

# Notebook UNTreeCiclo

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## 1 C++

### 1.1 C++ plantilla

```
#include <bits/stdc++.h>
using namespace std;
#define all(v) v.begin(), v.end()
#define sz(arr) ((int) arr.size())
#define rep(i, a, b) for(int i = a; i < (b); ++i)
```

```
typedef pair<int, int> ii;
typedef vector<int> vi;
typedef long long ll;
const char ln = '\n';

#define watch(x) cout<<#x<<"="<<x<<'\n'
#define print(arr) for(auto& x:arr) cout<<x<<" "; cout<<"\n"

typedef long double ld;
typedef vector<ii> vii;
typedef vector<long long> vll;
typedef pair<ll, ll> pll;
typedef vector<pll> vpll;
const int INF = 1e9;
const ll INFL = 1e18;
const int MOD = 1e9+7;
const double EPS = 1e-9;
const ld PI = acosl(-1);
int dirx[4] = {0,-1,1,0};
int diry[4] = {-1,0,0,1};
int dr[] = {1, 1, 0, -1, -1, -1, 0, 1};
int dc[] = {0, 1, 1, 1, 0, -1, -1, -1};
const string ABC = "abcdefghijklmnopqrstuvwxyz";

void main2() {
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout << setprecision(20) << fixed;
    // freopen("file.in", "r", stdin);
    // freopen("file.out", "w", stdout);
    clock_t start = clock();
    main2();
    cerr<<double(clock()-start)/CLOCKS_PER_SEC<<" s\n"
    ";
    return 0;
}
```

### 1.2 Librerías

```
// En caso de que no sirva #include <bits/stdc++.h>
#include <algorithm>
#include <iostream>
#include <iterator>
#include <sstream>
#include <fstream>
#include <cassert>
#include <climits>
#include <cstdlib>
#include <cstring>
#include <string>
#include <cstdio>
```

```
#include <vector>
#include <cmath>
#include <queue>
#include <deque>
#include <stack>
#include <list>
#include <map>
#include <set>
#include <bitset>
#include <iomanip>
#include <unordered_map>
/////
#include <tuple>
#include <random>
#include <chrono>
```

### 1.3 Create

```
import os
def folder(problem):
    os.makedirs(problem, exist_ok=True)
    with open(os.path.join(problem, "main.cpp"), "w")
        as f:
            f.write("")
    with open(os.path.join(problem, "in.txt"), "w")
        as f:
            f.write("")
    with open("plantilla.cpp", "w") as f:
        f.write("")
    with open("out.txt", "w") as f:
        f.write("")
    for i in range(ord('A'), ord('P') + 1):
        folder(chr(i))
```

### 1.4 Bitmask

\* Operaciones a nivel de bits. Si n es ll usar lll<< en los corrimientos.

```
x & 1          -> Verifica si x es impar
x & (1<<i)      -> Verifica si el i-esimo bit esta
                encendido
x = x | (1<<i)  -> Enciende el i-esimo bit
x = x & ~(1<<i) -> Apaga el i-esimo bit
x = x ^ (1<<i)  -> Invierte el i-esimo bit
x = ~x         -> Invierte todos los bits
x & -x         -> Devuelve el bit encendido mas a la
                derecha (potencia de 2, no el indice)
~x & (x+1)     -> Devuelve el bit apagado mas a la
                derecha (potencia de 2, no el indice)
```

```
x = x | (x+1)   -> Enciende el bit apagado mas a la
                derecha
x = x & (x-1)   -> Apaga el bit encendido mas a la
                derecha
x = x & ~y      -> Apaga en x los bits encendidos de y
* Funciones del compilador gcc. Si n es ll agregar el
  sufijo ll, por ej: __builtin_clzll(n).
__builtin_clz(x)      -> Cantidad de bits apagados por la
  izquierda
__builtin_ctz(x)      -> Cantidad de bits apagados por la
  derecha. Indice del bit encendido mas a la derecha
__builtin_popcount(x) -> Cantida de bits encendidos
__builtin_ffs(x)      -> Posicion del primer bit
  prendido (lsb+1)
* Logaritmo en base 2 (entero). Indice del bit encendido
  mas a la izquierda. Si x es ll usar 63 y clzll(x).
// 0(1)
int lg2(const int &x) { return 31-__builtin_clz(x); }
* Itera, con indices, los bits encendidos de una mascara.
// 0(#bits_encendidos)
for (int x = mask; x; x &= x-1) {
    int i = __builtin_ctz(x);
}
* Itera todas las submascaras de una mascara. (Iterar
  todas las submascaras de todas las mascaras es O(3^n))
// 0(2^(#bits_encendidos))
for (int sub = mask; ; sub = (sub-1)&mask) {
    // ...
    if (sub == 0) break;
}
// Ascendente
for(int sub = 0; ; sub = (sub-mask)&mask) {
    // ...
    if (sub == mask) break;
}
* retorna la siguiente mask con la misma cantidad
  encendida
ll nextMask(ll x){
    ll c = x & -x;
    ll r = x + c;
    return ((r ^ x) >> 2) / c | r;
}
// optimiza el .count de los bitsets y el popcount
#pragma GCC target("popcnt")
// Formulas
a | b = a ^ b + a & b
a ^ (a & b) = (a | b) ^ b
```

```

b ^ (a & b) = (a | b) ^ a
(a & b) ^ (a | b) = a ^ b
a + b = a | b + a & b
a + b = a ^ b + 2 * (a & b)
a - b = (a ^ (a & b)) - ((a | b) ^ a)
a - b = ((a | b) ^ b) - ((a | b) ^ a)
a - b = (a ^ (a & b)) - (b ^ (a & b))
a - b = ((a | b) ^ b) - (b ^ (a & b))
a ^ b = ~(a & b) & (a | b)
si (x < y < z) entonces min(x^y, y^z) < (x^z)

