

Notebook UNTreeCiclo

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1 C++

1.1 C++ plantilla

```
#include <bits/stdc++.h>
using namespace std;
#define watch(x) cout<<#x<<"="<<x<<'\n'
#define sz(arr) ((int) arr.size())
#define all(v) v.begin(), v.end()
typedef long long ll;
typedef long double ld;
typedef pair<int, int> ii;
typedef vector<ii> vii;
typedef vector<int> vi;
typedef vector<long long> vl;
typedef pair<ll, ll> pll;
typedef vector<pll> vll;
const int INF = 1e9;
const ll INFL = 1e18;
const int MOD = 1e9+7;
const double EPS = 1e-9;
```

```

const ld PI = acosl(-1);
int dirx[4] = {0,-1,1,0};
int diry[4] = {-1,0,0,1};
int dr[] = {1, 1, 0, -1, -1, -1, 0, 1};
int dc[] = {0, 1, 1, 1, 0, -1, -1, -1};
const string ABC = "abcdefghijklmnopqrstuvwxyz";
const char ln = '\n';

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout << setprecision(20) << fixed;
    // freopen("file.in", "r", stdin);
    // freopen("file.out", "w", stdout);

    return 0;
}

```

1.2 Librerías

```

// En caso de que no sirva #include <bits/stdc++.h>
#include <algorithm>
#include <iostream>
#include <iterator>
#include <sstream>
#include <fstream>
#include <cassert>
#include <climits>
#include <cstdlib>
#include <cstring>
#include <string>
#include <cstdio>
#include <vector>
#include <cmath>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <map>
#include <set>
#include <bitset>
#include <iomanip>
#include <unordered_map>
////
#include <tuple>
#include <random>
#include <chrono>

```

1.3 Bitmask

* Operaciones a nivel de bits. Si n es ll usar $lll<<$ en los corrimientos.

```

x & 1          -> Verifica si x es impar
x & (1<<i)     -> Verifica si el i-esimo bit esta
                encendido
x = x | (1<<i)  -> Enciende el i-esimo bit
x = x & ~(1<<i) -> Apaga el i-esimo bit
x = x ^ (1<<i)  -> Invierte el i-esimo bit
x = ~x         -> Invierte todos los bits
x & -x         -> Devuelve el bit encendido mas a la
                derecha (potencia de 2, no el indice)
~x & (x+1)     -> Devuelve el bit apagado mas a la
                derecha (potencia de 2, no el indice)
x = x | (x+1)   -> Enciende el bit apagado mas a la
                derecha
x = x & (x-1)   -> Apaga el bit encendido mas a la
                derecha
x = x & ~y      -> Apaga en x los bits encendidos de y

* Funciones del compilador gcc. Si n es ll agregar el
  sufijo ll, por ej: __builtin_clzll(n).

__builtin_clz(x)      -> Cantidad de bits apagados por la
  izquierda
__builtin_ctz(x)      -> Cantidad de bits apagados por la
  derecha. Indice del bit encendido mas a la derecha
__builtin_popcount(x) -> Cantida de bits encendidos

* Logaritmo en base 2 (entero). Indice del bit encendido
  mas a la izquierda. Si x es ll usar 63 y clzll(x).
// 0(1)
int lg2(const int &x) { return 31-__builtin_clz(x); }

* Itera, con indices, los bits encendidos de una mascara.
// 0(#bits_encendidos)
for (int x = mask; x; x &= x-1) {
    int i = __builtin_ctz(x);
}

* Itera todas las submascaras de una mascara. (Iterar
  todas las submascaras de todas las mascaras es 0(3^n))
// 0(2^(#bits_encendidos))
for (int sub = mask; sub; sub = (sub-1)&mask) {}

* retorna la siguiente mask con la misma cantidad
  encendida
ll nextMask(ll x){
    ll c = x & -x;
    ll r = x + c;
    return ((r ^ x) >> 2) / c | r;
}

```

1.4 Cosas de strings

```

int conv(char ch){return ((ch>='a' && ch<='z')?ch-'a':ch-
'A'+26);}
vector<string> split(string& s, char c=' '){
    vector<string> res;
    stringstream ss(s);
    string sub;
    while(getline(ss, sub, c))res.push_back(sub);
    return res;
}

for(char& c:s)c=toupper(c);
for(char& c:s)c=tolower(c);
int n=stoi(s); // de string a entero
int n=stoi(s, nullptr, 2); // base 2
double d=stod(s); // de string a double
string s=to_string(n); // de entero a string

```

1.5 Custom Hashing

```

struct custom_hash {
    static long long splitmix64(long long x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(long long x) const {
        static const long long FIXED_RANDOM =
            chrono::steady_clock::now().
            time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }

    size_t operator()(const pair<int,int>& x) const {
        return (size_t) x.first * 37U + (size_t)
            x.second;
    }

    size_t operator()(const vector<int>& v) const {
        size_t s = 0;
        for(auto &e : v)
            s ^= hash<int>() (e) + 0x9e3779b9 + (s
                <<6) + (s>>2);
        return s;
    }
};

unordered_map<long long, int, custom_hash> safe_map;
gp_hash_table<int, int, custom_hash> table;

```

1.6 Random

```

typedef unsigned long long u64;
mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());
u64 hash=rng();

mt19937 rng (chrono::steady_clock::now().time_since_epoch
    ().count());
int rand(int a, int b){return uniform_int_distribution<
    int>(a, b)(rng);} // uniform_real_distribution

```

2 Arboles

2.1 Centroid Decomposition

```

// O(nlog(n))
struct CentroidDecomposition{
    int dad[maxn],sz[maxn];
    set<int> adj[maxn]; // check, proc

    int operator[](int i){return dad[i];}
    void addEdge(int x,int y){adj[x].insert(y);adj[y]
        .insert(x);}

    void build(int v=0, int p=-1){
        int n=dfsSz(v, p);
        int centroid=dfsCentroid(v, p, n);
        dad[centroid]=p;
        // add dfs for paths
        for(int u:adj[centroid]){
            adj[u].erase(centroid);
            build(u,centroid);
        }
        adj[centroid].clear();
    }

    int dfsSz(int v,int p){
        sz[v]=1;
        for(int u:adj[v]){
            if(u==p)continue;
            sz[v]+=dfsSz(u, v);
        }
        return sz[v];
    }

    int dfsCentroid(int v, int p, int n){
        for(int u:adj[v]){
            if(u==p)continue;
            if(sz[u]>n/2) return dfsCentroid(u
                ,v,n);
        }
        return v;
    }
};

// for(int b=a;b!=-1;b=cd[b])

```

2.2 Heavy Light Decomposition

```

typedef long long T;
T null;
T oper(T a, T b);
// Segment tree
const int maxn=1e5+1; // >= 2e5, remove struct
bool edges=false; // arista padre
struct HLD{
    int par[maxn], root[maxn], dep[maxn];
    int sz[maxn], pos[maxn], ti;
    vector<int> adj[maxn];
    SegTree st;
    void addEdge(int x, int y){adj[x].push_back(y);
        adj[y].push_back(x);}
    void dfsSz(int x){
        sz[x]=0;
        for(int& y:adj[x]){
            if(y==par[x]) continue;
            par[y]=x; dep[y]=dep[x]+1;
            dfsSz(y);
            sz[x]+=sz[y]+1;
            if(sz[y]>sz[adj[x][0]]) swap(y, adj
                [x][0]);
        }
    }
    void dfsHld(int x){
        pos[x]=ti++;
        for(int y:adj[x]){
            if(y==par[x]) continue;
            root[y]=(y==adj[x][0]?root[x]:y);
            dfsHld(y);
        }
    }
    void build(int n, int v=0){
        root[v]=par[v]=v;
        dep[v]=ti=0;
        dfsSz(v);
        dfsHld(v);
        // vl palst(n);
        // for(int i=0; i<n; ++i) palst[pos[i]]=a[i]
        // l;
        // st.build(palst, n);
        st.build(n);
    }
    // O(log^2(n))
    template <class Oper>
    void processPath(int x, int y, Oper op){
        for(; root[x]!=root[y]; y=par[root[y]]){
            if(dep[root[x]]>dep[root[y]]) swap
                (x, y);
            op(pos[root[y]], pos[y]);
        }
    }
};

```

```

        if(dep[x]>dep[y]) swap(x, y);
        op(pos[x]+edges, pos[y]);
    }
    void modifyPath(int x, int y, int v){
        processPath(x, y, [this, &v](int l, int r){
            st.upd(l, r, v);
        });
    }
    T queryPath(int x, int y){
        T res=null;
        processPath(x, y, [this, &res](int l, int r)
            {
                res=oper(res, st.get(l, r));
            });
        return res;
    }
    void modifySubtree(int x, int v){st.upd(pos[x]+
        edges, pos[x]+sz[x], v);}
    int querySubtree(int x){return st.get(pos[x]+
        edges, pos[x]+sz[x]);}
    void modify(int x, int v){st.set(pos[x], v);}
    void modifyEdge(int x, int y, int v){
        if(dep[x]<dep[y]) swap(x, y);
        modify(x, v);
    }
};

```

2.3 LCA

```

const int maxn = 2e5+5, maxlog = 20+5;
int up[maxn][maxlog], dep[maxn]; // memset -1 (up)
vi adj[maxn];

void dfs(int v=0, int p=-1){
    up[v][0]=p;
    for(int u:adj[v]){
        if(u!=p){
            dep[u]=dep[v]+1;
            dfs(u, v);
        }
    }
}

void build(int n){
    for(int l=1; l<maxlog; ++l){
        for(int i=0; i<n; ++i){
            if(up[i][l-1]!=-1){
                up[i][l]=up[up[i][l-1]][l-1];
            }
        }
    }
}

int kth(int node, int k){

```

```

    for(int l=maxlog-1;l>=0;--l){
        if(node!=-1 && k&(1<<l)){
            node=up[node][l];
        }
    }
    return node;
}

int lca(int a, int b){
    a=kth(a, dep[a]-min(dep[a], dep[b]));
    b=kth(b, dep[b]-min(dep[a], dep[b]));
    if(a==b) return a;
    for(int l=maxlog-1;l>=0;--l){
        if(up[a][l]!=up[b][l]){
            a=up[a][l];
            b=up[b][l];
        }
    }
    return up[a][0];
}

```

2.4 Sack

```

const int maxn = 1e5+5;
int st[maxn], ft[maxn], ver[2*maxn];
int len[maxn], n, q, pos=0;
vi adj[maxn];

bool vis[maxn];
void ask(int v, bool add){
    if(vis[v] && !add){
        vis[v]=false;
        // delete node
    }else if(!vis[v] && add){
        vis[v]=true;
        // add node
    }
}

// O(nlogn)
void dfs(int v=0, int p=-1, bool keep=true){
    int mx=0, id=-1;
    for(int u:adj[v]){
        if(u==p) continue;
        if(len[u]>mx){
            mx=len[u];
            id=u;
        }
    }
    for(int u:adj[v]){
        if(u!=p && u!=id)
            dfs(u, v, 0);
    }
    if(id!=-1) dfs(id, v, 1);
    for(int u:adj[v]){

```

```

        if(u==p || u==id) continue;
        for(int p=st[u]; p<ft[u]; ++p)
            ask(ver[p], 1);
    }
    ask(v, 1);
    // answer queries
    if(keep) return;
    for(int p=st[v]; p<ft[v]; ++p)
        ask(ver[p], 0);
}

```

2.5 Virtual Tree

```

const int maxn = 2e5+5;
vi adjVT[maxn], adj[maxn];
int st[maxn], ft[maxn], n, pos=0;
bool important[maxn];

bool upper(int v, int u){return st[v]<=st[u] && ft[v]>=ft[u];}
bool cmp(int v, int u){return st[v]<st[u];}

// O(klogk)
int virtualTree(vi nodes){
    sort(all(nodes), cmp);
    int m=sz(nodes);
    for(int i=0; i<m-1; ++i){
        int v=lca(nodes[i], nodes[i+1]);
        nodes.push_back(v);
    }

    sort(all(nodes), cmp);
    nodes.erase(unique(all(nodes), nodes.end()));
    for(int u:nodes) adjVT[u].clear();

    vi s;
    s.push_back(nodes[0]);
    m=sz(nodes);
    for(int i=1; i<m; ++i){
        int v=nodes[i];
        while(sz(s)>=2 && !upper(s.back(), v)){
            adjVT[s[sz(s)-2]].push_back(s.back());
            s.pop_back();
        }
        s.push_back(v);
    }
    while(sz(s)>=2){
        adjVT[s[sz(s)-2]].push_back(s.back());
        s.pop_back();
    }
    return s[0];
}

// vi nodes(k);

```

```
// for(int& x:nodes)important[x]=true;
// int root=virtualTree(nodes);
// dp(root) - output answer - reset
```

3 Estructuras de Datos

3.1 Disjoint Set Union

```
struct dsu{
    vi p,size;
    int sets,maxSize;

    dsu(int n){
        p.assign(n,0);
        size.assign(n,1);
        sets = n;
        for (int i = 0; i<n; i++) p[i] = i;
    }

    int find_set(int i) {return (p[i] == i) ? i : (p[
        i] = find_set(p[i]));}

    bool is_same_set(int i, int j) {return find_set(i)
        == find_set(j);}

    void unionSet(int i, int j){
        if (!is_same_set(i, j)){
            int a = find_set(i), b = find_set
                (j);
            if (size[a] < size[b]) swap(a, b)
            p[b] = a;
            size[a] += size[b];
            maxSize = max(size[a], maxSize);
            sets--;
        }
    }
};
```

3.2 Dynamic Connectivity Offline

```
struct dsu{
    vi p,rank,h;
    int sets;
    dsu(int n){
        sets=n;
        p.assign(n,0);
        rank.assign(n,1);
        for(int i=0;i<n;++i)p[i]=i;
    }
    int get(int a){return (a==p[a]?a:get(p[a]));}
    void unite(int a, int b){
```

```
        a=get(a);b=get(b);
        if(a==b)return;
        if(rank[a]>rank[b])swap(a,b);
        rank[b]+=rank[a];
        h.push_back(a);
        p[a]=b;sets--;
    }
    void rollback(int x){
        int len=h.size();
        while(len>x){
            int a=h.back();
            h.pop_back();
            rank[p[a]]-=rank[a];
            p[a]=a;sets++;len--;
        }
    }
};

enum { ADD, DEL, QUERY };
struct Query{int type, u, v;};
struct DynCon{
    vector<Query> q;
    dsu uf;vi mt;
    map<pair<int,int>, int> prv;
    DynCon(int n): uf(n){}
    void add(int i, int j){
        if(i>j)swap(i, j);
        q.push_back({ADD, i, j});
        mt.push_back(-1);
        prv[{i,j}]=sz(q)-1;
    }
    void remove(int i, int j){
        if(i > j) swap(i, j);
        q.push_back({DEL, i, j});
        int pr=prv[{i, j}];
        mt[pr]=sz(q)-1;
        mt.push_back(pr);
    }
    void query(){q.push_back({QUERY, -1, -1});mt.
        push_back(-1);}
    void process(){ // answers all queries in order
        if(!sz(q))return;
        for(int i=0;i<sz(q);++i){
            if(q[i].type==ADD && mt[i]<0)mt[i]
                =sz(q);
            }go(0, sz(q));
        }
    void go(int s, int e){
        if(s+1==e){
            if(q[s].type == QUERY)cout<<uf.sets<<"\n"
                ;
            return;
        }int k=sz(uf.h),m=(s+e)/2;
        for(int i=e-1;i>=m;--i){
            if(mt[i]>=0 && mt[i]<s)uf.unite(q[i].u, q
                [i].v);
```

```

    }go(s, m);
    uf.rollback(k);
    for(int i=m-1;i>=s;--i){
        if(mt[i]>=e)uf.unite(q[i].u, q[i].v);
    }go(m, e);
    uf.rollback(k);
}
};

```

3.3 Dynamic Segment Tree

```

T null=0,nolz=0;
T oper(T a, T b);
struct Node{
    T val,lz;
    int l,r;
    Node *pl,*pr;
    Node(int ll, int rr){
        val=null;lz=nolz;
        pl=pr=nullptr;
        l=ll;r=rr;
    }
};

typedef Node* PNode;
void update(PNode x){
    if(x->r-x->l==1) return;
    x->val=oper(x->pl->val,x->pr->val);
}

void extends(PNode x){
    if(x->r-x->l!=1 && !x->pl){
        int m=(x->r+x->l)/2;
        x->pl=new Node(x->l, m);
        x->pr=new Node(m, x->r);
    }
}

void propagate(PNode x){
    if(x->r-x->l==1) return;
    if(x->lz==nolz) return;
    int m=(x->r+x->l)/2;
    // pl, pr
    x->lz=nolz;
}

struct SegTree{
    PNode root;
    void upd(PNode x, int l, int r, T v){
        int lx=x->l,rx=x->r;
        if(lx>=r || l>=rx) return;
        if(lx>=l && rx<=r){
            // val, lz
            return;
        }
    }
}

```

```

    extends(x);
    propagate(x);
    upd(x->pl,l,r,v);
    upd(x->pr,l,r,v);
    update(x);
}

T get(PNode x, int l, int r){
    int lx=x->l,rx=x->r;
    if(lx>=r || l>=rx) return null;
    if(lx>=l && rx<=r) return x->val;
    extends(x);
    propagate(x);
    T v1=get(x->pl,l,r);
    T v2=get(x->pr,l,r);
    return oper(v1,v2);
}

T get(int l, int r){return get(root,l,r+1);}
void upd(int l, int r, T v){upd(root,l,r+1,v);}
void build(int l, int r){root=new Node(l, r+1);}
};

```

3.4 Fenwick Tree

```

typedef long long T;
struct FwTree{
    int n;
    vector<T> bit;
    FwTree(int n):n(n),bit(n+1){}
    T get(int r){
        T sum=0;
        for(++r;r>=r&-r) sum+=bit[r];
        return sum;
    }
    T get(int l, int r){return get(r)-(l==0?0:get(l-1));}
    void upd(int r, T v){
        for(++r;r<=n;r+=r&-r) bit[r]+=v;
    }
};

struct FwTree2d{
    int n, m;
    vector<vector<T>> bit;
    FwTree2d(){}
    FwTree2d(int n, int m):n(n),m(m),bit(n+1, vector<T>(m+1,0)){}
    T get(int x, int y){
        T v=0;
        for(int i=x+1;i;i=i&-i)
            for(int j=y+1;j;j=j&-j) v+=bit[i][j];
        return v;
    }
}

```



```

T get(int x, int y, int x2, int y2){return get(x2
,y2)-get(x-1,y2)-get(x2,y-1)+get(x-1,y-1);}
void upd(int x, int y, T dt){
    for(int i=x+1;i<=n;i+=i&-i)
        for(int j=y+1;j<=m;j+=j&-j)bit[i][j]+=dt;
}
};

```

3.5 Li Chao

```

// inf max abs value that the function may take
typedef long long ty;
struct Line {
    ty m, b;
    Line(){}
    Line(ty m, ty b): m(m), b(b){}
    ty eval(ty x){return m * x + b;}
};
struct nLiChao{
    // see coments for min
    nLiChao *left = nullptr, *right = nullptr;
    ty l, r;
    Line line;
    nLiChao(ty l, ty r): l(l), r(r){
        line = {0, -inf}; // change to {0, inf};
    }
    // T(Log(Rango)) M(Log(rango))
    void addLine(Line nline){
        ty m = (l + r) >> 1;
        bool lef = nline.eval(l) > line.eval(l);
        // change > to <
        bool mid = nline.eval(m) > line.eval(m); //
        // change > to <
        if (mid) swap(nline, line);
        if (r == l) return;
        if (lef != mid){
            if (!left){
                left = new nLiChao(l, m);
                left -> line = nline;
            }
            else left -> addLine(nline);
        }
        else{
            if (!right){
                right = new nLiChao(m +
                    1, r);
            }
            else right -> addLine(nline);
        }
    }
};

```

```

        right -> line = nline;
    }
    else right -> addLine(nline);
}
// T(Log(Rango))
ty get(ty x) {
    ty m = (l + r) >> 1;
    ty opl = -inf, op2 = -inf; // change to
    // inf
    if(l == r) return line.eval(x);
    else if(x < m){
        if (left) opl = left -> get(x);
        return max(line.eval(x), opl); //
        // change max to min
    }
    else{
        if (right) op2 = right -> get(x);
        return max(line.eval(x), op2); //
        // change max to min
    }
}
};

int main() {
    // (rango superior) * (pendiente maxima) puede
    // desbordarse
    // usar double o long double en el eval para
    // estos casos
    // (puede dar problemas de precision)
    nLiChao liChao(0, 1e18);
}

```

3.6 Link Cut Tree

```

typedef long long T;
struct SplayTree{
    struct Node{
        int ch[2]={0, 0},p=0;
        T val=0,path=0,sz=1; // Path
        T sub=0,vir=0,ssz=0,vsz=0; // Subtree
        bool flip=0;T lz=0; // Lazy
    };
    vector<Node> ns;
    SplayTree(int n):ns(n+1){}
    T path(int u){return (u?ns[u].path:0);}
    T size(int u){return (u?ns[u].sz:0);}
    T subsize(int u){return (u?ns[u].ssz:0);}
    T subsum(int u){return (u?ns[u].sub:0);}
    void push(int x){

