Notebook UNTreeCiclo

Contents

1 C++

1.1 C++ plantilla

```cpp
#include <bits/stdc++.h>
using namespace std;
#define all(v) v.begin(), v.end()
#define sz(arr) ((int) arr.size())
typedef vector<int> vi;
typedef long long ll;
typedef pair<int, int> ii;
const char ln = '\n';

#define watch(x) cout<<#x<<"="<<x<<'\n'
typedef long double ld;
typedef vector<ii> vii;
typedef vector<long long> vl;
typedef pair<ll, ll> pll;
typedef vector<pll> vll;
const int INF = 1e9;
```

```
const ll INFL = 1e18;
const int MOD = 1e9+7;
const double EPS  = 1e-9;
const ld PI = acosl(-1);
int dirx[4] = {0,-1,1,0};
int diry[4] = {-1,0,0,1};
int dr[] = {1, 1, 0, -1, -1, -1, 0, 1};
int dc[] = {0, 1, 1, 1, 0, -1, -1, -1};
const string ABC = "abcdefghijklmnopqrstuvwxyz";

int main() {
        ios::sync_with_stdio(false);
        cin.tie(0);
        cout << setprecision(20) << fixed;
    // freopen("file.in", "r", stdin);
    // freopen("file.out", "w", stdout);

        return 0;
}
```

1.2 Librerias

```
// En caso de que no sirva #include <bits/stdc++.h>
#include <algorithm>
#include <iostream>
#include <iterator>
#include <sstream>
#include <fstream>
#include <cassert>
#include <climits>
#include <cstdlib>
#include <cstring>
#include <string>
#include <cstdio>
#include <vector>
#include <cmath>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <map>
#include <set>
#include <bitset>
#include <iomanip>
#include <unordered_map>
////
#include <tuple>
#include <random>
#include <chrono>
```

1.3 Bitmask

```
* Operaciones a nivel de bits. Si n es ll usar 1ll<< en
   los corrimientos.

x & 1          -> Verifica si x es impar
x & (1<<i)     -> Verifica si el i-esimo bit esta
   encendido
x = x | (1<<i)  -> Enciende el i-esimo bit
x = x & ~(1<<i) -> Apaga el i-esimo bit
x = x ^ (1<<i)  -> Invierte el i-esimo bit
x = ~x          -> Invierte todos los bits
x & -x          -> Devuelve el bit encendido mas a la
   derecha (potencia de 2, no el indice)
~x & (x+1)      -> Devuelve el bit apagado mas a la
   derecha (potencia de 2, no el indice)
x = x | (x+1)   -> Enciende el bit apagado mas a la
   derecha
x = x & (x-1)   -> Apaga el bit encendido mas a la
   derecha
x = x & ~y      -> Apaga en x los bits encendidos de y

* Funciones del compilador gcc. Si n es ll agregar el
   sufijo ll, por ej: __builtin_clzll(n).

__builtin_clz(x)      -> Cantidad de bits apagados por la
    izquierda
__builtin_ctz(x)      -> Cantidad de bits apagados por la
    derecha. Indice del bit encendido mas a la derecha
__builtin_popcount(x) -> Cantida de bits encendidos

* Logaritmo en base 2 (entero). Indice del bit encendido
   mas a la izquierda. Si x es ll usar 63 y clzll(x).
// O(1)
int lg2(const int &x) { return 31-__builtin_clz(x); }

* Itera, con indices, los bits encendidos de una mascara.
// O(#bits_encendidos)
for (int x = mask; x; x &= x-1) {
    int i = __builtin_ctz(x);
}

* Itera todas las submascaras de una mascara. (Iterar
   todas las submascaras de todas las mascaras es O(3^n))
   .
// O(2^(#bits_encendidos))
for (int sub = mask; sub; sub = (sub-1)&mask) {}

* retorna la siguiente mask con la misma cantidad
   encendida
ll nextMask(ll x){
    ll c = x & -x;
    ll r = x + c;
    return (((r ^ x) >> 2) / c) | r;
}
```

1.4 Cosas de strings

```
int conv(char ch){return ((ch>='a' && ch<='z')?ch-'a':ch-
    'A'+26);}
vector<string> split(string& s, char c=' '){
        vector<string> res;
        stringstream ss(s);
        string sub;
        while(getline(ss, sub, c))res.push_back(sub);
        return res;
}

for(char& c:s)c=toupper(c);
for(char& c:s)c=tolower(c);
int n=stoi(s); // de string a entero
int n=stoi(s, nullptr, 2); // base 2
double d=stod(s); // de string a double
string s=to_string(n); // de entero a string
```

1.5 Custom Hashing

```
struct custom_hash {
        static long long splitmix64(long long x) {
                x += 0x9e3779b97f4a7c15;
                x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
                x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
                return x ^ (x >> 31);
        }

        size_t operator()(long long x) const {
                static const long long FIXED_RANDOM =
                    chrono::steady_clock::now().
                    time_since_epoch().count();
                return splitmix64(x + FIXED_RANDOM);
        }

        size_t operator()(const pair<int,int>& x) const {
                return (size_t) x.first * 37U + (size_t)
                    x.second;
        }

        size_t operator()(const vector<int>& v) const {
                size_t s = 0;
                for(auto &e : v)
                        s^=hash<int>()(e)+0x9e3779b9+(s
                            <<6)+(s>>2);
                return s;
        }
};

unordered_map<long long, int, custom_hash> safe_map;
gp_hash_table<int, int, custom_hash> table;
```

1.6 Random

```
typedef unsigned long long u64;
mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());
u64 hash=rng();

mt19937 rng (chrono::steady_clock::now().time_since_epoch
    ().count());
int rand(int a, int b){return uniform_int_distribution<
    int>(a, b)(rng);} // uniform_real_distribution
```

2 Arboles

2.1 Centroid Decomposition

```
// O(nlog(n))
struct CentroidDecomposition{
        int dad[maxn],sz[maxn];
        set<int> adj[maxn]; // check, proc

        int operator[](int i){return dad[i];}
        void addEdge(int x,int y){adj[x].insert(y);adj[y
            ].insert(x);}

        void build(int v=0, int p=-1){
                int n=dfsSz(v, p);
                int centroid=dfsCentroid(v, p, n);
                dad[centroid]=p;
                // add dfs for paths
                for(int u:adj[centroid]){
                        adj[u].erase(centroid);
                        build(u,centroid);
                }
                adj[centroid].clear();
        }

        int dfsSz(int v,int p){
                sz[v]=1;
                for(int u:adj[v]){
                        if(u==p)continue;
                        sz[v]+=dfsSz(u, v);
                }
                return sz[v];
        }

        int dfsCentroid(int v, int p, int n){
                for(int u:adj[v]){
                        if(u==p)continue;
                        if(sz[u]>n/2)return dfsCentroid(u
                            ,v,n);
                }
                return v;
        }
```

```
};
// for(int b=a;b!=-1;b=cd[b])
```

2.2 Heavy Light Decomposition

```
typedef long long T;
T null;
T oper(T a, T b);
// Segment tree
const int maxn=1e5+1; // >= 2e5, remove struct
bool edges=false; // arista padre
struct HLD{
	int par[maxn], root[maxn], dep[maxn];
	int sz[maxn], pos[maxn], ti;
	vector<int> adj[maxn];
	SegTree st;
	void addEdge(int x, int y){adj[x].push_back(y);
	    adj[y].push_back(x);}
	void dfsSz(int x){
		sz[x]=0;
		for(int& y:adj[x]){
			if(y==par[x])continue;
			par[y]=x;dep[y]=dep[x]+1;
			dfsSz(y);
			sz[x]+=sz[y]+1;
			if(sz[y]>sz[adj[x][0]])swap(y,adj
			    [x][0]);
		}
	}
	void dfsHld(int x){
		pos[x]=ti++;
		for(int y:adj[x]){
			if(y==par[x])continue;
			root[y]=(y==adj[x][0]?root[x]:y);
			dfsHld(y);
		}
	}
	void build(int n,int v=0){
		root[v]=par[v]=v;
		dep[v]=ti=0;
		dfsSz(v);
		dfsHld(v);
		// vl palst(n);
		// for(int i=0;i<n;++i)palst[pos[i]]=a[i
		    ];
		// st.build(palst, n);
		st.build(n);
	}
	// O(log^2(n))
	template <class Oper>
	void processPath(int x, int y, Oper op){
		for(;root[x]!=root[y];y=par[root[y]]){
			if(dep[root[x]]>dep[root[y]])swap
			    (x,y);
				op(pos[root[y]],pos[y]);
			}
			if(dep[x]>dep[y])swap(x,y);
			op(pos[x]+edges,pos[y]);
	}
	void modifyPath(int x, int y, int v){
		processPath(x,y,[this,&v](int l, int r){
			st.upd(l,r,v);
		});
	}
	T queryPath(int x, int y){
		T res=null;
		processPath(x,y,[this,&res](int l, int r)
		    {
			res=oper(res, st.get(l,r));
		});
		return res;
	}
	void modifySubtree(int x, int v){st.upd(pos[x]+
	    edges,pos[x]+sz[x],v);}
	int querySubtree(int x){return st.get(pos[x]+
	    edges,pos[x]+sz[x]);}
	void modify(int x, int v){st.set(pos[x],v);}
	void modifyEdge(int x, int y, int v){
		if(dep[x]<dep[y])swap(x,y);
		modify(x,v);
	}
};
```

2.3 LCA

```
const int maxn = 2e5+5, maxlog = 20+5;
int up[maxn][maxlog], dep[maxn]; // memset -1 (up)
vi adj[maxn];

void dfs(int v=0, int p=-1){
	up[v][0]=p;
	for(int u:adj[v]){
		if(u!=p){
			dep[u]=dep[v]+1;
			dfs(u, v);
		}
	}
}

void build(int n){
	for(int l=1;l<maxlog;++l){
		for(int i=0;i<n;++i){
			if(up[i][l-1]!=-1){
				up[i][l]=up[up[i][l-1]][l
				    -1];
			}
		}
	}
}
```

```cpp
int kth(int node, int k){
	for(int l=maxlog-1;l>=0;--l){
		if(node!=-1 && k&(1<<l)){
			node=up[node][l];
		}
	}
	return node;
}
int lca(int a, int b){
	a=kth(a, dep[a]-min(dep[a], dep[b]));
	b=kth(b, dep[b]-min(dep[a], dep[b]));
	if(a==b)return a;
	for(int l=maxlog-1;l>=0;--l){
		if(up[a][l]!=up[b][l]){
			a=up[a][l];
			b=up[b][l];
		}
	}
	return up[a][0];
}
```

2.4 Sack

```cpp
const int maxn = 1e5+5;
int st[maxn], ft[maxn], ver[2*maxn];
int len[maxn], n, q, pos=0;
vi adj[maxn];

bool vis[maxn];
void ask(int v, bool add){
	if(vis[v] && !add){
		vis[v]=false;
		// delete node
	}else if(!vis[v] && add){
		vis[v]=true;
		// add node
	}
}

// O(nlogn)
void dfs(int v=0, int p=-1, bool keep=true){
	int mx=0,id=-1;
	for(int u:adj[v]){
		if(u==p)continue;
		if(len[u]>mx){
			mx=len[u];
			id=u;
		}
	}
	for(int u:adj[v]){
		if(u!=p && u!=id)
			dfs(u,v,0);
	}
	if(id!=-1)dfs(id,v,1);
	for(int u:adj[v]){
		if(u==p || u==id)continue;
		for(int p=st[u];p<ft[u];++p)
			ask(ver[p], 1);
	}
	ask(v, 1);
	// answer queries
	if(keep)return;
	for(int p=st[v];p<ft[v];++p)
		ask(ver[p], 0);
}
```

2.5 Virtual Tree

```cpp
const int maxn = 2e5+5;
vi adjVT[maxn], adj[maxn];
int st[maxn], ft[maxn], n, pos=0;
bool important[maxn];

bool upper(int v, int u){return st[v]<=st[u] && ft[v]>=ft
	[u];}
bool cmp(int v, int u){return st[v]<st[u];}

// O(klogk)
int virtualTree(vi nodes){
	sort(all(nodes), cmp);
	int m=sz(nodes);
	for(int i=0;i<m-1;++i){
		int v=lca(nodes[i], nodes[i+1]);
		nodes.push_back(v);
	}

	sort(all(nodes), cmp);
	nodes.erase(unique(all(nodes)), nodes.end());
	for(int u:nodes)adjVT[u].clear();

	vi s;
	s.push_back(nodes[0]);
	m=sz(nodes);
	for(int i=1;i<m;++i){
		int v=nodes[i];
		while(sz(s)>=2 && !upper(s.back(), v)){
			adjVT[s[sz(s)-2]].push_back(s.
				back());
			s.pop_back();
		}
		s.push_back(v);
	}
	while(sz(s)>=2){
		adjVT[s[sz(s)-2]].push_back(s.back());
		s.pop_back();
	}
	return s[0];
}
```

```
// vi nodes(k);
// for(int& x:nodes)important[x]=true;
// int root=virtualTree(nodes);
// dp(root) - output answer - reset
```

3 Estructuras de Datos

3.1 Disjoint Set Union

```
struct dsu{
        vi p,size;
        int sets,maxSize;

        dsu(int n){
                p.assign(n,0);
                size.assign(n,1);
                sets = n;
                for (int i = 0; i<n; i++) p[i] = i;
        }

        int find_set(int i) {return (p[i] == i) ? i : (p[
            i] = find_set(p[i]));}

        bool is_same_set(int i, int j) {return find_set(i
            ) == find_set(j);}

        void unionSet(int i, int j){
                if (!is_same_set(i, j)){
                        int a = find_set(i), b = find_set
                            (j);
                        if (size[a] < size[b]) swap(a, b)
                            ;
                        p[b] = a;
                        size[a] += size[b];
                        maxSize = max(size[a], maxSize);
                        sets--;
                }
        }
};
```

3.2 Dynamic Connectivity Offline

```
struct dsu{
        vi p,rank,h;
        int sets;
        dsu(int n){
                sets=n;
                p.assign(n,0);
                rank.assign(n,1);
                for(int i=0;i<n;++i)p[i]=i;
        }
        int get(int a){return (a==p[a]?a:get(p[a]));}
        void unite(int a, int b){
                a=get(a);b=get(b);
                if(a==b)return;
                if(rank[a]>rank[b])swap(a,b);
                rank[b]+=rank[a];
                h.push_back(a);
                p[a]=b;sets--;
        }
        void rollback(int x){
                int len=h.size();
                while(len>x){
                        int a=h.back();
                        h.pop_back();
                        rank[p[a]]-=rank[a];
                        p[a]=a;sets++;len--;
                }
        }
};

enum { ADD, DEL, QUERY };
struct Query{int type, u, v;};
struct DynCon{
        vector<Query> q;
        dsu uf;vi mt;
        map<pair<int,int>, int> prv;
        DynCon(int n): uf(n){}
        void add(int i, int j){
                if(i>j)swap(i, j);
                q.push_back({ADD, i, j});
                mt.push_back(-1);
                prv[{i,j}]=sz(q)-1;
        }
        void remove(int i, int j){
                if(i > j) swap(i, j);
                q.push_back({DEL, i, j});
                int pr=prv[{i, j}];
                mt[pr]=sz(q)-1;
                mt.push_back(pr);
        }
        void query(){q.push_back({QUERY, -1, -1});mt.
            push_back(-1);}
        void process(){ // answers all queries in order
                if(!sz(q))return;
                for(int i=0;i<sz(q);++i){
                        if(q[i].type==ADD && mt[i]<0)mt[i
                            ]=sz(q);
                }go(0, sz(q));
        }
        void go(int s, int e){
                if(s+1==e){
                if(q[s].type == QUERY)cout<<uf.sets<<"\n"
                    ;
                return;
                }int k=sz(uf.h),m=(s+e)/2;
                for(int i=e-1;i>=m;--i){
```

```
                if(mt[i]>=0 && mt[i]<s)uf.unite(q[i].u, q
                   [i].v);
                }go(s, m);
                uf.rollback(k);
                for(int i=m-1;i>=s;--i){
                if(mt[i]>=e)uf.unite(q[i].u, q[i].v);
                }go(m, e);
                uf.rollback(k);
        }
};
```

3.3 Dynamic Segment Tree

```
T null=0,nolz=0;
T oper(T a, T b);
struct Node{
        T val,lz;
        int l,r;
        Node *pl,*pr;
        Node(int ll, int rr){
                val=null;lz=nolz;
                pl=pr=nullptr;
                l=ll;r=rr;
        }
};

typedef Node* PNode;
void update(PNode x){
        if(x->r-x->l==1)return;
        x->val=oper(x->pl->val,x->pr->val);
}

void extends(PNode x){
        if(x->r-x->l!=1 && !x->pl){
                int m=(x->r+x->l)/2;
                x->pl=new Node(x->l, m);
                x->pr=new Node(m, x->r);
        }
}

void propagate(PNode x){
        if(x->r-x->l==1)return;
        if(x->lz==nolz)return;
        int m=(x->r+x->l)/2;
        // pl, pr
        x->lz=nolz;
}

struct SegTree{
        PNode root;
        void upd(PNode x, int l, int r, T v){
                int lx=x->l,rx=x->r;
                if(lx>=r || l>=rx)return;
                if(lx>=l && rx<=r){
                        // val, lz
                        return;
                }
                extends(x);
                propagate(x);
                upd(x->pl,l,r,v);
                upd(x->pr,l,r,v);
                update(x);
        }

        T get(PNode x, int l, int r){
                int lx=x->l,rx=x->r;
                if(lx>=r || l>=rx)return null;
                if(lx>=l && rx<=r)return x->val;
                extends(x);
                propagate(x);
                T v1=get(x->pl,l,r);
                T v2=get(x->pr,l,r);
                return oper(v1,v2);
        }

        T get(int l, int r){return get(root,l,r+1);}
        void upd(int l, int r, T v){upd(root,l,r+1,v);}
        void build(int l, int r){root=new Node(l, r+1);}
};
```

3.4 Fenwick Tree

```
typedef long long T;
struct FwTree{
        int n;
        vector<T> bit;
        FwTree(int n): n(n),bit(n+1){}
        T get(int r){
                T sum=0;
                for(++r;r;r-=r&-r)sum+=bit[r];
                return sum;
        }
        T get(int l, int r){return get(r)-(l==0?0:get(l
           -1));}
        void upd(int r, T v){
                for(++r;r<=n;r+=r&-r)bit[r]+=v;
        }
};

struct FwTree2d{
        int n, m;
        vector<vector<T>> bit;
        FwTree2d(){}
        FwTree2d(int n, int m):n(n),m(m),bit(n+1, vector<
           T>(m+1,0)){}
        T get(int x, int y){
                T v=0;
                for(int i=x+1;i;i-=i&-i)
                for(int j=y+1;j;j-=j&-j)v+=bit[i][j];
                return v;
```

```
		}
		T get(int x, int y, int x2, int y2){return get(x2
		    ,y2)-get(x-1,y2)-get(x2,y-1)+get(x-1,y-1);}
		void upd(int x, int y, T dt){
			for(int i=x+1;i<=n;i+=i&-i)
			for(int j=y+1;j<=m;j+=j&-j)bit[i][j]+=dt;
		}
};
```

3.5 Li Chao

```
// inf max abs value that the function may take
typedef long long ty;

struct Line {
  ty m, b;

  Line(){}
  Line(ty m, ty b): m(m), b(b){}

  ty eval(ty x){return m * x + b;}
};

struct nLiChao{
	// see coments for min

	nLiChao *left = nullptr, *right = nullptr;
	ty l, r;
	Line line;

	nLiChao(ty l, ty r): l(l), r(r){
		line = {0, -inf}; // change to {0, inf};
	}

	// T(Log(Rango)) M(Log(rango))
	void addLine(Line nline){
		ty m = (l + r) >> 1;
		bool lef = nline.eval(l) > line.eval(l);
		    // change > to <
	bool mid = nline.eval(m) > line.eval(m); //
	    change > to <

		if (mid) swap(nline, line);

		if (r == l) return;

	if (lef != mid){
			if (!left){
				left = new nLiChao(l, m);
				left -> line = nline;
			}

			else left -> addLine(nline);
		}
	else{
			if (!right){
				right = new nLiChao(m +
				    1, r);
				right -> line = nline;
			}

			else right -> addLine(nline);
		}
	}

	// T(Log(Rango))
	ty get(ty x) {
		ty m = (l + r) >> 1;
		ty op1 = -inf, op2 = -inf; // change to
		    inf

		if(l == r) return line.eval(x);
		else if(x < m){
			if (left) op1 = left -> get(x);
			return max(line.eval(x), op1); //
			    change max to min
		}
		else{
			if (right) op2 = right -> get(x);
			return max(line.eval(x), op2); //
			    change max to min
		}
	}
};

int main() {
	// (rango superior) * (pendiente maxima) puede
	    desbordarse
	// usar double o long double en el eval para
	    estos casos
	// (puede dar problemas de precision)
	nLiChao liChao(0, 1e18);
}
```