```

## 1.5 Custom Hashing

```

struct custom_hash {
    static long long splitmix64(long long x) {
        x += 0x9e3779b97f4a7c15;
        x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
        return x ^ (x >> 31);
    }

    size_t operator()(long long x) const {
        static const long long FIXED_RANDOM =
            chrono::steady_clock::now().
            time_since_epoch().count();
        return splitmix64(x + FIXED_RANDOM);
    }

    size_t operator()(const pair<int,int>& x) const {
        return (size_t) x.first * 37U + (size_t)
            x.second;
    }

    size_t operator()(const vector<int>& v) const {
        size_t s = 0;
        for(auto &e : v)
            s ^= hash<int>()(e) + 0x9e3779b9 + (s
                <<6) + (s >>2);
        return s;
    }
};

unordered_map<long long, int, custom_hash> safe_map; //
    unordered_map or gp_hash_table

safe_map.max_load_factor(0.25);
safe_map.reserve(1024); // potencia de 2 mas cercana

multitest - no usar reserve (por el clear, es pesado)

```

## 1.6 Random

```
typedef unsigned long long u64;
```

```

mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());
u64 xor_hash=rng();

// return random numbers in the range [l,r]
mt19937 rng (chrono::steady_clock::now().time_since_epoch
    ().count());
double rand(double l, double r){return
    uniform_real_distribution<double>(l, r)(rng);}
int rand(int l, int r){return uniform_int_distribution<
    int>(l, r)(rng);}

shuffle(all(vector), rng);

```

## 1.7 Cosas de strings

```

// si el caracter que separa el texto es distinto al
    espacio
// entonces descomentar el segundo parametro y cambiar el
    while por el otro
vector<string> split(const string &s/*, char c */) {
    vector<string> v;
    stringstream ss(s);
    string sub;
    while(ss>>sub)v.push_back(sub);
    // while(getline(ss,sub,c))if(sz(sub))v.push_back
        (sub);
    return v;
}

string s;
for(char& c:s)c=toupper(c);
for(char& c:s)c=tolower(c);
int n=stoi(s); // string -> int
int n=stoi(s, nullptr, 2); // bin string -> int
double d=stod(s); // string -> double
string s=to_string(n); // int -> string
cout<<"\U0001F600"; // emojis

Quitar repetidos (lo pongo aca porque no se donde mas
    ponerlo)
sort(all(bs));
bs.resize(unique(all(bs)) - bs.begin());