```

```

    if(!x) return;
    int l=ns[x].ch[0], r=ns[x].ch[1];
    if(ns[x].flip){
        ns[l].flip^=1, ns[r].flip^=1;
        swap(ns[x].ch[0], ns[x].ch[1]);
        // check with st oper
        ns[x].flip=0;
    }
    if(ns[x].lz){
        // ...
        ns[x].sub+=ns[x].lz*ns[x].ssz;
        ns[x].vir+=ns[x].lz*ns[x].vsz;
        // ...
    }
}

void pull(int x){
    int l=ns[x].ch[0], r=ns[x].ch[1];
    push(l); push(r);
    ns[x].sz=size(l)+size(r)+1;
    ns[x].path=max({path(l), path(r), ns[x].
        val});
    ns[x].sub=ns[x].vir+subsum(l)+subsum(r)+
        ns[x].val;
    ns[x].ssz=ns[x].vsz+subsize(l)+subsize(r)
        +1;
}

void set(int x, int d, int y){ ns[x].ch[d]=y; ns[y]
    ].p=x; pull(x); }
void splay(int x){
    auto dir=[&](int x){
        int p=ns[x].p; if(!p) return -1;
        return ns[p].ch[0]==x?0:ns[p].ch
            [1]==x?1:-1;
    };
    auto rotate=[&](int x){
        int y=ns[x].p, z=ns[y].p, dx=dir(x)
            , dy=dir(y);
        set(y, dx, ns[x].ch[!dx]);
        set(x, !dx, y);
        if(~dy) set(z, dy, x);
        ns[x].p=z;
    };
    for(push(x); ~dir(x);){
        int y=ns[x].p, z=ns[y].p;
        push(z); push(y); push(x);
        int dx=dir(x), dy=dir(y);
        if(~dy) rotate(dx!=dy?x:y);
        rotate(x);
    }
}

struct LinkCut:SplayTree{ // 1-indexed
    LinkCut(int n):SplayTree(n){}
};

```

```

int root(int u){
    access(u); splay(u); push(u);
    while(ns[u].ch[0]){ u=ns[u].ch[0]; push(u)
        ; }
    return splay(u), u;
}

int parent(int u){
    access(u); splay(u); push(u);
    u=ns[u].ch[0]; push(u);
    while(ns[u].ch[1]){ u=ns[u].ch[1]; push(u)
        ; }
    return splay(u), u;
}

int access(int x){
    int u=x, v=0;
    for(; u; v=u, u=ns[u].p){
        splay(u);
        int& ov=ns[u].ch[1];
        ns[u].vir+=ns[ov].sub;
        ns[u].vsz+=ns[ov].ssz;
        ns[u].vir-=ns[v].sub;
        ns[u].vsz-=ns[v].ssz;
        ov=v; pull(u);
    }
    return splay(x), v;
}

void reroot(int x){
    access(x); ns[x].flip^=1; push(x);
}

void link(int u, int v){ // u->v
    reroot(u);
    access(v);
    ns[v].vir+=ns[u].sub;
    ns[v].vsz+=ns[u].ssz;
    ns[u].p=v; pull(v);
}

void cut(int u, int v){
    int r=root(u);
    reroot(u);
    access(v);
    ns[v].ch[0]=ns[u].p=0; pull(v);
    reroot(r);
}

void cut(int u){ // cut parent
    access(u);
    ns[ns[u].ch[0]].p=0;
    ns[u].ch[0]=0; pull(u);
}

int lca(int u, int v){
    if(root(u)!=root(v)) return -1;
}

```

```

        access(u); return access(v);
    }

    int depth(int u){
        access(u); splay(u); push(u);
        return ns[u].sz;
    }

    T path(int u, int v){
        int r=root(u);
        reroot(u); access(v); pull(v);
        T ans=ns[v].path;
        return reroot(r), ans;
    }

    void set(int u, T val){access(u); ns[u].val=val;
        pull(u);}
    void upd(int u, int v, T val){
        int r=root(u);
        reroot(u); access(v); splay(v);
        // lazy
        reroot(r);
    }

    T comp_size(int u){return ns[root(u)].ssz;}
    T subtree_size(int u){
        int p=parent(u);
        if(!p) return comp_size(u);
        cut(u); int ans=comp_size(u);
        link(u,p); return ans;
    }

    T subtree_size(int u, int v){
        int r=root(u);
        reroot(v); access(u);
        T ans=ns[u].vsz+1;
        return reroot(r), ans;
    }

    T comp_sum(int u){return ns[root(u)].sub;}
    T subtree_sum(int u){
        int p=parent(u);
        if(!p) return comp_sum(u);
        cut(u); T ans=comp_sum(u);
        link(u,p); return ans;
    }

    T subtree_sum(int u, int v){ // subtree of u, v
        father
        int r=root(u);
        reroot(v); access(u);
        T ans=ns[u].vir+ns[u].val; // por el
        reroot
        return reroot(r), ans;
    }

};

```

3.7 Mos Algorithm

```

// O((n+q)*s), s=n^(1/2)
// O(q*(s+(n/s)^2) => O(q*(n^(2/3))), s=(2*(n^2))^(1/3) -
// s=n^(2/3)
int s,n;
struct upd{int i,old,cur;};
struct query {int l,r,t,idx;};
bool cmp(query& a, query& b){
    int x=a.l/s;
    if(a.l/s!=b.l/s) return a.l/s<b.l/s;
    if(a.r/s!=b.r/s) return (x&1?a.r<b.r:a.r>b.r);
    return a.t<b.t;
}

vector<int> ans;
vector<query> qu;
vector<upd> up;

int act();
void add(int i);
void remove(int i);
void update(int i,int v,int l,int r){
    if(l<=i && i<=r); // add, remove
}

void solve(){
    s=(int)ceil(sqrt(n));
    sort(all(qu), cmp);
    int l=0,r=-1,t=0;
    for(int i=0;i<sz(qu);++i){
        while(t<qu[i].t) update(up[t].i,up[t].cur,
            l,r,++t);
        while(t>qu[i].t)--t, update(up[t].i,up[t].
            old,l,r);
        while(r<qu[i].r) add(++r);
        while(l>qu[i].l) add(--l);
        while(r>qu[i].r) remove(r--);
        while(l<qu[i].l) remove(l++);
        ans[qu[i].idx]=act();
    }

    // tree
    int st[maxn],ft[maxn],ver[maxn*2];
    bool vis[maxn];

    void ask(int v){
        vis[v]=!vis[v];
        if(vis[v]) add(v);
        else remove(v);
    }

    // query[i] = {st[a]+1, st[b], i} + lca

```

3.8 Ordered set

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<typename T> using ordered_set = tree<T,
    null_type,less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
template<typename T> using ordered_multiset = tree<T,
    null_type,less_equal<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// ----- CONSTRUCTOR ----- //
// 1. Para ordenar por MAX cambiar less<int> por greater<
    int>
// 2. Para multiset cambiar less<int> por less_equal<int>
//     Para borrar siendo multiset:
//     int idx = st.order_of_key(value);
//     st.erase(st.find_by_order(idx));
// ----- METHODS ----- //
st.find_by_order(k) // returns pointer to the k-th
    smallest element
st.order_of_key(x) // returns how many elements are
    smaller than x
st.find_by_order(k) == st.end() // true, if element does
    not exist
```

3.9 Persistent Segment Tree

```
typedef long long T;
struct Node{T val;int l,r};
struct SegTree{
    vector<Node> ns;
    int act=0,size;
    vi roots;

    T null=0;
    T oper(T a, T b);
    void update(int x){
        ns[x].val=oper(ns[ns[x].l].val, ns[ns[x].
            r].val);
    }

    int newNode(T x){
        Node tmp={x,-1,-1};
        ns.push_back(tmp);
        return act++;
    }

    int newNode(int l, int r){
        Node tmp={null,l,r};
        ns.push_back(tmp);
        update(act);
        return act++;
    }
}
```

```
int build(vector<T>& a, int l, int r){
    if(r-l==1){return newNode(a[l]);}
    int m=(l+r)/2;
    return newNode(build(a, l, m),build(a, m,
        r));
}

int set(int x, int i, T v, int l, int r){
    if(r-l==1){return newNode(v);
    int m=(l+r)/2;
    if(i<m){return newNode(set(ns[x].l, i, v,
        l, m), ns[x].r);
    else return newNode(ns[x].l, set(ns[x].r,
        i, v, m, r));
}

T get(int x, int lx, int rx, int l, int r){
    if(lx>=r || l>=rx){return null;
    if(lx>=l && rx<=r){return ns[x].val;
    int m=(lx+rx)/2;
    T v1=get(ns[x].l, lx, m, l, r);
    T v2=get(ns[x].r, m, rx, l, r);
    return oper(v1,v2);
}

T get(int l, int r, int time){return get(roots[
    time], 0, size, l, r+1);}
void set(int i, T v, int time){roots.push_back(
    set(roots[time], i, v, 0, size));}
void build(vector<T>& a, int n){size=n;roots.
    push_back(build(a, 0, size));}
};
```

3.10 RMQ

```
typedef long long T;
T oper(T a, T b); // max, min, gcd ...
struct RMQ {
    vector<vector<T>> table;
    void build(vector<T>& v){
        int n=sz(v);
        table.assign(20, vector<T>(n)); // log2(n)
        for(int i=0;i<n;++i)table[0][i]=v[i];
        for(int j=1;(1<<j)<=n;++j)
            for(int i=0;i+(1<<(j-1))<n;++i)
                table[j][i]=oper(table[j-1][i
                    -1][i],table[j-1][i
                    +(1<<(j-1))]);
    }
    T get(int l, int r){
        int j=31-__builtin_clz(r-l+1);
        return oper(table[j][l], table[j][r-(1<<j
            )+1]);
    }
}
```

```
    }
};
```

3.11 Segment Tree Iterativo

```
struct segtree{
    int n; vl v; ll nulo = 0;
    ll op(ll a, ll b) {return a + b;}
    segtree(int n) : n(n), v(2*n, nulo){}
    segtree(vl &a) : n(sz(a)), v(2*n){
        for(int i = 0; i<n; i++) v[n + i] = a[i];
        for (int i = n-1; i>=1; --i) v[i] = op(v[i<<1], v
            [i<<1|1]);
    }
    void upd(int k, ll nv){
        for (v[k += n] = nv; k > 1; k >= 1) v[k>>1] = op
            (v[k], v[k^1]);
    }
    ll get(int l, int r){
        ll vl = nulo, vr = nulo;
        for (l += n, r += n+1; l < r; l >= 1, r >= 1){
            if (l&1) vl = op(vl, v[l++]);
            if (r&1) vr = op(v[--r], vr);
        }
        return op(vl, vr);
    }
};
```

3.12 Segment Tree Recursivo

```
typedef long long T;
struct SegTree{
    vector<T> vals,lazy;
    T null=0,nolz=0;
    int size;
    T oper(T a, T b);
    void build(vector<T>& a, int x, int lx, int rx){
        if(rx-lx==1){
            if(lx<sz(a))vals[x]=a[lx];
            return;
        }
        int m=(lx+rx)/2;
        build(a, 2*x+1, lx, m);
        build(a, 2*x+2, m, rx);
        vals[x]=oper(vals[2*x+1], vals[2*x+2]);
    }
    void build(vector<T>& a,int n){
```

```
        size=1;
        while(size<n) size*=2;
        vals.resize(2*size);
        lazy.assign(2*size, nolz);
        build(a, 0, 0, size);
    }
    void propagate(int x, int lx, int rx){
        if(rx-lx==1)return;
        if(lazy[x]==nolz)return;
        int m=(lx+rx)/2;
        // 2*x+1, 2*x+2 (lazy, vals)
        lazy[x]=nolz;
    }
    void upd(int l, int r, T v,int x, int lx, int rx)
    {
        if(lx>=r || l>=rx)return;
        if(lx>=l && rx<=r){
            // lazy, vals
            return;
        }
        propagate(x,lx,rx);
        int m=(lx+rx)/2;
        upd(l,r,v,2*x+1,lx,m);
        upd(l,r,v,2*x+2,m,rx);
        vals[x]=oper(vals[2*x+1], vals[2*x+2]);
    }
    void set(int i, T v, int x, int lx, int rx){
        if(rx-lx==1){
            vals[x]=v;
            return;
        }
        propagate(x,lx,rx);
        int m=(lx+rx)/2;
        if(i<m)set(i,v,2*x+1,lx,m);
        else set(i,v,2*x+2,m,rx);
        vals[x]=oper(vals[2*x+1], vals[2*x+2]);
    }
    T get(int l, int r, int x, int lx, int rx){
        if(lx>=r || l>=rx)return null;
        if(lx>=l && rx<=r)return vals[x];
        propagate(x,lx,rx);
        int m=(lx+rx)/2;
        T v1=get(l,r,2*x+1,lx,m);
        T v2=get(l,r,2*x+2,m,rx);
        return oper(v1,v2);
    }
    T get(int l, int r){return get(l,r+1,0,0,size);}
    void upd(int l, int r, T v){upd(l,r+1,v,0,0,size)
        ;}
    void set(int i, T val){set(i,val,0,0,size);}
};
```

3.13 Segment Tree 2D

```

const int N=1000+1;
ll st[2*N][2*N];
struct SegTree{
    int n,m,neutro=0;
    inline ll op(ll a, ll b){return a+b;}

    SegTree(int n, int m): n(n), m(m){
        for(int i=0;i<2*n;++i)for(int j=0;j<2*m;
            ++j)st[i][j]=neutro;
    }
    SegTree(vector<vi>& a): n(sz(a)), m(n ? sz(a[0])
        : 0){ build(a); }

    void build(vector<vi>& a){
        for(int i=0;i<n;++i)for(int j=0;j<m;++j)
            st[i+n][j+m]=a[i][j];
        for(int i=0;i<n;++i)for(int j=m-1;j>=1;--
            j)st[i+n][j]=op(st[i+n][j<<1], st[i+n
            ][j<<1|1]);
        for(int i=n-1;i>=1;--i)for(int j=0;j<2*m
            ;++j)st[i][j]=op(st[i<<1][j], st[i
            ][j<<1|1]);
    }

    void upd(int x, int y, ll v){
        st[x+n][y+m]=v;
        for(int j=y+m;j>1;j>>=1)st[x+n][j>>1]=op(
            st[x+n][j], st[x+n][j^1]);
        for(int i=x+n;i>1;i>>=1)for(int j=y+m;j;
            >>=1)st[i>>1][j]=op(st[i][j], st[i^1][
            j]);
    }

    ll get(int x0, int y0, int x1, int y1){
        ll r=neutro;
        for(int i0=x0+n,i1=x1+n+1;i0<i1;i0>>=1,i1
            >>=1){
            int t[4],q=0;
            if(i0&1)t[q++]=i0++;
            if(i1&1)t[q++]--i1;
            for(int k=0;k<q;++k)for(int j0=y0
                +m,j1=y1+m+1;j0<j1;j0>>=1,j1
                >>=1){
                if(j0&1)r=op(r,st[t[k]][
                    j0++]);
                if(j1&1)r=op(r,st[t[k]
                    ][--j1]);
            }
        }
        return r;
    }
};

```

3.14 Segment Tree Beats

```

typedef long long T;
T null=0,noVal=0;
T INF=1e18;
struct Node{
    T sum,lazy;
    T max1,max2,maxc;
    T min1,min2,minc;
};
struct SegTree{
    vector<Node> vals;int size;
    void oper(int a, int b, int c); // node c, left a
    , right b;
    Node single(T x){
        Node tmp;
        tmp.sum=tmp.max1=tmp.min1=x;
        tmp.maxc=tmp.minc=1;
        tmp.lazy=noVal;
        tmp.max2=-INF;
        tmp.min2=INF;
        return tmp;
    }

    void build(vector<T>& a,int n);
    void propagateMin(T v, int x, int lx, int rx){
        if(vals[x].max1<=v)return;
        vals[x].sum-=vals[x].max1*vals[x].maxc;
        vals[x].max1=v;
        vals[x].sum+=vals[x].max1*vals[x].maxc;
        if(rx-lx==1){
            vals[x].min1=v;
        }else{
            if(v<=vals[x].min1){
                vals[x].min1=v;
            }else if(v<vals[x].min2){
                vals[x].min2=v;
            }
        }
    }

    void propagateAdd(T v, int x, int lx, int rx){
        vals[x].sum+=v*((T)(rx-lx));
        vals[x].lazy+=v;
        vals[x].max1+=v;
        vals[x].min1+=v;
        if(vals[x].max2!=-INF)vals[x].max2+=v;
        if(vals[x].min2!=-INF)vals[x].min2+=v;
    }

    void propagate(int x, int lx, int rx){
        if(rx-lx==1)return;
        int m=(lx+rx)/2;
        if(vals[x].lazy!=noVal){

```

```

        propagateAdd(vals[x].lazy, 2*x+1,
                     lx, m);
        propagateAdd(vals[x].lazy, 2*x+2,
                     m, rx);
        vals[x].lazy=noVal;
    }
    propagateMin(vals[x].max1, 2*x+1, lx, m);
    propagateMin(vals[x].max1, 2*x+2, m, rx);
}

void updAdd(int l, int r, T v, int x, int lx, int rx) {
    if(lx>=r || l>=rx) return;
    if(lx>=l && rx<=r) {
        propagateAdd(v, x, lx, rx);
        return;
    }
    propagate(x, lx, rx);
    int m=(lx+rx)/2;
    updAdd(l, r, v, 2*x+1, lx, m);
    updAdd(l, r, v, 2*x+2, m, rx);
    oper(2*x+1, 2*x+2, x);
}

void updMin(int l, int r, T v, int x, int lx, int rx) {
    if(lx>=r || l>=rx || vals[x].max1<v)
        return;
    if(lx>=l && rx<=r && vals[x].max2<v) {
        propagateMin(v, x, lx, rx);
        return;
    }
    propagate(x, lx, rx);
    int m=(lx+rx)/2;
    updMin(l, r, v, 2*x+1, lx, m);
    updMin(l, r, v, 2*x+2, m, rx);
    oper(2*x+1, 2*x+2, x);
}

void updAdd(int l, int r, T v) {updAdd(l, r+1, v,
    0, 0, size);}
void updMin(int l, int r, T v) {updMin(l, r+1, v,
    0, 0, size);}

};

```

3.15 Sparse Table 2D

```

const int MAX_N = 100;
const int MAX_M = 100;
const int KN = log2(MAX_N)+1;
const int KM = log2(MAX_M)+1;
int table[KN][MAX_N][KM][MAX_M];
int _log2N[MAX_N+1];
int _log2M[MAX_M+1];

```

```

int MAT[MAX_N][MAX_M];
int n, m, ic, ir, jc, jr;

void calc_log2() {
    _log2N[1] = 0;
    _log2M[1] = 0;
    for (int i = 2; i <= MAX_N; i++) _log2N[i] = _log2N[i/2] + 1;
    for (int i = 2; i <= MAX_M; i++) _log2M[i] = _log2M[i/2] + 1;
}

void build() {
    for (ir = 0; ir < n; ir++) {
        for (ic = 0; ic < m; ic++)
            table[0][ir][0][ic] = MAT[ir][ic];
        for (jc = 1; jc < KM; jc++)
            for (ic = 0; ic + (1 << (jc-1)) < m; ic++)
                table[0][ir][jc][ic] = min(table[0][ir][jc-1][ic + (1 << (jc-1))]);
    }

    for (jr = 1; jr < KN; jr++)
        for (ir = 0; ir < n; ir++)
            for (jc = 0; jc < KM; jc++)
                for (ic = 0; ic < m; ic++)
                    table[jr][ir][jc][ic] = min(table[jr-1][ir + (1 << (jr-1))][jc][ic]);
}

int rmq(int x1, int y1, int x2, int y2) {
    int lenx = x2-x1+1;
    int kx = _log2N[lenx];
    int leny = y2-y1+1;
    int ky = _log2M[leny];

    int min_R1 = min(table[kx][x1][ky][y1], table[kx][x1 + (1 << ky)][ky][y2 + 1 - (1 << ky)]);
    int min_R2 = min(table[kx][x2+1-(1 << kx)][ky][y1], table[kx][x2+1-(1 << kx)][ky][y2 + 1 - (1 << ky)]);
    return min(min_R1, min_R2);
}

```

3.16 Sqrt Decomposition

```

typedef long long T;
struct Sqrt { // O(n/b+b)
    int b; // check b
    vector<T> nums, blocks;
    void build(vector<T>& arr, int n) {
        b=(int)ceil(sqrt(n)); nums=arr;
        blocks.assign(b, 0);
        for (int i=0; i<n; ++i) {

```

```

        blocks[i/b]+=nums[i];
    }
}

void set(int x, int v){
    blocks[x/b]-=nums[x];
    nums[x]=v;
    blocks[x/b]+=nums[x];
}

T get(int r){
    T res=0;
    for(int i=0;i<r/b;++i){res+=blocks[i];}
    for(int i=(r/b)*b;i<r;++i){res+=nums[i];}
    return res;
}

T get(int l, int r){return get(r+1)-get(l);}
};

```

3.17 Treap

```

// treap => order asc, implicit treap => order array
typedef long long T;
struct Treap{
    Treap *l,*r,*dad;
    u64 prior;
    T sz,value,sum,lz;
    Treap(T v){
        l=r=nullptr;
        lz=0;sz=1;
        prior=rng();
        value=sum=v;
    }
    ~Treap(){delete l;delete r;}
};

typedef Treap* PTreap;
T cnt(PTreap x){return (!x?0:x->sz);}
T sum(PTreap x){return (!x?0:x->sum);}

void propagate(PTreap x){
    if(x && x->lz){
        if(x->l); // lz, value, sum ...
        if(x->r); // lz, value, sum ...
        x->lz=0;
    }
}

void update(PTreap x){
    propagate(x->l);
    propagate(x->r);
    x->sz=cnt(x->l)+cnt(x->r)+1;
    x->sum=sum(x->l)+sum(x->r)+x->value;
    if(x->l)x->l->dad=x;
}

```

```

        if(x->r)x->r->dad=x;
    }
}

void upd(PTreap x, T v){
    if(!x)return;
    update(x);
    // lz, value, sum ...
}