3.6 Link Cut Tree

```
typedef long long T;
struct SplayTree{
	struct Node{
		int ch[2]={0, 0},p=0;
		T val=0,path=0,sz=1;     // Path
		T sub=0,vir=0,ssz=0,vsz=0; // Subtree
		bool flip=0;T lz=0;      // Lazy
	};
	vector<Node> ns;

	SplayTree(int n):ns(n+1){}

	T path(int u){return (u?ns[u].path:0);}
	T size(int u){return (u?ns[u].sz:0);}
	T subsize(int u){return (u?ns[u].ssz:0);}
```

```
	T subsum(int u){return (u?ns[u].sub:0);}
	void push(int x){
		if(!x)return;
		int l=ns[x].ch[0],r=ns[x].ch[1];
		if(ns[x].flip){
			ns[l].flip^=1,ns[r].flip^=1;
			swap(ns[x].ch[0], ns[x].ch[1]);
			   // check with st oper
			ns[x].flip=0;
		}
		if(ns[x].lz){
			// ...
			ns[x].sub+=ns[x].lz*ns[x].ssz;
			ns[x].vir+=ns[x].lz*ns[x].vsz;
			// ...
		}
	}

	void pull(int x){
		int l=ns[x].ch[0],r=ns[x].ch[1];
		push(l);push(r);
		ns[x].sz=size(l)+size(r)+1;
		ns[x].path=max({path(l), path(r), ns[x].
		    val});
		ns[x].sub=ns[x].vir+subsum(l)+subsum(r)+
		    ns[x].val;
		ns[x].ssz=ns[x].vsz+subsize(l)+subsize(r)
		    +1;
	}

	void set(int x, int d, int y){ns[x].ch[d]=y;ns[y
	    ].p=x;pull(x);}
	void splay(int x){
		auto dir=[&](int x){
			int p=ns[x].p;if(!p)return -1;
			return ns[p].ch[0]==x?0:ns[p].ch
			    [1]==x?1:-1;
		};
		auto rotate=[&](int x){
			int y=ns[x].p,z=ns[y].p,dx=dir(x)
			    ,dy=dir(y);
			set(y,dx,ns[x].ch[!dx]);
			set(x,!dx,y);
			if(~dy)set(z,dy,x);
			ns[x].p=z;
		};
		for(push(x);~dir(x);){
			int y=ns[x].p, z=ns[y].p;
			push(z);push(y);push(x);
			int dx=dir(x),dy=dir(y);
			if(~dy)rotate(dx!=dy?x:y);
			rotate(x);
		}
	}
};

struct LinkCut:SplayTree{ // 1-indexed
	LinkCut(int n):SplayTree(n){}
	int root(int u){
		access(u);splay(u);push(u);
		while(ns[u].ch[0]){u=ns[u].ch[0];push(u)
		    ;}
		return splay(u),u;
	}
	int parent(int u){
		access(u);splay(u);push(u);
		u=ns[u].ch[0];push(u);
		while(ns[u].ch[1]){u=ns[u].ch[1];push(u)
		    ;}
		return splay(u),u;
	}
	int access(int x){
		int u=x,v=0;
		for(;u;v=u,u=ns[u].p){
			splay(u);
			int& ov=ns[u].ch[1];
			ns[u].vir+=ns[ov].sub;
			ns[u].vsz+=ns[ov].ssz;
			ns[u].vir-=ns[v].sub;
			ns[u].vsz-=ns[v].ssz;
			ov=v;pull(u);
		}
		return splay(x),v;
	}
	void reroot(int x){
		access(x);ns[x].flip^=1;push(x);
	}
	void link(int u, int v){ // u->v
		reroot(u);
		access(v);
		ns[v].vir+=ns[u].sub;
		ns[v].vsz+=ns[u].ssz;
		ns[u].p=v;pull(v);
	}
	void cut(int u, int v){
		int r=root(u);
		reroot(u);
		access(v);
		ns[v].ch[0]=ns[u].p=0;pull(v);
		reroot(r);
	}
	void cut(int u){ // cut parent
		access(u);
		ns[ns[u].ch[0]].p=0;
		ns[u].ch[0]=0;pull(u);
	}
```

```
	int lca(int u, int v){
		if(root(u)!=root(v))return -1;
		access(u);return access(v);
	}

	int depth(int u){
		access(u);splay(u);push(u);
		return ns[u].sz;
	}

	T path(int u, int v){
		int r=root(u);
		reroot(u);access(v);pull(v);
		T ans=ns[v].path;
		return reroot(r),ans;
	}

	void set(int u, T val){access(u);ns[u].val=val;
	    pull(u);}
	void upd(int u, int v, T val){
		int r=root(u);
		reroot(u);access(v);splay(v);
		// lazy
		reroot(r);
	}

	T comp_size(int u){return ns[root(u)].ssz;}
	T subtree_size(int u){
		int p=parent(u);
		if(!p)return comp_size(u);
		cut(u);int ans=comp_size(u);
		link(u,p);return ans;
	}
	T subtree_size(int u, int v){
		int r=root(u);
		reroot(v);access(u);
		T ans=ns[u].vsz+1;
		return reroot(r),ans;
	}

	T comp_sum(int u){return ns[root(u)].sub;}
	T subtree_sum(int u){
		int p=parent(u);
		if(!p)return comp_sum(u);
		cut(u);T ans=comp_sum(u);
		link(u,p);return ans;
	}
	T subtree_sum(int u, int v){ // subtree of u, v
	    father
		int r=root(u);
		reroot(v);access(u);
		T ans=ns[u].vir+ns[u].val; // por el
		    reroot
		return reroot(r),ans;
	}
};
```

3.7 Mos Algorithm

```
// O((n+q)*s), s=n^(1/2)
// O(q*(s+(n/s)^2) => O(q*(n^(2/3))), s=(2*(n^2))^(1/3) -
    s=n^(2/3)
int s,n;
struct upd{int i,old,cur;};
struct query {int l,r,t,idx;};
bool cmp(query& a, query& b){
	int x=a.l/s;
	if(a.l/s!=b.l/s)return a.l/s<b.l/s;
	if(a.r/s!=b.r/s)return (x&1?a.r<b.r:a.r>b.r);
	return a.t<b.t;
}

vector<int> ans;
vector<query> qu;
vector<upd> up;

int act();
void add(int i);
void remove(int i);
void update(int i,int v,int l,int r){
	if(l<=i && i<=r); // add, remove
}

void solve(){
	s=(int)ceil(sqrt(n));
	sort(all(qu), cmp);
	int l=0,r=-1,t=0;
	for(int i=0;i<sz(qu);++i){
		while(t<qu[i].t)update(up[t].i,up[t].cur,
		    l,r),++t;
		while(t>qu[i].t)--t,update(up[t].i,up[t].
		    old,l,r);
		while(r<qu[i].r)add(++r);
		while(l>qu[i].l)add(--l);
		while(r>qu[i].r)remove(r--);
		while(l<qu[i].l)remove(l++);
		ans[qu[i].idx]=act();
	}
}

// tree
int st[maxn],ft[maxn],ver[maxn*2];
bool vis[maxn];

void ask(int v){
	vis[v]=!vis[v];
	if(vis[v])add(v);
	else remove(v);
}

// query[i] = {st[a]+1, st[b], i} + lca
```

3.8 Ordered set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<typename T> using ordered_set = tree<T,
   null_type,less<T>, rb_tree_tag,
   tree_order_statistics_node_update>;
template<typename T> using ordered_multiset = tree<T,
   null_type,less_equal<T>, rb_tree_tag,
   tree_order_statistics_node_update>;
// -------- CONSTRUCTOR -------- //
// 1. Para ordenar por MAX cambiar less<int> por greater<
   int>
// 2. Para multiset cambiar less<int> por less_equal<int>
//      Para borrar siendo multiset:
//      int idx = st.order_of_key(value);
//      st.erase(st.find_by_order(idx));
// -------- METHODS --------- //
st.find_by_order(k) // returns pointer to the k-th
   smallest element
st.order_of_key(x)  // returns how many elements are
   smaller than x
st.find_by_order(k) == st.end() // true, if element does
   not exist
```

3.9 Persistent Segment Tree

```
typedef long long T;
struct Node{T val;int l,r;};
struct SegTree{
        vector<Node> ns;
        int act=0,size;
        vi roots;

        T null=0;
        T oper(T a, T b);
        void update(int x){
                ns[x].val=oper(ns[ns[x].l].val, ns[ns[x].
                    r].val);
        }

        int newNode(T x){
                Node tmp={x,-1,-1};
                ns.push_back(tmp);
                return act++;
        }

        int newNode(int l, int r){
                Node tmp={null,l,r};
                ns.push_back(tmp);
                update(act);
                return act++;
        }

        int build(vector<T>& a, int l, int r){
                if(r-l==1){return newNode(a[l]);}
                int m=(l+r)/2;
                return newNode(build(a, l, m),build(a, m,
                    r));
        }

        int set(int x, int i, T v, int l, int r){
                if(r-l==1)return newNode(v);
                int m=(l+r)/2;
                if(i<m)return newNode(set(ns[x].l, i, v,
                   l, m), ns[x].r);
                else return newNode(ns[x].l, set(ns[x].r,
                    i, v, m, r));
        }

        T get(int x, int lx, int rx, int l, int r){
                if(lx>=r || l>=rx)return null;
                if(lx>=l && rx<=r)return ns[x].val;
                int m=(lx+rx)/2;
                T v1=get(ns[x].l, lx, m, l, r);
                T v2=get(ns[x].r, m, rx, l, r);
                return oper(v1,v2);
        }

        T get(int l, int r, int time){return get(roots[
           time], 0, size, l, r+1);}
        void set(int i, T v, int time){roots.push_back(
           set(roots[time], i, v, 0, size));}
        void build(vector<T>& a, int n){size=n;roots.
           push_back(build(a, 0, size));}
};
```

3.10 RMQ

```
typedef long long T;
T oper(T a, T b); // max, min, gcd ...
struct RMQ {
        vector<vector<T>> table;
        void build(vector<T>& v){
                int n=sz(v);
                table.assign(20, vector<T>(n)); // log2(n
                    )
                for(int i=0;i<n;++i)table[0][i]=v[i];
                for(int j=1;(1<<j)<=n;++j)
                        for(int i=0;i+(1<<(j-1))<n;++i)
                                table[j][i]=oper(table[j
                                   -1][i],table[j-1][i
                                   +(1<<(j-1))]);
        }
        T get(int l, int r){
                int j=31-__builtin_clz(r-l+1);
```

```
            return oper(table[j][l], table[j][r-(1<<j
                )+1]);
        }
};
```

3.11 Segment Tree Iterativo

```
struct segtree{
    int n; vl v; ll nulo = 0;
    ll op(ll a, ll b) {return a + b;}
    segtree(int n) : n(n), v(2*n, nulo){}
    segtree(vl &a) : n(sz(a)), v(2*n){
        for(int i = 0; i<n; i++) v[n + i] = a[i];
        for (int i = n-1; i>=1; --i) v[i] = op(v[i<<1], v
            [i<<1|1]);
    }
    void upd(int k, ll nv){
        for (v[k += n] = nv; k > 1; k >>= 1) v[k>>1] = op
            (v[k], v[k^1]);
    }
    ll get(int l, int r){
        ll vl = nulo, vr = nulo;
        for (l += n, r += n+1; l < r; l >>= 1, r >>= 1){
            if (l&1) vl = op(vl, v[l++]);
            if (r&1) vr = op(v[--r], vr);
        }
        return op(vl, vr);
    }
};
```

3.12 Segment Tree Recursivo

```
typedef long long T;
struct SegTree{
        vector<T> vals,lazy;
        T null=0,nolz=0;
        int size;
        T oper(T a, T b);
        void build(vector<T>& a, int x, int lx, int rx){
                if(rx-lx==1){
                        if(lx<sz(a))vals[x]=a[lx];
                        return;
                }
                int m=(lx+rx)/2;
                build(a, 2*x+1, lx, m);
                build(a, 2*x+2, m, rx);
                vals[x]=oper(vals[2*x+1], vals[2*x+2]);
        }
        void build(vector<T>& a,int n){
                size=1;
                while(size<n)size*=2;
                vals.resize(2*size);
                lazy.assign(2*size, nolz);
                build(a, 0, 0, size);
        }
        void propagate(int x, int lx, int rx){
                if(rx-lx==1)return;
                if(lazy[x]==nolz)return;
                int m=(lx+rx)/2;
                // 2*x+1, 2*x+2 (lazy, vals)
                lazy[x]=nolz;
        }
        void upd(int l, int r, T v,int x, int lx, int rx)
            {
                if(lx>=r || l>=rx)return;
                if(lx>=l && rx<=r){
                        // lazy, vals
                        return;
                }
                propagate(x,lx,rx);
                int m=(lx+rx)/2;
                upd(l,r,v,2*x+1,lx,m);
                upd(l,r,v,2*x+2,m,rx);
                vals[x]=oper(vals[2*x+1], vals[2*x+2]);
        }
        void set(int i, T v, int x, int lx, int rx){
                if(rx-lx==1){
                        vals[x]=v;
                        return;
                }
                propagate(x,lx,rx);
                int m=(lx+rx)/2;
                if(i<m)set(i,v,2*x+1,lx,m);
                else set(i,v,2*x+2,m,rx);
                vals[x]=oper(vals[2*x+1], vals[2*x+2]);
        }
        T get(int l, int r, int x, int lx, int rx){
                if(lx>=r || l>=rx)return null;
                if(lx>=l && rx<=r)return vals[x];
                propagate(x,lx,rx);
                int m=(lx+rx)/2;
                T v1=get(l,r,2*x+1,lx,m);
                T v2=get(l,r,2*x+2,m,rx);
                return oper(v1,v2);
        }
        T get(int l, int r){return get(l,r+1,0,0,size);}
        void upd(int l, int r, T v){upd(l,r+1,v,0,0,size)
            ;}
```

```
        void set(int i, T val){set(i,val,0,0,size);}
};
```

3.13 Segment Tree 2D

```
const int N=1000+1;
ll st[2*N][2*N];
struct SegTree{
        int n,m,neutro=0;
        inline ll op(ll a, ll b){return a+b;}

        SegTree(int n, int m): n(n), m(m){
                for(int i=0;i<2*n;++i)for(int j=0;j<2*m
                    ;++j)st[i][j]=neutro;
        }
        SegTree(vector<vi>& a): n(sz(a)), m(n ? sz(a[0])
            : 0){ build(a); }

        void build(vector<vi>& a){
                for(int i=0;i<n;++i)for(int j=0;j<m;++j)
                    st[i+n][j+m]=a[i][j];
                for(int i=0;i<n;++i)for(int j=m-1;j>=1;--
                    j)st[i+n][j]=op(st[i+n][j<<1], st[i+n
                    ][j<<1|1]);
                for(int i=n-1;i>=1;--i)for(int j=0;j<2*m
                    ;++j)st[i][j]=op(st[i<<1][j], st[i
                    <<1|1][j]);
        }

        void upd(int x, int y, ll v){
                st[x+n][y+m]=v;
                for(int j=y+m;j>1;j>>=1)st[x+n][j>>1]=op(
                    st[x+n][j], st[x+n][j^1]);
                for(int i=x+n;i>1;i>>=1)for(int j=y+m;j;j
                    >>=1)st[i>>1][j]=op(st[i][j], st[i^1][
                    j]);
        }

        ll get(int x0, int y0, int x1, int y1){
                ll r=neutro;
                for(int i0=x0+n,i1=x1+n+1;i0<i1;i0>>=1,i1
                    >>=1){
                        int t[4],q=0;
                        if(i0&1)t[q++]=i0++;
                        if(i1&1)t[q++]=--i1;
                        for(int k=0;k<q;++k)for(int j0=y0
                            +m,j1=y1+m+1;j0<j1;j0>>=1,j1
                            >>=1){
                                if(j0&1)r=op(r,st[t[k]][
                                    j0++]);
                                if(j1&1)r=op(r,st[t[k
                                    ]][--j1]);
                        }
                }
                        return r;
        }
};
```

3.14 Segment Tree Beats

```
typedef long long T;
T null=0,noVal=0;
T INF=1e18;
struct Node{
        T sum,lazy;
        T max1,max2,maxc;
        T min1,min2,minc;
};
struct SegTree{
        vector<Node> vals;int size;
        void oper(int a, int b, int c); // node c, left a
            , right b;
        Node single(T x){
                Node tmp;
                tmp.sum=tmp.max1=tmp.min1=x;
                tmp.maxc=tmp.minc=1;
                tmp.lazy=noVal;
                tmp.max2=-INF;
                tmp.min2=INF;
                return tmp;
        }

        void build(vector<T>& a,int n);
        void propagateMin(T v, int x, int lx, int rx){
                if(vals[x].max1<=v)return;
                vals[x].sum-=vals[x].max1*vals[x].maxc;
                vals[x].max1=v;
                vals[x].sum+=vals[x].max1*vals[x].maxc;
                if(rx-lx==1){
                        vals[x].min1=v;
                }else{
                        if(v<=vals[x].min1){
                                vals[x].min1=v;
                        }else if(v<vals[x].min2){
                                vals[x].min2=v;
                        }
                }
        }

        void propagateAdd(T v, int x, int lx, int rx){
                vals[x].sum+=v*((T)(rx-lx));
                vals[x].lazy+=v;
                vals[x].max1+=v;
                vals[x].min1+=v;
                if(vals[x].max2!=-INF)vals[x].max2+=v;
                if(vals[x].min2!=INF)vals[x].min2+=v;
        }

        void propagate(int x, int lx, int rx){
```

```
                if(rx-lx==1)return;
                int m=(lx+rx)/2;
                if(vals[x].lazy!=noVal){
                        propagateAdd(vals[x].lazy, 2*x+1,
                            lx, m);
                        propagateAdd(vals[x].lazy, 2*x+2,
                            m, rx);
                        vals[x].lazy=noVal;
                }
                propagateMin(vals[x].max1, 2*x+1, lx, m);
                propagateMin(vals[x].max1, 2*x+2, m, rx);
        }

        void updAdd(int l, int r, T v,int x, int lx, int
            rx){
                if(lx>=r || l>=rx)return;
                if(lx>=l && rx<=r){
                        propagateAdd(v, x, lx, rx);
                        return;
                }
                propagate(x,lx,rx);
                int m=(lx+rx)/2;
                updAdd(l,r,v,2*x+1,lx,m);
                updAdd(l,r,v,2*x+2,m,rx);
                oper(2*x+1, 2*x+2, x);
        }

        void updMin(int l, int r, T v,int x, int lx, int
            rx){
                if(lx>=r || l>=rx || vals[x].max1<v)
                    return;
                if(lx>=l && rx<=r && vals[x].max2<v){
                        propagateMin(v, x, lx, rx);
                        return;
                }
                propagate(x,lx,rx);
                int m=(lx+rx)/2;
                updMin(l,r,v,2*x+1,lx,m);
                updMin(l,r,v,2*x+2,m,rx);
                oper(2*x+1, 2*x+2, x);
        }

        void updAdd(int l, int r, T v){updAdd(l,r+1,v
            ,0,0,size);}
        void updMin(int l, int r, T v){updMin(l,r+1,v
            ,0,0,size);}
};
```

3.15 Sparse Table 2D

```
const int MAX_N = 100;
const int MAX_M = 100;
const int KN = log2(MAX_N)+1;
const int KM = log2(MAX_M)+1;
int table[KN][MAX_N][KM][MAX_M];
int _log2N[MAX_N+1];
int _log2M[MAX_M+1];

int MAT[MAX_N][MAX_M];
int n, m, ic, ir, jc, jr;

void calc_log2() {
    _log2N[1] = 0;
    _log2M[1] = 0;
    for (int i = 2; i <= MAX_N; i++) _log2N[i] = _log2N[i
        /2] + 1;
    for (int i = 2; i <= MAX_M; i++) _log2M[i] = _log2M[i
        /2] + 1;
}

void build() {
    for (ir = 0; ir < n; ir++) {
        for (ic = 0; ic < m; ic++)
            table[0][ir][0][ic] = MAT[ir][ic];
        for (jc = 1; jc < KM; jc++)
            for (ic = 0; ic + (1 <<(jc-1)) < m; ic++)
                table[0][ir][jc][ic] = min(table[0][ir][
                    jc-1][ic], table[0][ir][jc-1][ic + (1
                    << (jc-1))]);
    }

    for (jr = 1; jr < KN; jr++)
        for (ir = 0; ir < n; ir++)
            for (jc = 0; jc < KM; jc++)
                for (ic = 0; ic < m; ic++)
                    table[jr][ir][jc][ic] = min(table[jr
                        -1][ir][jc][ic], table[jr-1][ir
                        +(1<<(jr-1))][jc][ic]);
}

int rmq(int x1, int y1, int x2, int y2) {
    int lenx = x2-x1+1;
    int kx = _log2N[lenx];
    int leny = y2-y1+1;
    int ky = _log2M[leny];

    int min_R1 = min(table[kx][x1][ky][y1], table[kx][x1
        ][ky][y2 + 1 - (1<<ky)]);
    int min_R2 = min(table[kx][x2+1-(1<<kx)][ky][y1],
        table[kx][x2+1- (1<<kx)][ky][y2 + 1 - (1<<ky)]);
    return min(min_R1, min_R2);
}
```

3.16 Sqrt Descomposition

```
typedef long long T;
struct Sqrt { // O(n/b+b)
        int b; // check b
        vector<T> nums,blocks;
```

```
	void build(vector<T>& arr, int n){
		b=(int)ceil(sqrt(n));nums=arr;
		blocks.assign(b, 0);
		for(int i=0;i<n;++i){
			blocks[i/b]+=nums[i];
		}
	}
	void set(int x, int v){
		blocks[x/b]-=nums[x];
		nums[x]=v;
		blocks[x/b]+=nums[x];
	}
	T get(int r){
		T res=0;
		for(int i=0;i<r/b;++i){res+=blocks[i];}
		for(int i=(r/b)*b;i<r;++i){res+=nums[i];}
		return res;
	}
	T get(int l, int r){return get(r+1)-get(l);}
};
```

3.17 Treap