```

## 2 Arboles

### 2.1 Centroid Decomposition

```

// O(n*log(n))
// 1) init(adj,n);
struct CentroidDecomposition{
    vector<vi> adj;

```

```

vi dad,sz,proc;
int operator[] (int i){return dad[i];}
void init(vector<vi>& adj2, int n){
    proc.assign(n,false);
    dad.resize(n);
    sz.resize(n);
    adj=adj2;
    build();
}

void build(int v=0, int p=-1){
    int n=dfsSz(v, p);
    int centroid=dfsCentroid(v, p, n);
    dad[centroid]=p;
    // anadir dfs para el conteo de caminos
    proc[centroid]=true;
    for(int u:adj[centroid]){
        if(u==p || proc[u])continue;
        build(u,centroid);
    }
}

int dfsSz(int v,int p){
    sz[v]=1;
    for(int u:adj[v]){
        if(u==p || proc[u])continue;
        sz[v]+=dfsSz(u, v);
    }
    return sz[v];
}

int dfsCentroid(int v, int p, int n){
    for(int u:adj[v]){
        if(u==p || proc[u])continue;
        if(sz[u]>n/2) return dfsCentroid(u, v,n);
    }
    return v;
}

};
// para el arbol de centroides
// for(int b=a;b!=-1;b=cd[b])

```

## 2.2 Hash Tree

```

const int MOD=1e9+97;
const int P[2]={998244353,1000000007};
const int Q[2]={10000000033,10000000021};
const int R[2]={123456789,987654321};

int add(int a, int b){return a+b>=MOD?a+b-MOD:a+b;}
int mul(int a, int b){return ll(a)*b%MOD;}
int binpow(int a, int b){

```

```

int res=1;a%=MOD;
while(b>0){
    if(b&1)res=mul(res,a);
    a=mul(a,a);
    b>>=1;
}
return res%MOD;
}

// O(n), 1-indexed
struct Tree{
    vector<vi> g;
    int n;

    Tree(int _n):n(_n){g.resize(n+1);}
    void add_edge(int u, int v){
        g[u].push_back(v);
        g[v].push_back(u);
    }

    ii hash(int u, int pre=0){
        vector<vi> nw(2,vi());
        for(int v:g[u])
            if(v!=pre){
                ii tmp=hash(v,u);
                nw[0].push_back(tmp.first);
                nw[1].push_back(tmp.second);
            }
        ii ans={0,0};
        for(int i=0;i<2;++i){
            int& tmp=(i?ans.second:ans.first);
            for(int x:nw[i])tmp=add(tmp,
                binpow(P[i], x));
            tmp=add(mul(tmp,Q[i]),R[i]);
        }
        return ans;
    }

    // Isomorphism
    bool iso(Tree& t){
        vi a=get_centers();
        vi b=t.get_centers();
        for(int x:a)for(int y:b)if(hash(x)==t.
            hash(y))return 1;
        return 0;
    }

    vi get_centers(){
        auto du=bfs(1);
        int v=max_element(all(du))-du.begin();
        auto dv=bfs(v);
        int u=max_element(all(dv))-dv.begin();
        du=bfs(u);
        vi ans;

```

```

        for(int i=1;i<=n;++i){
            if(du[i]+dv[i]==du[v] && du[i]>=
                du[v]/2 && dv[i]>=du[v]/2){
                ans.push_back(i);
            }
        }
        return ans;
    }
}

vi bfs(int s){
    queue<int> q;
    vi d(n+1, n*2);
    d[0]=-1;
    q.push(s);
    d[s]=0;
    while(!q.empty()){
        int u=q.front();
        q.pop();
        for(int v:g[u])
            if(d[u]+1<d[v]){
                d[v]=d[u]+1;
                q.push(v);
            }
    }
    return d;
}

};

```

## 2.3 Heavy Light Decomposition

```

typedef long long T;
T oper(T a, T b){return max(a,b);}
T null=-1e18;
struct SegTree{}; // Add Segment tree

// O(nlog(n)) build
// O(log(n)^2) (query - update) path
// O(log(n)) (query - update) subtree, node

// 1) call build(adj,n,root)
struct HLD{
    SegTree st;
    vector<vi> adj;
    vi dad,root,dep,sz,pos;
    int time;
    bool edges=false; // if the values are on edges
                        // instead of nodes

    void build(vector<vi>& adj2, int n, int v=0){ //
        v is the root
        adj=adj2;
        dad.resize(n);
        root.resize(n);
        dep.resize(n);
        sz.resize(n);
    }
}