// pair<PTreap, PTreap> split(PTreap x, T key){ // f <=
//     key < s
pair<PTreap, PTreap> split(PTreap x, int left){ // cnt(f)
    == left
    if(!x)return {nullptr, nullptr};
    propagate(x);
    if(cnt(x->l)>=left){ // if(x->value>key){
        auto got=split(x->l, left); // , key);
        x->l=got.second;
        update(x);
        return {got.first, x};
    }else{
        auto got=split(x->r, left-cnt(x->l)-1);
        // , key);
        x->r=got.first;
        update(x);
        return {x, got.second};
    }
}

PTreap merge(PTreap x, PTreap y){
    if(!x)return y;
    if(!y)return x;
    propagate(x);
    propagate(y);
    if(x->prior<=y->prior){
        x->r=merge(x->r, y);
        update(x);
        return x;
    }else{
        y->l=merge(x, y->l);
        update(y);
        return y;
    }
}

PTreap combine(PTreap x, PTreap y){
    if(!x)return y;
    if(!y)return x;
    if(x->prior<y->prior)swap(x, y);
    auto z=split(y, x->value);
    x->r=combine(x->r, z.second);
    x->l=combine(z.first, x->l);
    return x;
}

T kth(PTreap& x, int k){ // indexed 0
    if(!x)return null;
}

```



```

    if(k==cnt(x->l)) return x->value;
    if(k<cnt(x->l)) return kth(x->l, k);
    return kth(x->r, k-cnt(x->l)-1);
}

pair<int, T> lower_bound(PTreap x, T key){ // index,
    value
    if(!x) return {0, null};
    if(x->value<key){
        auto y=lower_bound(x->r, key);
        y.first+=cnt(x->l)+1;
        return y;
    }
    auto y=lower_bound(x->l, key);
    if(y.first==cnt(x->l)) y.second=x->value;
    return y;
}

void dfs(PTreap x){
    if(!x) return;
    propagate(x);
    dfs(x->l); cout<<x->value<<" "; dfs(x->r);
}

// PTreap root=nullptr;
// PTreap act=new Treap(c);
// root=merge(root, act);

```

3.18 Two Stacks

```

typedef long long T;
struct Node{T val, acum;};
struct TwoStacks{
    stack<Node> s1,s2;

    void add(T x){
        Node tmp={x,x};
        if(!s2.empty()){
            // tmp.acum + s2.top().acum
        }
        s2.push(tmp);
    }

    void remove(){
        if(s1.empty()){
            while(!s2.empty()){
                Node tmp=s2.top();
                if(s1.empty()){
                    // tmp.acum = tmp.val
                }else{
                    // tmp.acum + s1.top().acum
                }
                s1.push(tmp);
                s2.pop();
            }
        }
    }
}

```

```

    }
    s1.pop();
}

bool good(){
    if(s1.empty() && s2.empty()) return false;
    else if(!s1.empty() && s2.empty()){
        return true; // eval s1.top();
    }else if(s1.empty() && !s2.empty()){
        return true; // eval s2.top();
    }else{
        return true; // eval s1.top() +
        s2.top()
    }
}

};

```

3.19 Wavelet Tree

```

const int maxn = 1e5+5, maxv = 1e9, minv = -1e9;
struct WaveletTree{ // indexed 1 - O(nlogn)
    int lo, hi;
    WaveletTree *l, *r;
    int *b, bsz, csz;
    ll *c;

    WaveletTree() {
        hi=bsz=csz=0;
        l=r=NULL;
        lo=1;
    }

    void build(int *from, int *to, int x, int y){
        lo=x, hi=y;
        if(from>=to) return;
        int mid=lo+(hi-lo)/2;
        auto f=[mid](int x){return x<=mid;};
        b=(int*)malloc((to-from+2)*sizeof(int));
        bsz=0;
        b[bsz++]=0;
        c=(ll*)malloc((to-from+2)*sizeof(ll));
        csz=0;
        c[csz++]=0;
        for(auto it=from; it!=to; ++it){
            b[bsz]=(b[bsz-1]+f(*it));
            c[csz]=(c[csz-1]+(*it));
            bsz++; csz++;
        }
        if(hi==lo) return;
        auto pivot=stable_partition(from, to, f);
        l=new WaveletTree();
        l->build(from, pivot, lo, mid);
        r=new WaveletTree();
        r->build(pivot, to, mid+1, hi);
    }
}

```

```

//kth smallest element in [l, r]
int kth(int l, int r, int k){
    if(l>r) return 0;
    if(lo==hi) return lo;
    int inLeft=b[r]-b[l-1], lb=b[l-1], rb=b[r];
    if(k<=inLeft) return this->l->kth(lb+1, rb, k);
    return this->r->kth(l-lb, r-rb, k-inLeft);
}

//count of numbers in [l, r] Less than or equal to k
int lte(int l, int r, int k){
    if(l>r || k<lo) return 0;
    if(hi<=k) return r-l+1;
    int lb=b[l-1], rb=b[r];
    return this->l->lte(lb+1, rb, k)+this->r->lte(l-lb, r-rb, k);
}

//count of numbers in [l, r] equal to k
int count(int l, int r, int k){
    if(l>r || k<lo || k>hi) return 0;
    if(lo==hi) return r-l+1;
    int lb=b[l-1], rb=b[r];
    int mid=(lo+hi)>>1;
    if(k<=mid) return this->l->count(lb+1, rb, k);
    return this->r->count(l-lb, r-rb, k);
}

//sum of numbers in [l, r] less than or equal to k
ll sum(int l, int r, int k){
    if(l>r || k<lo) return 0;
    if(hi<=k) return c[r]-c[l-1];
    int lb=b[l-1], rb=b[r];
    return this->l->sum(lb+1, rb, k)+this->r->sum(l-lb, r-rb, k);
}

~WaveletTree(){
    delete l;
    delete r;
}

};

// int a[maxn];
// WaveletTree wt;
// for(int i=1;i<=n;++i) cin>>a[i];
// wt.build(a+1, a+n+1, minv, maxv);

```

3.20 Trie Bit

```

struct node{
    int childs[2]{-1, -1};
};

struct TrieBit{
    vector<node> nds;
    vi passNums;

    TrieBit(){
        nds.pb(node());
        passNums.pb(0);
    }

    void insert(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;

            if(nds[cur].childs[bit] == -1){
                nds[cur].childs[bit] = nds.size();
                nds.pb(node());
                passNums.pb(0);
            }

            passNums[cur]++;
            cur = nds[cur].childs[bit];
        }

        passNums[cur]++;
    }

    void remove(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;
            passNums[cur]--;
            cur = nds[cur].childs[bit];
        }

        passNums[cur]--;
    }

    int maxXor(int num){
        int ans = 0;
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;

            int n1 = nds[cur].childs[!bit];
            if (n1 != -1 && passNums[n1]){
                ans += (1 << i);
                bit = !bit;
            }

            cur = nds[cur].childs[bit];
        }
    }
}

```

```

        return ans;
    }
};

```

4 Flujos

4.1 Blossom

```

/// Complexity:  $O(|E||V|^2)$ 
/// Tested: https://tinyurl.com/oe5rnpk
struct network {
    struct struct_edge { int v; struct_edge * n; };
    typedef struct_edge* edge;
    int n;
    struct_edge pool[MAXE]; ///2*n*n;
    edge top;
    vector<edge> adj;
    queue<int> q;
    vector<int> f, base, inq, inb, inp, match;
    vector<vector<int>> ed;
    network(int n) : n(n), match(n, -1), adj(n), top(pool),
        f(n), base(n),
            inq(n), inb(n), inp(n), ed(n, vector<
                int>(n)) {}
    void add_edge(int u, int v) {
        if(ed[u][v]) return;
        ed[u][v] = 1;
        top->v = v, top->n = adj[u], adj[u] = top++;
        top->v = u, top->n = adj[v], adj[v] = top++;
    }
    int get_lca(int root, int u, int v) {
        fill(inp.begin(), inp.end(), 0);
        while(1) {
            inp[u = base[u]] = 1;
            if(u == root) break;
            u = f[ match[u] ];
        }
        while(1) {
            if(inp[v = base[v]]) return v;
            else v = f[ match[v] ];
        }
    }
    void mark(int lca, int u) {
        while(base[u] != lca) {
            int v = match[u];
            inb[ base[u] ] = 1;
            inb[ base[v] ] = 1;
            u = f[v];
            if(base[u] != lca) f[u] = v;
        }
    }
    void blossom_contraction(int s, int u, int v) {

```

```

        int lca = get_lca(s, u, v);
        fill(inb.begin(), inb.end(), 0);
        mark(lca, u); mark(lca, v);
        if(base[u] != lca) f[u] = v;
        if(base[v] != lca) f[v] = u;
        for(int u = 0; u < n; u++)
            if(inb[base[u]]) {
                base[u] = lca;
                if(!inq[u]) {
                    inq[u] = 1;
                    q.push(u);
                }
            }
    }
    int bfs(int s) {
        fill(inq.begin(), inq.end(), 0);
        fill(f.begin(), f.end(), -1);
        for(int i = 0; i < n; i++) base[i] = i;
        q = queue<int>();
        q.push(s);
        inq[s] = 1;
        while(q.size()) {
            int u = q.front(); q.pop();
            for(edge e = adj[u]; e; e = e->n) {
                int v = e->v;
                if(base[u] != base[v] && match[u] != v) {
                    if((v == s) || (match[v] != -1 && f[match[v]]
                        != -1))
                        blossom_contraction(s, u, v);
                    else if(f[v] == -1) {
                        f[v] = u;
                        if(match[v] == -1) return v;
                        else if(!inq[match[v]]) {
                            inq[match[v]] = 1;
                            q.push(match[v]);
                        }
                    }
                }
            }
        }
        return -1;
    }
    int doit(int u) {
        if(u == -1) return 0;
        int v = f[u];
        doit(match[v]);
        match[v] = u; match[u] = v;
        return u != -1;
    }
    /// (i < net.match[i]) => means match
    int maximum_matching() {
        int ans = 0;
        for(int u = 0; u < n; u++)
            ans += (match[u] == -1) && doit(bfs(u));
        return ans;
    }

```

```
    }
};
```

4.2 Dinic

```
// O(|E|*|V|^2)
struct edge { ll v, cap, inv, flow, ori; };
struct network {
    ll n, s, t;
    vector<ll> lvl;
    vector<vector<edge>> g;
    network(ll n) : n(n), lvl(n), g(n) {}
    void add_edge(int u, int v, ll c) {
        g[u].push_back({v, c, sz(g[v]), 0, 1});
        g[v].push_back({u, 0, sz(g[u])-1, c, 0});
    }
    bool bfs() {
        fill(lvl.begin(), lvl.end(), -1);
        queue<ll> q;
        lvl[s] = 0;
        for(q.push(s); q.size(); q.pop()) {
            ll u = q.front();
            for(auto &e : g[u]) {
                if(e.cap > 0 && lvl[e.v] == -1) {
                    lvl[e.v] = lvl[u]+1;
                    q.push(e.v);
                }
            }
        }
        return lvl[t] != -1;
    }
    ll dfs(ll u, ll nf) {
        if(u == t) return nf;
        ll res = 0;
        for(auto &e : g[u]) {
            if(e.cap > 0 && lvl[e.v] == lvl[u]+1) {
                ll tf = dfs(e.v, min(nf, e.cap));
                res += tf; nf -= tf; e.cap -= tf;
                g[e.v][e.inv].cap += tf;
                g[e.v][e.inv].flow -= tf;
                e.flow += tf;
                if(nf == 0) return res;
            }
        }
        if(!res) lvl[u] = -1;
        return res;
    }
    ll max_flow(ll so, ll si, ll res = 0) {
        s = so; t = si;
        while(bfs()) res += dfs(s, LONG_LONG_MAX);
        return res;
    }
    void min_cut() {
        queue<ll> q;
```

```
vector<bool> vis(n, 0);
vis[s] = 1;
for(q.push(s); q.size(); q.pop()) {
    ll u = q.front();
    for(auto &e : g[u]) {
        if(e.cap > 0 && !vis[e.v]) {
            q.push(e.v);
            vis[e.v] = 1;
        }
    }
}
vii ans;
for (int i = 0; i<n; i++){
    for (auto &e : g[i]){
        if (vis[i] && !vis[e.v] && e.ori){
            ans.push_back({i+1, e.v+1});
        }
    }
    for (auto [x, y] : ans) cout << x << ' ' << y << ln;
}
bool dfs2(vi &path, vector<bool> &vis, int u){
    vis[u] = 1;
    for (auto &e : g[u]){
        if (e.flow > 0 && e.ori && !vis[e.v]){
            if (e.v == t || dfs2(path, vis, e.v)){
                path.push_back(e.v);
                e.flow = 0;
                return 1;
            }
        }
    }
    return 0;
}
void disjoint_paths(){
    vi path;
    vector<bool> vis(n, 0);
    while (dfs2(path, vis, s)){
        path.push_back(s);
        reverse(all(path));
        cout << sz(path) << ln;
        for (int v : path) cout << v+1 << ' ';
        cout << ln;
        path.clear(); vis.assign(n, 0);
    }
};
```

4.3 Edmonds Karp

```
//O(V * E^2)
ll bfs(vector<vi> &adj, vector<vl> &capacity, int s, int
t, vi& parent) {
```

```

fill(parent.begin(), parent.end(), -1);
parent[s] = -2;
queue<pll> q;
q.push({s, INFL});
while (!q.empty()) {
    int cur = q.front().first;
    ll flow = q.front().second;
    q.pop();
    for (int next : adj[cur]) {
        if (parent[next] == -1LL && capacity[cur][
            next]) {
            parent[next] = cur;
            ll new_flow = min(flow, capacity[cur][
                next]);
            if (next == t)
                return new_flow;
            q.push({next, new_flow});
        }
    }
    return 0;
}

ll maxflow(vector<vi> &adj, vector<vl> &capacity, int s,
int t, int n) {
    ll flow = 0;
    vi parent(n);
    ll new_flow;
    while ((new_flow = bfs(adj, capacity, s, t, parent)))
    {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return flow;
}

```

4.4 Hopcroft Karp

```

// Complexity:  $O(|E| \cdot \sqrt{|V|})$ 
struct mbm {
    vector<vector<int>> g;
    vector<int> d, match;
    int nil, l, r;
    /// u -> 0 to l, v -> 0 to r

```

```

mbm(int l, int r) : g(l+r), d(l+l+r, INF), match(l+r, l
    +r),
                    nil(l+r), l(l), r(r) {}
void add_edge(int a, int b) {
    g[a].push_back(l+b);
    g[l+b].push_back(a);
}
bool bfs() {
    queue<int> q;
    for(int u = 0; u < l; u++) {
        if(match[u] == nil) {
            d[u] = 0;
            q.push(u);
        } else d[u] = INF;
    }
    d[nil] = INF;
    while(q.size()) {
        int u = q.front(); q.pop();
        if(u == nil) continue;
        for(auto v : g[u]) {
            if(d[ match[v] ] == INF) {
                d[ match[v] ] = d[u]+1;
                q.push(match[v]);
            }
        }
    }
    return d[nil] != INF;
}
bool dfs(int u) {
    if(u == nil) return true;
    for(int v : g[u]) {
        if(d[ match[v] ] == d[u]+1 && dfs(match[v])) {
            match[v] = u; match[u] = v;
            return true;
        }
    }
    d[u] = INF;
    return false;
}
int max_matching() {
    int ans = 0;
    while(bfs()) {
        for(int u = 0; u < l; u++) {
            ans += (match[u] == nil && dfs(u));
        }
    }
    return ans;
}
void matchs() {
    for (int i = 0; i < l; i++) {
        if (match[i] == l+r) continue;
        cout << i+1 << ' ' << match[i]+1-l << ln;
    }
}
};

```

4.5 Maximum Bipartite Matching

```
// O(|E|*|V|)
struct mbm {
    int l, r;
    vector<vector<int>> g;
    vector<int> match, seen;
    mbm(int l, int r) : l(l), r(r), g(l), match(r), seen(r) {}
    void add_edge(int l, int r) { g[l].push_back(r); }
    bool dfs(int u) {
        for(auto v : g[u]) {
            if(seen[v]++) continue;
            if(match[v] == -1 || dfs(match[v])) {
                match[v] = u;
                return true;
            }
        }
        return false;
    }
    int max_matching() {
        int ans = 0;
        fill(match.begin(), match.end(), -1);
        for(int u = 0; u < l; ++u) {
            fill(seen.begin(), seen.end(), 0);
            ans += dfs(u);
        }
        return ans;
    }
    void matches() {
        for (int i = 0; i < r; i++) {
            if (match[i] == -1) continue;
            cout << match[i]+1 << ' ' << i+1 << ln;
        }
    }
};
```

4.6 Minimum Cost Maximum Flow

```
/// Complexity: O(|V|*|E|^2*log(|E|))
template <class type>
struct mcmf {
    struct edge { int u, v, cap, flow; type cost; };
    int n;
    vector<edge> ed;
    vector<vector<int>> g;
    vector<int> p;
    vector<type> d, phi;
    mcmf(int n) : n(n), g(n), p(n), d(n), phi(n) {}
    void add_edge(int u, int v, int cap, type cost) {
        g[u].push_back(ed.size());
```

```
        ed.push_back({u, v, cap, 0, cost});
        g[v].push_back(ed.size());
        ed.push_back({v, u, 0, 0, -cost});
    }
    bool dijkstra(int s, int t) {
        fill(d.begin(), d.end(), INF_TYPE);
        fill(p.begin(), p.end(), -1);
        set<pair<type, int>> q;
        d[s] = 0;
        for(q.insert({d[s], s}); q.size(); ) {
            int u = (*q.begin()).second; q.erase(q.begin());
            for(auto v : g[u]) {
                auto &e = ed[v];
                type nd = d[e.u]+e.cost+phi[e.u]-phi[e.v];
                if(0 < (e.cap-e.flow) && nd < d[e.v]) {
                    q.erase({d[e.v], e.v});
                    d[e.v] = nd; p[e.v] = v;
                    q.insert({d[e.v], e.v});
                }
            }
        }
        for(int i = 0; i < n; i++) phi[i] = min(INF_TYPE, phi[i]+d[i]);
        return d[t] != INF_TYPE;
    }
    pair<int, type> max_flow(int s, int t) {
        type mc = 0;
        int mf = 0;
        fill(phi.begin(), phi.end(), 0);
        while(dijkstra(s, t)) {
            int flow = INF;
            for(int v = p[t]; v != -1; v = p[ed[v].u])
                flow = min(flow, ed[v].cap-ed[v].flow);
            for(int v = p[t]; v != -1; v = p[ed[v].u]) {
                edge &e1 = ed[v];
                edge &e2 = ed[v^1];
                mc += e1.cost*flow;
                e1.flow += flow;
                e2.flow -= flow;
            }
            mf += flow;
        }
        return {mf, mc};
    }
};
```

4.7 Weighted Matching

```
/// Complexity: O(|V|^3)
typedef int type;
struct matching_weighted {
    int l, r;
    vector<vector<type>> c;
```

```

matching_weighted(int l, int r) : l(l), r(r), c(l,
    vector<type>(r)) {
    assert(l <= r);
}
void add_edge(int a, int b, type cost) { c[a][b] = cost; }
type matching() {
    vector<type> v(r), d(r); // v: potential
    vector<int> ml(l, -1), mr(r, -1); // matching pairs
    vector<int> idx(r), prev(r);
    iota(idx.begin(), idx.end(), 0);
    auto residue = [&](int i, int j) { return c[i][j] - v[j]; };
    for(int f = 0; f < l; ++f) {
        for(int j = 0; j < r; ++j) {
            d[j] = residue(f, j);
            prev[j] = f;
        }
    }
    type w;
    int j, l;
    for (int s = 0, t = 0;;) {
        if(s == t) {
            l = s;
            w = d[ idx[t++] ];
            for(int k = t; k < r; ++k) {
                j = idx[k];
                type h = d[j];
                if (h <= w) {
                    if (h < w) t = s, w = h;
                    idx[k] = idx[t];
                    idx[t++] = j;
                }
            }
            for (int k = s; k < t; ++k) {
                j = idx[k];
                if (mr[j] < 0) goto aug;
            }
        }
        int q = idx[s++], i = mr[q];
        for (int k = t; k < r; ++k) {
            j = idx[k];
            type h = residue(i, j) - residue(i, q) + w;
            if (h < d[j]) {
                d[j] = h;
                prev[j] = i;
                if(h == w) {
                    if(mr[j] < 0) goto aug;
                    idx[k] = idx[t];
                    idx[t++] = j;
                }
            }
        }
    }
    aug: for (int k = 0; k < l; ++k)
        v[ idx[k] ] += d[ idx[k] ] - w;
}

```

```

    int i;
    do {
        mr[j] = i = prev[j];
        swap(j, ml[i]);
    } while (i != f);
}
type opt = 0;
for (int i = 0; i < l; ++i)
    opt += c[i][ml[i]]; // (i, ml[i]) is a solution
return opt;
}
};

```

4.8 Hungarian

```

const int N = 509;
/* Complexity:  $O(n^3)$  but optimized
It finds minimum cost maximum matching.
For finding maximum cost maximum matching
add -cost and return -matching()
1-indexed */
struct Hungarian {
    long long c[N][N], fx[N], fy[N], d[N];
    int l[N], r[N], arg[N], trace[N];
    queue<int> q;
    int start, finish, n;
    const long long inf = 1e18;
    Hungarian() {}
    Hungarian(int n1, int n2) : n(max(n1, n2)) {
        for (int i = 1; i <= n; ++i) {
            fy[i] = l[i] = r[i] = 0;
            for (int j = 1; j <= n; ++j) c[i][j] = inf; // make
                it 0 for maximum cost matching (not necessarily
                with max count of matching)
        }
    }
    void add_edge(int u, int v, long long cost) {
        c[u][v] = min(c[u][v], cost);
    }
    inline long long getC(int u, int v) {
        return c[u][v] - fx[u] - fy[v];
    }
    void initBFS() {
        while (!q.empty()) q.pop();
        q.push(start);
        for (int i = 0; i <= n; ++i) trace[i] = 0;
        for (int v = 1; v <= n; ++v) {
            d[v] = getC(start, v);
            arg[v] = start;
        }
        finish = 0;
    }
    void findAugPath() {

```