```
// treap => order asc, implicit treap => order array
typedef long long T;
struct Treap{
	Treap *l,*r,*dad;
	u64 prior;
	T sz,value,sum,lz;
	Treap(T v){
		l=r=nullptr;
		lz=0;sz=1;
		prior=rng();
		value=sum=v;
	}
	~Treap(){delete l;delete r;}
};
typedef Treap* PTreap;
T cnt(PTreap x){return (!x?0:x->sz);}
T sum(PTreap x){return (!x?0:x->sum);}
void propagate(PTreap x){
	if(x && x->lz){
		if(x->l); // lz, value, sum ...
		if(x->r); // lz, value, sum ...
		x->lz=0;
	}
}
void update(PTreap x){
	propagate(x->l);
	propagate(x->r);
	x->sz=cnt(x->l)+cnt(x->r)+1;
	x->sum=sum(x->l)+sum(x->r)+x->value;
	if(x->l)x->l->dad=x;
	if(x->r)x->r->dad=x;
}
void upd(PTreap x, T v){
	if(!x)return;
	update(x);
	// lz, value, sum ...
}
// pair<PTreap, PTreap> split(PTreap x, T key){ // f <=
    key < s
pair<PTreap, PTreap> split(PTreap x, int left){ // cnt(f)
     == left
	if(!x)return {nullptr, nullptr};
	propagate(x);
	if(cnt(x->l)>=left){ // if(x->value>key){
		auto got=split(x->l, left); // , key);
		x->l=got.second;
		update(x);
		return {got.first, x};
	}else{
		auto got=split(x->r, left-cnt(x->l)-1);
		    // , key);
		x->r=got.first;
		update(x);
		return {x, got.second};
	}
}
PTreap merge(PTreap x, PTreap y){
	if(!x)return y;
	if(!y)return x;
	propagate(x);
	propagate(y);
	if(x->prior<=y->prior){
		x->r=merge(x->r, y);
		update(x);
		return x;
	}else{
		y->l=merge(x, y->l);
		update(y);
		return y;
	}
}
PTreap combine(PTreap x, PTreap y){
	if(!x)return y;
	if(!y)return x;
	if(x->prior<y->prior)swap(x, y);
	auto z=split(y, x->value);
	x->r=combine(x->r, z.second);
	x->l=combine(z.first, x->l);
	return x;
```

```
}

T kth(PTreap& x, int k){ // indexed 0
        if(!x)return null;
        if(k==cnt(x->l))return x->value;
        if(k<cnt(x->l))return kth(x->l, k);
        return kth(x->r, k-cnt(x->l)-1);
}

pair<int, T> lower_bound(PTreap x, T key){ // index,
    value
        if(!x)return {0, null};
        if(x->value<key){
                auto y=lower_bound(x->r, key);
                y.first+=cnt(x->l)+1;
                return y;
        }
        auto y=lower_bound(x->l, key);
        if(y.first==cnt(x->l))y.second=x->value;
        return y;
}

void dfs(PTreap x){
        if(!x)return;
        propagate(x);
        dfs(x->l);cout<<x->value<<" ";dfs(x->r);
}

// PTreap root=nullptr;
// PTreap act=new Treap(c);
// root=merge(root, act);
```

3.18 Two Stacks

```
typedef long long T;
struct Node{T val,acum;};
struct TwoStacks{
        stack<Node> s1,s2;

        void add(T x){
                Node tmp={x,x};
                if(!s2.empty()){
                // tmp.acum + s2.top().acum
                }
                s2.push(tmp);
        }

        void remove(){
                if(s1.empty()){
                        while(!s2.empty()){
                                Node tmp=s2.top();
                                if(s1.empty()){
                                // tmp.acum = tmp.val
                                }else{
                                // tmp.acum + s1.top().
                                    acum
                                }
                                s1.push(tmp);
                                s2.pop();
                        }
                }
                s1.pop();
        }

        bool good(){
                if(s1.empty() && s2.empty())return false;
                else if(!s1.empty() && s2.empty()){
                        return true; // eval s1.top();
                }else if(s1.empty() && !s2.empty()){
                        return true; // eval s2.top();
                }else{
                        return true; // eval s1.top() +
                            s2.top()
                }
        }
};
```

3.19 Wavelet Tree

```
const int maxn = 1e5+5, maxv = 1e9, minv = -1e9;
struct WaveletTree{ // indexed 1 - O(nlogn)
        int lo, hi;
        WaveletTree *l, *r;
        int *b, bsz, csz;
        ll *c;

        WaveletTree() {
                hi=bsz=csz=0;
                l=r=NULL;
                lo=1;
        }

        void build(int *from, int *to, int x, int y){
                lo=x,hi=y;
                if(from>=to)return;
                int mid=lo+(hi-lo)/2;
                auto f=[mid](int x){return x<=mid;};
                b=(int*)malloc((to-from+2)*sizeof(int));
                bsz=0;
                b[bsz++]=0;
                c=(ll*)malloc((to-from+2)*sizeof(ll));
                csz=0;
                c[csz++]=0;
                for(auto it=from;it!=to;++it){
                        b[bsz]=(b[bsz-1]+f(*it));
                        c[csz]=(c[csz-1]+(*it));
                        bsz++;csz++;
                }
                if(hi==lo)return;
                auto pivot=stable_partition(from, to, f);
                l=new WaveletTree();
```

```cpp
            l->build(from, pivot, lo, mid);
            r=new WaveletTree();
            r->build(pivot, to, mid+1, hi);
        }

        //kth smallest element in [l, r]
        int kth(int l, int r, int k){
            if(l>r)return 0;
            if(lo==hi)return lo;
            int inLeft=b[r]-b[l-1], lb=b[l-1], rb=b[r];
            if(k<=inLeft)return this->l->kth(lb+1, rb, k);
            return this->r->kth(l-lb, r-rb, k-inLeft);
        }

        //count of numbers in [l, r] Less than or equal to k
        int lte(int l, int r, int k){
            if(l>r || k<lo)return 0;
            if(hi<=k)return r-l+1;
            int lb=b[l-1], rb=b[r];
            return this->l->lte(lb+1, rb, k)+this->r->lte(l-lb, r-rb, k);
        }

        //count of numbers in [l, r] equal to k
        int count(int l, int r, int k){
            if(l>r || k<lo || k>hi)return 0;
            if(lo==hi)return r-l+1;
            int lb=b[l-1], rb=b[r];
            int mid=(lo+hi)>>1;
            if(k<=mid)return this->l->count(lb+1, rb, k);
            return this->r->count(l-lb, r-rb, k);
        }

        //sum of numbers in [l ,r] less than or equal to k
        ll sum(int l, int r, int k){
            if(l>r || k<lo)return 0;
            if(hi<=k)return c[r]-c[l-1];
            int lb=b[l-1], rb=b[r];
            return this->l->sum(lb+1, rb, k)+this->r->sum(l-lb, r-rb, k);
        }

        ~WaveletTree(){
            delete l;
            delete r;
        }
};

// int a[maxn];
// WaveletTree wt;
// for(int i=1;i<=n;++i)cin>>a[i];
// wt.build(a+1, a+n+1, minv, maxv);
```

3.20 Trie Bit

```cpp
struct node{
  int childs[2]{-1, -1};
};

struct TrieBit{
    vector<node> nds;
        vi passNums;

        TrieBit(){
        nds.pb(node());
        passNums.pb(0);
        }

    void insert(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;

            if(nds[cur].childs[bit] == -1){
                nds[cur].childs[bit] = nds.size();
                nds.pb(node());
                passNums.pb(0);
            }

            passNums[cur]++;
            cur = nds[cur].childs[bit];
        }

        passNums[cur]++;
    }

    void remove(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;
            passNums[cur]--;
            cur = nds[cur].childs[bit];
        }

        passNums[cur]--;
    }

    int maxXor(int num){
        int ans = 0;
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;

            int n1 = nds[cur].childs[!bit];
            if (n1 != -1 && passNums[n1]){
                ans += (1 << i);
                bit = !bit;
            }
```

```
            cur = nds[cur].childs[bit];
        }
        return ans;
    }
};
```

4 Flujos

4.1 Blossom

```
/// Complexity: O(|E||V|^2)
/// Tested: https://tinyurl.com/oe5rnpk
struct network {
  struct struct_edge { int v; struct_edge * n; };
  typedef struct_edge* edge;
  int n;
  struct_edge pool[MAXE]; ///2*n*n;
  edge top;
  vector<edge> adj;
  queue<int> q;
  vector<int> f, base, inq, inb, inp, match;
  vector<vector<int>> ed;
  network(int n) : n(n), match(n, -1), adj(n), top(pool),
      f(n), base(n),
                   inq(n), inb(n), inp(n), ed(n, vector<
                      int>(n)) {}
  void add_edge(int u, int v) {
    if(ed[u][v]) return;
    ed[u][v] = 1;
    top->v = v, top->n = adj[u], adj[u] = top++;
    top->v = u, top->n = adj[v], adj[v] = top++;
  }
  int get_lca(int root, int u, int v) {
    fill(inp.begin(), inp.end(), 0);
    while(1) {
      inp[u = base[u]] = 1;
      if(u == root) break;
      u = f[ match[u] ];
    }
    while(1) {
      if(inp[v = base[v]]) return v;
      else v = f[ match[v] ];
    }
  }
  void mark(int lca, int u) {
    while(base[u] != lca) {
      int v = match[u];
      inb[ base[u ]] = 1;
      inb[ base[v] ] = 1;
      u = f[v];
      if(base[u] != lca) f[u] = v;
    }
  }
  void blossom_contraction(int s, int u, int v) {
    int lca = get_lca(s, u, v);
    fill(inb.begin(), inb.end(), 0);
    mark(lca, u); mark(lca, v);
    if(base[u] != lca) f[u] = v;
    if(base[v] != lca) f[v] = u;
    for(int u = 0; u < n; u++)
      if(inb[base[u]]) {
        base[u] = lca;
        if(!inq[u]) {
            inq[u] = 1;
            q.push(u);
        }
      }
  }
  int bfs(int s) {
    fill(inq.begin(), inq.end(), 0);
    fill(f.begin(), f.end(), -1);
    for(int i = 0; i < n; i++) base[i] = i;
    q = queue<int>();
    q.push(s);
    inq[s] = 1;
   while(q.size()) {
      int u = q.front(); q.pop();
      for(edge e = adj[u]; e; e = e->n) {
        int v = e->v;
        if(base[u] != base[v] && match[u] != v) {
          if((v == s) || (match[v] != -1 && f[match[v]]
              != -1))
            blossom_contraction(s, u, v);
          else if(f[v] == -1) {
            f[v] = u;
            if(match[v] == -1) return v;
            else if(!inq[match[v]]) {
              inq[match[v]] = 1;
              q.push(match[v]);
            }
          }
        }
      }
    }
    return -1;
  }
  int doit(int u) {
    if(u == -1) return 0;
    int v = f[u];
    doit(match[v]);
    match[v] = u; match[u] = v;
    return u != -1;
  }
  /// (i < net.match[i]) => means match
  int maximum_matching() {
    int ans = 0;
```

```
    for(int u = 0; u < n; u++)
      ans += (match[u] == -1) && doit(bfs(u));
    return ans;
  }
};
```

4.2 Dinic

```
// O(|E|*|V|^2)
struct edge { ll v, cap, inv, flow, ori; };
struct network {
  ll n, s, t;
  vector<ll> lvl;
  vector<vector<edge>> g;
  network(ll n) : n(n), lvl(n), g(n) {}
  void add_edge(int u, int v, ll c) {
    g[u].push_back({v, c, sz(g[v]), 0, 1});
    g[v].push_back({u, 0, sz(g[u])-1, c, 0});
  }
  bool bfs() {
    fill(lvl.begin(), lvl.end(), -1);
    queue<ll> q;
    lvl[s] = 0;
    for(q.push(s); q.size(); q.pop()) {
      ll u = q.front();
      for(auto &e : g[u]) {
        if(e.cap > 0 && lvl[e.v] == -1) {
          lvl[e.v] = lvl[u]+1;
          q.push(e.v);
        }
      }
    }
    return lvl[t] != -1;
  }
  ll dfs(ll u, ll nf) {
    if(u == t) return nf;
    ll res = 0;
    for(auto &e : g[u]) {
      if(e.cap > 0 && lvl[e.v] == lvl[u]+1) {
        ll tf = dfs(e.v, min(nf, e.cap));
        res += tf; nf -= tf; e.cap -= tf;
        g[e.v][e.inv].cap += tf;
        g[e.v][e.inv].flow -= tf;
        e.flow += tf;
        if(nf == 0) return res;
      }
    }
    if(!res) lvl[u] = -1;
    return res;
  }
  ll max_flow(ll so, ll si, ll res = 0) {
    s = so; t = si;
    while(bfs()) res += dfs(s, LONG_LONG_MAX);
    return res;
  }
  void min_cut(){
    queue<ll> q;
    vector<bool> vis(n, 0);
    vis[s] = 1;
    for(q.push(s); q.size(); q.pop()) {
      ll u = q.front();
      for(auto &e : g[u]) {
        if(e.cap > 0 && !vis[e.v]) {
          q.push(e.v);
          vis[e.v] = 1;
        }
      }
    }
    vii ans;
    for (int i = 0; i<n; i++){
      for (auto &e : g[i]){
          if (vis[i] && !vis[e.v] && e.ori){
            ans.push_back({i+1, e.v+1});
          }
      }
    }
    for (auto [x, y] : ans) cout << x << ' ' << y << ln;
  }
  bool dfs2(vi &path, vector<bool> &vis, int u){
    vis[u] = 1;
    for (auto &e : g[u]){
      if (e.flow > 0 && e.ori && !vis[e.v]){
        if (e.v == t || dfs2(path, vis, e.v)){
          path.push_back(e.v);
          e.flow = 0;
          return 1;
        }
      }
    }
    return 0;
  }
  void disjoint_paths(){
    vi path;
    vector<bool> vis(n, 0);
    while (dfs2(path, vis, s)){
      path.push_back(s);
      reverse(all(path));
      cout << sz(path) << ln;
      for (int v : path) cout << v+1 << ' ';
      cout << ln;
      path.clear(); vis.assign(n, 0);
    }
  }
};
```

4.3 Edmonds Karp

```
//O(V * E^2)
ll bfs(vector<vi> &adj, vector<vl> &capacity, int s, int
    t, vi& parent) {
     fill(parent.begin(), parent.end(), -1);
     parent[s] = -2;
     queue<pll> q;
     q.push({s, INFL});

     while (!q.empty()) {
          int cur = q.front().first;
          ll flow = q.front().second;
          q.pop();

          for (int next : adj[cur]) {
               if (parent[next] == -1LL && capacity[cur][
                   next]) {
                    parent[next] = cur;
                    ll new_flow = min(flow, capacity[cur][
                        next]);
                    if (next == t)
                         return new_flow;
                    q.push({next, new_flow});
               }
          }
     }

     return 0;
}

ll maxflow(vector<vi> &adj, vector<vl> &capacity, int s,
    int t, int n) {
     ll flow = 0;
     vi parent(n);
     ll new_flow;

     while ((new_flow = bfs(adj, capacity, s, t, parent)))
          {
          flow += new_flow;
          int cur = t;
          while (cur != s) {
               int prev = parent[cur];
               capacity[prev][cur] -= new_flow;
               capacity[cur][prev] += new_flow;
               cur = prev;
          }
     }

     return flow;
}
```

4.4 Hopcroft Karp

```
// Complexity: O(|E|*sqrt(|V|))
struct mbm {
  vector<vector<int>> g;
  vector<int> d, match;
  int nil, l, r;
  /// u -> 0 to l, v -> 0 to r
  mbm(int l, int r) : g(l+r), d(1+l+r, INF), match(l+r, l
      +r),
                      nil(l+r), l(l), r(r) {}
  void add_edge(int a, int b) {
    g[a].push_back(l+b);
    g[l+b].push_back(a);
  }
  bool bfs() {
    queue<int> q;
    for(int u = 0; u < l; u++) {
      if(match[u] == nil) {
        d[u] = 0;
        q.push(u);
      } else d[u] = INF;
    }
    d[nil] = INF;
    while(q.size()) {
      int u = q.front(); q.pop();
      if(u == nil) continue;
      for(auto v : g[u]) {
        if(d[ match[v] ] == INF) {
          d[ match[v] ] = d[u]+1;
          q.push(match[v]);
        }
      }
    }
    return d[nil] != INF;
  }
  bool dfs(int u) {
    if(u == nil) return true;
    for(int v : g[u]) {
      if(d[ match[v] ] == d[u]+1 && dfs(match[v])) {
        match[v] = u; match[u] = v;
        return true;
      }
    }
    d[u] = INF;
    return false;
  }
  int max_matching() {
    int ans = 0;
    while(bfs()) {
      for(int u = 0; u < l; u++) {
        ans += (match[u] == nil && dfs(u));
      }
    }
    return ans;
  }
  void matchs(){
    for (int i = 0; i<l; i++){
        if (match[i] == l+r) continue;
        cout << i+1 << ' ' << match[i]+1-l << ln;
    }
```

```
    }
};
```

4.5 Maximum Bipartite Matching

```
// O(|E|*|V|)
struct mbm {
  int l, r;
  vector<vector<int>> g;
  vector<int> match, seen;
  mbm(int l, int r) : l(l), r(r), g(l), match(r), seen(r)
      {}
  void add_edge(int l, int r) { g[l].push_back(r); }
  bool dfs(int u) {
    for(auto v : g[u]) {
      if(seen[v]++) continue;
      if(match[v] == -1 || dfs(match[v])) {
        match[v] = u;
        return true;
      }
    }
    return false;
  }
  int max_matching() {
    int ans = 0;
    fill(match.begin(), match.end(), -1);
    for(int u = 0; u < l; ++u) {
      fill(seen.begin(), seen.end(), 0);
      ans += dfs(u);
    }
    return ans;
  }
  void matchs(){
    for (int i = 0; i<r; i++){
        if (match[i] == -1) continue;
        cout << match[i]+1  << ' ' << i+1 << ln;
    }
  }
};
```

4.6 Minimum Cost Maximum Flow

```
/// Complexity: O(|V|*|E|^2*log(|E|))
template <class type>
struct mcmf {
  struct edge { int u, v, cap, flow; type cost; };
  int n;
  vector<edge> ed;
  vector<vector<int>> g;
  vector<int> p;
  vector<type> d, phi;
  mcmf(int n) : n(n), g(n), p(n), d(n), phi(n) {}
  void add_edge(int u, int v, int cap, type cost) {
    g[u].push_back(ed.size());
    ed.push_back({u, v, cap, 0, cost});
    g[v].push_back(ed.size());
    ed.push_back({v, u, 0, 0, -cost});
  }
  bool dijkstra(int s, int t) {
    fill(d.begin(), d.end(), INF_TYPE);
    fill(p.begin(), p.end(), -1);
    set<pair<type, int>> q;
    d[s] = 0;
    for(q.insert({d[s], s}); q.size();) {
      int u = (*q.begin()).second; q.erase(q.begin());
      for(auto v : g[u]) {
        auto &e = ed[v];
        type nd = d[e.u]+e.cost+phi[e.u]-phi[e.v];
        if(0 < (e.cap-e.flow) && nd < d[e.v]) {
          q.erase({d[e.v], e.v});
          d[e.v] = nd; p[e.v] = v;
          q.insert({d[e.v], e.v});
        }
      }
    }
    for(int i = 0; i < n; i++) phi[i] = min(INF_TYPE, phi
        [i]+d[i]);
    return d[t] != INF_TYPE;
  }
  pair<int, type> max_flow(int s, int t) {
    type mc = 0;
    int mf = 0;
    fill(phi.begin(), phi.end(), 0);
    while(dijkstra(s, t)) {
      int flow = INF;
      for(int v = p[t]; v != -1; v = p[ ed[v].u ])
        flow = min(flow, ed[v].cap-ed[v].flow);
      for(int v = p[t]; v != -1; v = p[ ed[v].u ]) {
        edge &e1 = ed[v];
        edge &e2 = ed[v^1];
        mc += e1.cost*flow;
        e1.flow += flow;
        e2.flow -= flow;
      }
      mf += flow;
    }
    return {mf, mc};
  }
};
```

4.7 Weighted Matching

```
/// Complexity: O(|V|^3)
typedef int type;
```

```
struct matching_weighted {
  int l, r;
  vector<vector<type>> c;
  matching_weighted(int l, int r) : l(l), r(r), c(l,
      vector<type>(r)) {
    assert(l <= r);
  }
  void add_edge(int a, int b, type cost) { c[a][b] = cost
      ; }
  type matching() {
    vector<type> v(r), d(r); // v: potential
    vector<int> ml(l, -1), mr(r, -1); // matching pairs
    vector<int> idx(r), prev(r);
    iota(idx.begin(), idx.end(), 0);
    auto residue = [&](int i, int j) { return c[i][j]-v[j
        ]; };
    for(int f = 0; f < l; ++f) {
      for(int j = 0; j < r; ++j) {
        d[j] = residue(f, j);
        prev[j] = f;
      }
      type w;
      int j, l;
      for (int s = 0, t = 0;;) {
        if(s == t) {
          l = s;
          w = d[ idx[t++] ];
          for(int k = t; k < r; ++k) {
            j = idx[k];
            type h = d[j];
            if (h <= w) {
              if (h < w) t = s, w = h;
              idx[k] = idx[t];
              idx[t++] = j;
            }
          }
          for (int k = s; k < t; ++k) {
            j = idx[k];
            if (mr[j] < 0) goto aug;
          }
        }
        int q = idx[s++], i = mr[q];
        for (int k = t; k < r; ++k) {
          j = idx[k];
          type h = residue(i, j) - residue(i, q) + w;
          if (h < d[j]) {
            d[j] = h;
            prev[j] = i;
            if(h == w) {
              if(mr[j] < 0) goto aug;
              idx[k] = idx[t];
              idx[t++] = j;
            }
          }
        }
      }
      aug: for (int k = 0; k < l; ++k)
        v[ idx[k] ] += d[ idx[k] ] - w;
      int i;
      do {
        mr[j] = i = prev[j];
        swap(j, ml[i]);
      } while (i != f);
    }
    type opt = 0;
    for (int i = 0; i < l; ++i)
      opt += c[i][ml[i]]; // (i, ml[i]) is a solution
    return opt;
  }
};
```

4.8 Hungarian