```

```

pos.resize(n);
root[v]=dad[v]=v;
dep[v]=time=0;

dfsSz(v);
dfsHld(v);
// vector<T> palst(n);
// for(int i=0;i<n;++i)palst[pos[i]]=vals[i];
// st.build(palst);
st.build(n);
}

void dfsSz(int x){
    sz[x]=0;
    for(int& y:adj[x]){
        if(y==dad[x])continue;
        dad[y]=x;dep[y]=dep[x]+1;
        dfsSz(y);
        sz[x]+=sz[y]+1;
        if(sz[y]>sz[adj[x][0]])swap(y,adj[x][0]);
    }
}

void dfsHld(int x){
    pos[x]=time++;
    for(int y:adj[x]){
        if(y==dad[x])continue;
        root[y]=(y==adj[x][0]?root[x]:y);
        dfsHld(y);
    }
}

// O(log(n)^2)
template <class Oper>
void processPath(int x, int y, Oper op){
    for(;root[x]!=root[y];y=dad[root[y]]){
        if(dep[root[x]]>dep[root[y]])swap(x,y);
        op(pos[root[y]],pos[y]);
    }
    if(dep[x]>dep[y])swap(x,y);
    op(pos[x]+edges,pos[y]);
}

void modifyPath(int x, int y, int v){
    processPath(x,y,[this,&v](int l, int r){
        st.upd(l,r,v);
    });
}

T queryPath(int x, int y){
    T res=null;
    processPath(x,y,[this,&res](int l, int r){
        res=oper(res, st.get(l,r));
    });
}

```

```

    });
    return res;
}
// O(log(n))
void modifySubtree(int x, int v){
    st.upd(pos[x]+edges, pos[x]+sz[x], v);
}
T querySubtree(int x){
    return st.get(pos[x]+edges, pos[x]+sz[x]);
}
void modify(int x, int v){st.set(pos[x], v);}
void modifyEdge(int x, int y, int v){
    if(dep[x]<dep[y]) swap(x, y);
    modify(x, v);
}
};

```

## 2.4 Kruskal Reconstruction Tree

*// Kruskal Reconstruction Tree (KRT)*  
*// the main idea is to build a tree to efficiently answer queries*  
*// about the minimum or maximum edge weight between two nodes.*  
*// each edge will be represented as a node in the tree.*  
*// query(a,b) = lca(a,b)*  
*// Add LCA*

```

const int maxn = 1e5+5;
const int maxm = 2e5+5;
vector<vi> adj;

// sometimes it is useful
int ver[2*(maxn+maxm)]; // node at position i in euler tour
int st[maxn+maxm]; // start time of v
int ft[maxn+maxm]; // finish time of v
struct DSU{
    vi p, size;
    vector<bool> roots; // if the graph is a forest
    DSU(int n){
        p.assign(n, 0);
        size.assign(n, 1);
        roots.assign(n, true);
        for(int i=0; i<n; ++i) p[i]=i;
    }
    int get(int a){return (a==p[a]?a:p[a]=get(p[a]));}
    // unite node a and node b with the edge m => node m
    void unite(int a, int b, int m){

```

```

        a=get(a); b=get(b);
        if(a==b) return;
        size[m]=size[a]+size[b];
        p[a]=p[b]=m;
        roots[a]=false;
        roots[b]=false;
        adj[m].push_back(a);
        adj[m].push_back(b);
    }
};

```

## 2.5 LCA Binary Lifting

*// O(n\*log(n)) build*  
*// O(log(n)) kth, lca, dist*

```

struct LCA{
    vector<vi> up;
    vi dep;
    int n, maxlog;

    void build(vector<vi>& adj, int root){
        n=sz(adj);
        maxlog=ceil(log2(n))+3;
        up.assign(n, vi(maxlog, -1));
        dep.assign(n, 0);
        dfs(adj, root);
        calc(n);
    }

    void dfs(vector<vi>& adj, int v=0, int p=-1){
        up[v][0]=p;
        for(int u:adj[v]){
            if(u==p) continue;
            dep[u]=dep[v]+1;
            dfs(adj, u, v);
        }
    }

    void calc(int n){
        for(int l=1; l<maxlog; ++l){
            for(int i=0; i<n; ++i){
                if(up[i][l-1]!=-1){
                    up[i][l]=up[up[i][l-1]][l-1];
                }
            }
        }
    }

    // kth ancestor, return -1 if it doesn't exist
    int kth(int u, int k){
        for(int l=maxlog-1; l>=0; --l){
            if(u!=-1 && k&(1<<l)){
                u=up[u][l];
            }
        }
    }
}