```

while (!q.empty()) {
    int u = q.front();
    q.pop();
    for (int v = 1; v <= n; ++v) if (!trace[v]) {
        long long w = getC(u, v);
        if (!w) {
            trace[v] = u;
            if (!r[v]) {
                finish = v;
                return;
            }
            q.push(r[v]);
        }
        if (d[v] > w) {
            d[v] = w;
            arg[v] = u;
        }
    }
}

void subX_addY() {
    long long delta = inf;
    for (int v = 1; v <= n; ++v) if (trace[v] == 0 && d[v] < delta) {
        delta = d[v];
    }
    // Rotate
    fx[start] += delta;
    for (int v = 1; v <= n; ++v) if (trace[v]) {
        int u = r[v];
        fy[v] -= delta;
        fx[u] += delta;
    } else d[v] -= delta;
    for (int v = 1; v <= n; ++v) if (!trace[v] && !d[v]) {
        trace[v] = arg[v];
        if (!r[v]) {
            finish = v;
            return;
        }
        q.push(r[v]);
    }
}

void Enlarge() {
    do {
        int u = trace[finish];
        int nxt = l[u];
        l[u] = finish;
        r[finish] = u;
        finish = nxt;
    } while (finish);
}

long long maximum_matching() {
    for (int u = 1; u <= n; ++u) {
        fx[u] = c[u][1];

```

```

        for (int v = 1; v <= n; ++v) {
            fx[u] = min(fx[u], c[u][v]);
        }
    }
    for (int v = 1; v <= n; ++v) {
        fy[v] = c[1][v] - fx[1];
        for (int u = 1; u <= n; ++u) {
            fy[v] = min(fy[v], c[u][v] - fx[u]);
        }
    }
    for (int u = 1; u <= n; ++u) {
        start = u;
        initBFS();
        while (!finish) {
            findAugPath();
            if (!finish) subX_addY();
        }
        Enlarge();
    }
    long long ans = 0;
    for (int i = 1; i <= n; ++i) {
        if (c[i][l[i]] != inf) ans += c[i][l[i]];
        else l[i] = 0;
    }
    return ans;
}
};

```

5 Geometria

5.1 Puntos

```

typedef long double lf;
const lf EPS = 1e-9;
const lf E0 = 0.0L; //Keep = 0 for integer coordinates,
                    otherwise = EPS
const lf PI = acos(-1);

struct pt {
    lf x, y;
    pt() {}
    pt(lf a, lf b): x(a), y(b) {}
    pt(lf ang): x(cos(ang)), y(sin(ang)) {} // Polar unit
    point: ang(RAD)
    pt operator - (const pt &q) const { return {x - q.x,
        y - q.y}; }
    pt operator + (const pt &q) const { return {x + q.x,
        y + q.y}; }
    pt operator * (pt p) { return {x * p.x - y * p.y, x *
        p.y + y * p.x}; }
    pt operator * (const lf &t) const { return {x * t, y
        * t}; }
    pt operator / (const lf &t) const { return {x / t, y
        / t}; }

```



```

    bool operator == (pt p){ return abs(x - p.x) <= EPS
        && abs(y - p.y) <= EPS; }
    bool operator != (pt p){ return !operator==(p); }
    bool operator < (const pt & q) const { // set / sort
        if(fabs1(x - q.x) > E0) return x < q.x;
        return y < q.y;
    }
    void print(){ cout << x << " " << y << "\n"; }
};

pt normalize(pt p){
    lf norm = hypot1(p.x, p.y);
    if(fabs1(norm) > EPS) return {p.x /= norm, p.y /=
        norm};
    else return p;
}

int cmp(lf a, lf b){ return (a + EPS < b ? -1 : (b + EPS <
    a ? 1 : 0)); } // float comparator

// rota ccw
pt rot90(pt p){ return {-p.y, p.x}; }
// w(RAD)
pt rot(pt p, lf w){ return {cos1(w) * p.x - sin1(w) * p.y
    , sin1(w) * p.x + cos1(w) * p.y}; }

lf norm2(pt p){ return p.x * p.x + p.y * p.y; }
lf norm(pt p){ return hypot1(p.x, p.y); }

lf dis2(pt p, pt q){ return norm2(p - q); }
lf dis(pt p, pt q){ return norm(p - q); }

lf arg(pt a){ return atan2(a.y, a.x); } // ang(RAD) a x-
pos
lf dot(pt a, pt b){ return a.x * b.x + a.y * b.y; } // x
= 90 -> cos = 0
lf cross(pt a, pt b){ return a.x * b.y - a.y * b.x; } //
x = 180 -> sin = 0
lf orient(pt a, pt b, pt c){ return cross(b - a, c - a);
    } // AB clockwise = -
int sign(lf x){ return (EPS < x) - (x < -EPS); }

// p inside angle abc (center in a)
bool in_angle(pt a, pt b, pt c, pt p) {
    //assert(fabs1(orient(a, b, c)) > E0);
    if(orient(a, b, c) < -E0)
        return orient(a, b, p) >= -E0 || orient(a, c, p)
            <= E0;
    return orient(a, b, p) >= -E0 && orient(a, c, p) <=
        E0;
}

lf min_angle(pt a, pt b){ return acos(max((lf)-1.0, min((
    lf)1.0, dot(a, b)/norm(a)/norm(b)))); } // ang(RAD)
lf angle(pt a, pt b){ return atan2(cross(a, b), dot(a, b)
    ); } // ang(RAD)
lf angle(pt a, pt b, pt c){ // ang(RAD) AB AC ccw
    lf ang = angle(b - a, c - a);

```

```

    if (ang < 0) ang += 2 * PI;
    return ang;
}

bool half(pt p){ // true if is in (0, 180] (line is x
    axis)
    // assert(p.x != 0 || p.y != 0); // the argument of
    (0, 0) is undefined
    return p.y > 0 || (p.y == 0 && p.x < 0);
}

bool half_from(pt p, pt v = {1, 0}) {
    return cross(v, p) < 0 || (cross(v, p) == 0 && dot(v, p) <
        0);
}

// polar sort
bool polar_cmp(const pt &a, const pt &b){
    return make_tuple(half(a), 0) < make_tuple(half(b),
        cross(a, b));
}

void polar_sort(vector<pt> &v, pt o){ // sort points in
    counterclockwise with respect to point o
    sort(v.begin(), v.end(), [&](pt a, pt b) {
        return make_tuple(half(a - o), 0.0, norm2((a - o)
            )) < make_tuple(half(b - o), cross(a - o, b -
            o), norm2((b - o)));
    });
}

int cuad(pt p){ // REVISAR
    if(p.x > 0 && p.y >= 0) return 0;
    if(p.x <= 0 && p.y > 0) return 1;
    if(p.x < 0 && p.y <= 0) return 2;
    if(p.x >= 0 && p.y < 0) return 3;
    return -1; // x == 0 && y == 0
}

bool cmp(pt p1, pt p2){
    int c1 = cuad(p1), c2 = cuad(p2);
    return c1 == c2 ? p1.y * p2.x < p1.x * p2.y : c1 < c2;
}

```

5.2 Lineas

```

// add points operators
struct line {
    pt v; lf c; // v: dir, c: mov y
    line(pt v, lf c) : v(v), c(c) {}
    line(lf a, lf b, lf c) : v({b, -a}), c(c) {} // ax +
        by = c
    line(pt p, pt q) : v(q - p), c(cross(v, p)) {}
    bool operator < (line l){ return cross(v, l.v) > 0; }
}

```

```

bool operator == (line l){ return (abs(cross(v, l.v))
    <= E0) && c == l.c; } // abs(c) == abs(l.c)

lf side(pt p){ return cross(v, p) - c; }
lf dist(pt p){ return abs(side(p)) / norm(v); }
lf dist2(pt p){ return side(p) * side(p) / (lf)norm2(
    v); }
line perp_through(pt p){ return {p, p + rot90(v)}; }
// line perp to v passing through p
bool cmp_proj(pt p, pt q){ return dot(v, p) < dot(v,
    q); } // order for points over the line
// use: auto fsort = [&l1](const pt &a, const pt &b){
    return l1.cmp_proj(a, b); };
line translate(pt t){ return {v, c + cross(v, t)}; }
line shift_left(lf d){ return {v, c + d*norm(v)}; }
pt proj(pt p){ return p - rot90(v) * side(p) / norm2(
    v); } // pt projected on the line
pt refl(pt p){ return p - rot90(v) * 2 * side(p) /
    norm2(v); } // pt reflected on the other side of
    the line
bool has(pt p){ return abs(cross(v, p) - c) <= E0; };
// pt on line

lf evalx(lf x){
    assert(fabs1(v.x) > EPS);
    return (c + v.y * x) / v.x;
}

};

pt inter_ll(line l1, line l2) {
    if (abs(cross(l1.v, l2.v)) <= EPS) return {INF, INF};
    // parallel
    return {l2.v * l1.c - l1.v * l2.c) / cross(l1.v, l2.v
    ); } // floating points

// bisector divides the angle in 2 equal angles
// interior line goes on the same direction as l1 and l2
line bisector(line l1, line l2, bool interior) {
    // assert(cross(l1.v, l2.v) != 0); // l1 and l2
    // cannot be parallel
    lf sign = interior ? 1 : -1;
    return {l2.v / norm(l2.v) + l1.v / norm(l1.v) * sign,
        l2.c / norm(l2.v) + l1.c / norm(l1.v) * sign
        };
}

```

5.3 Polígonos

```

// add Points Lines Segments Circles
// points in polygon(vector<pt>) ccw or cw
enum {OUT, IN, ON};

lf area(vector<pt> &p){

```

```

    lf r = 0.;
    for(int i = 0, n = p.size(); i < n; ++i){
        r += cross(p[i], p[(i + 1) % n]);
    }
    return r / 2; // negative if CW, positive if CCW
}

lf perimeter(vector<pt> &p) {
    lf per = 0;
    for (int i = 0, n = p.size(); i < n; ++i){
        per += norm(p[i] - p[(i + 1) % n]);
    }
    return per;
}

bool is_convex(vector<pt> &p) {
    bool pos = 0, neg = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int o = orient(p[i], p[(i + 1) % n], p[(i + 2) %
            n]);
        if (o > 0) pos = 1;
        if (o < 0) neg = 1;
    }
    return !(pos && neg);
}

int point_in_polygon(vector<pt> &pol, pt &p){
    int wn = 0;
    for(int i = 0, n = pol.size(); i < n; ++i) {
        lf c = orient(p, pol[i], pol[(i + 1) % n]);
        if(fabs1(c) <= E0 && dot(pol[i] - p, pol[(i + 1)
            % n] - p) <= E0) return ON; // on segment

        if(c > 0 && pol[i].y <= p.y + E0 && pol[(i + 1) %
            n].y - p.y > E0) ++wn;
        if(c < 0 && pol[(i + 1) % n].y <= p.y + E0 && pol
            [i].y - p.y > E0) --wn;
    }
    return wn ? IN : OUT;
}

// O(logn) polygon CCW, remove collinear
int point_in_convex_polygon(const vector<pt> &pol, const
    pt &p){
    int low = 1, high = pol.size() - 1;
    while(high - low > 1){
        int mid = (low + high) / 2;
        if(orient(pol[0], pol[mid], p) >= -E0)
            low = mid;
        else high = mid;
    }
    if(orient(pol[0], pol[low], p) < -E0) return OUT;
    if(orient(pol[low], pol[high], p) < -E0) return
        OUT;
    if(orient(pol[high], pol[0], p) < -E0) return OUT
        ;
}

```

```

    if(low == 1 && orient(pol[0], pol[low], p) <= E0)
        return ON;
    if(orient(pol[low], pol[high], p) <= E0) return
    ON;
    if(high == (int) pol.size() - 1 && orient(pol[high]
    ], pol[0], p) <= E0) return ON;
    return IN;
}

// convex polygons in some order (CCW, CW)
vector<pt> minkowski(vector<pt> P, vector<pt> Q) {
    rotate(P.begin(), min_element(P.begin(), P.end())
    , P.end());
    rotate(Q.begin(), min_element(Q.begin(), Q.end())
    , Q.end());

    P.push_back(P[0]), P.push_back(P[1]);
    Q.push_back(Q[0]), Q.push_back(Q[1]);

    vector<pt> ans;
    size_t i = 0, j = 0;
    while(i < P.size() - 2 || j < Q.size() - 2){
        ans.push_back(P[i] + Q[j]);
        lf dt = cross(P[i + 1] - P[i], Q[j + 1] -
        Q[j]);
        if(dt >= E0 && i < P.size() - 2) ++i;
        if(dt <= E0 && j < Q.size() - 2) ++j;
    }
    return ans;
}

pt centroid(vector<pt>& p){
    pt c{0, 0};
    lf scale = 6. * area(p);
    for (int i = 0, n = p.size(); i < n; ++i){
        c = c + (p[i] + p[(i + 1) % n]) * cross(p[i], p[(
        i + 1) % n]);
    }
    return c / scale;
}

void normalize(vector<pt>& p) { // polygon CCW
    int bottom = min_element(p.begin(), p.end()) - p.
    begin();
    vector<pt> tmp(p.begin() + bottom, p.end());
    tmp.insert(tmp.end(), p.begin(), p.begin()+bottom);
    p.swap(tmp);
    bottom = 0;
}

void remove_col(vector<pt>& p){
    vector<pt> s;
    for(int i = 0, n = p.size(); i < n; i++){
        if(!on_segment(p[(i - 1 + n) % n], p[(i + 1) % n
        ], p[i])) s.push_back(p[i]);
    }
    p.swap(s);
}

```

```

}

void delete_repetead(vector<pt>& p){
    vector<pt> aux;
    sort(p.begin(), p.end());
    for (pt &pi : p){
        if (aux.empty() || aux.back() != pi) aux.
        push_back(pi);
    }
    p.swap(aux);
}

pt farthest(vector<pt>& p, pt v){ // O(log(n)) only
    CONVEX, v: dir
    int n = p.size();
    if(n < 10){
        int k = 0;
        for(int i = 1; i < n; i++) if(dot(v, (p[i] - p[k
        ])) > EPS) k = i;
        return p[k];
    }
    pt a = p[1] - p[0];
    int s = 0, e = n, ua = dot(v, a) > EPS;
    if(!ua && dot(v, (p[n - 1] - p[0])) <= EPS) return p
    [0];
    while(1){
        int m = (s + e) / 2;
        pt c = p[(m + 1) % n] - p[m];
        int uc = dot(v, c) > EPS;
        if(!uc && dot(v, (p[(m - 1 + n) % n] - p[m])) <=
        EPS) return p[m];
        if(ua && (!uc || dot(v, (p[s] - p[m])) > EPS)) e
        = m;
        else if(ua || uc || dot(v, (p[s] - p[m])) >= -EPS
        ) s = m, a = c, ua = uc;
        else e = m;
        assert(e > s + 1);
    }
}

vector<pt> cut(vector<pt>& p, line l){
    // cut CONVEX polygon by line l
    // returns part at left of l.pq
    vector<pt> q;
    for(int i = 0, n = p.size(); i < n; i++) {
        int d0 = sign(l.side(p[i]));
        int d1 = sign(l.side(p[(i + 1) % n]));
        if(d0 >= 0) q.push_back(p[i]);

        line m(p[i], p[(i + 1) % n]);
        if(d0 * d1 < 0 && !(abs(cross(l.v, m.v)) <= EPS))
        {
            q.push_back((inter_ll(l, m)));
        }
    }
    return q;
}

```

```

}
// O(n)
vector<pair<int, int>> antipodal(vector<pt>& p){
    vector<pair<int, int>> ans;
    int n = p.size();
    if (n == 2) ans.push_back({0, 1});
    if (n < 3) return ans;
    auto nxt = [&](int x){ return (x + 1 == n ? 0 : x + 1); };
    auto area2 = [&](pt a, pt b, pt c){ return cross(b - a, c - a); };
    int b0 = 0;
    while (abs(area2(p[n - 1], p[0], p[nxt(b0)])) > abs(area2(p[n - 1], p[0], p[b0]))) ++b0;
    for (int b = b0, a = 0; b != 0 && a <= b0; ++a) {
        ans.push_back({a, b});
        while (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) > abs(area2(p[a], p[nxt(a)], p[b]))) {
            b = nxt(b);
            if (a != b0 || b != 0) ans.push_back({a, b});
            else return ans;
        }
        if (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) == abs(area2(p[a], p[nxt(a)], p[b]))) {
            if (a != b0 || b != n - 1) ans.push_back({a, nxt(b)});
            else ans.push_back({nxt(a), b});
        }
    }
    return ans;
}

// O(n)
// square distance of most distant points, prereq: convex, ccw, NO COLLINEAR POINTS
lf callipers(vector<pt>& p){
    int n = p.size();
    lf r = 0;
    for(int i = 0, j = n < 2 ? 0 : 1; i < j; ++i){
        for(;; j = (j + 1) % n){
            r = max(r, norm2(p[i] - p[j]));
            if(cross((p[(i + 1) % n] - p[i]), (p[(j + 1) % n] - p[j])) <= EPS) break;
        }
    }
    return r;
}

// O(n + m) max_dist between 2 points (pa, pb) of 2 Convex polygons (a, b)
lf rotating_callipers(vector<pt>& a, vector<pt>& b){ // REVISAR
    if (a.size() > b.size()) swap(a, b); // <- del or add pair<ll, int> start = {-1, -1};

```

```

    if(a.size() == 1) swap(a, b);
    for(int i = 0; i < a.size(); i++) start = max(start, {norm2(b[0] - a[i]), i});
    if(b.size() == 1) return start.first;

    lf r = 0;
    for(int i = 0, j = start.second; i < b.size(); ++i){
        for(;; j = (j + 1) % a.size()){
            r = max(r, norm2(b[i] - a[j]));
            if(cross((b[(i + 1) % b.size()] - b[i]), (a[(j + 1) % a.size()] - a[j])) <= EPS) break;
        }
    }
    return r;
}

lf intercircle(vector<pt>& p, circle c){ // area of intersection with circle
    lf r=0.;
    for(int i = 0, n = p.size(); i < n; i++){
        int j = (i + 1) % n;
        lf w = intertriangle(c, p[i], p[j]);
        if(cross((p[j] - c.center), (p[i] - c.center)) > 0) r += w;
        else r -= w;
    }
    return abs(r);
}

ll pick(vector<pt>& p){
    ll boundary = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int j = (i + 1 == n ? 0 : i + 1);
        boundary += __gcd((ll)abs(p[i].x - p[j].x), (ll)abs(p[i].y - p[j].y));
    }
    return abs(area(p)) + 1 - boundary / 2;
}

```

5.4 Circulos

```

using namespace std;
#include <bits/stdc++.h>
#define all(v) v.begin(), v.end()
const char ln = '\n';

#include "Points.cpp"
#include "Lines.cpp"
// add Lines Points

enum {OUT, IN, ON};

struct circle {
    pt center; lf r;
    // (x - xo)^2 + (y - yo)^2 = r^2

```

```

circle(pt c, lf r): center(c), r(r){};
// circle that passes through abc
circle(pt a, pt b, pt c) {
    b = b - a, c = c - a;
    assert(cross(b, c) != 0); // no circumcircle if A
    , B, C aligned
    pt cen = a + rot90(b * norm2(c) - c * norm2(b)) /
        cross(b, c) / 2;
    center = cen;
    r = norm(a - cen);
}

// diameter = segment pq
circle(pt p, pt q) {
    center = (p + q) * 0.5L;
    r = dis(p, q) * 0.5L;
}

int contains(pt &p) {
    lf det = r * r - dis2(center, p);
    if(fabs1(det) <= EPS) return ON;
    return (det > EPS ? IN : OUT);
}

bool in(circle c){ return norm(center - c.center) + r
    <= c.r + EPS; } // non strict
};

// centers of the circles that pass through ab and has
// radius r
vector<pt> centers(pt a, pt b, lf r) {
    if (norm(a - b) > 2 * r + EPS) return {};
    pt m = (a + b) / 2;
    double f = sqrt(r * r / norm2(a - m) - 1);
    pt c = rot90(a - m) * f;
    return {m - c, m + c};
}

vector<pt> inter_cl(circle c, line l){
    vector<pt> s;
    pt p = l.proj(c.center);
    lf d = norm(p - c.center);
    if(d - EPS > c.r) return s;
    if(abs(d - c.r) <= EPS){ s.push_back(p); return s
        ; }
    d=sqrt(c.r * c.r - d * d);
    s.push_back(p + normalize(l.v) * d);
    s.push_back(p - normalize(l.v) * d);
    return s;
}

vector<pt> inter_cc(circle c1, circle c2) {
    pt dir = c2.center - c1.center;
    lf d2 = dis2(c1.center, c2.center);

    if(d2 <= E0) {

```

```

        //assert( fabs1( c1.r - c2.r ) > E0 );
        return {};
    }

    lf td = 0.5L * ( d2 + c1.r * c1.r - c2.r * c2.r );
    lf h2 = c1.r * c1.r - td / d2 * td;

    pt p = c1.center + dir * (td / d2);
    if(fabs1( h2 ) < EPS) return {p};
    if(h2 < 0.0L) return {};

    pt dir_h = rot90(dir) * sqrt1(h2 / d2);

    return {p + dir_h, p - dir_h};
}

// circle-line inter = 1, inner: 1 = oxo 0 = o=o
vector<pair<pt, pt>> tangents(circle c1, circle c2, bool
    inner){
    vector<pair<pt, pt>> out;
    if (inner) c2.r = -c2.r; // inner tangent
    pt d = c2.center - c1.center;
    double dr = c1.r - c2.r, d2 = norm2(d), h2 = d2 - dr
        * dr;
    if (d2 == 0 || h2 < 0) { assert(h2 != 0); return {};
    } // (identical)
    for (double s : {-1, 1}) {
        pt v = (d * dr + rot90(d) * sqrt(h2) * s) / d2;
        out.push_back({c1.center + v * c1.r, c2.center +
            v * c2.r});
    }
    return out; // if size 1: circle are tangent
}