```
const int N = 509;
/* Complexity: O(n^3) but optimized
It finds minimum cost maximum matching.
For finding maximum cost maximum matching
add -cost and return -matching()
1-indexed */
struct Hungarian {
  long long c[N][N], fx[N], fy[N], d[N];
  int l[N], r[N], arg[N], trace[N];
  queue<int> q;
  int start, finish, n;
  const long long inf = 1e18;
  Hungarian() {}
  Hungarian(int n1, int n2): n(max(n1, n2)) {
    for (int i = 1; i <= n; ++i) {
      fy[i] = l[i] = r[i] = 0;
      for (int j = 1; j <= n; ++j) c[i][j] = inf; // make
          it 0 for maximum cost matching (not necessarily
          with max count of matching)
    }
  }
  void add_edge(int u, int v, long long cost) {
    c[u][v] = min(c[u][v], cost);
  }
  inline long long getC(int u, int v) {
    return c[u][v] - fx[u] - fy[v];
  }
  void initBFS() {
    while (!q.empty()) q.pop();
    q.push(start);
    for (int i = 0; i <= n; ++i) trace[i] = 0;
    for (int v = 1; v <= n; ++v) {
      d[v] = getC(start, v);
      arg[v] = start;
    }
```

```
    finish = 0;
  }
  void findAugPath() {
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int v = 1; v <= n; ++v) if (!trace[v]) {
          long long w = getC(u, v);
          if (!w) {
            trace[v] = u;
            if (!r[v]) {
              finish = v;
              return;
            }
            q.push(r[v]);
          }
          if (d[v] > w) {
            d[v] = w;
            arg[v] = u;
          }
        }
    }
  }
  void subX_addY() {
    long long delta = inf;
    for (int v = 1; v <= n; ++v) if (trace[v] == 0 && d[v
       ] < delta) {
        delta = d[v];
      }
    // Rotate
    fx[start] += delta;
    for (int v = 1; v <= n; ++v) if(trace[v]) {
        int u = r[v];
        fy[v] -= delta;
        fx[u] += delta;
      } else d[v] -= delta;
    for (int v = 1; v <= n; ++v) if (!trace[v] && !d[v])
       {
        trace[v] = arg[v];
        if (!r[v]) {
          finish = v;
          return;
        }
        q.push(r[v]);
      }
  }
  void Enlarge() {
    do {
      int u = trace[finish];
      int nxt = l[u];
      l[u] = finish;
      r[finish] = u;
      finish = nxt;
    } while (finish);
  }
  long long maximum_matching() {
    for (int u = 1; u <= n; ++u) {
      fx[u] = c[u][1];
      for (int v = 1; v <= n; ++v) {
        fx[u] = min(fx[u], c[u][v]);
      }
    }
    for (int v = 1; v <= n; ++v) {
      fy[v] = c[1][v] - fx[1];
      for (int u = 1; u <= n; ++u) {
        fy[v] = min(fy[v], c[u][v] - fx[u]);
      }
    }
    for (int u = 1; u <= n; ++u) {
      start = u;
      initBFS();
      while (!finish) {
        findAugPath();
        if (!finish) subX_addY();
      }
      Enlarge();
    }
    long long ans = 0;
    for (int i = 1; i <= n; ++i) {
      if (c[i][l[i]] != inf) ans += c[i][l[i]];
      else l[i] = 0;
    }
    return ans;
  }
};
```

5 Geometria

5.1 Puntos

```
typedef long double lf;
const lf EPS = 1e-9;
const lf E0 = 0.0L; //Keep = 0 for integer coordinates,
    otherwise = EPS
const lf PI = acos(-1);

struct pt {
    lf x, y;
    pt(){}
    pt(lf a, lf b): x(a), y(b){}
    pt(lf ang): x(cos(ang)), y(sin(ang)){}  // Polar unit
        point: ang(RAD)
    pt operator - (const pt &q) const { return {x - q.x ,
        y - q.y }; }
    pt operator + (const pt &q) const { return {x + q.x ,
        y + q.y }; }
    pt operator * (pt p){ return {x * p.x - y * p.y, x *
        p.y + y * p.x}; }
```

```
    pt operator * (const lf &t) const { return {x * t , y * t }; }
    pt operator / (const lf &t) const { return {x / t , y / t }; }
    bool operator == (pt p){ return abs(x - p.x) <= EPS && abs(y - p.y) <= EPS; }
    bool operator != (pt p){ return !operator==(p); }
    bool operator < (const pt & q) const { // set / sort
        if(fabsl(x - q.x) > E0) return x < q.x;
        return y < q.y;
    }
    void print(){ cout << x << " " << y << "\n"; }
};

pt normalize(pt p){
    lf norm = hypotl(p.x, p.y);
    if(fabsl(norm) > EPS) return {p.x /= norm, p.y /= norm};
    else return p;
}

int cmp(lf a, lf b){ return (a + EPS < b ? -1 :(b + EPS < a ? 1 : 0)); } // float comparator

// rota ccw
pt rot90(pt p){ return {-p.y, p.x}; }
// w(RAD)
pt rot(pt p, lf w){ return {cosl(w) * p.x - sinl(w) * p.y, sinl(w) * p.x + cosl(w) * p.y}; }

lf norm2(pt p){ return p.x * p.x + p.y * p.y; }
lf norm(pt p){ return hypotl(p.x, p.y); }

lf dis2(pt p, pt q){ return norm2(p - q); }
lf dis(pt p, pt q){ return norm(p - q); }

lf arg(pt a){return atan2(a.y, a.x); } // ang(RAD) a x-pos
lf dot(pt a, pt b){ return a.x * b.x + a.y * b.y; } // x = 90 -> cos = 0
lf cross(pt a, pt b){ return a.x * b.y - a.y * b.x; } // x = 180 -> sin = 0
lf orient(pt a, pt b, pt c){ return cross(b - a, c - a); } // AB clockwise = -
int sign(lf x){ return (EPS < x) - (x < -EPS); }

// p inside angle abc (center in a)
bool in_angle(pt a, pt b, pt c, pt p) {
    //assert(fabsl(orient(a, b, c)) > E0);
    if(orient(a, b, c) < -E0)
        return orient(a, b, p) >= -E0 || orient(a, c, p) <= E0;
    return orient(a, b, p) >= -E0 && orient(a, c, p) <= E0;
}

lf min_angle(pt a, pt b){ return acos(max((lf)-1.0, min((lf)1.0, dot(a, b)/norm(a)/norm(b)))); } // ang(RAD)
lf angle(pt a, pt b){ return atan2(cross(a, b), dot(a, b)); } // ang(RAD)
lf angle(pt a, pt b, pt c){ // ang(RAD) AB AC ccw
    lf ang = angle(b - a, c - a);
    if (ang < 0) ang += 2 * PI;
    return ang;
}

bool half(pt p){ // true if is in (0, 180] (line is x axis)
    // assert(p.x != 0 || p.y != 0); // the argument of (0, 0) is undefined
    return p.y > 0 || (p.y == 0 && p.x < 0);
}

bool half_from(pt p, pt v = {1, 0}) {
  return cross(v,p) < 0 || (cross(v,p) == 0 && dot(v,p) < 0);
}

// polar sort
bool polar_cmp(const pt &a, const pt &b){
  return make_tuple(half(a), 0) < make_tuple(half(b), cross(a,b));
}

void polar_sort(vector<pt> &v, pt o){ // sort points in counterclockwise with respect to point o
    sort(v.begin(), v.end(), [&](pt a,pt b) {
        return make_tuple(half(a - o), 0.0, norm2((a - o))) < make_tuple(half(b - o), cross(a - o, b - o), norm2((b - o)));
    });
}

int cuad(pt p){ // REVISAR
    if(p.x > 0 && p.y >= 0) return 0;
    if(p.x <= 0 && p.y > 0) return 1;
    if(p.x < 0 && p.y <= 0) return 2;
    if(p.x >= 0 && p.y < 0) return 3;
    return -1; // x == 0 && y == 0
}

bool cmp(pt p1, pt p2){
  int c1 = cuad(p1), c2 = cuad(p2);
  return c1 == c2 ? p1.y * p2.x < p1.x * p2.y : c1 < c2;
}
```

5.2 Lineas

```
// add points operators
struct line {
    pt v; lf c; // v: dir, c: mov y
    line(pt v, lf c) : v(v), c(c) {}
    line(lf a, lf b, lf c) : v({b, -a}), c(c) {} // ax +
```

```
        by = c
    line(pt p, pt q) : v(q - p), c(cross(v, p)) {}

    bool operator < (line l){ return cross(v, l.v) > 0; }
    bool operator == (line l){ return (abs(cross(v, l.v))
         <= E0) && c == l.c; } // abs(c) == abs(l.c)

    lf side(pt p){ return cross(v, p) - c; }
    lf dist(pt p){ return abs(side(p)) / norm(v); }
    lf dist2(pt p){ return side(p) * side(p) / (lf)norm2(
        v); }
    line perp_through(pt p){ return {p, p + rot90(v)}; }
        // line perp to v passing through p
    bool cmp_proj(pt p, pt q){ return dot(v, p) < dot(v,
        q); } // order for points over the line
    // use: auto fsort = [&l1](const pt &a, const pt &b){
         return l1.cmp_proj(a, b); };
    line translate(pt t){ return {v, c + cross(v, t)}; }
    line shift_left(lf d){ return {v, c + d*norm(v)}; }
    pt proj(pt p){ return p - rot90(v) * side(p) / norm2(
        v); } // pt proyected on the line
    pt refl(pt p){ return p - rot90(v) * 2 * side(p) /
        norm2(v); } // pt reflected on the other side of
        the line
    bool has(pt p){ return abs(cross(v, p) - c) <= E0; };
         // pt on line

    lf evalx(lf x){
         assert(fabsl(v.x) > EPS);
         return (c + v.y * x) / v.x;
    }
};

pt inter_ll(line l1, line l2) {
    if (abs(cross(l1.v, l2.v)) <= EPS) return {INF, INF};
         // parallel
    return (l2.v * l1.c - l1.v * l2.c) / cross(l1.v, l2.v
        ); // floating points
}

// bisector divides the angle in 2 equal angles
// interior line goes on the same direction as l1 and l2
line bisector(line l1, line l2, bool interior) {
    // assert(cross(l1.v, l2.v) != 0); // l1 and l2
        cannot be parallel
    lf sign = interior ? 1 : -1;
    return {l2.v / norm(l2.v) + l1.v / norm(l1.v) * sign,
            l2.c / norm(l2.v) + l1.c / norm(l1.v) * sign
               };
}
```

5.3 Poligonos

```
// add Points Lines Segments Circles
// points in polygon(vector<pt>) ccw or cw
enum {OUT, IN, ON};

lf area(vector<pt>& p){
    lf r = 0.;
    for(int i = 0, n = p.size(); i < n; ++i){
        r += cross(p[i], p[(i + 1) % n]);
    }
    return r / 2; // negative if CW, positive if CCW
}

lf perimeter(vector<pt>& p) {
    lf per = 0;
    for (int i = 0, n = p.size(); i < n; ++i){
        per += norm(p[i] - p[(i + 1) % n]);
    }
    return per;
}

bool is_convex(vector<pt>& p) {
    bool pos = 0, neg = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int o = orient(p[i], p[(i + 1) % n], p[(i + 2) %
            n]);
        if (o > 0) pos = 1;
        if (o < 0) neg = 1;
    }
    return !(pos && neg);
}

int point_in_polygon(vector<pt>& pol, pt& p){
    int wn = 0;
    for(int i = 0, n = pol.size(); i < n; ++i) {
        lf c = orient(p, pol[i], pol[(i + 1) % n]);
        if(fabsl(c) <= E0 && dot(pol[i] - p, pol[(i + 1)
            % n] - p) <= E0) return ON; // on segment

        if(c > 0 && pol[i].y <= p.y + E0 && pol[(i + 1) %
             n].y - p.y > E0) ++wn;
        if(c < 0 && pol[(i + 1) % n].y <= p.y + E0 && pol
            [i].y - p.y > E0) --wn;
    }
    return wn ? IN : OUT;
}

// O(logn) polygon CCW, remove collinear
int point_in_convex_polygon(const vector<pt> &pol, const
    pt &p){
        int low = 1, high = pol.size() - 1;
        while(high - low > 1){
                int mid = (low + high) / 2;
                if(orient(pol[0], pol[mid], p) >= -E0)
                    low = mid;
                else high = mid;
        }
        if(orient(pol[0], pol[low], p) < -E0) return OUT;
```

```
        if(orient(pol[low], pol[high], p) < -E0) return OUT;
        if(orient(pol[high], pol[0], p) < -E0) return OUT;

        if(low == 1 && orient(pol[0], pol[low], p) <= E0) return ON;
        if(orient(pol[low], pol[high], p) <= E0) return ON;
        if(high == (int) pol.size() -1 && orient(pol[high], pol[0], p) <= E0) return ON;
        return IN;
}

// convex polygons in some order (CCW, CW)
vector<pt> minkowski(vector<pt> P, vector<pt> Q) {
        rotate(P.begin(), min_element(P.begin(), P.end()), P.end());
        rotate(Q.begin(), min_element(Q.begin(), Q.end()), Q.end());

        P.push_back(P[0]), P.push_back(P[1]);
        Q.push_back(Q[0]), Q.push_back(Q[1]);

        vector<pt> ans;
        size_t i = 0, j = 0;
        while(i < P.size() - 2 || j < Q.size() - 2){
                ans.push_back(P[i] + Q[j]);
                lf dt = cross(P[i + 1] - P[i], Q[j + 1] - Q[j]);
                if(dt >= E0 && i < P.size() - 2) ++i;
                if(dt <= E0 && j < Q.size() - 2) ++j;
        }
        return ans;
}

pt centroid(vector<pt>& p){
    pt c{0, 0};
    lf scale = 6. * area(p);
    for (int i = 0, n = p.size(); i < n; ++i){
        c = c + (p[i] + p[(i + 1) % n]) * cross(p[i], p[(i + 1) % n]);
    }
    return c / scale;
}

void normalize(vector<pt>& p) { // polygon CCW
    int bottom = min_element(p.begin(), p.end()) - p.begin();
    vector<pt> tmp(p.begin() + bottom, p.end());
    tmp.insert(tmp.end(), p.begin(), p.begin()+bottom);
    p.swap(tmp);
    bottom = 0;
}

void remove_col(vector<pt>& p){
    vector<pt> s;
    for(int i = 0, n = p.size(); i < n; i++){
        if(!on_segment(p[(i - 1 + n) % n], p[(i + 1) % n], p[i])) s.push_back(p[i]);
    }
    p.swap(s);
}

void delete_repetead(vector<pt>& p){
    vector<pt> aux;
    sort(p.begin(), p.end());
    for (pt &pi : p){
        if (aux.empty() || aux.back() != pi) aux.push_back(pi);
    }
    p.swap(aux);
}

pt farthest(vector<pt>& p, pt v){ // O(log(n)) only CONVEX, v: dir
    int n = p.size();
    if(n < 10){
        int k = 0;
        for(int i = 1; i < n; i++) if(dot(v, (p[i] - p[k])) > EPS) k = i;
        return p[k];
    }
    pt a = p[1] - p[0];
    int s = 0, e = n, ua = dot(v, a) > EPS;
    if(!ua && dot(v, (p[n - 1] - p[0])) <= EPS) return p[0];
    while(1){
        int m = (s + e) / 2;
        pt c = p[(m + 1) % n] - p[m];
        int uc = dot(v, c) > EPS;
        if(!uc && dot(v, (p[(m - 1 + n) % n] - p[m])) <= EPS) return p[m];
        if(ua && (!uc || dot(v, (p[s] - p[m])) > EPS)) e = m;
        else if(ua || uc || dot(v, (p[s] - p[m])) >= -EPS) s = m, a = c, ua = uc;
        else e = m;
        assert(e > s + 1);
    }
}

vector<pt> cut(vector<pt>& p, line l){
    // cut CONVEX polygon by line l
    // returns part at left of l.pq
    vector<pt> q;
    for(int i = 0, n = p.size(); i < n; i++) {
        int d0 = sign(l.side(p[i]));
        int d1 = sign(l.side(p[(i + 1) % n]));
        if(d0 >= 0) q.push_back(p[i]);

        line m(p[i], p[(i + 1) % n]);
        if(d0 * d1 < 0 && !(abs(cross(l.v, m.v)) <= EPS)) {
```

```
            q.push_back((inter_ll(l, m)));
        }
    }
    return q;
}

// O(n)
vector<pair<int, int>> antipodal(vector<pt>& p){
    vector<pair<int, int>> ans;
    int n = p.size();
    if (n == 2) ans.push_back({0, 1});
    if (n < 3) return ans;
    auto nxt = [&](int x){ return (x + 1 == n ? 0 : x +
        1); };
    auto area2 = [&](pt a, pt b, pt c){ return cross(b -
        a, c - a); };
    int b0 = 0;
    while (abs(area2(p[n - 1], p[0], p[nxt(b0)])) > abs(
        area2(p[n - 1], p[0], p[b0]))) ++b0;
    for (int b = b0, a = 0; b != 0 && a <= b0; ++a) {
        ans.push_back({a, b});
        while (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) >
            abs(area2(p[a], p[nxt(a)], p[b]))){
            b = nxt(b);
            if (a != b0 || b != 0) ans.push_back({a, b});
            else return ans;
        }
        if (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) == abs
            (area2(p[a], p[nxt(a)], p[b]))){
            if (a != b0 || b != n - 1) ans.push_back({a,
                nxt(b)});
            else ans.push_back({nxt(a), b});
        }
    }
    return ans;
}

// O(n)
// square distance of most distant points, prereq: convex
    , ccw, NO COLLINEAR POINTS
lf callipers(vector<pt>& p){
    int n = p.size();
    lf r = 0;
    for(int i = 0, j = n < 2 ? 0 : 1; i < j; ++i){
        for(;;j = (j + 1) % n){
            r = max(r, norm2(p[i] - p[j]));
            if(cross((p[(i + 1) % n] - p[i]), (p[(j + 1)
                % n] - p[j])) <= EPS) break;
        }
    }

    return r;
}

// O(n + m) max_dist between 2 points (pa, pb) of 2
    Convex polygons (a, b)
lf rotating_callipers(vector<pt>& a, vector<pt>& b){ //
    REVISAR
    if (a.size() > b.size()) swap(a, b); // <- del or add
    pair<ll, int> start = {-1, -1};
    if(a.size() == 1) swap(a, b);
    for(int i = 0; i < a.size(); i++) start = max(start,
        {norm2(b[0] - a[i]), i});
    if(b.size() == 1) return start.first;

    lf r = 0;
    for(int i = 0, j = start.second; i < b.size(); ++i){
        for(;; j = (j + 1) % a.size()){
            r = max(r, norm2(b[i] - a[j]));
            if(cross((b[(i + 1) % b.size()] - b[i]), (a[(
                j + 1) % a.size()] - a[j])) <= EPS) break;
        }
    }
    return r;
}

lf intercircle(vector<pt>& p, circle c){ // area of
    intersection with circle
    lf r=0.;
    for(int i = 0, n = p.size(); i < n; i++){
        int j = (i + 1) % n;
        lf w = intertriangle(c, p[i], p[j]);
        if(cross((p[j] - c.center), (p[i] - c.center)) >
            0) r += w;
        else r -= w;
    }
    return abs(r);
}

ll pick(vector<pt>& p){
    ll boundary = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int j = (i + 1 == n ? 0 : i + 1);
        boundary += __gcd((ll)abs(p[i].x - p[j].x), (ll)
            abs(p[i].y - p[j].y));
    }
    return abs(area(p)) + 1 - boundary / 2;
}
```

5.4 Circulos

```
// add Lines Points
enum {OUT, IN, ON};

struct circle {
    pt center; lf r;
    // (x - xo)^2 + (y - yo)^2 = r^2

    circle(pt c, lf r): center(c), r(r){};

    // circle that passes through abc
```

```
    circle(pt a, pt b, pt c) {
        b = b - a, c = c - a;
        assert(cross(b, c) != 0); // no circumcircle if A
            , B, C aligned
        pt cen = a + rot90(b * norm2(c) - c * norm2(b)) /
             cross(b, c) / 2;
        center = cen;
        r = norm(a - cen);
    }

    // diameter = segment pq
    circle(pt p, pt q) {
        center = (p + q) * 0.5L;
        r = dis(p, q) * 0.5L;
    }

    int contains(pt &p) {
        lf det = r * r - dis2(center, p);
        if(fabsl(det) <= EPS) return ON;
        return (det > EPS ? IN : OUT);
    }

    bool in(circle c){ return norm(center - c.center) + r
         <= c.r + EPS; }  // non strict
};

// centers of the circles that pass through ab and has
    radius r
vector<pt> centers(pt a, pt b, lf r) {
    if (norm(a - b) > 2 * r + EPS) return {};
    pt m = (a + b) / 2;
    double f = sqrt(r * r / norm2(a - m) - 1);
    pt c = rot90(a - m) * f;
    return {m - c, m + c};
}

vector<pt> inter_cl(circle c, line l){
        vector<pt> s;
        pt p = l.proj(c.center);
        lf d = norm(p - c.center);
        if(d - EPS > c.r) return s;
        if(abs(d - c.r) <= EPS){ s.push_back(p); return s
            ; }
        d=sqrt(c.r * c.r - d * d);
        s.push_back(p + normalize(l.v) * d);
        s.push_back(p - normalize(l.v) * d);
        return s;
}

vector<pt> inter_cc(circle c1, circle c2) {
    pt dir = c2.center - c1.center;
    lf d2 = dis2(c1.center, c2.center);

    if(d2 <= E0) {
        //assert( fabsl( c1.r - c2.r ) > E0 );
        return {};
    }
    lf td = 0.5L * ( d2 + c1.r * c1.r - c2.r * c2.r );
    lf h2 = c1.r * c1.r - td / d2 * td;

    pt p = c1.center + dir * (td / d2);
    if(fabsl( h2 ) < EPS) return {p};
    if(h2 < 0.0L) return {};

    pt dir_h = rot90(dir) * sqrtl(h2 / d2);

    return {p + dir_h, p - dir_h};
}

// circle-line inter = 1, inner: 1 = oxo 0 = o=o
vector<pair<pt, pt>> tangents(circle c1, circle c2, bool
    inner){
    vector<pair<pt, pt>> out;
    if (inner) c2.r = -c2.r; // inner tangent
    pt d = c2.center - c1.center;
    double dr = c1.r - c2.r, d2 = norm2(d), h2 = d2 - dr
        * dr;
    if (d2 == 0 || h2 < 0) { assert(h2 != 0); return {};
        } // (identical)
    for (double s : {-1, 1}) {
        pt v = (d * dr + rot90(d) * sqrt(h2) * s) / d2;
        out.push_back({c1.center + v * c1.r, c2.center +
            v * c2.r});
    }
    return out; // if size 1: circle are tangent
}