```



```

    }
    return u;
}

int lca(int a, int b){
    // if(kth(a, dep[a])!=kth(b, dep[b]))
    // return -1; // forest
    a=kth(a, dep[a]-min(dep[a], dep[b]));
    b=kth(b, dep[b]-min(dep[a], dep[b]));
    if(a==b) return a;
    for(int l=maxlog-1; l>=0; --l){
        if(up[a][l]!=up[b][l]){
            a=up[a][l];
            b=up[b][l];
        }
    }
    return up[a][0];
}

int dist(int a, int b){
    return dep[a]+dep[b]-2*dep[lca(a,b)];
}
};

```

## 2.6 LCA RMQ

```

// Add RMQ - Min
typedef int T;
struct Table{
    void build(vector<T>& a);
    int get(int l, int r);
};

// O(n*log(n)) build
// O(1) lca
struct LCA{
    Table rmq;
    vi time, path, tmp;
    int n, ti;

    void build(vector<vi>& adj, int root){
        path.clear(); tmp.clear();
        n=sz(adj); ti=0;
        time.resize(n);
        dfs(adj, root);
        rmq.build(tmp);
    }

    void dfs(vector<vi>& adj, int u, int p=-1){
        time[u]=ti++;
        for(int v:adj[u]){
            if(v==p) continue;
            path.push_back(u);
            tmp.push_back(time[u]);
            dfs(adj, v, u);
        }
    }
};

```

```

    }
}

int lca(int a, int b){ // check forest
    if(a==b) return a;
    a=time[a], b=time[b];
    if(a>b) swap(a,b);
    return path[rmq.get(a,b-1)];
}
};

```

## 2.7 Sack

```

const int maxn = 1e5+5;
vi adj[maxn];
int ver[2*maxn]; // nodo en la posicion i del euler tour
int len[maxn]; // tamaño del subarbol de v
int st[maxn]; // tiempo inicial de v
int ft[maxn]; // tiempo final de v
int pos=0;

// O(n*log(n))
// 1) dfs0(root);
// 2) dfs1(root);

void dfs0(int v=0, int p=-1){
    len[v]=1;
    ver[pos]=v;
    st[v]=pos++;
    for(int u:adj[v]){
        if(u==p) continue;
        dfs0(u, v);
        len[v]+=len[u];
    }
    ver[pos]=v;
    ft[v]=pos++;
}

bool vis[maxn];
void ask(int v, bool add){
    if(vis[v] && !add){
        vis[v]=false;
        // eliminar nodo v
        // ...
    }else if(!vis[v] && add){
        vis[v]=true;
        // anadir nodo v
        // ...
    }
}

void dfs1(int v=0, int p=-1, bool keep=true){
    int mx=0, id=-1;
    for(int u:adj[v]){
        if(u==p) continue;
        dfs1(u, v, keep);
        if(len[u]>mx){
            mx=len[u];
            id=u;
        }
    }
    if(!keep) return;
    path.push_back(id);
    tmp.push_back(time[id]);
    dfs1(v, id, keep);
}

```

```

        if(len[u]>mx){
            mx=len[u];
            id=u;
        }
    }
    for(int u:adj[v]){
        if(u!=p && u!=id)
            dfs1(u,v,0);
    }
    if(id!=-1)dfs1(id,v,1);
    for(int u:adj[v]){
        if(u==p || u==id)continue;
        for(int p=st[u];p<ft[u];++p)
            ask(ver[p], 1);
    }
    ask(v, 1);
    // responder las consultas relacionadas con el
    // subarbol de v
    // ...
    if(keep) return;
    for(int p=st[v];p<ft[v];++p)
        ask(ver[p], 0);
}