// circle tangent passing through pt p
pair<pt, pt> tangent_through_pt(circle c, pt p){
    pair<pt, pt> out;
    double d = norm2(p - c.center);
    if (d < c.r) return {};
    pt base = c.center - p;
    double w = sqrt(norm2(base) - c.r * c.r);
    pt a = {w, c.r}, b = {w, -c.r};
    pt s = p + base * a / norm2(base) * w;
    pt t = p + base * b / norm2(base) * w;
    out = {s, t};
    return out;
}

lf safeAcos(lf x) {
    if (x < -1.0) x = -1.0;
    if (x > 1.0) x = 1.0;
    return acos(x);
}

lf areaOfIntersectionOfTwoCircles(circle c1, circle c2){
    lf r1 = c1.r, r2 = c2.r, d = dis(c1.center, c2.center
    );
    if(d >= r1 + r2) return 0.0L;

```

```

    if(d <= fabs1(r2 - r1)) return PI * (r1 < r2 ? r1 *
        r1 : r2 * r2);
    lf alpha = safeAcos((r1 * r1 - r2 * r2 + d * d) /
        (2.0L * d * r1));
    lf betha = safeAcos((r2 * r2 - r1 * r1 + d * d) /
        (2.0L * d * r2));
    lf a1 = r1 * r1 * (alpha - sin1(alpha) * cos1(alpha));
    lf a2 = r2 * r2 * (betha - sin1(betha) * cos1(betha));
    return a1 + a2;
};

lf intertriangle(circle& c, pt a, pt b){ // area of
    intersection with oab
    if(abs(cross((c.center - a), (c.center - b))) <= EPS)
        return 0.;
    vector<pt> q = {a}, w = inter_cl(c, line(a, b));
    if(w.size() == 2) for(auto p: w) if(dot((a - p), (b -
        p)) < -EPS) q.push_back(p);
    q.push_back(b);
    if(q.size() == 4 && dot((q[0] - q[1]), (q[2] - q[1]))
        > EPS) swap(q[1], q[2]);
    lf s = 0;
    for(int i = 0; i < q.size() - 1; ++i){
        if(!c.contains(q[i]) || !c.contains(q[i + 1])) s
            += c.r * c.r * min_angle((q[i] - c.center), q[
                i+1] - c.center) / 2;
        else s += abs(cross((q[i] - c.center), (q[i + 1]
            - c.center)) / 2);
    }
    return s;
}

bool circumcircle_contains(vector<pt> tr, pt D) { //
    triange CCW
    pt A = tr[0] - D, B = tr[1] - D, C = tr[2] - D;

    lf norm_a = norm2(tr[0]) - norm2(D);
    lf norm_b = norm2(tr[1]) - norm2(D);
    lf norm_c = norm2(tr[2]) - norm2(D);

    lf det1 = A.x * (B.y * norm_c - norm_b * C.y);
    lf det2 = B.x * (C.y * norm_a - norm_c * A.y);
    lf det3 = C.x * (A.y * norm_b - norm_a * B.y);

    return det1 + det2 + det3 > E0;
}

// r[k]: area covered by at least k circles
// O(n^2 log n) (high constant)
vector<lf> intercircles(vector<circle> c){
    vector<lf> r(c.size() + 1);
    for(int i = 0; i < c.size(); ++i){
        int k = 1; pt O = c[i].center;
        vector<pair<pt, int>> p = {
            {c[i].center + pt(1,0) * c[i].r,
                0},

```

```

            {c[i].center - pt(1,0) * c[i].r,
                0}};
        for(int j = 0; j < c.size(); ++j) if(j !=
            i){
            bool b0 = c[i].in(c[j]), b1 = c[j]
                .in(c[i]);
            if(b0 && (!b1 || i < j)) ++k;
            else if(!b0 && !b1){
                auto v = inter_cc(c[i], c
                    [j]);
                if(v.size() == 2){
                    swap(v[0], v[1]);
                    p.push_back({v
                        [0], 1});
                    p.push_back({v[1], -1});
                    if(polar_cmp(v[1]
                        - O, v[0] - O
                        )) ++k;
                }
            }
        }
    }
    sort(all(p), [&](auto& a, auto& b){
        return polar_cmp(a.first - O, b.first
            - O); });
    for(int j = 0; j < p.size(); ++j){
        pt p0 = p[j ? j - 1 : p.size()
            - 1].first, p1 = p[j].first;
        lf a = min_angle((p0 - c[i].
            center), (p1 - c[i].center));
        r[k] += (p0.x - p1.x) * (p0.y +
            p1.y) / 2 + c[i].r * c[i].r *
            (a - sin(a)) / 2;
        k += p[j].second;
    }
    return r;
}

```

5.5 Semiplanos

```

const lf INF = 1e100;
struct Halfplane {
    pt p, pq; // p: point on line, pq: dir, take left
    lf angle;
    Halfplane(){}
    Halfplane(pt& a, pt& b): p(a), pq(b - a){
        angle = atan21(pq.y, pq.x);
    }

    bool out(const pt& r){ return cross(pq, r - p) < -EPS;
    }; // checks if p is inside the half plane
    bool operator < (const Halfplane& e) const { return
        angle < e.angle; }
}

```

```

};
// intersection pt of the lines of 2 halfplanes
pt inter(const Halfplane& s, const Halfplane& t){
    if (abs(cross(s.pq, t.pq)) <= EPS) return {INF, INF};
    lf alpha = cross((t.p - s.p), t.pq) / cross(s.pq, t.pq);
    return s.p + (s.pq * alpha);
}
// O(nlogn) return CCW polygon
vector<pt> hp_intersect(vector<Halfplane>& H) {
    pt box[4] = {pt(INF, INF), pt(-INF, INF), pt(-INF, -INF), pt(INF, -INF)};

    for(int i = 0; i < 4; ++i) {
        Halfplane aux(box[i], box[(i + 1) % 4]);
        H.push_back(aux);
    }

    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); ++i){
        while (len > 1 && H[i].out(inter(dq[len - 1], dq[len - 2]))){
            dq.pop_back();
            --len;
        }
        while (len > 1 && H[i].out(inter(dq[0], dq[1]))){
            dq.pop_front();
            --len;
        }
        if (len > 0 && fabs1(cross(H[i].pq, dq[len - 1].pq)) < EPS){
            if (dot(H[i].pq, dq[len - 1].pq) < 0.0)
                return vector<pt>();

            if (H[i].out(dq[len - 1].p)){
                dq.pop_back();
                --len;
            } else continue;
        }
        dq.push_back(H[i]);
        ++len;
    }
    while (len > 2 && dq[0].out(inter(dq[len - 1], dq[len - 2]))){
        dq.pop_back();
        --len;
    }
    while (len > 2 && dq[len - 1].out(inter(dq[0], dq[1]))

```

```

    ){
        dq.pop_front();
        --len;
    }
    if (len < 3) return vector<pt>();
    vector<pt> ret(len);
    for(int i = 0; i + 1 < len; ++i) ret[i] = inter(dq[i], dq[i + 1]);
    ret.back() = inter(dq[len - 1], dq[0]);
    // remove repeated points if needed
    return ret;
}

// -----
// intersection of halfplanes
vector<pt> hp_intersect(vector<halfplane>& b){
    vector<pt> box = {{inf, inf}, {-inf, inf}, {-inf, -inf}, {inf, -inf}};
    for(int i = 0; i < 4; ++i){
        b.push_back({box[i], box[(i + 1) % 4]});
    }
    sort(b.begin(), b.end());
    int n = b.size(), q = 1, h = 0;
    vector<halfplane> c(n + 10);
    for(int i = 0; i < n; ++i){
        while (q < h && b[i].out(inter(c[h], c[h - 1]))) h--;
        while (q < h && b[i].out(inter(c[q], c[q + 1]))) q++;
        c[++h] = b[i];
        if (q < h && abs(cross(c[h].pq, c[h - 1].pq)) < EPS)
            if (dot(c[h].pq, c[h - 1].pq) <= 0) return {};
            h--;
        if (b[i].out(c[h].p)) c[h] = b[i];
    }
    while (q < h - 1 && c[q].out(inter(c[h], c[h - 1]))) h--;
    while (q < h - 1 && c[h].out(inter(c[q], c[q + 1]))) q++;
    if (h - q <= 1) return {};
    c[h + 1] = c[q];
    vector<pt> s;
    for(int i = q; i < h + 1; ++i) s.pb(inter(c[i], c[i + 1]));
    return s;
}

```

5.6 Segmentos

// add Lines Points


```

bool in_disk(pt a, pt b, pt p){ // pt p inside ab disk
    return dot(a - p, b - p) <= E0;
}

bool on_segment(pt a, pt b, pt p) { // p on ab
    return orient(a, b, p) == 0 && in_disk(a, b, p);
}

// ab crossing cd
bool proper_inter(pt a, pt b, pt c, pt d, pt& out) {
    if oa = orient(c, d, a);
    if ob = orient(c, d, b);
    if oc = orient(a, b, c);
    if od = orient(a, b, d);
    // Proper intersection exists iff opposite signs
    if (oa * ob < 0 && oc * od < 0) {
        out = (a * ob - b * oa) / (ob - oa);
        return true;
    }
    return false;
}

// intersection bwn segments
set<pt> inter_ss(pt a, pt b, pt c, pt d) {
    pt out;
    if (proper_inter(a, b, c, d, out)) return {out}; //
    if cross -> 1
    set<pt> s;
    if (on_segment(c, d, a)) s.insert(a); // a in cd
    if (on_segment(c, d, b)) s.insert(b); // b in cd
    if (on_segment(a, b, c)) s.insert(c); // c in ab
    if (on_segment(a, b, d)) s.insert(d); // d in ab
    return s; // 0, 2
}

if pt_to_seg(pt a, pt b, pt p) { // p to ab
    if (a != b) {
        line l(a, b);
        if (l.cmp_proj(a, p) && l.cmp_proj(p, b)) // if
            closest to projection = (a, p, b)
            return l.dist(p); // output distance to line
    }
    return min(norm(p - a), norm(p - b)); // otherwise
    distance to A or B
}

if seg_to_seg(pt a, pt b, pt c, pt d) {
    pt dummy;
    if (proper_inter(a, b, c, d, dummy)) return 0; // ab
    intersects cd
    return min({pt_to_seg(a, b, c), pt_to_seg(a, b, d),
        pt_to_seg(c, d, a), pt_to_seg(c, d, b)}); // try
    the 4 pts
}

int length_union(vector<pt>& a) { // REVISAR

```

```

int n = a.size();
vector<pair<int, bool>> x(n * 2);
for (int i = 0; i < n; i++) {
    x[i * 2] = {a[i].x, false};
    x[i * 2 + 1] = {a[i].y, true};
}
sort(x.begin(), x.end());
int result = 0;
int c = 0;
for (int i = 0; i < n * 2; i++) {
    if (i > 0 && x[i].first > x[i - 1].first && c >
        0) result += x[i].first - x[i - 1].first;
    if (x[i].second) c--;
    else c++;
}
return result;
}

```

5.7 Convex Hull

```

// CCW order
// if colinear are needed, use > in orient and remove
// repeated points
vector<pt> chull(vector<pt>& p) {
    if (p.size() < 3) return p;

    vector<pt> r; //r.reserve(p.size());
    sort(p.begin(), p.end()); // first x, then y

    for (int i = 0; i < p.size(); i++) { // lower hull
        while (r.size() >= 2 && orient(r[r.size() - 2], p[i], r.back()) >= 0) r.pop_back();
        r.pb(p[i]);
    }
    r.pop_back();

    int k = r.size();
    for (int i = p.size() - 1; i >= 0; --i) { // upper hull
        while (r.size() >= k + 2 && orient(r[r.size() - 2], p[i], r.back()) >= 0) r.pop_back();
        r.pb(p[i]);
    }
    r.pop_back();
    return r;
}

```

5.8 Closest Points

// $O(n \log n)$


```

pair<pt, pt> closest_points(vector<pt> v){
    sort(v.begin(), v.end());
    pair<pt, pt> ans;
    lf d2 = INF;

    function<void( int, int )> solve = [&](int l, int r)
    {
        if(l == r) return;

        int mid = (l + r) / 2;
        lf x_mid = v[mid].x;
        solve(l, mid);
        solve(mid + 1, r);

        vector<pt> aux;
        int p1 = l, p2 = mid + 1;
        while (p1 <= mid && p2 <= r) {
            if(v[p1].y < v[p2].y) aux.push_back(v[p1++]);
            else aux.push_back(v[p2++]);
        }
        while(p1 <= mid) aux.push_back(v[p1++]);
        while(p2 <= r) aux.push_back(v[p2++]);

        vector<pt> nb;
        for(int i = l; i <= r; ++i){
            v[i] = aux[i - 1];
            lf dx = (x_mid - v[i].x);
            if(dx * dx < d2)
                nb.push_back(v[i]);
        }

        for(int i = 0; i < (int) nb.size(); ++i){
            for(int k = i + 1; k < (int) nb.size(); ++k){
                lf dy = (nb[k].y - nb[i].y);
                if(dy * dy > d2) break;
                lf nd2 = dis2(nb[i], nb[k]);
                if(nd2 < d2) d2 = nd2, ans = {nb[i], nb[k]};
            }
        }
    };
    solve(0, v.size() - 1);
    return ans;
}

```

5.9 Min Circle

```

// minimo circulo que encierra todos los puntos
// Promedio: O(n), Peor: O(n^2)
Circle min_circle(vector<pt> v){
    random_shuffle(v.begin(), v.end()); // shuffle(all(
    vec), rng);
    auto f2 = [&](int a, int b){
        Circle ans(v[a], v[b]);
        for(int i = 0; i < a; ++ i)
            if(ans.contains(v[i]) == OUT) ans = Circle(v[i],

```

```

        v[a], v[b]);
        return ans;
    };

    auto f1 = [&](int a){
        Circle ans(v[a], 0.0L);
        for(int i = 0; i < a; ++i)
            if(ans.contains(v[i]) == OUT) ans = f2(i, a);
        return ans;
    };

    Circle ans(v[0], 0.0L);
    for(int i = 1; i < (int) v.size(); ++i)
        if(ans.contains(v[i]) == OUT) ans = f1(i);

    return ans;
}

```

5.10 3D

```

typedef double lf;
struct p3 {
    lf x, y, z;
    p3(lf x, lf y, lf z): x(x), y(y), z(z){}
    p3 operator + (p3 p){ return {x + p.x, y + p.y, z + p
        .z}; }
    p3 operator - (p3 p){ return {x - p.x, y - p.y, z - p
        .z}; }
    p3 operator * (lf d){ return {x * d, y * d, z * d}; }
    p3 operator / (lf d){ return {x / d, y / d, z / d}; }
    // only for floating point
    // Some comparators
    bool operator == (p3 p){ return tie(x, y, z) == tie(p
        .x, p.y, p.z); }
    bool operator != (p3 p){ return !operator == (p); }
    void print(){ cout << x << " " << y << " " << z
        << "\n"; }
    // scale: (newnorm / norm) * p3
};

lf dot(p3 v, p3 w){ return v.x * w.x + v.y * w.y + v.z *
    w.z; }

p3 cross(p3 v, p3 w){
    return { v.y * w.z - v.z * w.y, v.z * w.x - v.x * w.z
        , v.x * w.y - v.y * w.x };
}

lf norm2(p3 v){ return dot(v, v); }
lf norm(p3 v){ return sqrt(norm2(v)); }
p3 unit(p3 v){ return v / norm(v); }

// ang(RAD)
double angle(p3 v, p3 w){
    double cos_theta = dot(v, w) / norm(v) / norm(w);
    return acos(max(-1.0, min(1.0, cos_theta)));
}

```

```

}
// orient s, pqr form a triangle pos: 'up', zero = on,
// neg = 'dow'
lf orient(p3 p, p3 q, p3 r, p3 s){
    return dot(cross((q - p), (r - p)), (s - p));
}
// same as 2D but in n-normal direction
lf orient_by_normal(p3 p, p3 q, p3 r, p3 n){
    return dot(cross((q - p), (r - p)), n);
}
struct plane {
    p3 n; lf d; // n: normal d: dist to zero
    // From normal n and offset d
    plane(p3 n, lf d): n(n), d(d) {}
    // From normal n and point P
    plane(p3 n, p3 p): n(n), d(dot(n, p)) {}
    // From three non-collinear points P,Q,R
    plane(p3 p, p3 q, p3 r): plane(cross((q - p), (r - p)
    ), p){}
    // - these work with lf = int
    lf side(p3 p) { return dot(n, p) - d; }
    double dist(p3 p) { return abs(side(p)) / norm(n); }
    plane translate(p3 t) { return {n, d + dot(n, t)}; }
    /// - these require lf = double
    plane shift_up(double dist){ return {n, d + dist *
    norm(n)}; }
    p3 proj(p3 p){ return p - n * side(p) / norm2(n); }
    p3 refl(p3 p){ return p - n * 2 * side(p) / norm2(n);
    }
};
struct line3d {
    p3 d, o; // d: dir o: point on line
    // From two points P, Q
    line3d(p3 p, p3 q): d(q - p), o(p){}
    // From two planes p1, p2 (requires lf = double)
    line3d(plane p1, plane p2){
        d = cross(p1.n, p2.n);
        o = cross((p2.n * p1.d - p1.n * p2.d), d)
        / norm2(d);
    }
    // - these work with lf = int
    double dist2(p3 p){ return norm2(cross(d, (p - o)
    )) / norm2(d); }
    double dist(p3 p){ return sqrt(dist2(p)); }
    bool cmp_proj(p3 p, p3 q){ return dot(d, p) < dot
    (d, q); }
    // - these require lf = double
    p3 proj(p3 p){ return o + d * dot(d, (p - o)) /
    norm2(d); }
    p3 refl(p3 p){ return proj(p) * 2 - p; }
    p3 inter(plane p){ return o - d * p.side(o) / dot
    (p.n, d); }
};

```

```

// get other point: pl.o + pl.d * t;
};
double dist(line3d l1, line3d l2) {
    p3 n = cross(l1.d, l2.d);
    if(n == p3(0, 0, 0)) return l1.dist(l2.o); //
    parallel
    return abs(dot((l2.o - l1.o), n)) / norm(n);
}
// closest point on l1 to l2
p3 closest_on_line1(line3d l1, line3d l2) {
    p3 n2 = cross(l2.d, cross(l1.d, l2.d));
    return l1.o + l1.d * (dot((l2.o - l1.o), n2)) /
    dot(l1.d, n2);
}
double small_angle(p3 v, p3 w){ return acos(min(abs(dot(v
, w)) / norm(v) / norm(w), 1.0)); } // 0 90
double angle(plane p1, plane p2){ return small_angle(p1.n
, p2.n); }
bool is_parallel(plane p1, plane p2){ return cross(p1.n,
p2.n) == p3(0, 0, 0); }
bool is_perpendicular(plane p1, plane p2){ return dot(p1.
n, p2.n) == 0; }
double angle(line3d l1, line3d l2){ return small_angle(l1
.d, l2.d); }
bool is_parallel(line3d l1, line3d l2){ return cross(l1.d
, l2.d) == p3(0, 0, 0); }
bool is_perpendicular(line3d l1, line3d l2){ return dot(
l1.d, l2.d) == 0; }
double angle(plane p, line3d l){ return M_PI / 2 -
small_angle(p.n, l.d); }
bool is_parallel(plane p, line3d l){ return dot(p.n, l.d)
== 0; }
bool is_perpendicular(plane p, line3d l){ return cross(p.
n, l.d) == p3(0, 0, 0); }
line3d perp_through(plane p, p3 o){ return line3d(o, o +
p.n); }
plane perp_through(line3d l, p3 o){ return plane(l.d, o);
}

```

5.11 KD Tree

```

// given a set of points, answer queries of nearest point
in O(log(n))
bool onx(pt a, pt b){return a.x < b.x;}
bool ony(pt a, pt b){return a.y < b.y;}
struct Node {
    pt pp;
    lf x0 = inf, x1 = -inf, y0 = inf, y1 = -inf;
    Node *first = 0, *second = 0;
    lf distance(pt p){
        lf x = min(max(x0, p.x), x1);
    }
};

```

```

        ll y = min(max(y0, p.y), y1);
        return norm2(pt(x, y) - p);
    }
    Node(vector<pt>&& vp) : pp(vp[0]){
        for(pt p : vp){
            x0 = min(x0, p.x);
            x1 = max(x1, p.x);
            y0 = min(y0, p.y);
            y1 = max(y1, p.y);
        }
        if(vp.size() > 1){
            sort(all(vp), x1 - x0 >= y1 - y0
                 ? onx : ony);
            int m = vp.size() / 2;
            first = new Node({vp.begin(), vp.
                             begin() + m});
            second = new Node({vp.begin() + m
                               , vp.end()});
        }
    };
    struct KDTree {
        Node* root;
        KDTree(const vector<pt>& vp): root(new Node({all(
            vp)})){}
        pair<ll, pt> search(pt p, Node *node){
            if(!node->first){
                // avoid query point as answer
                // if(p.x == node->pp.x && p.y ==
                //   node->pp.y) return {inf, pt()}
                return {norm2(p-node->pp), node->
                        pp};
            }
            Node *f = node->first, *s = node->second;
            ll bf = f->distance(p), bs = s->
                distance(p);
            if(bf > bs) swap(bf, bs), swap(f, s);
            auto best = search(p, f);
            if(bs < best.ff) best = min(best, search(
                p, s));
            return best;
        }
        pair<ll, pt> nearest(pt p){ return search(p, root
        ); }
    };

```

6 Grafos

6.1 Puentes

// $O(n+m)$