// circle targent passing through pt p
pair<pt, pt> tangent_through_pt(circle c, pt p){
    pair<pt, pt> out;
    double d = norm2(p - c.center);
    if (d < c.r) return {};
    pt base = c.center - p;
    double w = sqrt(norm2(base) - c.r * c.r);
    pt a = {w, c.r}, b = {w, -c.r};
    pt s = p + base * a / norm2(base) * w;
    pt t = p + base * b / norm2(base) * w;
    out = {s, t};
    return out;
}

lf safeAcos(lf x) {
    if (x < -1.0) x = -1.0;
    if (x > 1.0) x = 1.0;
    return acos(x);
}

lf areaOfIntersectionOfTwoCircles(circle c1, circle c2){
    lf r1 = c1.r, r2 = c2.r, d = dis(c1.center, c2.center
        );
    if(d >= r1 + r2) return 0.0L;
    if(d <= fabsl(r2 - r1)) return PI * (r1 < r2 ? r1 *
        r1 : r2 * r2);
```

```
    lf alpha = safeAcos((r1 * r1 - r2 * r2 + d * d) /
        (2.0L * d * r1));
    lf betha = safeAcos((r2 * r2 - r1 * r1 + d * d) /
        (2.0L * d * r2));
    lf a1 = r1 * r1 * (alpha - sinl(alpha) * cosl(alpha))
        ;
    lf a2 = r2 * r2 * (betha - sinl(betha) * cosl(betha))
        ;
    return a1 + a2;
};

lf intertriangle(circle& c, pt a, pt b){ // area of
    intersection with oab
    if(abs(cross((c.center - a), (c.center - b))) <= EPS)
        return 0.;
    vector<pt> q = {a}, w = inter_cl(c, line(a, b));
    if(w.size() == 2) for(auto p: w) if(dot((a - p), (b -
        p)) < -EPS) q.push_back(p);
    q.push_back(b);
    if(q.size() == 4 && dot((q[0] - q[1]), (q[2] - q[1]))
        > EPS) swap(q[1], q[2]);
    lf s = 0;
    for(int i = 0; i < q.size() - 1; ++i){
        if(!c.contains(q[i]) || !c.contains(q[i + 1])) s
            += c.r * c.r * min_angle((q[i] - c.center), q[
            i+1] - c.center) / 2;
        else s += abs(cross((q[i] - c.center), (q[i + 1]
            - c.center)) / 2);
    }
    return s;
}

bool circumcircle_contains(vector<pt> tr, pt D) { //
    triange CCW
  pt A = tr[0] - D, B = tr[1] - D, C = tr[2] - D;

  lf norm_a = norm2(tr[0]) - norm2(D);
  lf norm_b = norm2(tr[1]) - norm2(D);
  lf norm_c = norm2(tr[2]) - norm2(D);

  lf det1 = A.x * (B.y * norm_c - norm_b * C.y);
  lf det2 = B.x * (C.y * norm_a - norm_c * A.y);
  lf det3 = C.x * (A.y * norm_b - norm_a * B.y);

  return det1 + det2 + det3 > E0;
}

// r[k]: area covered by at least k circles
// O(n^2 log n) (high constant)
vector<lf> intercircles(vector<circle> c){
        vector<lf> r(c.size() + 1);
        for(int i = 0; i < c.size(); ++i){
                int k = 1;  pt O = c[i].center;
                vector<pair<pt, int>> p = {
                        {c[i].center + pt(1,0) * c[i].r,
                            0},
                        {c[i].center - pt(1,0) * c[i].r,
                            0}};
                for(int j = 0; j < c.size(); ++j) if(j !=
                    i){
                        bool b0 = c[i].in(c[j]), b1 = c[j
                            ].in(c[i]);
                        if(b0 && (!b1 || i < j)) ++k;
                        else if(!b0 && !b1){
                                auto v = inter_cc(c[i], c
                                    [j]);
                                if(v.size() == 2){
        swap(v[0], v[1]);
                                        p.push_back({v
                                            [0], 1});
        p.push_back({v[1], -1});
                                        if(polar_cmp(v[1]
                                            - O, v[0] - O
                                            )) ++k;
                                }
                        }
                }

                sort(all(p), [&](auto& a, auto& b){
                    return polar_cmp(a.first - O, b.first
                    - O); });
                for(int j = 0; j < p.size(); ++j){
                        pt p0 = p[j ? j - 1 : p.size()
                            -1].first, p1 = p[j].first;
                        lf a = min_angle((p0 - c[i].
                            center), (p1 - c[i].center));
                        r[k] += (p0.x - p1.x) * (p0.y +
                            p1.y) / 2 + c[i].r * c[i].r *
                            (a - sin(a)) / 2;
                        k += p[j].second;
                }
        }
        return r;
}
```

5.5 Semiplanos

```
const lf INF = 1e100;
struct Halfplane {
    pt p, pq; // p: point on line, pq: dir, take left
    lf angle;
    Halfplane(){}
    Halfplane(pt& a, pt& b): p(a), pq(b - a){
        angle = atan2l(pq.y, pq.x);
    }

    bool out(const pt& r){ return cross(pq, r - p) < -EPS
        ;} // checks if p is inside the half plane
    bool operator < (const Halfplane& e) const { return
        angle < e.angle; }
```

```
};
// intersection pt of the lines of 2 halfplanes
pt inter(const Halfplane& s, const Halfplane& t){
    if (abs(cross(s.pq, t.pq)) <= EPS) return {INF, INF};
    lf alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.pq);
    return s.p + (s.pq * alpha);
}
// O(nlogn) return CCW polygon
vector<pt> hp_intersect(vector<Halfplane>& H) {
    pt box[4] = {pt(INF, INF), pt(-INF, INF), pt(-INF, -INF), pt(INF, -INF)};

    for(int i = 0; i < 4; ++i ) {
        Halfplane aux(box[i], box[(i + 1) % 4]);
        H.push_back(aux);
    }

    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); ++i){

        while (len > 1 && H[i].out(inter(dq[len - 1], dq[len - 2]))){
            dq.pop_back();
            --len;
        }

        while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
            dq.pop_front();
            --len;
        }

        if (len > 0 && fabsl(cross(H[i].pq, dq[len - 1].pq)) < EPS){
            if (dot(H[i].pq, dq[len - 1].pq) < 0.0)
                return vector<pt>();

            if (H[i].out(dq[len - 1].p)){
                dq.pop_back();
                --len;
            } else continue;
        }

        dq.push_back(H[i]);
        ++len;
    }

    while (len > 2 && dq[0].out(inter(dq[len - 1], dq[len - 2]))){
        dq.pop_back();
        --len;
    }

    while (len > 2 && dq[len - 1].out(inter(dq[0], dq[1]))){
        dq.pop_front();
        --len;
    }

    if (len < 3) return vector<pt>();

    vector<pt> ret(len);
    for(int i = 0; i + 1 < len; ++i) ret[i] = inter(dq[i], dq[i + 1]);

    ret.back() = inter(dq[len - 1], dq[0]);
    // remove repeated points if needed
    return ret;
}

// ----------------------------------------------------

// intersection of halfplanes
vector<pt> hp_intersect(vector<halfplane>& b){
    vector<pt> box = {{inf, inf}, {-inf, inf}, {-inf, -inf}, {inf, -inf}};
    for(int i = 0; i < 4; i++){
        b.push_back({box[i], box[(i + 1) % 4]});
    }
    sort(b.begin(), b.end());
    int n = b.size(), q = 1, h = 0;
    vector<halfplane> c(n + 10);
    for(int i = 0; i < n; i++){
        while(q < h && b[i].out(inter(c[h], c[h - 1]))) h--;
        while(q < h && b[i].out(inter(c[q], c[q + 1]))) q++;
        c[++h] = b[i];
        if(q < h && abs(cross(c[h].pq, c[h-1].pq)) < EPS){
            if(dot(c[h].pq, c[h - 1].pq) <= 0) return {};
            h--;
            if(b[i].out(c[h].p)) c[h] = b[i];
        }
    }
    while(q < h - 1 && c[q].out(inter(c[h], c[h - 1]))) h--;
    while(q < h - 1 && c[h].out(inter(c[q], c[q + 1]))) q++;
    if(h - q <= 1) return {};
    c[h + 1] = c[q];
    vector<pt> s;
    for(int i = q; i < h + 1; i++) s.pb(inter(c[i], c[i + 1]));
    return s;
}
```

5.6 Segmentos

```
// add Lines Points
```

```
bool in_disk(pt a, pt b, pt p){ // pt p inside ab disk
    return dot(a - p, b - p) <= E0;
}

bool on_segment(pt a, pt b, pt p) { // p on ab
    return orient(a, b, p) == 0 && in_disk(a, b, p);
}

// ab crossing cd
bool proper_inter(pt a, pt b, pt c, pt d, pt& out) {
    lf oa = orient(c, d, a);
    lf ob = orient(c, d, b);
    lf oc = orient(a, b, c);
    lf od = orient(a, b, d);
    // Proper intersection exists iff opposite signs
    if (oa * ob < 0 && oc * od < 0) {
        out = (a * ob - b * oa) / (ob - oa);
        return true;
    }
    return false;
}

// intersection bwn segments
set<pt> inter_ss(pt a, pt b, pt c, pt d) {
    pt out;
    if (proper_inter(a, b, c, d, out)) return {out}; //
        if cross -> 1
    set<pt> s;
    if (on_segment(c, d, a)) s.insert(a); // a in cd
    if (on_segment(c, d, b)) s.insert(b); // b in cd
    if (on_segment(a, b, c)) s.insert(c); // c in ab
    if (on_segment(a, b, d)) s.insert(d); // d in ab
    return s; // 0, 2
}

lf pt_to_seg(pt a, pt b, pt p) { // p to ab
    if (a != b) {
        line l(a, b);
        if (l.cmp_proj(a, p) && l.cmp_proj(p, b)) // if
            closest to projection = (a, p, b)
            return l.dist(p); // output distance to line
    }
    return min(norm(p - a), norm(p - b)); // otherwise
        distance to A or B
}

lf seg_to_seg(pt a, pt b, pt c, pt d) {
    pt dummy;
    if (proper_inter(a, b, c, d, dummy)) return 0; // ab
        intersects cd
    return min({pt_to_seg(a, b, c), pt_to_seg(a, b, d),
        pt_to_seg(c, d, a), pt_to_seg(c, d, b)}); // try
        the 4 pts
}

int length_union(vector<pt>& a){ // REVISAR
    int n = a.size();
    vector<pair<int, bool>> x(n * 2);
    for (int i = 0; i < n; i++) {
        x[i * 2] = {a[i].x, false};
        x[i * 2 + 1] = {a[i].y, true};
    }
    sort(x.begin(), x.end());
    int result = 0;
    int c = 0;
    for (int i = 0; i < n * 2; i++) {
        if (i > 0 && x[i].first > x[i - 1].first && c >
            0) result += x[i].first - x[i - 1].first;
        if (x[i].second) c--;
        else c++;
    }
    return result;
}
```

5.7 Convex Hull

```
// CCW order
// if colineal are needed, use > in orient and remove
    repeated points
vector<pt> chull(vector<pt>& p){
        if(p.size() < 3) return p;

        vector<pt> r; //r.reserve(p.size());
        sort(p.begin(), p.end()); // first x, then y

        for(int i = 0; i < p.size(); i++){ // lower hull
                while(r.size() >= 2 && orient(r[r.size()
                    - 2], p[i], r.back()) >= 0) r.pop_back
                    ();
                r.pb(p[i]);
        }
        r.pop_back();

        int k = r.size();
        for(int i = p.size() - 1; i >= 0; --i){ // upper
            hull
                while(r.size() >= k + 2 && orient(r[r.
                    size() - 2], p[i], r.back()) >= 0) r.
                    pop_back();
                r.pb(p[i]);
        }
        r.pop_back();

        return r;
}
```

5.8 Closest Points

```
// O(nlogn)
pair<pt, pt> closest_points(vector<pt> v){
```

```
    sort(v.begin(), v.end());
    pair<pt, pt> ans;
    lf d2 = INF;

    function<void( int, int )> solve = [&](int l, int r)
        {
        if(l == r) return;

        int mid = (l + r) / 2;
        lf x_mid = v[mid].x;
        solve(l, mid);
        solve(mid + 1, r);

        vector<pt> aux;
        int p1 = l, p2 = mid + 1;
        while (p1 <= mid && p2 <= r) {
            if(v[p1].y < v[p2].y) aux.push_back(v[p1++]);
            else aux.push_back(v[p2++]);
        }
        while(p1 <= mid) aux.push_back(v[p1++]);
        while(p2 <= r) aux.push_back(v[p2++]);

        vector<pt> nb;
        for(int i = l; i <= r; ++i){
        v[i] = aux[i - l];
        lf dx = (x_mid - v[i].x);
        if(dx * dx < d2)
            nb.push_back(v[i]);
        }

        for(int i = 0; i < (int) nb.size(); ++i){
        for(int k = i + 1; k < (int) nb.size(); ++k){
            lf dy = (nb[k].y - nb[i].y);
            if(dy * dy > d2) break;
            lf nd2 = dis2(nb[i], nb[k]);
            if(nd2 < d2) d2 = nd2, ans = {nb[i], nb[k]};
        }
        }
    };
    solve(0, v.size() -1);
    return ans;
}
```

5.9 Min Circle

```
// minimo circulo que encierra todos los puntos
// Promedio: O(n), Peor: O(n^2)
Circle min_circle(vector<pt> v){
    random_shuffle(v.begin(), v.end()); // shuffle(all(
        vec), rng);
    auto f2 = [&](int a, int b){
        Circle ans(v[a], v[b]);
        for(int i = 0; i < a; ++ i)
        if(ans.contains(v[i]) == OUT) ans = Circle(v[i],
            v[a], v[b]);
        return ans;
    };

    auto f1 = [&]( int a ){
        Circle ans(v[a], 0.0L);
        for(int i = 0; i < a; ++i)
        if(ans.contains(v[i]) == OUT) ans = f2( i, a );
        return ans;
    };

    Circle ans( v[0], 0.0L );
    for(int i = 1; i < (int) v.size(); ++i)
        if(ans.contains(v[i]) == OUT ) ans = f1(i);

    return ans;
}
```

5.10 3D

```
typedef double lf;
struct p3 {
    lf x, y, z;
        p3(){}
        p3(lf x, lf y, lf z): x(x), y(y), z(z){}
    p3 operator + (p3 p){ return {x + p.x, y + p.y, z + p
        .z}; }
    p3 operator - (p3 p){ return {x - p.x, y - p.y, z - p
        .z}; }
    p3 operator * (lf d){ return {x * d, y * d, z * d}; }
    p3 operator / (lf d){ return {x / d, y / d, z / d}; }
        // only for floating point
    // Some comparators
    bool operator == (p3 p){ return tie(x, y, z) == tie(p
        .x, p.y, p.z); }
    bool operator != (p3 p){ return !operator == (p); }
        void print(){ cout << x << " " << y << " " << z
            << "\n"; }
        // scale: (newnorm / norm) * p3
};

lf dot(p3 v, p3 w){ return v.x * w.x + v.y * w.y + v.z *
    w.z; }

p3 cross(p3 v, p3 w){
    return { v.y * w.z - v.z * w.y, v.z * w.x - v.x * w.z
        , v.x * w.y - v.y * w.x };
}

lf norm2(p3 v){ return dot(v, v); }
lf norm(p3 v){ return sqrt(norm2(v)); }
p3 unit(p3 v){ return v / norm(v); }

// ang(RAD)
double angle(p3 v, p3 w){
    double cos_theta = dot(v, w) / norm(v) / norm(w);
    return acos(max(-1.0, min(1.0, cos_theta)));
}
```

```cpp
// orient s, pqr form a triangle pos: 'up', zero = on,
   neg = 'dow'
lf orient(p3 p, p3 q, p3 r, p3 s){
        return dot(cross((q - p), (r - p)), (s - p));
}

// same as 2D but in n-normal direction
lf orient_by_normal(p3 p, p3 q, p3 r, p3 n){
        return dot(cross((q - p), (r - p)), n);
}

struct plane {
    p3 n; lf d; // n: normal d: dist to zero
    // From normal n and offset d
    plane(p3 n, lf d): n(n), d(d) {}
    // From normal n and point P
    plane(p3 n, p3 p): n(n), d(dot(n, p)) {}
    // From three non-collinear points P,Q,R
    plane(p3 p, p3 q, p3 r): plane(cross((q - p), (r - p)
        ), p){}
    // - these work with lf = int
    lf side(p3 p) { return dot(n, p) - d; }
    double dist(p3 p) { return abs(side(p)) / norm(n); }
    plane translate(p3 t) {return {n, d + dot(n, t)}; }
    /// - these require lf = double
    plane shift_up(double dist){ return {n, d + dist *
        norm(n)}; }
    p3 proj(p3 p){ return p - n * side(p) / norm2(n); }
    p3 refl(p3 p){ return p - n * 2 * side(p) / norm2(n);
        }
};

struct line3d {
        p3 d, o; // d: dir o: point on line
        // From two points P, Q
        line3d(p3 p, p3 q): d(q - p), o(p){}
        // From two planes p1, p2 (requires lf = double)
        line3d(plane p1, plane p2){
                d = cross(p1.n, p2.n);
                o = cross((p2.n * p1.d - p1.n * p2.d), d)
                    / norm2(d);
        }
        // - these work with lf = int
        double dist2(p3 p){ return norm2(cross(d, (p - o)
            )) / norm2(d); }
        double dist(p3 p){ return sqrt(dist2(p)); }
        bool cmp_proj(p3 p, p3 q){ return dot(d, p) < dot
            (d, q); }
        // - these require lf = double
        p3 proj(p3 p){ return o + d * dot(d, (p - o)) /
            norm2(d); }
        p3 refl(p3 p){ return proj(p) * 2 - p; }
        p3 inter(plane p){ return o - d * p.side(o) / dot
            (p.n, d); }
        // get other point: pl.o + pl.d * t;
};

double dist(line3d l1, line3d l2) {
        p3 n = cross(l1.d, l2.d);
        if(n == p3(0, 0, 0)) return l1.dist(l2.o); //
            parallel
        return abs(dot((l2.o - l1.o), n)) / norm(n);
}

// closest point on l1 to l2
p3 closest_on_line1(line3d l1, line3d l2) {
        p3 n2 = cross(l2.d, cross(l1.d, l2.d));
        return l1.o + l1.d * (dot((l2.o - l1.o), n2)) /
            dot(l1.d, n2);
}

double small_angle(p3 v, p3 w){ return acos(min(abs(dot(v
    , w)) / norm(v) / norm(w), 1.0)); } // 0 90
double angle(plane p1, plane p2){ return small_angle(p1.n
    , p2.n); }
bool is_parallel(plane p1, plane p2){ return cross(p1.n,
    p2.n) == p3(0, 0, 0); }
bool is_perpendicular(plane p1, plane p2){ return dot(p1.
    n, p2.n) == 0; }
double angle(line3d l1, line3d l2){ return small_angle(l1
    .d, l2.d); }
bool is_parallel(line3d l1, line3d l2){ return cross(l1.d
    , l2.d) == p3(0, 0, 0); }
bool is_perpendicular(line3d l1, line3d l2){ return dot(
    l1.d, l2.d) == 0; }
double angle(plane p, line3d l){ return M_PI / 2 -
    small_angle(p.n, l.d); }
bool is_parallel(plane p, line3d l){ return dot(p.n, l.d)
     == 0; }
bool is_perpendicular(plane p, line3d l){ return cross(p.
    n, l.d) == p3(0, 0, 0); }
line3d perp_through(plane p, p3 o){ return line3d(o, o +
    p.n); }
plane perp_through(line3d l, p3 o){ return plane(l.d, o);
     }
```

5.11 KD Tree

```cpp
// given a set of points, answer queries of nearest point
   in O(log(n))
bool onx(pt a, pt b){return a.x < b.x;}
bool ony(pt a, pt b){return a.y < b.y;}
struct Node {
        pt pp;
        lf x0 = inf, x1 = -inf, y0 = inf, y1 = -inf;
        Node *first = 0, *second = 0;

        ll distance(pt p){
                ll x = min(max(x0, p.x), x1);
                ll y = min(max(y0, p.y), y1);
```