```

## 2.8 Virtual Tree

```

// O(k*log(k))
// 1) build(n,root,adj);
// 2) query(nodes);

LCA g; // Add LCA
int lca(int a, int b){return g.lca(a,b);};
struct VirtualTree{
    vector<vi> adj,adjVT;
    vector<int> st,ft;
    vector<bool> important;
    int pos=0;

    void build(vector<vi>& adj2, int n, int root){
        important.assign(n,false);
        adjVT.assign(n,vi());
        st.resize(n);
        ft.resize(n);
        adj=adj2;pos=0;
        dfs(root);
    }

    void dfs(int v, int p=-1){
        st[v]=pos++;
        for(int u:adj[v]){
            if(u==p)continue;
            dfs(u, v);
        }
        ft[v]=pos++;
    }
}

```

```

}

bool upper(int v, int u){return st[v]<=st[u] &&
    ft[v]>=ft[u];}

int getRootVirtualTree(vi nodes){
    sort(all(nodes), [&](int v, int u){
        return st[v] < st[u]; });
    int m=sz(nodes);
    for(int i=0;i<m-1;++i){
        int v=lca(nodes[i], nodes[i+1]);
        nodes.push_back(v);
    }

    sort(all(nodes), [&](int v, int u){
        return st[v] < st[u]; });
    nodes.erase(unique(all(nodes)), nodes.end()
    ());
    for(int u:nodes)adjVT[u].clear();

    vi s;
    s.push_back(nodes[0]);
    m=sz(nodes);
    for(int i=1;i<m;++i){
        int v=nodes[i];
        while(sz(s)>=2 && !upper(s.back()
            , v)){
            adjVT[s[sz(s)-2]].
                push_back(s.back());
            s.pop_back();
        }
        s.push_back(v);
    }
    while(sz(s)>=2){
        adjVT[s[sz(s)-2]].push_back(s.
            back());
        s.pop_back();
    }
    return s[0];
}

void dfs2(int u, int p=-1){
    if(important[u]){
        // pass
    }else{
        // pass
    }
    for(int v:adjVT[u]){
        if(v==p)continue;
        dfs2(v,u);
    }
}

void query(vi& nodes){
    for(int u:nodes)important[u]=true;
    int root=getRootVirtualTree(nodes);
    dfs2(root);
}

```

```

        // cout ans
        for(int u:nodes) important[u]=false;
    }
};

```

## 3 Estructuras de Datos

### 3.1 Bit

```

// O(n) build
// O(log(n)) get, upd
typedef long long T;
struct BIT{
    vector<T> t;
    int n;

    BIT(int _n){
        n=_n;
        t.assign(n+1,0);
    }
    void upd(int i, T v){ // add v to ith element
        for(int j=i+1;j<=n;j+=j&-j)t[j]+=v;
    }
    T get(int i){ // get sum of range [0,i0]
        T ans=0;
        for(int j=i;j;j-=j&-j)ans+=t[j];
        return ans;
    }
    T get(int l, int r){ // get sum of range [l,r]
        return get(r+1)-get(l);
    }
};

```

### 3.2 Bit 2D

```

// O(n*m) build
// O(log(n)*log(m)) get, upd
typedef long long T;
struct BIT2D{
    vector<vector<T>> bit;
    int n,m;

    BIT2D(int _n, int _m){
        n=_n;m=_m;
        bit.assign(n+1, vector<T>(m+1,0));
    }
    T get(int x, int y){
        if(x<0 || y<0)return 0;
        T v=0;
        for(int i=x+1;i;i-=i&-i)
            for(int j=y+1;j;j-=j&-j)v+=bit[i][j];
    }
};

```

```

        return v;
    }
    T get(int x, int y, int x2, int y2){
        return get(x2,y2)-get(x-1,y2)-get(x2,y-1)
            +get(x-1,y-1);
    }
    void upd(int x, int y, T dt){
        if(x<0 || y<0)return;
        for(int i=x+1;i<=n;i+=i&-i)
            for(int j=y+1;j<=m;j+=j&-j)bit[i][j]+=dt;
    }
};

```

### 3.3 Cartesian Tree

```

// O(n) build
typedef long long T;
struct CartesianTree{ // 1-indexed
    vector<int> l,r;
    int root,n;

    CartesianTree(vector<T>& a){
        reverse(all(a));
        a.push_back(0);
        reverse(all(a));
        int tot=0;n=sz(a)-1;
        l.assign(n+1,0);
        r.assign(n+1,0);
        vector<int> s(n+1,0);
        vector<bool> vis(n+1,false);
        for(int i=1;i<=n;++i){
            int k=tot;
            while(k>0 && a[s[k-1]]>a[i])k--;
            // < max heap
            if(k)r[s[k-1]]=i;
            if(k<tot)l[i]=s[k];
            s[k++]=i;
            tot=k;
        }
        for(int i=1;i<=n;++i)vis[l[i]]=vis[r[i]]
            =1;
        root=0;
        for(int i=1;i<=n;++i){
            if(!vis[i])root=i;
        }
    }
};