```

vector<bool> visited;
vi tin, low;
int timer;

void IS_BRIDGE(int u, int v, vii &puentes){
    puentes.push_back({min(u, v), max(u, v)});
}

void dfs(vector<vi> &adj, vii &puentes, int v, int p =
-1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);
        } else {
            dfs(adj, puentes, to, v);
            low[v] = min(low[v], low[to]);
            if (low[to] > tin[v])
                IS_BRIDGE(v, to, puentes);
        }
    }
}

void find_bridges(vector<vi> &adj, vii &puentes, int n) {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs(adj, puentes, i);
    }
}

```

6.2 Puntos de Articulacion

```

//  $O(n+m)$ 
int n;
vector<vector<int>> adj;

vector<bool> visited;
vector<int> tin, low;
int timer;

void dfs(int v, int p = -1) {
    visited[v] = true;
    tin[v] = low[v] = timer++;
    int children=0;
    for (int to : adj[v]) {
        if (to == p) continue;
        if (visited[to]) {
            low[v] = min(low[v], tin[to]);

```

```

    } else {
        dfs(to, v);
        low[v] = min(low[v], low[to]);
        if (low[to] >= tin[v] && p!=-1)
            IS_CUTPOINT(v);
        ++children;
    }
}
if(p == -1 && children > 1)
    IS_CUTPOINT(v);
}

void find_cutpoints() {
    timer = 0;
    visited.assign(n, false);
    tin.assign(n, -1);
    low.assign(n, -1);
    for (int i = 0; i < n; ++i) {
        if (!visited[i])
            dfs (i);
    }
}

```

6.3 Kosajaru

```

//Encontrar las componentes fuertemente conexas en un
//grafo dirigido
//Componente fuertemente conexas: es un grupo de nodos en
//el que hay
//un camino dirigido desde cualquier nodo hasta cualquier
//otro nodo dentro del grupo.
const int maxn = 1e5+5;
vi adj_rev[maxn], adj[maxn];
bool used[maxn];
vi order, comp;

// O(n+m)
void dfs1(int v) {
    used[v]=true;
    for(int u:adj[v])
        if(!used[u]) dfs1(u);
    order.push_back(v);
}

void dfs2(int v) {
    used[v]=true;
    comp.push_back(v);
    for(int u:adj_rev[v])
        if(!used[u]) dfs2(u);
}

void init(int n) {
    for(int i=0; i<n; ++i) if(!used[i]) dfs1(i);
    for(int i=0; i<n; ++i) used[i]=false;
    reverse(order.begin(), order.end());
}

```

```

        for(int v:order) {
            if(!used[v]) {
                dfs2(v);
                // comp
                comp.clear();
            }
        }

adj[a].push_back(b);
adj_rev[b].push_back(a);

```

6.4 Tarjan

```

// O(n+m) (?)
vi low, num, comp, g[nax];
int scc, timer;
stack<int> st;
void tjn(int u) {
    low[u] = num[u] = timer++; st.push(u); int v;
    for(int v: g[u]) {
        if(num[v]==-1) tjn(v);
        if(comp[v]==-1) low[u] = min(low[u], low[v]);
    }
    if(low[u]==num[u]) {
        do{ v = st.top(); st.pop(); comp[v]=scc;
        }while(u != v);
        ++scc;
    }
}

void callt(int n) {
    timer = scc = 0;
    num = low = comp = vector<int>(n, -1);
    for(int i = 0; i < n; i++) if(num[i]==-1) tjn(i);
}

```

6.5 Dijkstra

```

// O ((V+E)*log V)
vi dijkstra(vector<vii> &adj, int s, int V) {
    vi dist(V+1, INT_MAX); dist[s] = 0;
    priority_queue<ii, vii, greater<ii> > pq; pq.push(ii(0, s));
    while(!pq.empty()) {
        ii front = pq.top(); pq.pop();
        int d = front.first, u = front.second;
        if (d > dist[u]) continue;
        for (int j = 0; j < (int)adj[u].size(); j++) {
            ii v = adj[u][j];
            if (dist[u] + v.second < dist[v.first]) {
                dist[v.first] = dist[u] + v.second;
                pq.push(ii(dist[v.first], v.first));
            }
        }
    }
}

```

```

    }
}
return dist;
}

```

6.6 Bellman Ford

```

// O(V*E)
vi bellman_ford(vector<vii> &adj, int s, int n){
    vi dist(n, INF); dist[s] = 0;
    for (int i = 0; i < n-1; i++){
        bool modified = false;
        for (int u = 0; u < n; u++){
            if (dist[u] != INF)
                for (auto &[v, w] : adj[u]){
                    if (dist[v] >= dist[u] + w) continue;
                    dist[v] = dist[u] + w;
                    modified = true;
                }
            if (!modified) break;
        }
        bool negativeCicle = false;
        for (int u = 0; u < n; u++){
            if (dist[u] != INF)
                for (auto &[v, w] : adj[u]){
                    if (dist[v] > dist[u] + w) negativeCicle
                        = true;
                }
        }
        return dist;
    }
}

```

6.7 Floyd Warshall

```

// O(n^3)
vector<vi> adjMat(n+1, vi(n+1));
//Condicion previa: adjMat[i][j] contiene peso de la
//arista (i, j)
//o INF si no existe esa arista
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (adjMat[i][k] < INF && adjMat[k][j] < INF)
                adjMat[i][j] = min(adjMat[i][j], adjMat[i]
                    [k] + adjMat[k][j]);
        }
    }
}

```

6.8 MST Kruskal

```

//O(E*log V)
vector<tuple<int,int,int>> edges;
void kruskal(){
    ll ans=0;
    dsu uf(n);
    sort(all(edges));
    for(auto& [w,u,v]:edges){
        if(uf.get(u)!=uf.get(v)){
            uf.unite(u, v);
            ans+=w;
        }
    }
    if(uf.sets==1){
        cout<<ans<<"\n";
    }
}

```

6.9 MST Prim

```

// O(E * log V)
vector<vii> adj;
vi tomado;
priority_queue<ii> pq;
void process(int u){
    tomado[u] = 1;
    for (auto &[v, w] : adj[u]){
        if (!tomado[v]) pq.emplace(-w, -v);
    }
}

int prim(int v, int n){
    tomado.assign(n, 0);
    process(0);
    int mst_costo = 0, tomados = 0;
    while (!pq.empty()){
        auto [w, u] = pq.top(); pq.pop();
        w = -w; u = -u;
        if (tomado[u]) continue;
        mst_costo += w;
        process(u);
        tomados++;
        if (tomados == n-1) break;
    }
    return mst_costo;
}

```

6.10 Shortest Path Faster Algorithm

```

//Algoritmo mas rapido de ruta minima
//O(V*E) peor caso, O(E) en promedio.

```

```

bool spfa(vector<vii> &adj, vector<int> &d, int s, int n)
{
    d.assign(n, INF);
    vector<int> cnt(n, 0);
    vector<bool> inqueue(n, false);
    queue<int> q;

    d[s] = 0;
    q.push(s);
    inqueue[s] = true;
    while (!q.empty()) {
        int v = q.front();
        q.pop();
        inqueue[v] = false;

        for (auto& [to, len] : adj[v]) {
            if (d[v] + len < d[to]) {
                d[to] = d[v] + len;
                if (!inqueue[to]) {
                    q.push(to);
                    inqueue[to] = true;
                    cnt[to]++;
                    if (cnt[to] > n)
                        return false; //ciclo negativo
                }
            }
        }
    }
    return true;
}

```

6.11 Camino mas corto de longitud fija

```

/*
Modificar operacion * de matrix de esta forma:
En la exponenciacion binaria inicializar matrix ans = b
*/
matrix operator * (const matrix &b){
    matrix ans(this->r, b.c, vector<vl>(this->r, vl(b.c,
        INFL)));

    for (int i = 0; i<this->r; i++) {
        for (int k = 0; k<b.r; k++){
            for (int j = 0; j<b.c; j++){
                ans.m[i][j] = min(ans.m[i][j], m[i][k] +
                    b.m[k][j]);
            }
        }
    }
    return ans;
}

int main() {
    int n, m, k; cin >> n >> m >> k;

```

```

vector<vl> adj(n, vl(n, INFL));

for (int i = 0; i<m; i++){
    ll a, b, c; cin >> a >> b >> c; a--; b--;
    adj[a][b] = min(adj[a][b], c);
}

matrix graph(n, n, adj);
graph = pow(graph, k-1);

cout << (graph.m[0][n-1]==INFL ? -1 : graph.m[0][n
    -1]) << "\n";

return 0;
}

```

6.12 2sat

```

// O(n+m)
// l=(x1 or y1) and (x2 or y2) and ... and (xn or yn)
struct sat2 {
    int n;
    vector<vector<vi>> g;
    vector<bool> vis, val;
    vi comp;
    stack<int> st;

    sat2(int n):n(n),g(2, vector<vi>(2*n)),vis(2*n),
        val(2*n),comp(2*n){}

    int neg(int x){return 2*n-x-1;}
    void make_true(int u){add_edge(neg(u), u);}
    void make_false(int u){make_true(neg(u));}
    void add_or(int u, int v){implication(neg(u),v);}
    void diff(int u, int v){eq(u, neg(v));}
    void eq(int u, int v){
        implication(u, v);
        implication(v, u);
    }
    void implication(int u,int v){
        add_edge(u, v);
        add_edge(neg(v),neg(u));
    }

    void add_edge(int u, int v){
        g[0][u].PB(v);
        g[1][v].PB(u);
    }

    void dfs(int id, int u, int t=0){
        vis[u]=true;
        for(auto &v:g[id][u])
            if(!vis[v])dfs(id, v, t);
        if(id)comp[u]=t;
        else st.push(u);
    }
}

```

```

    }
    void kosaraju() {
        for(int u=0;u<n;++u){
            if(!vis[u])dfs(0, u);
            if(!vis[neg(u)])dfs(0, neg(u));
        }
        vis.assign(2*n, false);
        int t=0;
        while(!st.empty()){
            int u=st.top();st.pop();
            if(!vis[u])dfs(1, u, t++);
        }
    }
    bool check(){
        kosaraju();
        for(int i=0;i<n;++i){
            if(comp[i]==comp[neg(i)])return
                false;
            val[i]=comp[i]>comp[neg(i)];
        }
        return true;
    }
};

int m,n;
sat2 s(n);
char c1,c2;
for(int a,b,i=0;i<m;++i){
    cin>>c1>>a>>c2>>b;
    a--;b--;
    if(c1=='-')a=s.neg(a);
    if(c2=='-')b=s.neg(b);
    s.add_or(a,b);
}
if(s.check()){
    for(int i=0;i<n;++i)cout<<(s.val[i]?'+':'-')<<" ";
    cout<<"\n";
} else cout<<"IMPOSSIBLE\n";

```

7 Matematicas

7.1 De Bruijn sequences

```

// Given alphabet [0, k) constructs a cyclic string
// of length k^n that contains every length n string as
// substr.
vi deBruijnSeq(int k, int n, int lim){
    if (k == 1) return {0};
    vi seq, aux(n + 1);
    int cont = 0;
    function<void(int,int)> gen = [&](int t, int p) {

```

```

        if (t > n){
            if (n % p == 0) for(int i = 1; i
                < p + 1; i++){
                if (cont >= lim) return;
                seq.pb(aux[i]);
                cont++;
            }
        } else {
            aux[t] = aux[t - p];
            gen(t + 1, p);
            while (++aux[t] < k){
                if (cont >= lim) return;
                gen(t + 1, t);
            }
        }
    };
    gen(1, 1);
    return seq;
}

```

7.2 Chinese Remainder Theorem

```

// Complexity: |N|*log(|N|)
// Tested: Not yet.
// finds a suitable x that meets: x is congruent to a_i
// mod n_i
/** Works for non-coprime moduli.
    Returns {-1,-1} if solution does not exist or input is
    invalid.
    Otherwise, returns {x,L}, where x is the solution unique
    to mod L = LCM of mods
*/
pll crt( vl A, vl M ) {
    ll n = A.size(), a1 = A[0], m1 = M[0];
    for(ll i = 1; i < n; i++) {
        ll a2 = A[i], m2 = M[i];
        ll g = __gcd(m1, m2);
        if( a1 % g != a2 % g ) return {-1,-1};
        ll p, q;
        extended_euclid(m1/g, m2/g, p, q);
        ll mod = m1 / g * m2;
        q %= mod; p %= mod;
        ll x = ((1ll*(a1%mod)*(m2/g))%mod*q + (1ll*(a2%mod)*
            (m1/g))%mod*p) % mod; // if WA there is overflow
        a1 = x;
        if (a1 < 0) a1 += mod;
        m1 = mod;
    }
    return {a1, m1};
}

```

7.3 Totient y Divisores

```
vector<int> count_divisors_sieve() {
    bitset<mx> is_prime; is_prime.set();
    vector<int> cnt(mx, 1);
    is_prime[0] = is_prime[1] = 0;
    for(int i = 2; i < mx; i++) {
        if(!is_prime[i]) continue;
        cnt[i]++;
        for(int j = i+i; j < mx; j += i) {
            int n = j, c = 1;
            while( n%i == 0 ) n /= i, c++;
            cnt[j] *= c;
            is_prime[j] = 0;
        }
    }
    return cnt;
}

vector<int> euler_phi_sieve() {
    bitset<mx> is_prime; is_prime.set();
    vector<int> phi(mx);
    iota(phi.begin(), phi.end(), 0);
    is_prime[0] = is_prime[1] = 0;
    for(int i = 2; i < mx; i++) {
        if(!is_prime[i]) continue;
        for(int j = i; j < mx; j += i) {
            phi[j] -= phi[j]/i;
            is_prime[j] = 0;
        }
    }
    return phi;
}

ll euler_phi(ll n) {
    ll ans = n;
    for(ll i = 2; i * i <= n; ++i) {
        if(n % i == 0) {
            ans -= ans / i;
            while(n % i == 0) n /= i;
        }
    }
    if(n > 1) ans -= ans / n;
    return ans;
}
```

7.4 Ecuaciones Diofanticas

```
// O(log(n))
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
    ll xx = y = 0;
    ll yy = x = 1;
    while (b) {
        ll q = a / b;
        ll t = b; b = a % b; a = t;
```

```
        t = xx; xx = x - q * xx; x = t;
        t = yy; yy = y - q * yy; y = t;
    }
    return a;
}

// a*x+b*y=c. returns valid x and y if possible.
// all solutions are of the form (x0 + k * b / g, y0 - k
// * b / g)
bool find_any_solution (ll a, ll b, ll c, ll &x0, ll &y0,
    ll &g) {
    if (a == 0 and b == 0) {
        if (c) return false;
        x0 = y0 = g = 0;
        return true;
    }
    g = extended_euclid (abs(a), abs(b), x0, y0);
    if (c % g != 0) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 *= -1;
    if (b < 0) y0 *= -1;
    return true;
}

void shift_solution(ll &x, ll &y, ll a, ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}

// returns the number of solutions where x is in the
// range[minx, maxx] and y is in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c, ll minx, ll maxx,
    ll miny, ll maxy) {
    ll x, y, g;
    if (find_any_solution(a, b, c, x, y, g) == 0) return 0;
    if (a == 0 and b == 0) {
        assert(c == 0);
        return 1LL * (maxx - minx + 1) * (maxy - miny + 1);
    }
    if (a == 0) {
        return (maxx - minx + 1) * (miny <= c / b and c / b
            <= maxy);
    }
    if (b == 0) {
        return (maxy - miny + 1) * (minx <= c / a and c / a
            <= maxx);
    }
    a /= g, b /= g;
    ll sign_a = a > 0 ? +1 : -1;
    ll sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx) shift_solution(x, y, a, b, sign_b);
    if (x > maxx) return 0;
    ll lx1 = x;
    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx) shift_solution(x, y, a, b, -sign_b);
    ll rx1 = x;
```



```

shift_solution(x, y, a, b, -(miny - y) / a);
if (y < miny) shift_solution(x, y, a, b, -sign_a);
if (y > maxy) return 0;
ll lx2 = x;
shift_solution(x, y, a, b, -(maxy - y) / a);
if (y > maxy) shift_solution(x, y, a, b, sign_a);
ll rx2 = x;
if (lx2 > rx2) swap(lx2, rx2);
ll lx = max(lx1, lx2);
ll rx = min(rx1, rx2);
if (lx > rx) return 0;
return (rx - lx) / abs(b) + 1;
}

//finds the first k | x + b * k / gcd(a, b) >= val
ll greater_or_equal_than(ll a, ll b, ll x, ll val, ll g)
{
    ld got = 1.0 * (val - x) * g / b;
    return b > 0 ? ceil(got) : floor(got);
}

```

7.5 Exponenciación binaria

```

ll binpow(ll b, ll n, ll m) {
    b %= m;
    ll res = 1;
    while (n > 0) {
        if (n & 1)
            res = res * b % m;
        b = b * b % m;
        n >>= 1;
    }
    return res % m;
}

```

7.6 Exponenciación matricial

```

struct matrix {
    int r, c; vector<vl> m;
    matrix(int r, int c, const vector<vl> &m) : r(r), c(c), m(m) {}

    matrix operator * (const matrix &b) {
        matrix ans(this->r, b.c, vector<vl>(this->r, vl(b.c, 0)));
        for (int i = 0; i < this->r; i++) {
            for (int k = 0; k < b.c; k++) {
                if (m[i][k] == 0) continue;
                for (int j = 0; j < b.c; j++) {
                    ans.m[i][j] += mod(m[i][k], MOD) *
                        mod(b.m[k][j], MOD);
                }
            }
        }
    }
}

```

```

        ans.m[i][j] = mod(ans.m[i][j], MOD);
    }
}
return ans;
}
};

matrix pow(matrix &b, ll p) {
    matrix ans(b.r, b.c, vector<vl>(b.r, vl(b.c, 0)));
    for (int i = 0; i < b.r; i++) ans.m[i][i] = 1;
    while (p) {
        if (p & 1) {
            ans = ans * b;
        }
        b = b * b;
        p >>= 1;
    }
    return ans;
}

```

7.7 Fibonacci Fast Doubling

```

// O(log n) muy rapido
pair<int, int> fib(int n) {
    if (n == 0)
        return {0, 1};

    auto p = fib(n >> 1);
    int c = p.first * (2 * p.second - p.first);
    int d = p.first * p.first + p.second * p.second;
    if (n & 1)
        return {d, c + d};
    else
        return {c, d};
}

```

7.8 Freivalds algorithm

```

mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
// check if two n*n matrix a*b=c within complexity (iteration*n^2)
// probability of error 2^(-iteration)
// O(iter*n^2)
int Freivalds(matrix &a, matrix &b, matrix &c) {
    int n = a.r, iteration = 20;
    matrix zero(n, 1), r(n, 1);
    while (iteration--) {
        for (int i = 0; i < n; i++) r.m[i][0] = rnd() % 2;
        matrix ans = (a * (b * r)) - (c * r);
        if (ans.m != zero.m) return 0;
    }
}

```

```

    }
    return 1;
}

```

7.9 Gauss Jordan

```

// O(min(n, m) * n * m)
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be
                    // infinity or a big number

int gauss (vector < vector<double> > a, vector<double> &
ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

7.10 Gauss Jordan mod 2

```

// O(min(n, m) * n * m)
int gauss (vector < bitset<N> > &a, int n, int m, bitset<
N> &ans) {
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        for (int i=row; i<n; ++i)
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        if (! a[row][col])
            continue;
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        ++row;
    }

    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

7.11 GCD y LCM

```

//O(log10 n) n == max(a, b)
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a%b); }
int lcm(int a, int b) { return a / gcd(a, b) * b; }
//gcd(a, b, c) = gcd(a, gcd(b, c))

```

7.12 Integral Definida

```

const int steps = 1e6; // %2==0

```

```
double f(double x);
double simpson(double a, double b){
    double h=(b-a)/steps;
    double s=f(a)+f(b);
    for(int i=1;i<=steps-1;i++){
        double x=a+h*i;
        s+=f(x)*((i&1)?4:2);
    }
    s*=h/3;
    return s;
}
```

7.13 Inverso modular

```
ll mod(ll a, ll m){
    return ((a%m) + m) % m;
}

ll modInverse(ll b, ll m){
    ll x, y;
    ll d = extEuclid(b, m, x, y); //obtiene  $b*x + m*y == d$ 
    if (d != 1) return -1; //indica error
    //  $b*x + m*y == 1$ , ahora aplicamos (mod m) para
    // obtener  $b*x == 1 \pmod{m}$ 
    return mod(x, m);
}

// Otra forma
//  $O(\log MOD)$ 
ll inv(ll a){
    return binpow(a, MOD-2, MOD);
}

//Modulo constante
inv[1] = 1;
for(int i = 2; i < p; ++i)
    inv[i] = (p - (p / i) * inv[p % i] % p) % p;
```

7.14 Logaritmo Discreto

```
//  $O(\sqrt{m})$ 
// Returns minimum x for which  $a^x \pmod{m} = b \pmod{m}$ .
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 111 * a / g) % m;
    }
```