```
            return norm2(pt(x, y) - p);
        }
        Node(vector<pt>&& vp) : pp(vp[0]){
            for(pt p : vp){
                x0 = min(x0, p.x);
                x1 = max(x1, p.x);
                y0 = min(y0, p.y);
                y1 = max(y1, p.y);
            }
            if(vp.size() > 1){
                sort(all(vp), x1 - x0 >= y1 - y0
                    ? onx : ony);
                int m = vp.size() / 2;
                first = new Node({vp.begin(), vp.
                    begin() + m});
                second = new Node({vp.begin() + m
                    , vp.end()});
            }
        }
};
struct KDTree {
        Node* root;
        KDTree(const vector<pt>& vp): root(new Node({all(
            vp)})){}
        pair<ll, pt> search(pt p, Node *node){
            if(!node->first){
                // avoid query point as answer
                // if(p.x == node->pp.x && p.y ==
                    node->pp.y) return {inf, pt()
                    };
                return {norm2(p-node->pp), node->
                    pp};
            }
            Node *f = node->first, *s = node->second;
            ll bf = f->distance(p), bs = s ->
                distance(p);
            if(bf > bs) swap(bf, bs), swap(f, s);
            auto best = search(p, f);
            if(bs < best.ff) best = min(best, search(
                p, s));
            return best;
        }
        pair<ll, pt> nearest(pt p){ return search(p, root
            ); }
};
```

6 Grafos

6.1 Puentes

```
// O(n+m)
vector<bool> visited;
vi tin, low;
int timer;

void IS_BRIDGE(int u, int v, vii &puentes){
    puentes.push_back({min(u, v), max(u, v)});
}

void dfs(vector<vi> &adj, vii &puentes, int v, int p =
    -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(adj, puentes, to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] > tin[v])
                IS_BRIDGE(v, to, puentes);

        }
    }
}

void find_bridges(vector<vi> &adj, vii &puentes, int n) {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs(adj, puentes, i);
    }
}
```

6.2 Puntos de Articulacion

```
// O(n+m)
int n;
vector<vector<int>> adj;

vector<bool> visited;
vector<int> tin, low;
int timer;

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
```

```
                dfs(to, v);
                low[v] = min(low[v], low[to]);
                if (low[to] >= tin[v] && p!=-1)
                    IS_CUTPOINT(v);
                ++children;
            }
        }
        if(p == -1 && children > 1)
            IS_CUTPOINT(v);
    }

    void find_cutpoints() {
        timer = 0;
        visited.assign(n, false);
        tin.assign(n, -1);
        low.assign(n, -1);
        for (int i = 0; i < n; ++i) {
            if (!visited[i])
                dfs (i);
        }
    }
```

6.3 Kosajaru

```
//Encontrar las componentes fuertemente conexas en un
   grafo dirigido
//Componente fuertemente conexa: es un grupo de nodos en
   el que hay
//un camino dirigido desde cualquier nodo hasta cualquier
    otro nodo dentro del grupo.
const int maxn = 1e5+5;
vi adj_rev[maxn],adj[maxn];
bool used[maxn];
vi order,comp;

// O(n+m)
void dfs1(int v){
        used[v]=true;
        for(int u:adj[v])
                if(!used[u])dfs1(u);
        order.push_back(v);
}

void dfs2(int v){
        used[v]=true;
        comp.push_back(v);
        for(int u:adj_rev[v])
                if(!used[u])dfs2(u);
}

void init(int n){
        for(int i=0;i<n;++i)if(!used[i])dfs1(i);
        for(int i=0;i<n;++i)used[i]=false;
        reverse(order.begin(), order.end());
        for(int v:order){
                if(!used[v]){
                        dfs2(v);
                        // comp
                        comp.clear();
                }
        }
}

adj[a].push_back(b);
adj_rev[b].push_back(a);
```

6.4 Tarjan

```
// O(n+m) (?)
vi low, num, comp, g[nax];
int scc, timer;
stack<int> st;
void tjn(int u) {
  low[u] = num[u] = timer++; st.push(u); int v;
  for(int v: g[u]) {
    if(num[v]==-1) tjn(v);
    if(comp[v]==-1) low[u] = min(low[u], low[v]);
  }
  if(low[u]==num[u]) {
    do{ v = st.top(); st.pop(); comp[v]=scc;
    }while(u != v);
    ++scc;
  }
}
void callt(int n) {
  timer = scc= 0;
  num = low = comp = vector<int>(n,-1);
  for(int i = 0; i<n; i++) if(num[i]==-1) tjn(i);
}
```

6.5 Dijkstra

```
// O ((V+E)*log V)
vl dijkstra(vector<vector<pll>> &adj, int s, int n){
    vl dist(n, INFL); dist[s] = 0;
    priority_queue<pll, vector<pll>, greater<pll> > pq;
        pq.push(pll(0, s));
    while(!pq.empty()){
        pll front = pq.top(); pq.pop();
        ll d = front.first, u = front.second;
        if (d > dist[u]) continue;

        for (auto &[v, w] : adj[u]){
            if (dist[u] + w < dist[v]){
                dist[v] = dist[u] + w;
                pq.push(pll(dist[v], v));
            }
        }
```

```
    }
    return dist;
}
```

6.6 Bellman Ford

```
// O(V*E)
vi bellman_ford(vector<vii> &adj, int s, int n){
    vi dist(n, INF); dist[s] = 0;
    for (int i = 0; i<n-1; i++){
        bool modified = false;
        for (int u = 0; u<n; u++)
            if (dist[u] != INF)
                for (auto &[v, w] : adj[u]){
                    if (dist[v] <= dist[u] + w) continue;
                    dist[v] = dist[u] + w;
                    modified = true;
                }
        if (!modified) break;
    }

    bool negativeCicle = false;
    for (int u = 0; u<n; u++)
        if (dist[u] != INF)
            for (auto &[v, w] : adj[u]){
                if (dist[v] > dist[u] + w) negativeCicle
                    = true;
            }

    return dist;
}
```

6.7 Floyd Warshall

```
// O(n^3)
vector<vi> adjMat(n+1, vi(n+1));
//Condicion previa: adjMat[i][j] contiene peso de la
   arista (i, j)
//o INF si no existe esa arista
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (adjMat[i][k] < INF && adjMat[k][j] < INF)
                adjMat[i][j] = min(adjMat[i][j], adjMat[i
                    ][k] + adjMat[k][j]);
        }
    }
}
```

6.8 MST Kruskal

```
//O(E*log V)
vector<tuple<int,int,int>> edges;
void kruskal(){
        ll ans=0;
        dsu uf(n);
        sort(all(edges));
        for(auto& [w,u,v]:edges){
                if(uf.get(u)!=uf.get(v)){
                        uf.unite(u, v);
                        ans+=w;
                }
        }
        if(uf.sets==1){
                cout<<ans<<"\n";
        }
}
```

6.9 MST Prim

```
// O(E * log V)
vector<vii> adj;
vi tomado;
priority_queue<ii> pq;
void process(int u){
    tomado[u] = 1;
    for (auto &[v, w] : adj[u]){
        if (!tomado[v]) pq.emplace(-w, -v);
    }
}

int prim(int v, int n){
    tomado.assign(n, 0);
    process(0);
    int mst_costo = 0, tomados = 0;
    while (!pq.empty()){
        auto [w, u] = pq.top(); pq.pop();
        w = -w; u = -u;
        if (tomado[u]) continue;
        mst_costo += w;
        process(u);
        tomados++;
        if (tomados == n-1) break;
    }
    return mst_costo;
}
```

6.10 Shortest Path Faster Algorithm

```
//Algoritmo mas rapido de ruta minima
//O(V*E) peor caso, O(E) en promedio.
bool spfa(vector<vii> &adj, vector<int> &d, int s, int n)
    {
    d.assign(n, INF);
```

```
    vector<int> cnt(n, 0);
    vector<bool> inqueue(n, false);
    queue<int> q;

    d[s] = 0;
    q.push(s);
    inqueue[s] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        inqueue[v] = false;

        for (auto& [to, len] : adj[v]) {

            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                if (!inqueue[to]) {
                    q.push(to);
                    inqueue[to] = true;
                    cnt[to]++;
                    if (cnt[to] > n)
                        return false;//ciclo negativo
                }
            }
        }
    }
    return true;
}
```

6.11 Camino mas corto de longitud fija

```
/*
Modificar operacion * de matrix de esta forma:
En la exponenciacion binaria inicializar matrix ans = b
*/
matrix operator * (const matrix &b){
    matrix ans(this->r, b.c, vector<vl>(this->r, vl(b.c,
        INFL)));

    for (int i = 0; i<this->r; i++) {
        for (int k = 0; k<b.r; k++){
            for (int j = 0; j<b.c; j++){
                ans.m[i][j] = min(ans.m[i][j], m[i][k] +
                    b.m[k][j]);
            }
        }
    }
    return ans;
}

int main() {

    int n, m, k; cin >> n >> m >> k;

    vector<vl> adj(n, vl(n, INFL));

    for (int i = 0; i<m; i++){
        ll a, b, c; cin >> a >> b >> c; a--; b--;
        adj[a][b] = min(adj[a][b], c);
    }

    matrix graph(n, n, adj);
    graph = pow(graph, k-1);

    cout << (graph.m[0][n-1]==INFL ? -1 : graph.m[0][n
        -1]) << "\n";

    return 0;
}
```

6.12 2sat

```
// O(n+m)
// l=(x1 or y1) and (x2 or y2) and ... and (xn or yn)
struct sat2 {
        int n;
        vector<vector<vi>> g;
        vector<bool> vis, val;
        vi comp;
        stack<int> st;

        sat2(int n):n(n),g(2, vector<vi>(2*n)),vis(2*n),
           val(2*n),comp(2*n){}

        int neg(int x){return 2*n-x-1;}
        void make_true(int u){add_edge(neg(u), u);}
        void make_false(int u){make_true(neg(u));}
        void add_or(int u, int v){implication(neg(u),v);}
        void diff(int u, int v){eq(u, neg(v));}
        void eq(int u, int v){
                implication(u, v);
                implication(v, u);
        }
        void implication(int u,int v){
                add_edge(u, v);
                add_edge(neg(v),neg(u));
        }

        void add_edge(int u, int v){
                g[0][u].PB(v);
                g[1][v].PB(u);
        }

        void dfs(int id, int u, int t=0){
                vis[u]=true;
                for(auto &v:g[id][u])
                        if(!vis[v])dfs(id, v, t);
                if(id)comp[u]=t;
                else st.push(u);
        }

        void kosaraju() {
```

```
				for(int u=0;u<n;++u){
					if(!vis[u])dfs(0, u);
					if(!vis[neg(u)])dfs(0, neg(u));
				}
				vis.assign(2*n, false);
				int t=0;
				while(!st.empty()){
					int u=st.top();st.pop();
					if(!vis[u])dfs(1, u, t++);
				}
			}

			bool check(){
				kosaraju();
				for(int i=0;i<n;++i){
					if(comp[i]==comp[neg(i)])return
					    false;
					val[i]=comp[i]>comp[neg(i)];
				}
				return true;
			}
	};

	int m,n;
	sat2 s(n);
	char c1,c2;
	for(int a,b,i=0;i<m;++i){
		cin>>c1>>a>>c2>>b;
		a--;b--;
		if(c1=='-')a=s.neg(a);
		if(c2=='-')b=s.neg(b);
		s.add_or(a,b);
	}
	if(s.check()){
		for(int i=0;i<n;++i)cout<<(s.val[i]?'+':'-')<<" "
		    ;
		cout<<"\n";
	}else cout<<"IMPOSSIBLE\n";
```

7 Matematicas

7.1 De Bruijn sequences

```
	// Given alphabet [0, k) constructs a cyclic string
	// of length k^n that contains every length n string as
	    substr.
	vi deBruijnSeq(int k, int n, int lim){
		if (k == 1) return {0};
		vi seq, aux(n + 1);
		int cont = 0;
		function<void(int,int)> gen = [&](int t, int p) {
			if (t > n){
				if (n % p == 0) for(int i = 1; i
				    < p + 1; i++){
					if (cont >= lim) return;
					seq.pb(aux[i]);
					cont++;
				}
			} else {
				aux[t] = aux[t - p];
				gen(t + 1, p);
				while (++aux[t] < k){
					if (cont >= lim) return;
					gen(t + 1, t);
				}
			}
		};
		gen(1, 1);
		return seq;
	}
```

7.2 Chinese Remainder Theorem

```
	/// Complexity: |N|*log(|N|)
	/// Tested: Not yet.
	/// finds a suitable x that meets: x is congruent to a_i
	    mod n_i
	/** Works for non-coprime moduli.
	 Returns {-1,-1} if solution does not exist or input is
	    invalid.
	 Otherwise, returns {x,L}, where x is the solution unique
	     to mod L = LCM of mods
	*/
	pll crt( vl A, vl M ) {
	  ll n = A.size(), a1 = A[0], m1 = M[0];
	  for(ll i = 1; i < n; i++) {
	    ll a2 = A[i], m2 = M[i];
	    ll g = __gcd(m1, m2);
	    if( a1 % g != a2 % g ) return {-1,-1};
	    ll p, q;
	    extended_euclid(m1/g, m2/g, p, q);
	    ll mod = m1 / g * m2;
	    q %= mod; p %= mod;
	    ll x = ((1ll*(a1%mod)*(m2/g))%mod*q + (1ll*(a2%mod)*(
	        m1/g))%mod*p) % mod; // if WA there is overflow
	    a1 = x;
	    if (a1 < 0) a1 += mod;
	    m1 = mod;
	  }
	  return {a1, m1};
	}
```

7.3 Totient y Divisores

```
vector<int> count_divisors_sieve() {
  bitset<mx> is_prime; is_prime.set();
  vector<int> cnt(mx, 1);
  is_prime[0] = is_prime[1] = 0;
  for(int i = 2; i < mx; i++) {
    if(!is_prime[i]) continue;
    cnt[i]++;
    for(int j = i+i; j < mx; j += i) {
      int n = j, c = 1;
      while( n%i == 0 ) n /= i, c++;
      cnt[j] *= c;
      is_prime[j] = 0;
    }
  }
  return cnt;
}
vector<int> euler_phi_sieve() {
  bitset<mx> is_prime; is_prime.set();
  vector<int> phi(mx);
  iota(phi.begin(), phi.end(), 0);
  is_prime[0] = is_prime[1] = 0;
  for(int i = 2; i < mx; i++) {
    if(!is_prime[i]) continue;
    for(int j = i; j < mx; j += i) {
      phi[j] -= phi[j]/i;
      is_prime[j] = 0;
    }
  }
  return phi;
}
ll euler_phi(ll n) {
  ll ans = n;
  for(ll i = 2; i * i <= n; ++i) {
    if(n % i == 0) {
      ans -= ans / i;
      while(n % i == 0) n /= i;
    }
  }
  if(n > 1) ans -= ans / n;
  return ans;
}
```

7.4 Ecuaciones Diofanticas

```
// O(log(n))
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
  ll xx = y = 0;
  ll yy = x = 1;
  while (b) {
    ll q = a / b;
    ll t = b; b = a % b; a = t;
    t = xx; xx = x - q * xx; x = t;
    t = yy; yy = y - q * yy; y = t;
  }
  return a;
}
// a*x+b*y=c. returns valid x and y if possible.
// all solutions are of the form (x0 + k * b / g, y0 - k
   * b / g)
bool find_any_solution (ll a, ll b, ll c, ll &x0, ll &y0,
    ll &g) {
  if (a == 0 and b == 0) {
    if (c) return false;
    x0 = y0 = g = 0;
    return true;
  }
  g = extended_euclid (abs(a), abs(b), x0, y0);
  if (c % g != 0) return false;
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 *= -1;
  if (b < 0) y0 *= -1;
  return true;
}
void shift_solution(ll &x, ll &y, ll a, ll b, ll cnt) {
  x += cnt * b;
  y -= cnt * a;
}
// returns the number of solutions where x is in the
   range[minx, maxx] and y is in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c, ll minx, ll maxx,
    ll miny,ll maxy) {
  ll x, y, g;
  if (find_any_solution(a, b, c, x, y, g) == 0) return 0;
  if (a == 0 and b == 0) {
    assert(c == 0);
    return 1LL * (maxx - minx + 1) * (maxy - miny + 1);
  }
  if (a == 0) {
    return (maxx - minx + 1) * (miny <= c / b and c / b
       <= maxy);
  }
  if (b == 0) {
    return (maxy - miny + 1) * (minx <= c / a and c / a
       <= maxx);
  }
  a /= g, b /= g;
  ll sign_a = a > 0 ? +1 : -1;
  ll sign_b = b > 0 ? +1 : -1;
  shift_solution(x, y, a, b, (minx - x) / b);
  if (x < minx) shift_solution(x, y, a, b, sign_b);
  if (x > maxx) return 0;
  ll lx1 = x;
  shift_solution(x, y, a, b, (maxx - x) / b);
  if (x > maxx) shift_solution (x, y, a, b, -sign_b);
  ll rx1 = x;
  shift_solution(x, y, a, b, -(miny - y) / a);
  if (y < miny) shift_solution (x, y, a, b, -sign_a);
  if (y > maxy) return 0;
```

```
  ll lx2 = x;
  shift_solution(x, y, a, b, -(maxy - y) / a);
  if (y > maxy) shift_solution(x, y, a, b, sign_a);
  ll rx2 = x;
  if (lx2 > rx2) swap (lx2, rx2);
  ll lx = max(lx1, lx2);
  ll rx = min(rx1, rx2);
  if (lx > rx) return 0;
  return (rx - lx) / abs(b) + 1;
}

///finds the first k | x + b * k / gcd(a, b) >= val
ll greater_or_equal_than(ll a, ll b, ll x, ll val, ll g)
    {
  ld got = 1.0 * (val - x) * g / b;
  return b > 0 ? ceil(got) : floor(got);
}
```

7.5 Exponenciacion binaria

```
ll binpow(ll b, ll n, ll m) {
    b %= m;
    ll res = 1;
    while (n > 0) {
        if (n & 1)
            res = res * b % m;
        b = b * b % m;
        n >>= 1;
    }
    return res % m;
}
```

7.6 Exponenciacion matricial

```
struct matrix {
    int r, c; vector<vl> m;
    matrix(int r, int c, const vector<vl> &m) : r(r), c(c
        ), m(m){}

    matrix operator * (const matrix &b){
        matrix ans(this->r, b.c, vector<vl>(this->r, vl(b
            .c, 0)));

        for (int i = 0; i<this->r; i++) {
            for (int k = 0; k<b.r; k++){
                if (m[i][k] == 0) continue;
                for (int j = 0; j<b.c; j++){
                    ans.m[i][j] += mod(m[i][k], MOD) *
                        mod(b.m[k][j], MOD);
                    ans.m[i][j] = mod(ans.m[i][j], MOD);
                }
            }
        }
        return ans;
    }
};

matrix pow(matrix &b, ll p){
    matrix ans(b.r, b.c, vector<vl>(b.r, vl(b.c, 0)));
    for (int i = 0; i<b.r; i++) ans.m[i][i] = 1;
    while (p){
        if (p&1){
            ans = ans*b;
        }
        b = b*b;
        p >>= 1;
    }
    return ans;
}
```

7.7 Fibonacci Fast Doubling

```
// O(log n) muy rapido
pair<int, int> fib (int n) {
    if (n == 0)
        return {0, 1};

    auto p = fib(n >> 1);
    int c = p.first * (2 * p.second - p.first);
    int d = p.first * p.first + p.second * p.second;
    if (n & 1)
        return {d, c + d};
    else
        return {c, d};
}
```

7.8 Freivalds algorithm

```
mt19937 rnd(chrono::steady_clock::now().time_since_epoch
    ().count());
// check if two n*n matrix a*b=c within complexity (
    iteration*n^2)
// probability of error 2^(-iteration)
// O(iter*n^2)
int Freivalds(matrix &a, matrix &b, matrix &c) {
  int n = a.r, iteration = 20;
  matrix zero(n, 1), r(n, 1);
  while (iteration--) {
        for(int i = 0; i < n; i++) r.m[i][0] = rnd() % 2;
        matrix ans = (a * (b * r)) - (c * r);
        if(ans.m != zero.m) return 0;
  }
  return 1;
}
```

7.9 Gauss Jordan

```cpp
// O(min(n, m)*n*m)
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be
    infinity or a big number

int gauss (vector < vector<double> > a, vector<double> &
    ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}
```

7.10 Gauss Jordan mod 2

```cpp
// O(min(n, m)*n*m)
int gauss (vector < bitset<N> > &a, int n, int m, bitset<
    N> & ans) {
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        for (int i=row; i<n; ++i)
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        if (! a[row][col])
            continue;
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        ++row;
    }

    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;

}
```

7.11 GCD y LCM

```cpp
//O(log10 n) n == max(a, b)
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a%b);
    }
int lcm(int a, int b) { return a / gcd(a, b) * b; }
//gcd(a, b, c) = gcd(a, gcd(b, c))
```

7.12 Integral Definida

```cpp
const int steps = 1e6; // %2==0
double f(double x);
double simpson(double a, double b){
        double h=(b-a)/steps;
        double s=f(a)+f(b);
        for(int i=1;i<=steps-1;i++){
```

```
                double x=a+h*i;
                s+=f(x)*((i&1)?4:2);
        }
        s*=h/3;
        return s;
}
```

7.13 Inverso modular

```
ll mod(ll a, ll m){
    return ((a%m) + m) % m;
}

ll modInverse(ll b, ll m){
    ll x, y;
    ll d = extEuclid(b, m, x, y);  //obtiene b*x + m*y == d
    if (d != 1) return -1;          //indica error
    // b*x + m*y == 1, ahora aplicamos (mod m) para obtener b*x == 1 (mod m)
    return mod(x, m);
}

// Otra forma
// O(log MOD)
ll inv (ll a){
    return binpow(a, MOD-2, MOD);
}

//Modulo constante
inv[1] = 1;
for(int i = 2; i < p; ++i)
        inv[i] = (p - (p / i) * inv[p % i] % p) % p;
```

7.14 Logaritmo Discreto

```
// O(sqrt(m))
// Returns minimum x for which a ^ x % m = b % m.
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 1ll * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 1ll * a) % m;
    }

    for (int p = 1, cur = k; p <= n; ++p) {
        cur = (cur * 1ll * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur] + add;
            return ans;
        }
    }
    return -1;
}
```

7.15 Miller Rabin