```

### 3.4 Disjoint Set Union

```

struct dsu{

```

```

vi p, size;
int sets, maxSize;

dsu(int n){
    p.assign(n, 0);
    size.assign(n, 1);
    sets = n;
    for (int i = 0; i < n; i++) p[i] = i;
}

int find_set(int i) {return p[i] == i ? i : (p[i] = find_set(p[i]));}

bool is_same_set(int i, int j) {return find_set(i) == find_set(j);}

void unionSet(int i, int j){
    if (!is_same_set(i, j)){
        int a = find_set(i), b = find_set(j);
        if (size[a] < size[b]) swap(a, b);
        p[b] = a;
        size[a] += size[b];
        maxSize = max(size[a], maxSize);
        sets--;
    }
}

};

```

### 3.5 Disjoint Sparse Table

```

// lo mismo que sparse table, pero para st ops
// O(n*log(n)) build
// O(1) get
typedef int T;
T null = 0;
T op(T a, T b){return a^b;}
struct DST {
    vector<vector<T>> pre, suf;
    int k, n;
    DST(vector<T>& a) {
        n = sz(a);
        k = log2(n) + 2;
        pre.assign(k + 1, vector<T>(n));
        suf.assign(k + 1, vector<T>(n));
        for(int j = 0; (1 <= j) <= n; ++j) {
            int mask = (1 << j) - 1;
            T nw = null;
            for(int i = 0; i < n; ++i) {
                nw = op(nw, a[i]);
                pre[j][i] = nw;
                if((i & mask) == mask) nw = null;
            }
        }
    }
};

```

```

nw = null;
for(int i = n - 1; i >= 0; --i) {
    nw = op(a[i], nw);
    suf[j][i] = nw;
    if((i & mask) == 0) nw = null;
}
}

T get(int l, int r) {
    if(l == r) return pre[0][l];
    int i = 31 - __builtin_clz(l ^ r);
    return op(suf[i][l], pre[i][r]);
}

};

```

### 3.6 Dynamic Connectivity Offline

```

typedef pair<int, int> ii;
struct DSU {
    vector<int> p, size, h;
    int sets;
    void build(int n){
        sets = n;
        p.assign(n, 0);
        size.assign(n, 1);
        for(int i = 0; i < n; ++i) p[i] = i;
    }
    int get(int a){return (a == p[a] ? a : get(p[a]));}
    void unite(int a, int b){
        a = get(a); b = get(b);
        if(a == b) return;
        if(size[a] > size[b]) swap(a, b);
        h.push_back(a);
        size[b] += size[a];
        p[a] = b; sets--;
    }
    void rollback(int s){
        while(sz(h) > s){
            int a = h.back();
            h.pop_back();
            size[p[a]] -= size[a];
            p[a] = a; sets++;
        }
    }
};

// O(q*log(q)*log(n))
enum { ADD, DEL, QUERY };
struct Query { int type, u, v; };
struct DynCon {
    map<ii, int> edges; DSU uf;
    vector<Query> q;
    vector<int> t;
};

```

```

void add(int u, int v){
    if(u>v) swap(u,v);
    edges[{u,v}]=sz(q);
    q.push_back({ADD, u, v});
    t.push_back(-1);
}
void del(int u, int v){
    if(u>v) swap(u,v);
    int i=edges[{u,v}];
    t[i]=sz(q);
    q.push_back({DEL, u, v});
    t.push_back(i);
}
void query(){
    q.push_back({QUERY, -1, -1});
    t.push_back(-1);
}
void dnc(int l, int r){
    if(r-l==1){
        if(q[l].type==QUERY)
            cout<<uf.sets<<"\n";
        return;
    }
    int m=l+(r-l)/2,k=sz(uf.h);
    for(int i=m;i<r;++i)
        if(q[i].type==DEL && t[i]<l)
            uf.unite(q[i].u, q[i].v);

    dnc(l, m);
    uf.rollback(k);
    for(int i=l;i<m;++i)
        if(q[i].type==ADD && t[i]>=r)
            uf.unite(q[i].u, q[i].v);

    dnc(m, r);
    uf.rollback(k);
}
void init(int n){
    uf.build(n);
    if(!sz(q)) return;
    for(int& ti:t) if(ti==-1) ti=sz(q);
    dnc(0, sz(q));
}
};