```

    }

    int n = sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 111 * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 111 * a) % m;
    }

    for (int p = 1, cur = k; p <= n; ++p) {
        cur = (cur * 111 * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur] + add;
            return ans;
        }
    }
    return -1;
}
```

7.15 Miller Rabin

```
ll mul (ll a, ll b, ll mod) {
    ll ret = 0;
    for(a %= mod, b %= mod; b != 0;
        b >>= 1, a <= 1, a = a >= mod ? a - mod : a) {
        if (b & 1) {
            ret += a;
            if (ret >= mod) ret -= mod;
        }
    }
    return ret;
}

ll fpow (ll a, ll b, ll mod) {
    ll ans = 1;
    for (; b; b >>= 1, a = mul(a, a, mod))
        if (b & 1)
            ans = mul(ans, a, mod);
    return ans;
}

bool witness (ll a, ll s, ll d, ll n) {
    ll x = fpow(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int i = 0; i < s - 1; i++) {
        x = mul(x, x, n);
        if (x == 1) return true;
        if (x == n - 1) return false;
    }
    return true;
}

ll test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 0};
bool is_prime (ll n) {
```

```

if (n < 2) return false;
if (n == 2) return true;
if (n % 2 == 0) return false;
ll d = n - 1, s = 0;
while (d % 2 == 0) ++s, d /= 2;
for (int i = 0; test[i] && test[i] < n; ++i)
    if (witness(test[i], s, d, n))
        return false;
return true;
}

```

7.16 Miller Rabin Probabilístico

```

using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if
n is probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;

    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))

```

```

        return false;
    }
    return true;
}

```

7.17 Mobius

```

const int N = 1e6+1;
int mob[N];
void mobius() {
    mob[1] = 1;
    for (int i = 2; i < N; i++) {
        mob[i]--;
        for (int j = i + i; j < N; j += i) {
            mob[j] -= mob[i];
        }
    }
}

```

7.18 Pollard Rho

```

//O(n^(1/4)) (?)
ll pollard_rho(ll n, ll c) {
    ll x = 2, y = 2, i = 1, k = 2, d;
    while (true) {
        x = (mul(x, x, n) + c);
        if (x >= n) x -= n;
        d = __gcd(x - y, n);
        if (d > 1) return d;
        if (++i == k) y = x, k <= 1;
    }
    return n;
}

void factorize(ll n, vector<ll> &f) {
    if (n == 1) return;
    if (is_prime(n)) {
        f.push_back(n);
        return;
    }
    ll d = n;
    for (int i = 2; d == n; i++)
        d = pollard_rho(n, i);
    factorize(d, f);
    factorize(n/d, f);
}

```

7.19 Simplex

```

// Maximizar c1*x1 + c2*x2 + c3*x3 ...
// Restricciones a11*x1 + a12*x2 <= b1, a22*x2 + a23*x3
<= b2 ...

```

```

// Retorna valor optimo y valores de las variables
//  $O(c^2 \cdot b)$ ,  $O(c \cdot b)$  - variables  $c$ , restricciones  $b$ 
struct Simplex{
    vector<vector<double>> A;
    vector<double> B,C;
    vector<int> X,Y;
    double z;
    int n,m;

    Simplex(vector<vector<double>> _a, vector<double>
        _b, vector<double> _c){
        A=_a;B=_b;C=_c;
        n=B.size();m=C.size();z=0.;
        X=vector<int>(m);Y=vector<int>(n);
        for(int i=0;i<m;++i)X[i]=i;
        for(int i=0;i<n;++i)Y[i]=i+m;
    }

    void pivot(int x,int y){
        swap(X[Y],Y[X]);
        B[X]/=A[X][Y];
        for(int i=0;i<m;++i)if(i!=Y)A[X][i]/=A[X][Y];
        A[X][Y]=1/A[X][Y];
        for(int i=0;i<n;++i)if(i!=X&&abs(A[i][Y])>EPS){
            B[i]-=A[i][Y]*B[X];
            for(int j=0;j<m;++j)if(j!=Y)A[i][j]-=A[i][Y]*A[X][j];
            A[i][Y]=-A[i][Y]*A[X][Y];
        }
        z+=C[Y]*B[X];
        for(int i=0;i<m;++i)if(i!=Y)C[i]-=C[Y]*A[X][i];
        C[Y]=-C[Y]*A[X][Y];
    }

    pair<double, vector<double>> maximize(){
        while(1){
            int x=-1,y=-1;
            double mn=-EPS;
            for(int i=0;i<n;++i)if(B[i]<mn)mn=B[i],x=i;
            if(x<0)break;
            for(int i=0;i<m;++i)if(A[X][i]<-EPS){y=i;break;}
            //  $y < 0$ , no solution to  $Ax \leq B$ 
            pivot(x,y);
        }
        while(1){
            double mx=EPS;
            int x=-1,y=-1;
            for(int i=0;i<m;++i)if(C[i]>mx)mx=C[i],y=i;
            if(y<0)break;

```

```

            double mn=1e200;
            for(int i=0;i<n;++i)if(A[i][y]>EPS&&B[i]/A[i][y]<mn)mn=B[i]/A[i][y],x=i;
            //  $x < 0$ , unbounded
            pivot(x,y);
        }
        vector<double> r(m);
        for(int i=0;i<n;++i)if(Y[i]<m)r[Y[i]]=B[i];
        return {z,r};
    }
};

```

7.20 Fast Fourier Transform

```

//  $O(N \log N)$ 
const double PI = acos(-1);
struct base {
    double a, b;
    base(double a = 0, double b = 0) : a(a), b(b) {}
    const base operator + (const base &c) const {
        return base(a + c.a, b + c.b);
    }
    const base operator - (const base &c) const {
        return base(a - c.a, b - c.b);
    }
    const base operator * (const base &c) const {
        return base(a * c.a - b * c.b, a * c.b + b * c.a);
    }
};

void fft(vector<base> &p, bool inv = 0) {
    int n = p.size(), i = 0;
    for(int j = 1; j < n - 1; ++j) {
        for(int k = n >> 1; k > (i ^ k); k >>= 1);
        if(j < i) swap(p[i], p[j]);
    }
    for(int l = 1, m; (m = l << 1) <= n; l <= m) {
        double ang = 2 * PI / m;
        base wn = base(cos(ang), (inv ? 1. : -1.) * sin(ang));
        for(int i = 0, j, k; i < n; i += m) {
            for(w = base(1, 0), j = i, k = i + 1; j < k; ++j, w = w * wn) {
                base t = w * p[j + 1];
                p[j + 1] = p[j] - t;
                p[j] = p[j] + t;
            }
        }
    }
    if(inv) for(int i = 0; i < n; ++i) p[i].a /= n, p[i].b /= n;
}

vector<long long> multiply(vector<int> &a, vector<int> &b) {
    int n = a.size(), m = b.size(), t = n + m - 1, sz = 1;

```

```

while(sz < t) sz <= 1;
vector<base> x(sz), y(sz), z(sz);
for(int i = 0; i < sz; ++i) {
    x[i] = i < (int)a.size() ? base(a[i], 0) : base(0, 0);
    y[i] = i < (int)b.size() ? base(b[i], 0) : base(0, 0);
}
fft(x), fft(y);
for(int i = 0; i < sz; ++i) z[i] = x[i] * y[i];
fft(z, 1);
vector<long long> ret(sz);
for(int i = 0; i < sz; ++i) ret[i] = (long long) round(
    z[i].a);
// while((int)ret.size() > 1 && ret.back() == 0) ret.
// pop_back();
return ret;
}

```

7.21 Number Theoretic Transform

```

const int N = 1 << 20;
const int mod = 469762049; //998244353
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {
    for (; y >= 1; x = (long long) x * x % mod) if (y & 1) ans = (long long) ans * x % mod;
    return ans;
}
void precompute(int len) {
    lim = wn[0] = 1; int s = -1;
    while (lim < len) lim <= 1, ++s;
    for (int i = 0; i < lim; ++i) rev[i] = rev[i >> 1] >> 1 | (i & 1) << s;
    const int g = POW(root, (mod - 1) / lim);
    inv_lim = POW(lim, mod - 2);
    for (int i = 1; i < lim; ++i) wn[i] = (long long) wn[i - 1] * g % mod;
}
void ntt(vector<int> &a, int typ) {
    for (int i = 0; i < lim; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int i = 1; i < lim; i <= 1) {
        for (int j = 0, t = lim / i / 2; j < i; ++j) w[j] = wn[j * t];
        for (int j = 0; j < lim; j += i << 1) {
            for (int k = 0; k < i; ++k) {
                const int x = a[k + j], y = (long long) a[k + j + i] * w[k] % mod;
                reduce(a[k + j] += y - mod), reduce(a[k + j + i] = x - y);
            }
        }
    }
}

```

```

}
}
if (!typ) {
    reverse(a.begin() + 1, a.begin() + lim);
    for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] * inv_lim % mod;
}
}
vector<int> multiply(vector<int> &f, vector<int> &g) {
    int n = (int)f.size() + (int)g.size() - 1;
    precompute(n);
    vector<int> a = f, b = g;
    a.resize(lim); b.resize(lim);
    ntt(a, 1), ntt(b, 1);
    for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] * b[i] % mod;
    ntt(a, 0);
    a.resize(n + 1);
    return a;
}

```

8 Programacion dinamica

8.1 Bin Packing

```

int main() {
    ll n, capacidad;
    cin >> n >> capacidad;
    vl pesos(n, 0);
    forx(i, n) cin >> pesos[i];

    vector<pll> dp((1 << n));
    dp[0] = {1, 0};
    // dp[X] = {#numero de paquetes, peso de min paquete}
    // La idea es probar todos los subset y en cada uno preguntarnos
    // quien es mejor para subirse de ultimo buscando minimizar
    // primero el numero de paquetes
    for (int subset = 1; subset < (1 << n); subset++) {
        dp[subset] = {21, 0};

        for (int iPer = 0; iPer < n; iPer++) {
            if ((subset >> iPer) & 1) {
                pll ant = dp[subset ^ (1 << iPer)];
                ll k = ant.ff;
                ll w = ant.ss;

                if (w + pesos[iPer] > capacidad) {
                    k++;
                    w = min(pesos[iPer], w);
                } else {
                    w += pesos[iPer];
                }
            }
        }
    }
}

```

```

    }
    dp[subset] = min(dp[subset], {k, w});
}
}
cout << dp[(1 << n) - 1].ff << ln;
}

```

8.2 CHT

```

// - Me dan las pendientes ordenadas
// Caso 1: Me hacen las querys ordenadas
// O(N + Q)
// Caso 2: Me hacen querys arbitrarias
// O(N + QlogN)

struct CHT {
    // funciona tanto para min como para max, depende del
    // orden en que pasamos las lineas

    struct Line {
        int slope, yIntercept;

        Line(int slope, int yIntercept) : slope(slope),
            yIntercept(yIntercept){}
        int val(int x){ return slope * x + yIntercept; }
        int intersect(Line y) {
            return (y.yIntercept - yIntercept + slope - y
                .slope - 1) / (slope - y.slope);
        }
    };

    deque<pair<Line, int>> dq;

    void insert(int slope, int yIntercept){
        // lower hull si m1 < m2 < m3
        // upper hull si si m1 > m2 > m3
        Line newLine(slope, yIntercept);
        while (!dq.empty() && dq.back().second >= dq.back
            ().first.intersect(newLine)) dq.pop_back();
        if (dq.empty()) {
            dq.emplace_back(newLine, 0);
            return;
        }
        dq.emplace_back(newLine, dq.back().first.
            intersect(newLine));
    }

    int query(int x) { // cuando las consultas son
        // crecientes
        while (dq.size() > 1) {
            if (dq[1].second <= x) dq.pop_front();
            else break;
        }
    }
}

```

```

        return dq[0].first.val(x);
    }

    int query2(int x) { // cuando son arbitrarias
        auto qry = *lower_bound(dq.rbegin(), dq.rend(),
            make_pair(Line(0, 0), x),
            [&](const pair<Line, int>
                &a, const pair<Line,
                    int> &b) {
                return a.second > b.
                    second;
            });

        return qry.first.val(x);
    }
};

```

8.3 CHT Dynamic

```

// O((N+Q) log N) <- usando set para add y bs para q
// lineas de la forma mx + b
#pragma once

struct Line {
    mutable ll m, b, p;
    bool operator<(const Line& o) const { return m <
        o.m; }
    bool operator<(ll x) const { return p < x; }
};

struct CHT : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    static const bool mini = 0; // <---- 1 FOR MIN
    ll div(ll a, ll b){ // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y){
        if (y == end()) return x->p = inf, 0;
        if (x->m == y->m) x->p = x->b > y->b ?
            inf : -inf;
        else x->p = div(y->b - x->b, x->m - y->m)
            ;
        return x->p >= y->p;
    }

    void add(ll m, ll b){
        if (mini){ m *= -1, b *= -1; }
        auto z = insert({m, b, 0}), y = z++, x =
            y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(
            x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >=
            y->p)
            isect(x, erase(y));
    }
}

```

```

    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        if (mini) return -l.m * x + -l.b;
        else return l.m * x + l.b;
    }
};

```

8.4 Divide Conquer

```

// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c < d.
int m, n;
vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int opttr) {
    if (l > r)
        return;

    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, opttr); k++) {
        best = min(best, {(k ? dp_before[k - 1] : 0) + C(
            k, mid), k});
    }

    dp_cur[mid] = best.first;
    int opt = best.second;

    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, opttr);
}

int solve() {
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);

    for (int i = 1; i < m; i++) {
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }

    return dp_before[n - 1];
}

```

8.5 Edit Distances

```

int editDistances(string& wor1, string& wor2) {
    // O(tam1*tam2)
    // minimo de letras que debemos insertar, eliminar o
    // reemplazar

```

```

// de wor1 para obtener wor2
ll tam1=wor1.size();
ll tam2=wor2.size();
vector<vl> dp(tam2+1, vl(tam1+1, 0));
for(int i=0; i<=tam1; i++) dp[0][i]=i;
for(int i=0; i<=tam2; i++) dp[i][0]=i;
dp[0][0]=0;
for(int i=1; i<=tam2; i++) {
    for(int j=1; j<=tam1; j++) {
        ll op1 = min(dp[i-1][j], dp[i][j-1])+1;
        // el minimo entre eliminar o insertar
        ll op2 = dp[i-1][j-1]; // reemplazarlo
        if(wor1[j-1]!=wor2[i-1]) op2++;
        // si el reemplazo tiene efecto o quedo igual
        dp[i][j]=min(op1, op2);
    }
}

return dp[tam2][tam1];
}

```

8.6 Kadane 2D

```

int main() {
    ll fil, col; cin >> fil >> col;
    vector<vl> grid(fil, vl(col, 0));

    // Algoritmo de Kadane/DP para suma maxima de una matriz
    // 2D en o(n^3)
    for(int i=0; i<fil; i++) {
        for(int e=0; e<col; e++) {
            ll num; cin >> num;
            if (e>0) grid[i][e]=num+grid[i][e-1];
            else grid[i][e]=num;
        }
    }

    ll maxGlobal = LONG_LONG_MIN;
    for(int l=0; l<col; l++) {
        for(int r=l; r<col; r++) {
            ll maxLoc=0;
            for(int row=0; row<fil; row++) {
                if (l>0) maxLoc+=grid[row][r]-grid[row][l-1];
                else maxLoc+=grid[row][r];
                if (maxLoc<0) maxLoc=0;
                maxGlobal= max(maxGlobal, maxLoc);
            }
        }
    }
}

```

8.7 Knuth


```

// C[b][c] <= C[a][d]
// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c
< d.
int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];
    auto C = [&](int i, int j) {
        ... // Implement cost function C.
    };
    for (int i = 0; i < N; i++) {
        opt[i][i] = i;
        ... // Initialize dp[i][i] according to the
             problem
    }
    for (int i = N-2; i >= 0; i--) {
        for (int j = i+1; j < N; j++) {
            int mn = INT_MAX;
            int cost = C(i, j);
            for (int k = opt[i][j-1]; k <= min(j-1, opt[i+1][j]); k++) {
                if (mn >= dp[i][k] + dp[k+1][j] + cost) {
                    opt[i][j] = k;
                    mn = dp[i][k] + dp[k+1][j] + cost;
                }
            }
            dp[i][j] = mn;
        }
    }
    cout << dp[0][N-1] << endl;
}

```

8.8 LIS

```

// O(nlogn)
int lis(vi& a) {
    int n=sz(a), last=0;
    vi dp(n+1, INT_MAX), cnt(n, 0);
    dp[0]=INT_MIN;
    for (int i=0; i<n; ++i) {
        int j=lower_bound(all(dp), a[i])-dp.begin();
        // upper_bound
        if (dp[j-1]<a[i] && a[i]<dp[j]) { // dp[j-1]<=a[i]
            dp[j]=a[i];
            last=max(last, j);
        }
        cnt[i]=j;
    }
    int ans=0;
    for (int i=0; i<=n; i++) {

```

```

        if (dp[i]<INT_MAX) ans=i;
    }
    vi LIS(ans);
    int act=ans;
    for (int i=n-1; i>=0; --i) {
        if (cnt[i]==act) {
            LIS[act-1]=a[i];
            act--;
        }
    }
    return ans;
}

```

8.9 SOS

```

const int bits = 23;
int dp[1<<bits];
// O(n*2^n)
void SOS() {
    for (int i = 0; i < (1 << bits); ++i) dp[i] = A[i];
    // top - down
    for (int i = 0; i < bits; ++i) {
        for (int s = 0; s < (1 << bits); ++s) {
            if (s & (1 << i)) {
                dp[s] += dp[s ^ (1 << i)];
            }
        }
    }
    // bottom - up
    for (int i = 0; i < bits; ++i) {
        for (int s = (1 << bits) - 1; s >= 0; --s) {
            if (s & (1 << i)) {
                dp[s ^ (1 << i)] += dp[s];
            }
        }
    }
}

```

9 Strings

9.1 Hashing

```

1000234999, 1000567999, 1000111997, 1000777121,
1001265673, 1001864327, 999727999, 1070777777
const int mod[2] = { 1001864327, 1001265673 };
const ii base(257, 367), zero(0, 0), one(1, 1);

```

```

const int maxn = 1e6;
inline int add(int a, int b, int m){return a+b>=m?a+b-m:a+b;}
inline int sbt(int a, int b, int m){return a-b<0?a-b+m:a-b;}
inline int mul(int a, int b, int m){return ll(a)*b%m;}
inline ll operator ! (const ii a){return (ll(a.first)
<<32)|a.second;}
inline ii operator + (const ii& a, const ii& b){return {
add(a.first, b.first, mod[0]), add(a.second, b.second,
mod[1])};}
inline ii operator - (const ii& a, const ii& b){return {
sbt(a.first, b.first, mod[0]), sbt(a.second, b.second,
mod[1])};}
inline ii operator * (const ii& a, const ii& b){return {
mul(a.first, b.first, mod[0]), mul(a.second, b.second,
mod[1])};}

ii p[maxn+1];
void prepare(){ // Acordate del prepare()!!
    p[0]=one;
    for(int i=1;i<=maxn;i++)p[i]=p[i-1]*base;
}

template <class type>
struct hashing{
    vector<ii> h;
    hashing(type& t){
        h.resize((int)t.size()+1);
        h[0]=zero;
        for(int i=1;i<(int)h.size();++i)
            h[i]=h[i-1]*base + ii{t[i-1], t[i-1]};
    }
    ii get(int l, int r){return h[r+1]-h[l]*p[r-l+1];}
};
ii combine(ii a, ii b, int lenb){return a*p[lenb]+b;}

```

9.2 KMP

```

// O(n)
vi phi(string& s){
    int n=sz(s);
    vi tmp(n);
    for(int i=1,j=0;i<n;++i){
        while(j>0 && s[j]!=s[i])j=tmp[j-1];
        if(s[i]==s[j])j++;
        tmp[i]=j;
    }
    return tmp;
}

// O(n+m)

```

```

int kmp(string& s, string& p){
    int n=sz(s),m=sz(p),cnt=0;
    vi pi=phi(p);
    for(int i=0,j=0;i<n;++i){
        while(j && s[i]!=p[j])j=pi[j-1];
        if(s[i]==p[j])j++;
        if(j==m){
            cnt++;
            j=pi[j-1];
        }
    }
    return cnt;
}

```

9.3 KMP Automaton

```

const int maxn = 1e5+5, alpha = 26;
int to[maxn][alpha];

// O(n*alpha)
void build(string& s){
    to[0][conv(s[0])]=1;
    int n=sz(s);
    for(int i=1,p=0;i<n+1;++i){
        for(int j=0;j<alpha;++j)to[i][j]=to[p][j];
        if(i<n){
            to[i][conv(s[i])]=i+1;
            p=to[p][conv(s[i])];
        }
    }
}

```

9.4 Manacher

```

// O(n), par (raiz, izq, der) 1 - impar 0
vi manacher(string& s, int par){
    int l=0,r=-1,n=sz(s);vi m(n,0);
    for(int i=0;i<n;++i){
        int k=(i>r?(l-par):min(m[l+r-i+par], r-i+par))+par;
        while(i+k-par<n && i-k>=0 && s[i+k-par]==s[i-k])++k;
        m[i]=k-par;--k;
        if(i+k-par>r)l=i-k,r=i+k-par;
    }
    for(int i=0;i<n;++i)m[i]=(m[i]-l+par)*2+1-par;
    return m;
}

```

9.5 Minimum Expression

```
// O(n)
int minimum_expression(string s){
    s=s+s;int n=sz(s),i=0,j=1,k=0;
    while(i+k<n && j+k<n){
        if(s[i+k]==s[j+k])k++;
        else if(s[i+k]>s[j+k])i=i+k+1,k=0; //
            cambiar por < para max
        else j=j+k+1,k=0;
        if(i==j)j++;
    }
    return min(i, j);
}
```

9.6 Palindromic Tree