```
ll mul (ll a, ll b, ll mod) {
  ll ret = 0;
  for(a %= mod, b %= mod; b != 0;
    b >>= 1, a <<= 1, a = a >= mod ? a - mod : a) {
    if (b & 1) {
      ret += a;
      if (ret >= mod) ret -= mod;
    }
  }
  return ret;
}
ll fpow (ll a, ll b, ll mod) {
  ll ans = 1;
  for (; b; b >>= 1, a = mul(a, a, mod))
    if (b & 1)
      ans = mul(ans, a, mod);
  return ans;
}
bool witness (ll a, ll s, ll d, ll n) {
  ll x = fpow(a, d, n);
  if (x == 1 || x == n - 1) return false;
  for (int i = 0; i < s - 1; i++) {
    x = mul(x, x, n);
    if (x == 1) return true;
    if (x == n - 1) return false;
  }
  return true;
}
ll test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 0};
bool is_prime (ll n) {
  if (n < 2) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  ll d = n - 1, s = 0;
  while (d % 2 == 0) ++s, d /= 2;
```

```
  for (int i = 0; test[i] && test[i] < n; ++i)
    if (witness(test[i], s, d, n))
      return false;
  return true;
}
```

7.16 Miller Rabin Probabilistico

```
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if
   n is probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;

    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}
```

7.17 Mobius

```
const int N = 1e6+1;
int mob[N];
void mobius() {
  mob[1] = 1;
  for (int i = 2; i < N; i++){
    mob[i]--;
    for (int j = i + i; j < N; j += i) {
      mob[j] -= mob[i];
    }
  }
}
```

7.18 Pollard Rho

```
//O(n^(1/4)) (?)
ll pollard_rho(ll n, ll c) {
  ll x = 2, y = 2, i = 1, k = 2, d;
  while (true) {
    x = (mul(x, x, n) + c);
    if (x >= n) x -= n;
    d = __gcd(x - y, n);
    if (d > 1) return d;
    if (++i == k) y = x, k <<= 1;
  }
  return n;
}
void factorize(ll n, vector<ll> &f) {
  if (n == 1) return;
  if (is_prime(n)) {
    f.push_back(n);
    return;
  }
  ll d = n;
  for (int i = 2; d == n; i++)
    d = pollard_rho(n, i);
  factorize(d, f);
  factorize(n/d, f);
}
```

7.19 Simplex

```
// Maximizar c1*x1 + c2*x2 + c3*x3 ...
// Restricciones a11*x1 + a12*x2 <= b1, a22*x2 + a23*x3
   <= b2 ...
// Retorna valor optimo y valores de las variables

// O(c^2*b), O(c*b) - variables c, restricciones b
struct Simplex{
        vector<vector<double>> A;
        vector<double> B,C;
```

```
vector<int> X,Y;
double z;
int n,m;

Simplex(vector<vector<double>> _a, vector<double>
     _b, vector<double> _c){
        A=_a;B=_b;C=_c;
        n=B.size();m=C.size();z=0.;
        X=vector<int>(m);Y=vector<int>(n);
        for(int i=0;i<m;++i)X[i]=i;
        for(int i=0;i<n;++i)Y[i]=i+m;
}

void pivot(int x,int y){
        swap(X[y],Y[x]);
        B[x]/=A[x][y];
        for(int i=0;i<m;++i)if(i!=y)A[x][i]/=A[x
            ][y];
        A[x][y]=1/A[x][y];
        for(int i=0;i<n;++i)if(i!=x&&abs(A[i][y])
            >EPS){
                B[i]-=A[i][y]*B[x];
                for(int j=0;j<m;++j)if(j!=y)A[i][
                    j]-=A[i][y]*A[x][j];
                A[i][y]=-A[i][y]*A[x][y];
        }
        z+=C[y]*B[x];
        for(int i=0;i<m;++i)if(i!=y)C[i]-=C[y]*A[
            x][i];
        C[y]=-C[y]*A[x][y];
}

pair<double, vector<double>> maximize(){
        while(1){
                int x=-1,y=-1;
                double mn=-EPS;
                for(int i=0;i<n;++i)if(B[i]<mn)mn
                    =B[i],x=i;
                if(x<0)break;
                for(int i=0;i<m;++i)if(A[x][i]<-
                    EPS){y=i;break;}
                // y<0, no solution to Ax<=B
                pivot(x,y);
        }
        while(1){
                double mx=EPS;
                int x=-1,y=-1;
                for(int i=0;i<m;++i)if(C[i]>mx)mx
                    =C[i],y=i;
                if(y<0)break;
                double mn=1e200;
                for(int i=0;i<n;++i)if(A[i][y]>
                    EPS&&B[i]/A[i][y]<mn)mn=B[i]/A
                    [i][y],x=i;
                // x<0, unbounded
                pivot(x,y);
                }
                vector<double> r(m);
                for(int i=0;i<n;++i)if(Y[i]<m)r[Y[i]]=B[i
                    ];
                return {z,r};
        }
};
```

7.20 Fast Fourier Transform

```
// O(N log N)
const double PI = acos(-1);
struct base {
  double a, b;
  base(double a = 0, double b = 0) : a(a), b(b) {}
  const base operator + (const base &c) const
    { return base(a + c.a, b + c.b); }
  const base operator - (const base &c) const
    { return base(a - c.a, b - c.b); }
  const base operator * (const base &c) const
    { return base(a * c.a - b * c.b, a * c.b + b * c.a);
      }
};
void fft(vector<base> &p, bool inv = 0) {
  int n = p.size(), i = 0;
  for(int j = 1; j < n - 1; ++j) {
    for(int k = n >> 1; k > (i ^= k); k >>= 1);
    if(j < i) swap(p[i], p[j]);
  }
  for(int l = 1, m; (m = l << 1) <= n; l <<= 1) {
    double ang = 2 * PI / m;
    base wn = base(cos(ang), (inv ? 1. : -1.) * sin(ang))
      , w;
    for(int i = 0, j, k; i < n; i += m) {
      for(w = base(1, 0), j = i, k = i + l; j < k; ++j, w
          = w * wn) {
        base t = w * p[j + l];
        p[j + l] = p[j] - t;
        p[j] = p[j] + t;
      }
    }
  }
  if(inv) for(int i = 0; i < n; ++i) p[i].a /= n, p[i].b
     /= n;
}
vector<long long> multiply(vector<int> &a, vector<int> &b
    ) {
  int n = a.size(), m = b.size(), t = n + m - 1, sz = 1;
  while(sz < t) sz <<= 1;
  vector<base> x(sz), y(sz), z(sz);
  for(int i = 0 ; i < sz; ++i) {
    x[i] = i < (int)a.size() ? base(a[i], 0) : base(0, 0)
      ;
    y[i] = i < (int)b.size() ? base(b[i], 0) : base(0, 0)
      ;
```

```
  }
  fft(x), fft(y);
  for(int i = 0; i < sz; ++i) z[i] = x[i] * y[i];
  fft(z, 1);
  vector<long long> ret(sz);
  for(int i = 0; i < sz; ++i) ret[i] = (long long) round(
      z[i].a);
//    while((int)ret.size() > 1 && ret.back() == 0) ret.
    pop_back();
  return ret;
}
```

7.21 Number Theoretic Transform

```
const int N = 1 << 20;
const int mod = 469762049; //998244353
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {
  for (; y; y >>= 1, x = (long long) x * x % mod) if (y &
       1) ans = (long long) ans * x % mod;
  return ans;
}
void precompute(int len) {
  lim = wn[0] = 1; int s = -1;
  while (lim < len) lim <<= 1, ++s;
  for (int i = 0; i < lim; ++i) rev[i] = rev[i >> 1] >> 1
       | (i & 1) << s;
  const int g = POW(root, (mod - 1) / lim);
  inv_lim = POW(lim, mod - 2);
  for (int i = 1; i < lim; ++i) wn[i] = (long long) wn[i
      - 1] * g % mod;
}
void ntt(vector<int> &a, int typ) {
  for (int i = 0; i < lim; ++i) if (i < rev[i]) swap(a[i
      ], a[rev[i]]);
  for (int i = 1; i < lim; i <<= 1) {
    for (int j = 0, t = lim / i / 2; j < i; ++j) w[j] =
        wn[j * t];
    for (int j = 0; j < lim; j += i << 1) {
      for (int k = 0; k < i; ++k) {
        const int x = a[k + j], y = (long long) a[k + j +
            i] * w[k] % mod;
        reduce(a[k + j] += y - mod), reduce(a[k + j + i]
            = x - y);
      }
    }
  }
  if (!typ) {
    reverse(a.begin() + 1, a.begin() + lim);
    for (int i = 0; i < lim; ++i) a[i] = (long long) a[i]
        * inv_lim % mod;
  }
}
vector<int> multiply(vector<int> &f, vector<int> &g) {
  int n=(int)f.size() + (int)g.size() - 1;
  precompute(n);
  vector<int> a = f, b = g;
  a.resize(lim); b.resize(lim);
  ntt(a, 1), ntt(b, 1);
  for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] *
      b[i] % mod;
  ntt(a, 0);
  a.resize(n + 1);
  return a;
}
```

8 Programacion dinamica

8.1 Bin Packing

```
int main() {
    ll n, capacidad;
    cin >> n >> capacidad;
    vl pesos(n, 0);
    forx(i, n) cin >> pesos[i];

    vector<pll> dp((1 << n));
    dp[0] = {1, 0};
    // dp[X] = {#numero de paquetes, peso de min paquete}

    // La idea es probar todos los subset y en cada uno
       preguntarnos
    // quien es mejor para subirse de ultimo buscando
       minimizar
    // primero el numero de paquetes
    for (int subset = 1; subset < (1 << n); subset++) {
        dp[subset] = {21, 0};

        for (int iPer = 0; iPer < n; iPer++) {
            if ((subset >> iPer) & 1) {
                pll ant = dp[subset ^ (1 << iPer)];
                ll k = ant.ff;
                ll w = ant.ss;

                if (w + pesos[iPer] > capacidad) {
                    k++;
                    w = min(pesos[iPer], w);
                } else {
                    w += pesos[iPer];
                }

                dp[subset] = min(dp[subset], {k, w});
            }
        }
    }
```

```
        cout << dp[(1 << n) - 1].ff << ln;
    }
```

8.2 CHT

```
// - Me dan las pendientes ordenadas
// Caso 1: Me hacen las querys ordenadas
// O(N + Q)
// Caso 2: Me hacen querys arbitrarias
// O(N + QlogN)
struct CHT {
    // funciona tanto para min como para max, depende del
        orden en que pasamos las lineas
    struct Line {
        int slope, yIntercept;

        Line(int slope, int yIntercept) : slope(slope),
            yIntercept(yIntercept){}
        int val(int x){ return slope * x + yIntercept; }
        int intersect(Line y) {
            return (y.yIntercept - yIntercept + slope - y
                .slope - 1) / (slope - y.slope);
        }
    };

    deque<pair<Line, int>> dq;

    void insert(int slope, int yIntercept){
                // lower hull si m1 < m2 < m3
                // upper hull si si m1 > m2 > m3
        Line newLine(slope, yIntercept);
        while (!dq.empty() && dq.back().second >= dq.back
            ().first.intersect(newLine)) dq.pop_back();
        if (dq.empty()) {
            dq.emplace_back(newLine, 0);
            return;
        }
        dq.emplace_back(newLine, dq.back().first.
            intersect(newLine));
    }

    int query(int x) { // cuando las consultas son
        crecientes
        while (dq.size() > 1) {
            if (dq[1].second <= x) dq.pop_front();
            else break;
        }
        return dq[0].first.val(x);
    }

    int query2(int x) { // cuando son arbitrarias
        auto qry = *lower_bound(dq.rbegin(), dq.rend(),
                        make_pair(Line(0, 0), x),
                        [&](const pair<Line, int>
                            &a, const pair<Line,
                            int> &b) {
                            return a.second > b.
                                second;
                        });
        return qry.first.val(x);
    }
};
```

8.3 CHT Dynamic

```
// O((N+Q) log N) <- usando set para add y bs para q
// lineas de la forma mx + b
#pragma once

struct Line {
        mutable ll m, b, p;
        bool operator<(const Line& o) const { return m <
            o.m; }
        bool operator<(ll x) const { return p < x; }
};

struct CHT : multiset<Line, less<>> {
        // (for doubles, use inf = 1/.0, div(a,b) = a/b)
        static const ll inf = LLONG_MAX;
        static const bool mini = 0; // <---- 1 FOR MIN
        ll div(ll a, ll b){ // floored division
                return a / b - ((a ^ b) < 0 && a % b); }
        bool isect(iterator x, iterator y){
                if (y == end()) return x->p = inf, 0;
                if (x->m == y->m) x->p = x->b > y->b ?
                    inf : -inf;
                else x->p = div(y->b - x->b, x->m - y->m)
                    ;
                return x->p >= y->p;
        }
        void add(ll m, ll b){
        if (mini){ m *= -1, b *= -1; }
                auto z = insert({m, b, 0}), y = z++, x =
                    y;
                while (isect(y, z)) z = erase(z);
                if (x != begin() && isect(--x, y)) isect(
                    x, y = erase(y));
                while ((y = x) != begin() && (--x)->p >=
                    y->p)
                        isect(x, erase(y));
        }
        ll query(ll x) {
                assert(!empty());
                auto l = *lower_bound(x);
        if (mini) return -l.m * x + -l.b;
                else return l.m * x + l.b;
        }
```

```
};
```

8.4 Divide Conquer

```
// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c
    < d.
int m, n;
vector<long long> dp_before(n), dp_cur(n);

long long C(int i, int j);

// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr) {
    if (l > r)
        return;

    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};

    for (int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {(k ? dp_before[k - 1] : 0) + C(
            k, mid), k});
    }

    dp_cur[mid] = best.first;
    int opt = best.second;

    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}

int solve() {
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);

    for (int i = 1; i < m; i++) {
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }

    return dp_before[n - 1];
}
```

8.5 Edit Distances

```
int editDistances(string& wor1,string& wor2){
    // O(tam1*tam2)
    // minimo de letras que debemos insertar, elminar o
        reemplazar
    // de wor1 para obtener wor2
    ll tam1=wor1.size();
    ll tam2=wor2.size();
    vector<vl> dp(tam2+1,vl(tam1+1,0));
    for(int i=0;i<=tam1;i++)dp[0][i]=i;
    for(int i=0;i<=tam2;i++)dp[i][0]=i;
    dp[0][0]=0;
    for(int i=1;i<=tam2;i++){
        for(int j=1;j<=tam1;j++){
            ll op1 = min(dp[i-1][j],dp[i][j-1])+1;
            // el minimo entre eliminar o insertar
            ll op2 = dp[i-1][j-1]; // reemplazarlo
            if(wor1[j-1]!=wor2[i-1])op2++;
            // si el reemplazo tiene efecto o quedo igual
            dp[i][j]=min(op1,op2);
        }
    }

    return dp[tam2][tam1];
}
```

8.6 Kadane 2D

```
int main() {
    ll fil,col;cin>>fil>>col;
    vector<vl> grid(fil,vl(col,0));

// Algoritmo de Kadane/DP para suma maxima de una matriz
    2D en o(n^3)
    for(int i=0;i<fil;i++){
        for(int e=0;e<col;e++){
            ll num;cin>>num;
            if (e>0) grid[i][e]=num+grid[i][e-1];
            else grid[i][e]=num;
        }
    }

    ll maxGlobal = LONG_LONG_MIN;
    for(int l=0;l<col;l++){
        for(int r=l;r<col;r++){
            ll maxLoc=0;
            for(int row=0;row<fil;row++){
                if (l>0) maxLoc+=grid[row][r]-grid[row][l
                    -1];
                else maxLoc+=grid[row][r];
                if (maxLoc<0) maxLoc=0;
                maxGlobal= max(maxGlobal,maxLoc);
            }
        }
    }
}
```

8.7 Knuth

```
// C[b][c] <= C[a][d]
// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c
    < d.
int solve() {
    int N;
```

```
    ... // read N and input
    int dp[N][N], opt[N][N];

    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };

    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i] according to the
           problem
    }

    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1, opt[i
               +1][j]); k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] + dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }

    cout << dp[0][N-1] << endl;
}
```

8.8 LIS

```
// O(nlogn)
int lis(vi& a){
        int n=sz(a),last=0;
        vi dp(n+1,INT_MAX),cnt(n,0);
        dp[0]=INT_MIN;
        for(int i=0;i<n;++i){
                int j=lower_bound(all(dp), a[i])-dp.begin
                   (); // upper_bound
                if(dp[j-1]<a[i] && a[i]<dp[j]){ // dp[j
                   -1]<=a[i]
                        dp[j]=a[i];
                        last=max(last,j);
                }
                cnt[i]=j;
        }
        int ans=0;
        for(int i=0;i<=n;i++){
                if(dp[i]<INT_MAX)ans=i;
        }
        vi LIS(ans);
        int act=ans;
        for(int i=n-1;i>=0;--i){
                if(cnt[i]==act){
                        LIS[act-1]=a[i];
                        act--;
                }
        }
        return ans;
}
```

8.9 SOS

```
const int bits = 23;
int dp[1<<bits];

// O(n*2^n)
void SOS(){
        for(int i = 0; i < (1 << bits); ++i) dp[i] = A[i
           ];

        // top - down
        for(int i = 0; i < bits; ++i){
                for(int s = 0; s < (1 << bits); ++s){
                        if(s & (1 << i)){
                                dp[s] += dp[s ^ (1 << i)
                                   ];
                        }
                }
        }

        // bottom - up
        for(int i = 0; i < bits; ++i){
                for(int s = (1 << bits) - 1; s >= 0; --s)
                   {
                        if(s & (1 << i)){
                                dp[s ^ (1 << i)] += dp[s
                                   ];
                        }
                }
        }
}
```

9 Strings

9.1 Hashing

```
1000234999, 1000567999, 1000111997, 1000777121,
   1001265673, 1001864327, 999727999, 1070777777
const int mod[2] = { 1001864327, 1001265673 };
const ii base(257, 367), zero(0, 0), one(1, 1);
const int maxn = 1e6;

inline int add(int a, int b, int m){return a+b>=m?a+b-m:a
   +b;}
```

```
inline int sbt(int a, int b, int m){return a-b<0?a-b+m:a-b;}
inline int mul(int a, int b, int m){return ll(a)*b%m;}
inline ll operator ! (const ii a){return (ll(a.first)<<32)|a.second;}
inline ii operator + (const ii& a, const ii& b){return {add(a.first, b.first, mod[0]), add(a.second, b.second, mod[1])};}
inline ii operator - (const ii& a, const ii& b){return {sbt(a.first, b.first, mod[0]), sbt(a.second, b.second, mod[1])};}
inline ii operator * (const ii& a, const ii& b){return {mul(a.first, b.first, mod[0]), mul(a.second, b.second, mod[1])};}

ii p[maxn+1];
void prepare(){ // Acordate del prepare()!!
	p[0]=one;
	for(int i=1;i<=maxn;i++)p[i]=p[i-1]*base;
}

template <class type>
struct hashing{
	vector<ii> h;
	hashing(type& t){
		h.resize((int)t.size()+1);
		h[0]=zero;
		for(int i=1;i<(int)h.size();++i)
			h[i]=h[i-1]*base + ii{t[i-1], t[i-1]};
	}
	ii get(int l, int r){return h[r+1]-h[l]*p[r-l+1];}
};
ii combine(ii a, ii b, int lenb){return a*p[lenb]+b;}
```

9.2 KMP

```
// O(n)
vi phi(string& s){
	int n=sz(s);
	vi tmp(n);
	for(int i=1,j=0;i<n;++i){
		while(j>0 && s[j]!=s[i])j=tmp[j-1];
		if(s[i]==s[j])j++;
		tmp[i]=j;
	}
	return tmp;
}

// O(n+m)
int kmp(string& s, string& p){
	int n=sz(s),m=sz(p),cnt=0;
	vi pi=phi(p);
	for(int i=0,j=0;i<n;++i){
		while(j && s[i]!=p[j])j=pi[j-1];
		if(s[i]==p[j])j++;
		if(j==m){
			cnt++;
			j=pi[j-1];
		}
	}
	return cnt;
}
```

9.3 KMP Automaton

```
const int maxn = 1e5+5, alpha = 26;
int to[maxn][alpha];

// O(n*alpha)
void build(string& s){
	to[0][conv(s[0])]=1;
	int n=sz(s);
	for(int i=1,p=0;i<n+1;++i){
		for(int j=0;j<alpha;++j)to[i][j]=to[p][j];
		if(i<n){
			to[i][conv(s[i])]=i+1;
			p=to[p][conv(s[i])];
		}
	}
}
```

9.4 Manacher

```
// O(n), par (raiz, izq, der) 1 - impar 0
vi manacher(string& s, int par){
	int l=0,r=-1,n=sz(s);vi m(n,0);
	for(int i=0;i<n;++i){
		int k=(i>r?(1-par):min(m[l+r-i+ par], r-i+par))+par;
		while(i+k-par<n && i-k>=0 && s[i+k-par]==s[i-k])++k;
		m[i]=k-par;--k;
		if(i+k-par>r)l=i-k,r=i+k-par;
	}
	for(int i=0;i<n;++i)m[i]=(m[i]-1+par)*2+1-par;
	return m;
}
```

9.5 Minimum Expression

```
// O(n)
int minimum_expression(string s){
	s=s+s;int n=sz(s),i=0,j=1,k=0;
```

```
        while(i+k<n && j+k<n){
                if(s[i+k]==s[j+k])k++;
                else if(s[i+k]>s[j+k])i=i+k+1,k=0; //
                    cambiar por < para max
                else j=j+k+1,k=0;
                if(i==j)j++;
        }
        return min(i, j);
}
```

9.6 Palindromic Tree

```
const int alpha = 26;
const char fc = 'a';

// tree suf is the longest suffix palindrome
// tree dad is the palindrome add c to the right and left
struct Node{
        int next[alpha];
        int len,suf,dep,cnt,dad;
};

