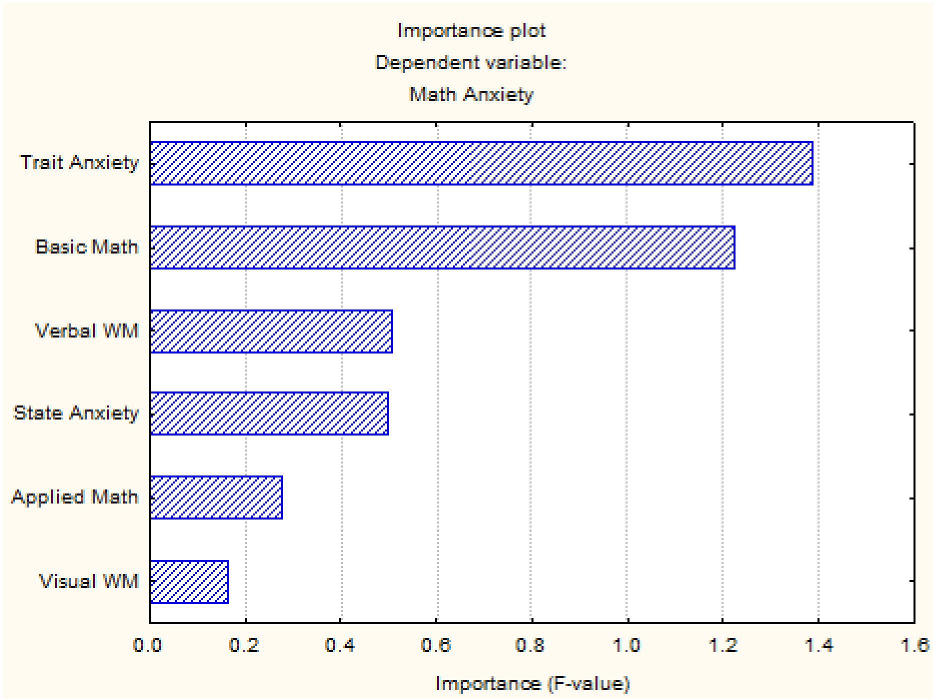

Figure 12: Predictor importance for male Cluster 1

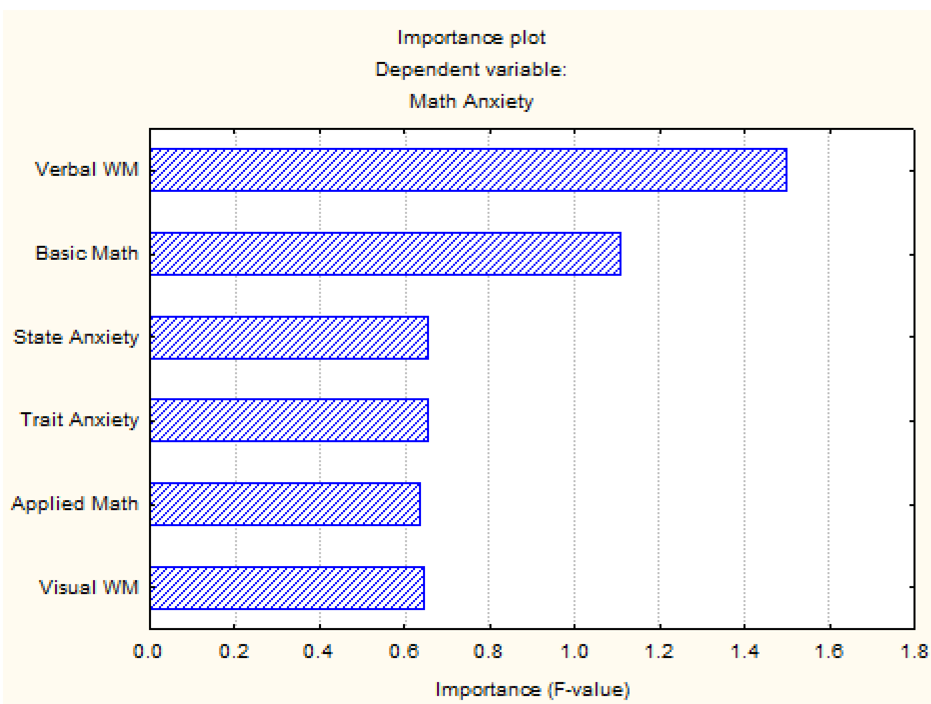

Figure 13: Predictor importance for male Cluster 2

Conclusions

In this article, we conducted a detailed cluster and statistical analysis study to determine the impact of visual and verbal working memory on math anxiety for males and females. Our analysis revealed interesting and previously unknown trends. Specifically, our analysis revealed that in males, verbal working memory has a greater importance towards of math anxiety. For females though, our analysis showed that it is the visual working memory that has the most importance to math anxiety. Our on-going work aims to design gender based intervention strategies for math anxiety by focusing on verbal relaxation techniques for males, and visual memory relaxation techniques for females.

References

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2. Mark S. Aldenderfer, *Cluster Analysis (Quantitative Applications in the Social Sciences)*, Sage Publications, 1984