```
const int alpha = 26;
const char fc = 'a';

// tree suf is the longest suffix palindrome
// tree dad is the palindrome add c to the right and left
struct Node{
    int next[alpha];
    int len,suf,dep,cnt,dad;
};

// O(nlogn)
struct PalindromicTree{
    vector<Node> tree;
    string s;
    int len,n;
    int size; // node 1 - root with len -1, node 2 -
        root with len 0
    int last; // max suffix palindrome
    bool addLetter(int pos){
        int cur=last,curlen=0;
        int let=s[pos]-fc;
        while(true){
            curlen=tree[cur].len;
            if(pos-1-curlen>=0 && s[pos-1-
                curlen]==s[pos])break;
            cur=tree[cur].suf;
        }
        if(tree[cur].next[let]){
            last=tree[cur].next[let];
            tree[last].cnt++;
            return false;
        }
        size++;
        last=size;
        tree[size].len=tree[cur].len+2;
        tree[cur].next[let]=size;
        tree[size].cnt=1;
    }
};
```

```
tree[size].dad=cur;
if(tree[size].len==1){
    tree[size].suf=2;
    tree[size].dep=1;
    return true;
}
while(true){
    cur=tree[cur].suf;
    curlen=tree[cur].len;
    if(pos-1-curlen>=0 && s[pos-1-
        curlen]==s[pos]){
        tree[size].suf=tree[cur].
            next[let];
        break;
    }
}
tree[size].dep=1+tree[tree[size].suf].dep;
return true;
}

PalindromicTree(string& s2, int n){
    tree.assign(n+4,Node());
    tree[1].len=-1;tree[1].suf=1;
    tree[2].len=0;tree[2].suf=1;
    size=2;last=2;s=s2;

    for(int i=0;i<n;i++){
        addLetter(i);
    }

    for(int i=size;i>=3;i--){
        tree[tree[i].suf].cnt+=tree[i].
            cnt;
    }
}

};
```

9.7 Suffix Array

```
// O(nlogn)
struct SuffixArray{
    const int alpha = 256;
    string s;int n;
    vi sa,rnk,lcp;

    SuffixArray(string& _s){
        s=_s;s.push_back('$'); // check
        n=sz(s);
        sa.assign(n, 0);
        rnk.assign(n, 0);
        lcp.assign(n-1, 0);
        buildSA();
    }
};
```

```

}
void buildSA() {
    vi cnt(max(alpha, n), 0);
    for(int i=0; i<n; ++i) cnt[s[i]]++;
    for(int i=1; i<max(alpha, n); ++i) cnt[i] += cnt[i-1];
    for(int i=n-1; i>=0; --i) sa[--cnt[s[i]]] = i;
    for(int i=1; i<n; ++i) rnk[sa[i]] = rnk[sa[i-1]] + (s[sa[i]] != s[sa[i-1]]);

    for(int k=1; k<n; k*=2) {
        vi nsa(n), nrnk(n), ncnt(n);
        for(int i=0; i<n; ++i) sa[i] = (sa[i] - k + n) % n;
        for(int i=0; i<n; ++i) ncnt[rnk[i]]++;
        for(int i=1; i<n; ++i) ncnt[i] += ncnt[i-1];
        for(int i=n-1; i>=0; --i) nsa[--ncnt[rnk[sa[i]]]] = sa[i];
        for(int i=1; i<n; ++i) {
            ii op1 = {rnk[nsa[i]], rnk[(nsa[i] + k) % n]};
            ii op2 = {rnk[nsa[i-1]], rnk[(nsa[i-1] + k) % n]};
            nrnk[nsa[i]] = nrnk[nsa[i-1]] + (op1 != op2);
        }
        swap(sa, nsa); swap(rnk, nrnk);
    }

    for(int i=0, k=0; i<n-1; ++i) {
        while(s[i+k] == s[sa[rnk[i]-1]+k]) k++;
        lcp[rnk[i]-1] = k;
        if(k) k--;
    }
}
};

```

9.8 Suffix Automaton

```

// O(n*log(alpha))
struct SuffixAutomaton{
    vector<map<char, int>> to;
    vector<bool> end;
    vi suf, len; // len, longest string
    int last;

    SuffixAutomaton(string& s){
        to.push_back(map<char, int>());
        suf.push_back(-1);
        len.push_back(0);
        last=0;
    }
};

```

```

for(int i=0; i<sz(s); i++){
    to.push_back(map<char, int>());
    suf.push_back(0);
    len.push_back(i+1);
    int r=sz(to)-1;

    int p=last;
    while(p>=0 && to[p].find(s[i]) == to[p].end()) {
        to[p][s[i]] = r;
        p=suf[p];
    }
    if(p!=-1){
        int q=to[p][s[i]];
        if(len[p]+1==len[q]){
            suf[r]=q;
        } else {
            to.push_back(to[q]);
            suf.push_back(suf[q]);
            len.push_back(len[p]+1);
            int qq=sz(to)-1;
            suf[q]=qq;
            suf[r]=qq;
            while(p>=0 && to[p][s[i]]==q){
                to[p][s[i]] = qq;
                p=suf[p];
            }
        }
        last=r;
    }
    end.assign(sz(to), false);
    int p=last;
    while(p){
        end[p]=true;
        p=suf[p];
    }
}
};

```

9.9 Suffix Tree

```

// O(n)
struct SuffixTree{
    vector<map<char, int>> to;
    vector<int> pos, len, link;
    const int inf = 1e9;
};

```

```

int size=0;
string s;

int make(int _pos, int _len){
    to.push_back(map<char,int>());
    pos.push_back(_pos);
    len.push_back(_len);
    link.push_back(-1);
    return size++;
}

void add(int& p, int& lef, char c){
    s+=c; ++lef; int lst=0;
    for(; lef; p=link[p]:lef--){
        while(lef>1 && lef>len[to[p][s[sz(s)-lef]]]){
            p=to[p][s[sz(s)-lef]], lef-=len[p];
        }
        char e=s[sz(s)-lef];
        int& q=to[p][e];
        if(!q){
            q=make(sz(s)-lef, inf),
            link[lst]=p, lst=0;
        } else {
            char t=s[pos[q]+lef-1];
            if(t==c){ link[lst]=p;
                return; }
            int u=make(pos[q], lef-1);
            to[u][c]=make(sz(s)-1, inf);
            to[u][t]=q;
            pos[q]+=lef-1;
            if(len[q]!=inf) len[q]-=lef-1;
            q=u, link[lst]=u, lst=u;
        }
    }
}

void build(string& _s){
    make(-1, 0); int p=0, lef=0;
    for(char c:_s) add(p, lef, c);
    add(p, lef, '$');
    s.pop_back();
}

int query(string& p){
    for(int i=0, u=0, n=sz(p); i<n; i++){
        if(i==n || !to[u].count(p[i]))
            return i;
        u=to[u][p[i]];
        for(int j=0; j<len[u]; ++j){
            if(i==n || s[pos[u]+j]!=p[j]) return i;
            i++;
        }
    }
}

```

```

    }
}

vector<int> sa;
void genSA(int x=0, int Len=0){
    if(!sz(to[x])) sa.push_back(pos[x]-Len);
    else for(auto t:to[x]) genSA(t.second, Len+len[x]);
}

};

```

9.10 Trie

```

const int maxn = 2e6+5, alpha = 26, bits = 30;
int to[maxn][alpha], cnt[maxn], act;

void init(){
    for(int i=0; i<=act; ++i){
        cnt[i]=0;
        // suf[i]=dad[i]=0;
        // adj[i].clear();
        memset(to[i], 0, sizeof(to[i]));
    }
    act=0;
}

int add(string& s){
    int u=0;
    for(char ch:s){
        int c=conv(ch);
        if(!to[u][c]) to[u][c]=++act;
        u=to[u][c];
    }
    cnt[u]++;
    return u;
}

// Aho-Corasick
vector<int> adj[maxn]; // dad or suf
int dad[maxn], suf[maxn];

// O(sum(n)*alpha)
void build(){
    queue<int> q{{0}};
    while(!q.empty()){
        int u=q.front(); q.pop();
        for(int i=0; i<alpha; ++i){
            int v=to[u][i];
            if(!v) to[u][i]=to[suf[u]][i];
            else q.push(v);
            if(!u || !v) continue;
            suf[v]=to[suf[u]][i];
            dad[v]=cnt[suf[v]]?suf[v]:dad[suf[v]];
        }
    }
}

```

```

    }
    for(int i=1;i<=act;++i){
        adj[i].push_back(dad[i]);
        adj[dad[i]].push_back(i);
    }
}

```

9.11 Z Algorithm

```

// O(n)
vi z_function(string& s){
    int n=sz(s),l=0,r=0;vi z(n);
    for(int i=1;i<n;i++){
        if(i<r) z[i]=min(r-i, z[i-l]);
        while(i+z[i]<n && s[z[i]]==s[i+z[i]]) z[i]++;
        if(i+z[i]>r){
            l=i;
            r=i+z[i];
        }
    }
    return z;
}

```

10 Misc

10.1 Counting Sort

```

// O(n+k)
void counting_sort(vi& a){
    int maxi=*max_element(all(a));
    int mini=*min_element(all(a));
    int k=maxi-mini+1,n=sz(a);
    vi cnt(k,0);
    for(int i=0;i<n;++i)++cnt[a[i]-mini];
    for(int i=0,j=0;i<k;++i)
        while(cnt[i]--){
            a[j++] = i+mini;
        }
}

```

10.2 Dates

```

int dateToInt(int y, int m, int d){
    return 1461*(y+4800+(m-14)/12)/4+367*(m-2-(m-14)/12*12)/12-
        3*((y+4900+(m-14)/12)/100)/4+d-32075;
}
void intToDate(int jd, int& y, int& m, int& d){
    int x,n,i,j;x=jd+68569;

```

```

n=4*x/146097;x-=(146097*n+3)/4;
i=(4000*(x+1))/1461001;x-=1461*i/4-31;
j=80*x/2447;d=x-2447*j/80;
x=j/11;m=j+2-12*x;y=100*(n-49)+i+x;
}
int DayOfWeek(int d, int m, int y){ //starting on
    Sunday
    static int ttt[]={0, 3, 2, 5, 0, 3, 5, 1, 4, 6,
        2, 4};
    y-=m<3;
    return (y+y/4-y/100+y/400+ttt[m-1]+d)%7;
}

```

10.3 Expression Parsing

```

// O(n) - En python es eval()
bool delim(char c){return c==' ';}
bool is_op(char c){return c=='+' || c=='-' || c=='*' || c
    =='/';}
bool is_unary(char c){return c=='+' || c=='-';}

int priority(char op){
    if(op<0) return 3;
    if(op=='+' || op=='-') return 1;
    if(op=='*' || op=='/') return 2;
    return -1;
}

void process_op(stack<int>& st, char op){
    if(op<0){
        int l=st.top();st.pop();
        switch(-op){
            case '+':st.push(l);break;
            case '-':st.push(-l);break;
        }
    }else{
        int r=st.top();st.pop();
        int l=st.top();st.pop();
        switch(op){
            case '+':st.push(l+r);break;
            case '-':st.push(l-r);break;
            case '*':st.push(l*r);break;
            case '/':st.push(l/r);break;
        }
    }
}

int evaluate(string& s){
    stack<int> st;
    stack<char> op;
    bool may_be_unary=true;
    for(int i=0;i<sz(s);++i){
        if(delim(s[i])) continue;
        if(s[i] == '('){

```

```

        op.push('(');
        may_be_unary=true;
    }else if (s[i]=='('){
        while (op.top()!='('){
            process_op(st, op.top());
            op.pop();
        }
        op.pop();
        may_be_unary=false;
    }else if (is_op(s[i])){
        char cur_op=s[i];
        if (may_be_unary && is_unary(
            cur_op))cur_op=-cur_op;
        while (!op.empty() && ((cur_op >=
            0 && priority(op.top()) >=
            priority(cur_op)) || (cur_op <
            0 && priority(op.top()) >
            priority(cur_op)))){
            process_op(st, op.top());
            op.pop();
        }
        op.push(cur_op);
        may_be_unary=true;
    }else{
        int number=0;
        while (i<sz(s) && isalnum(s[i]))
            number=number*10+s[i++]-'0';
        --i;
        st.push(number);
        may_be_unary=false;
    }
}
while (!op.empty()){
    process_op(st, op.top());
    op.pop();
}
return st.top();
}

```

10.4 Ternary Search

```

// O(log((r-l)/eps))
double ternary(){
    double l, r;

```

```

    while (r-l>eps){
        double m1=l+(r-l)/3.0;
        double m2=r-(r-l)/3.0;
        if (f(m1)<f(m2)) l=m1;
        else r=m2;
    }
    return max(f(l), f(r));
}

```

10.5 Prefix3D

```

const int N = 100;
int A[N][N][N];
int prefix[N + 1][N + 1][N + 1];

void build(int n){
    for (int x = 1; x <= n; x++){
        for (int y = 1; y <= n; y++){
            for (int z = 1; z <= n; z++){
                prefix[x][y][z] = A[x - 1][y - 1][z - 1]
                    + prefix[x - 1][y][z] + prefix[x][y - 1][z]
                    - prefix[x - 1][y - 1][z] + prefix[x][y][z - 1]
                    - prefix[x - 1][y - 1][z - 1] - prefix[x][y - 1][z - 1]
                    + prefix[x - 1][y - 1][z - 1];
            }
        }
    }

    ll query(int lx, int rx, int ly, int ry, int lz, int rz){
        ll ans = prefix[rx][ry][rz]
            - prefix[lx - 1][ry][rz] - prefix[rx][ly - 1][rz]
            - prefix[rx][ly - 1][rz] - prefix[rx][ry][lz - 1]
            + prefix[lx - 1][ly - 1][rz] + prefix[lx - 1][ry][lz - 1]
            + prefix[rx][ly - 1][lz - 1] + prefix[rx][ly - 1][lz - 1]
            - prefix[lx - 1][ly - 1][lz - 1];

        return ans;
    }
}

```

11 Teoría y miscelánea

11.1 Sumatorias

- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^n i^5 = \frac{(n(n+1))^2(2n^2+2n-1)}{12}$
- $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{i=0}^n x^i = \frac{x^{n+1}-1}{x-1}$ para $x \neq 1$

11.2 Teoría de Grafos

11.2.1 Teorema de Euler

En un grafo conectado planar, se cumple que $V - E + F = 2$, donde V es el número de vértices, E es el número de aristas y F es el número de caras. Para varios componentes la formula es: $V - E + F = 1 + C$, siendo C el número de componentes.

11.2.2 Planaridad de Grafos

Un grafo es planar si y solo si no contiene un subgrafo homeomorfo a K_5 (grafo completo con 5 vértices) ni a $K_{3,3}$ (grafo bipartito completo con 3 vértices en cada conjunto).

11.3 Teoría de Números

11.3.1 Ecuaciones Diofánticas Lineales

Una ecuación diofántica lineal es una ecuación en la que se buscan soluciones enteras x e y que satisfagan la relación lineal $ax + by = c$, donde a , b y c son constantes dadas.

Para encontrar soluciones enteras positivas en una ecuación diofántica lineal, podemos seguir el siguiente proceso:

1. Encontrar una solución particular: Encuentra una solución particular (x_0, y_0) de la ecuación. Esto puede hacerse utilizando el algoritmo de Euclides extendido.
2. Encontrar la solución general: Una vez que tengas una solución particular, puedes obtener la solución general utilizando la fórmula:

$$x = x_0 + \frac{b}{\text{mcd}(a, b)} \cdot t$$

$$y = y_0 - \frac{a}{\text{mcd}(a, b)} \cdot t$$

donde t es un parámetro entero.

3. Restringir a soluciones positivas: Si deseas soluciones positivas, asegúrate de que las soluciones generales satisfagan $x \geq 0$ y $y \geq 0$. Puedes ajustar el valor de t para cumplir con estas restricciones.

11.3.2 Pequeño Teorema de Fermat

Si p es un número primo y a es un entero no divisible por p , entonces $a^{p-1} \equiv 1 \pmod{p}$.

11.3.3 Teorema de Euler

Para cualquier número entero positivo n y un entero a coprimo con n , se cumple que $a^{\phi(n)} \equiv 1 \pmod{n}$, donde $\phi(n)$ es la función phi de Euler, que representa la cantidad de enteros positivos menores que n y coprimos con n .

11.4 Geometría

11.4.1 Teorema de Pick

Sea un polígono simple cuyos vertices tienen coordenadas enteras. Si B es el número de puntos enteros en el borde, I el número de puntos enteros en el interior del polígono, entonces el área A del polígono se puede calcular con la formula:

$$A = I + \frac{B}{2} - 1$$

11.4.2 Fórmula de Herón

Si los lados del triángulo tienen longitudes a , b y c , y s es el semiperímetro (es decir, $s = \frac{a+b+c}{2}$), entonces el área A del triángulo está dada por:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

11.4.3 Relación de Existencia Triangular

Para un triángulo con lados de longitud a , b , y c , la relación de existencia triangular se expresa como:

$$b - c < a < b + c, \quad a - c < b < a + c, \quad a - b < c < a + b$$

11.5 Combinatoria

11.5.1 Permutaciones

El número de permutaciones de n objetos distintos tomados de a r a la vez (sin repetición) se denota como $P(n, r)$ y se calcula mediante:

$$P(n, r) = \frac{n!}{(n-r)!}$$

11.5.2 Combinaciones

El número de combinaciones de n objetos distintos tomados de a r a la vez (sin repetición) se denota como $C(n, r)$ o $\binom{n}{r}$ y se calcula mediante:

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

11.5.3 Permutaciones con Repetición

El número de permutaciones de n objetos tomando en cuenta repeticiones se denota como $P_{\text{rep}}(n; n_1, n_2, \dots, n_k)$ y se calcula mediante:

$$P_{\text{rep}}(n; n_1, n_2, \dots, n_k) = \frac{n!}{n_1!n_2! \cdots n_k!}$$

11.5.4 Combinaciones con Repetición

El número de combinaciones de n objetos tomando en cuenta repeticiones se denota como $C_{\text{rep}}(n; n_1, n_2, \dots, n_k)$ y se calcula mediante:

$$C_{\text{rep}}(n; n_1, n_2, \dots, n_k) = \binom{n+k-1}{n} = \binom{n+k-1}{k-1}$$

11.5.5 Números de Catalan

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

Los números de Catalan también pueden calcularse utilizando la siguiente fórmula recursiva:

$$C_0 = 1$$

$$C_{n+1} = \frac{4n+2}{n+2} C_n$$

Usos:

- $\text{Cat}(n)$ cuenta el número de árboles binarios distintos con n vértices.
- $\text{Cat}(n)$ cuenta el número de expresiones que contienen n pares de paréntesis correctamente emparejados.
- $\text{Cat}(n)$ cuenta el número de formas diferentes en que se pueden colocar $n+1$ factores entre paréntesis, por ejemplo, para $n = 3$ y $3+1 = 4$ factores: a, b, c, d , tenemos: $(ab)(cd), a(b(cd)), ((ab)c)d$ y $a((bc)d)$.

- Los números de Catalan cuentan la cantidad de caminos no cruzados en una rejilla $n \times n$ que se pueden trazar desde una esquina de un cuadrado o rectángulo a la esquina opuesta, moviéndose solo hacia arriba y hacia la derecha.
- Los números de Catalan representan el número de árboles binarios completos con $n+1$ hojas.
- $\text{Cat}(n)$ cuenta el número de formas en que se puede triangular un polígono convexo de $n+2$ lados. Otra forma de decirlo es como la cantidad de formas de dividir un polígono convexo en triángulos utilizando diagonales no cruzadas.

11.5.6 Estrellas y barras

Número de soluciones de la ecuación $x_1 + x_2 + \cdots + x_k = n$.

- Con $x_i \geq 0$: $\binom{n+k-1}{n}$
- Con $x_i \geq 1$: $\binom{n-1}{k-1}$

Número de sumas de enteros con límite inferior:

Esto se puede extender fácilmente a sumas de enteros con diferentes límites inferiores. Es decir, queremos contar el número de soluciones para la ecuación:

$$x_1 + x_2 + \cdots + x_k = n$$

con $x_i \geq a_i$.

Después de sustituir $x'_i := x_i - a_i$ recibimos la ecuación modificada:

$$(x'_1 + a_i) + (x'_2 + a_i) + \cdots + (x'_k + a_k) = n$$

$$\Leftrightarrow x'_1 + x'_2 + \cdots + x'_k = n - a_1 - a_2 - \cdots - a_k$$

con $x'_i \geq 0$. Así que hemos reducido el problema al caso más simple con $x'_i \geq 0$ y nuevamente podemos aplicar el teorema de estrellas y barras.

11.6 DP Optimization Theory

Name	Original Recurrence	Sufficient Condition	From	To
CH 1	$dp[i] = \min_{j < i} \{dp[j] + b[j] * a[i]\}$	$b[j] \geq b[j+1]$ Optionally $a[i] \leq a[i+1]$	$O(n^2)$	$O(n)$
CH 2	$dp[i][j] = \min_{k < j} \{dp[i-1][k] + b[k] * a[j]\}$	$b[k] \geq b[k+1]$ Optionally $a[j] \leq a[j+1]$	$O(kn^2)$	$O(kn)$
D&Q	$dp[i][j] = \min_{k < j} \{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	$O(kn \log n)$
Knuth	$dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j]\} + C[i][j]$	$A[i, j-1] \leq A[i, j] \leq A[i+1, j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$ - the smallest k that gives the optimal answer, for example in $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$ - some given cost function
- We can generalize a bit in the following way $dp[i] = \min_{j < i} \{F[j] + b[j] * a[i]\}$, where $F[j]$ is computed from $dp[j]$ in constant time