// O(nlogn)
struct PalindromicTree{
        vector<Node> tree;
        string s;
        int len,n;
        int size; // node 1 - root with len -1, node 2 -
            root with len 0
        int last; // max suffix palindrome

        bool addLetter(int pos){
                int cur=last,curlen=0;
                int let=s[pos]-fc;
                while(true){
                        curlen=tree[cur].len;
                        if(pos-1-curlen>=0 && s[pos-1-
                            curlen]==s[pos])break;
                        cur=tree[cur].suf;
                }

                if(tree[cur].next[let]){
                        last=tree[cur].next[let];
                        tree[last].cnt++;
                        return false;
                }

                size++;
                last=size;
                tree[size].len=tree[cur].len+2;
                tree[cur].next[let]=size;
                tree[size].cnt=1;
                tree[size].dad=cur;

                if(tree[size].len==1){
                        tree[size].suf=2;
                        tree[size].dep=1;
                        return true;
                }

                while(true){
                        cur=tree[cur].suf;
                        curlen=tree[cur].len;
                        if(pos-1-curlen>=0 && s[pos-1-
                            curlen]==s[pos]){
                                tree[size].suf=tree[cur].
                                    next[let];
                                break;
                        }
                }

                tree[size].dep=1+tree[tree[size].suf].dep
                    ;
                return true;
        }

        PalindromicTree(string& s2, int n){
                tree.assign(n+4,Node());
                tree[1].len=-1;tree[1].suf=1;
                tree[2].len=0;tree[2].suf=1;
                size=2;last=2;s=s2;

                for(int i=0;i<n;i++){
                        addLetter(i);
                }

                for(int i=size;i>=3;i--){
                        tree[tree[i].suf].cnt+=tree[i].
                            cnt;
                }
        }
};
```

9.7 Suffix Array

```
// O(nlogn)
struct SuffixArray{
        const int alpha = 256;
        string s;int n;
        vi sa,rnk,lcp;

        SuffixArray(string& _s){
                s=_s;s.push_back('$'); // check
                n=sz(s);
                sa.assign(n, 0);
                rnk.assign(n, 0);
                lcp.assign(n-1, 0);
                buildSA();
        }

        void buildSA() {
```

```
			vi cnt(max(alpha, n),0);
			for(int i=0;i<n;++i)cnt[s[i]]++;
			for(int i=1;i<max(alpha,n);++i)cnt[i]+=cnt[i-1];
			for(int i=n-1;i>=0;--i)sa[--cnt[s[i]]]=i;
			for(int i=1;i<n;++i)rnk[sa[i]]=rnk[sa[i-1]]+(s[sa[i]]!=s[sa[i-1]]);

			for(int k=1;k<n;k*=2){
				vi nsa(n),nrnk(n),ncnt(n);
				for(int i=0;i<n;++i)sa[i]=(sa[i]-k+n)%n;
				for(int i=0;i<n;++i)ncnt[rnk[i]]++;
				for(int i=1;i<n;++i)ncnt[i]+=ncnt[i-1];
				for(int i=n-1;i>=0;--i)nsa[--ncnt[rnk[sa[i]]]]=sa[i];
				for(int i=1;i<n;++i){
					ii op1={rnk[nsa[i]], rnk[(nsa[i]+k)%n]};
					ii op2={rnk[nsa[i-1]], rnk[(nsa[i-1]+k)%n]};
					nrnk[nsa[i]]=nrnk[nsa[i-1]]+(op1!=op2);
				}
				swap(sa, nsa);swap(rnk, nrnk);
			}

			for(int i=0,k=0;i<n-1;++i){
				while(s[i+k]==s[sa[rnk[i]-1]+k])k++;
				lcp[rnk[i]-1]=k;
				if(k)k--;
			}
		}
};
```

9.8 Suffix Automaton

```
// O(n*log(alpha))
struct SuffixAutomaton{
	vector<map<char,int>> to;
	vector<bool> end;
	vi suf,len; // len, longest string
	int last;

	SuffixAutomaton(string& s){
		to.push_back(map<char,int>());
		suf.push_back(-1);
		len.push_back(0);
		last=0;

		for(int i=0;i<sz(s);i++){
			to.push_back(map<char,int>());
			suf.push_back(0);
			len.push_back(i+1);
			int r=sz(to)-1;

			int p=last;
			while(p>=0 && to[p].find(s[i])==to[p].end()){
				to[p][s[i]]=r;
				p=suf[p];
			}
			if(p!=-1){
				int q=to[p][s[i]];
				if(len[p]+1==len[q]){
					suf[r]=q;
				}else{
					to.push_back(to[q]);
					suf.push_back(suf[q]);
					len.push_back(len[p]+1);
					int qq=sz(to)-1;
					suf[q]=qq;
					suf[r]=qq;
					while(p>=0 && to[p][s[i]]==q){
						to[p][s[i]]=qq;
						p=suf[p];
					}
				}
			}
			last=r;
		}

		end.assign(sz(to), false);
		int p=last;
		while(p){
			end[p]=true;
			p=suf[p];
		}
	}
};
```

9.9 Suffix Tree

```
// O(n)
struct SuffixTree{
	vector<map<char,int>> to;
	vector<int> pos,len,link;
	const int inf = 1e9;
	int size=0;
	string s;

	int make(int _pos, int _len){
```

```
		to.push_back(map<char,int>());
		pos.push_back(_pos);
		len.push_back(_len);
		link.push_back(-1);
		return size++;
	}

	void add(int& p, int& lef, char c){
		s+=c;++lef;int lst=0;
		for(;lef;p?p=link[p]:lef--){
			while(lef>1 && lef>len[to[p][s[sz
			    (s)-lef]]]){
				p=to[p][s[sz(s)-lef]],lef
				    -=len[p];
			}
			char e=s[sz(s)-lef];
			int& q=to[p][e];
			if(!q){
				q=make(sz(s)-lef,inf),
				    link[lst]=p,lst=0;
			}else{
				char t=s[pos[q]+lef-1];
				if(t==c){link[lst]=p;
				    return;}
				int u=make(pos[q],lef-1);
				to[u][c]=make(sz(s)-1,inf
				    );
				to[u][t]=q;
				pos[q]+=lef-1;
				if(len[q]!=inf)len[q]-=
				    lef-1;
				q=u,link[lst]=u,lst=u;
			}
		}
	}

	void build(string& _s){
		make(-1,0);int p=0,lef=0;
		for(char c:_s)add(p,lef,c);
		add(p,lef,'$');
		s.pop_back();
	}

	int query(string& p){
		for(int i=0,u=0,n=sz(p);;){
			if(i==n || !to[u].count(p[i]))
			    return i;
			u=to[u][p[i]];
			for(int j=0;j<len[u];++j){
				if(i==n || s[pos[u]+j]!=p
				    [i])return i;
				i++;
			}
		}
	}

	vector<int> sa;
	void genSA(int x=0, int Len=0){
		if(!sz(to[x]))sa.push_back(pos[x]-Len);
		else for(auto t:to[x])genSA(t.second,Len+
		    len[x]);
	}
};
```

9.10 Trie

```
const int maxn = 2e6+5, alpha = 26, bits = 30;
int to[maxn][alpha],cnt[maxn],act;

void init(){
	for(int i=0;i<=act;++i){
		cnt[i]=0;
		// suf[i]=dad[i]=0;
		// adj[i].clear();
		memset(to[i], 0, sizeof(to[i]));
	}
	act=0;
}

int add(string& s){
	int u=0;
	for(char ch:s){
		int c=conv(ch);
		if(!to[u][c])to[u][c]=++act;
		u=to[u][c];
	}
	cnt[u]++;
	return u;
}

// Aho-Corasick
vector<int> adj[maxn]; // dad or suf
int dad[maxn],suf[maxn];

// O(sum(n)*alpha)
void build(){
	queue<int> q{{0}};
	while(!q.empty()){
		int u=q.front();q.pop();
		for(int i=0;i<alpha;++i){
			int v=to[u][i];
			if(!v)to[u][i]=to[suf[u]][i];
			else q.push(v);
			if(!u || !v)continue;
			suf[v]=to[suf[u]][i];
			dad[v]=cnt[suf[v]]?suf[v]:dad[suf
			    [v]];
		}
	}
	for(int i=1;i<=act;++i){
		adj[i].push_back(dad[i]);
```

```
            adj[dad[i]].push_back(i);
        }
}
```

9.11 Z Algorithm

```
// O(n)
vi z_function(string& s){
        int n=sz(s),l=0,r=0;vi z(n);
        for(int i=1;i<n;i++){
                if(i<r)z[i]=min(r-i, z[i-l]);
                while(i+z[i]<n && s[z[i]]==s[i+z[i]])z[i
                    ]++;
                if(i+z[i]>r){
                        l=i;
                        r=i+z[i];
                }
        }
        return z;
}
```

10 Misc

10.1 Counting Sort

```
// O(n+k)
void counting_sort(vi& a){
        int maxi=*max_element(all(a));
        int mini=*min_element(all(a));
        int k=maxi-mini+1,n=sz(a);
        vi cnt(k,0);
        for(int i=0;i<n;++i)++cnt[a[i]-mini];
        for(int i=0,j=0;i<k;++i)
                while(cnt[i]--)a[j++]=i+mini;
}
```

10.2 Dates

```
int dateToInt(int y, int m, int d){
        return 1461*(y+4800+(m-14)/12)/4+367*(m-2-(m-14)
            /12*12)/12-
                3*((y+4900+(m-14)/12)/100)/4+d-32075;
}
void intToDate(int jd, int& y, int& m, int& d){
        int x,n,i,j;x=jd+68569;
        n=4*x/146097;x-=(146097*n+3)/4;
        i=(4000*(x+1))/1461001;x-=1461*i/4-31;
        j=80*x/2447;d=x-2447*j/80;
        x=j/11;m=j+2-12*x;y=100*(n-49)+i+x;
}
int DayOfWeek(int d, int m, int y){     //starting on
    Sunday
        static int ttt[]={0, 3, 2, 5, 0, 3, 5, 1, 4, 6,
            2, 4};
        y-=m<3;
        return (y+y/4-y/100+y/400+ttt[m-1]+d)%7;
}
```

10.3 Expression Parsing

```
// O(n) - En python es eval()
bool delim(char c){return c==' ';}
bool is_op(char c){return c=='+' || c=='-' || c=='*' || c
    =='/';}
bool is_unary(char c){return c=='+' || c=='-';}

int priority(char op){
        if(op<0)return 3;
        if(op=='+' || op=='-')return 1;
        if(op=='*' || op=='/')return 2;
        return -1;
}

void process_op(stack<int>& st, char op){
        if(op<0){
                int l=st.top();st.pop();
                switch(-op){
                        case '+':st.push(l);break;
                        case '-':st.push(-l);break;
                }
        }else{
                int r=st.top();st.pop();
                int l=st.top();st.pop();
                switch(op){
                        case '+':st.push(l+r);break;
                        case '-':st.push(l-r);break;
                        case '*':st.push(l*r);break;
                        case '/':st.push(l/r);break;
                }
        }
}

int evaluate(string& s){
        stack<int> st;
        stack<char> op;
        bool may_be_unary=true;
        for(int i=0;i<sz(s);++i){
                if(delim(s[i]))continue;
                if(s[i] == '('){
                        op.push('(');
                        may_be_unary=true;
                }else if(s[i]==')'){
                        while(op.top()!='('){
```

```
                    process_op(st, op.top());
                    op.pop();
                }
                op.pop();
                may_be_unary=false;
            }else if(is_op(s[i])){
                char cur_op=s[i];
                if(may_be_unary && is_unary(
                    cur_op))cur_op=-cur_op;
                while(!op.empty() && ((cur_op >=
                    0 && priority(op.top()) >=
                    priority(cur_op)) || (cur_op <
                     0 && priority(op.top()) >
                    priority(cur_op)))){
                        process_op(st, op.top());
                        op.pop();
                }
                op.push(cur_op);
                may_be_unary=true;
            }else{
                int number=0;
                while(i<sz(s) && isalnum(s[i]))
                    number=number*10+s[i++]-'0';
                --i;
                st.push(number);
                may_be_unary=false;
            }
        }
        while(!op.empty()){
            process_op(st, op.top());
            op.pop();
        }
        return st.top();
}
```

10.4 Ternary Search

```
// O(log((r-l)/eps))
double ternary(){
        double l,r;
        while(r-l>eps){
                double m1=l+(r-l)/3.0;
                double m2=r-(r-l)/3.0;
                if(f(m1)<f(m2))l=m1;
                else r=m2;
        }
        return max(f(l),f(r));
}
```

10.5 Prefix3D

```
const int N = 100;
int A[N][N][N];
int preffix[N + 1][N + 1][N + 1];

void build(int n){
        for (int x = 1; x <= n; x++){
        for (int y = 1; y <= n; y++){
            for (int z = 1; z <= n; z++){
                preffix[x][y][z] = A[x - 1][y - 1][z - 1]
                    + preffix[x - 1][y][z] + preffix[x][y
                         - 1][z] + preffix[x][y][z - 1]
                    - preffix[x - 1][y - 1][z] - preffix[
                      x - 1][y][z - 1] - preffix[x][y -
                      1][z - 1]
                    + preffix[x - 1][y - 1][z - 1];
                }

            }

        }
}

ll query(int lx, int rx, int ly, int ry, int lz, int rz){
        ll ans = preffix[rx][ry][rz]
                - preffix[lx - 1][ry][rz] - preffix[rx][
                  ly - 1][rz] - preffix[rx][ry][lz - 1]
                + preffix[lx - 1][ly - 1][rz] + preffix[
                  lx - 1][ry][lz - 1] + preffix[rx][ly -
                   1][lz - 1]
                - preffix[lx - 1][ly - 1][lz - 1];

        return ans;
}
```

10.6 Hanoi

```
// hanoi(n) = 2 * hanoi(n-1) + 1
// hanoi(n, 1, 3)
vi ans;
void hanoi(int x, int start, int end){
    if(!x)return;
    hanoi(x-1, start, 6-start-end);
    ans.push_back({start, end});
    hanoi(x-1, 6-start-end, end);
}
```

11 Teoría y miscelánea

11.1 Sumatorias

- $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^{n} i^5 = \frac{(n(n+1))^2(2n^2+2n-1)}{12}$
- $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$ para $x \neq 1$

11.2 Teoría de Grafos

11.2.1 Teorema de Euler

En un grafo conectado planar, se cumple que $V - E + F = 2$, donde V es el número de vértices, E es el número de aristas y F es el número de caras. Para varios componentes la formula es: $V - E + F = 1 + C$, siendo C el número de componentes.

11.2.2 Planaridad de Grafos

Un grafo es planar si y solo si no contiene un subgrafo homeomorfo a K_5 (grafo completo con 5 vértices) ni a $K_{3,3}$ (grafo bipartito completo con 3 vértices en cada conjunto).

11.3 Teoría de Números

11.3.1 Ecuaciones Diofánticas Lineales

Una ecuación diofántica lineal es una ecuación en la que se buscan soluciones enteras x e y que satisfagan la relación lineal $ax+by = c$, donde a, b y c son constantes dadas.

Para encontrar soluciones enteras positivas en una ecuación diofántica lineal, podemos seguir el siguiente proceso:

1. Encontrar una solución particular: Encuentra una solución particular (x_0, y_0) de la ecuación. Esto puede hacerse utilizando el algoritmo de Euclides extendido.

2. Encontrar la solución general: Una vez que tengas una solución particular, puedes obtener la solución general utilizando la fórmula:

$$x = x_0 + \frac{b}{\mathrm{mcd}(a,b)} \cdot t$$

$$y = y_0 - \frac{a}{\mathrm{mcd}(a,b)} \cdot t$$

donde t es un parámetro entero.

3. Restringir a soluciones positivas: Si deseas soluciones positivas, asegúrate de que las soluciones generales satisfagan $x \geq 0$ y $y \geq 0$. Puedes ajustar el valor de t para cumplir con estas restricciones.

11.3.2 Pequeño Teorema de Fermat

Si p es un número primo y a es un entero no divisible por p, entonces $a^{p-1} \equiv 1 \pmod{p}$.

11.3.3 Teorema de Euler

Para cualquier número entero positivo n y un entero a coprimo con n, se cumple que $a^{\phi(n)} \equiv 1 \pmod{n}$, donde $\phi(n)$ es la función phi de Euler, que representa la cantidad de enteros positivos menores que n y coprimos con n.

11.4 Geometría

11.4.1 Teorema de Pick

Sea un poligono simple cuyos vertices tienen coordenadas enteras. Si B es el numero de puntos enteros en el borde, I el numero de puntos enteros en el interior del poligono, entonces el area A del poligono se puede calcular con la formula:

$$A = I + \frac{B}{2} - 1$$

11.4.2 Fórmula de Herón

Si los lados del triángulo tienen longitudes a, b y c, y s es el semiperímetro (es decir, $s = \frac{a+b+c}{2}$), entonces el área A del triángulo está dada por:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

11.4.3 Relación de Existencia Triangular

Para un triángulo con lados de longitud a, b, y c, la relación de existencia triangular se expresa como:

$$b - c < a < b + c, \quad a - c < b < a + c, \quad a - b < c < a + b$$

11.5 Combinatoria

11.5.1 Permutaciones

El número de permutaciones de n objetos distintos tomados de a r a la vez (sin repetición) se denota como $P(n,r)$ y se calcula mediante:

$$P(n,r) = \frac{n!}{(n-r)!}$$

11.5.2 Combinaciones

El número de combinaciones de n objetos distintos tomados de a r a la vez (sin repetición) se denota como $C(n,r)$ o $\binom{n}{r}$ y se calcula mediante:

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

11.5.3 Permutaciones con Repetición

El número de permutaciones de n objetos tomando en cuenta repeticiones se denota como $P_{\text{rep}}(n;n_1,n_2,\ldots,n_k)$ y se calcula mediante:

$$P_{\text{rep}}(n;n_1,n_2,\ldots,n_k) = \frac{n!}{n_1!n_2!\cdots n_k!}$$

11.5.4 Combinaciones con Repetición

El número de combinaciones de n objetos tomando en cuenta repeticiones se denota como $C_{\text{rep}}(n;n_1,n_2,\ldots,n_k)$ y se calcula mediante:

$$C_{\text{rep}}(n;n_1,n_2,\ldots,n_k) = \binom{n+k-1}{n} = \binom{n+k-1}{k-1}$$

11.5.5 Números de Catalan

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

Los números de Catalan también pueden calcularse utilizando la siguiente fórmula recursiva:

$$C_0 = 1$$

$$C_{n+1} = \frac{4n+2}{n+2}C_n$$

Usos:

- $\mathrm{Cat}(n)$ cuenta el número de árboles binarios distintos con n vértices.
- $\mathrm{Cat}(n)$ cuenta el número de expresiones que contienen n pares de paréntesis correctamente emparejados.
- $\mathrm{Cat}(n)$ cuenta el número de formas diferentes en que se pueden colocar $n+1$ factores entre paréntesis, por ejemplo, para $n=3$ y $3+1=4$ factores: a,b,c,d, tenemos: $(ab)(cd), a(b(cd)), ((ab)c)d$ y $a((bc)d)$.
- Los números de Catalan cuentan la cantidad de caminos no cruzados en una rejilla n x n que se pueden trazar desde una esquina de un cuadrado o rectángulo a la esquina opuesta, moviéndose solo hacia arriba y hacia la derecha.
- Los números de Catalan representan el número de árboles binarios completos con $n+1$ hojas.
- $\mathrm{Cat}(n)$ cuenta el número de formas en que se puede triangular un poligono convexo de $n+2$ lados. Otra forma de decirlo es como la cantidad de formas de dividir un polígono convexo en triángulos utilizando diagonales no cruzadas.

11.5.6 Estrellas y barras

Número de soluciones de la ecuación $x_1+x_2+\cdots+x_k=n$.

- Con $x_i \geq 0$: $\binom{n+k-1}{n}$
- Con $x_i \geq 1$: $\binom{n-1}{k-1}$

Número de sumas de enteros con límite inferior:

Esto se puede extender fácilmente a sumas de enteros con diferentes límites inferiores. Es decir, queremos contar el número de soluciones para la ecuación:

$$x_1+x_2+\cdots+x_k=n$$

con $x_i \geq a_i$.

Después de sustituir $x_i' := x_i - a_i$ recibimos la ecuación modificada:

$$(x_1'+a_i)+(x_2'+a_i)+\cdots+(x_k'+a_k)=n$$

$$\Leftrightarrow \quad x_1'+x_2'+\cdots+x_k' = n-a_1-a_2-\cdots-a_k$$

con $x_i' \geq 0$. Así que hemos reducido el problema al caso más simple con $x_i' \geq 0$ y nuevamente podemos aplicar el teorema de estrellas y barras.

11.6 DP Optimization Theory

Name	Original Recurrence	Sufficient Condition	From	To
CH 1	$dp[i] = min_{j<i}\{dp[j] + b[j] * a[i]\}$	$b[j] \geq b[j+1]$ Optionally $a[i] \leq a[i+1]$	$O(n^2)$	$O(n)$
CH 2	$dp[i][j] = min_{k<j}\{dp[i-1][k] + b[k] * a[j]\}$	$b[k] \geq b[k+1]$ Optionally $a[j] \leq a[j+1]$	$O(kn^2)$	$O(kn)$
D&Q	$dp[i][j] = min_{k<j}\{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	$O(kn \log n)$
Knuth	$dp[i][j] = min_{i<k<j}\{dp[i][k] + dp[k][j]\} + C[i][j]$	$A[i, j-1] \leq A[i, j] \leq A[i+1, j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$ - the smallest k that gives the optimal answer, for example in $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$ - some given cost function
- We can generalize a bit in the following way $dp[i] = \min_{j<i}\{F[j] + b[j] * a[i]\}$, where $F[j]$ is computed from $dp[j]$ in constant time