```

### 3.7 DSU Bipartite

```

// Bipartite graph
// get return the leader and the parity of the distance
// to the leader
typedef pair<int, int> ii;
struct DSU{
    vector<int> p,size,len;
    DSU(int n){
        p.assign(n,0);

```

```

        len.assign(n,0);
        size.assign(n,1);
        for(int i=0;i<n;++i) p[i]=i;
    }
    ii get(int a){
        if(a==p[a]) return {a, 0};
        ii va=get(p[a]);
        p[a]=va.first;
        len[a]=(len[a]+va.second)%2;
        return {p[a], len[a]};
    }
    void unite(int a, int b){
        ii va=get(a);
        ii vb=get(b);
        if(va.first==vb.first) return;
        if(size[va.first]>size[vb.first]) swap(va,
            vb);
        p[va.first]=vb.first;
        len[va.first]=(va.second+vb.second+1)%2;
        size[vb.first]+=size[va.first];
    }
};

```

### 3.8 Dynamic Segment Tree

```

// O(q*log(n)), q => queries
typedef long long T;
T null=0, noVal=0;
T oper(T a, T b){return a+b;}
struct Node{
    T val,lz;
    int l,r;
    Node *pl,*pr;
    Node(int ll, int rr){
        val=null;lz=noVal;
        pl=pr=nullptr;
        l=ll;r=rr;
    }
    void update(){
        if(r-l==1) return;
        val=oper(pl->val, pr->val);
    }
    void update(T v){
        val+=(T)(r-l)*v;
        lz+=v;
    }
    void extends(){
        if(r-l!=1 && !pl){
            int m=(r+l)/2;
            pl=new Node(l, m);
            pr=new Node(m, r);
        }
    }
};

```

```

void propagate() {
    if (r-l==1) return;
    if (lz==noVal) return;
    pl->update(lz);
    pr->update(lz);
    lz=noVal;
}

};

typedef Node* PNode;
struct SegTree{
    PNode root;
    SegTree(int l, int r){root=new Node(l, r+1);}

    void upd(PNode x, int l, int r, T v){
        int lx=x->l, rx=x->r;
        if (lx>=r || l>=rx) return;
        if (lx>=l && rx<=r){
            x->update(v);
            return;
        }
        x->extends();
        x->propagate();
        upd(x->pl, l, r, v);
        upd(x->pr, l, r, v);
        x->update();
    }

    T get(PNode x, int l, int r){
        int lx=x->l, rx=x->r;
        if (lx>=r || l>=rx) return null;
        if (lx>=l && rx<=r) return x->val;
        x->extends();
        x->propagate();
        T v1=get(x->pl, l, r);
        T v2=get(x->pr, l, r);
        return oper(v1, v2);
    }

    T get(int l, int r){return get(root, l, r+1);}
    void upd(int l, int r, T v){upd(root, l, r+1, v);}
};

```

### 3.9 Implicit Treap

```

// Treap => Binary Search Tree + Binary Heap
// 1. create a empty root (PTreap root=nullptr);
// 2. Append the nodes in order (left -> right)
// PTreap tmp=new Treap(x);
// root=merge(root, tmp);

typedef long long T;
typedef unsigned long long u64;
mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());

```

```

T null = 0;
struct Treap{
    Treap *l,*r; // left child, right child
    u64 prior; // random
    T val,sum,lz; // value, sum subtree, lazy
    int sz; // size subtree
    Treap(T v){
        l=r=nullptr;
        prior=rng();
        val=sum=v;
        lz=0;sz=1;
    }
    ~Treap(){
        delete l;
        delete r;
    }
};

typedef Treap* PTreap;
int cnt(PTreap x){return (!x?0:x->sz);}
T sum(PTreap x){return (!x?0:x->sum);}

void update_helper(PTreap x, T v){
    // lz += v
    // val += v
    // sum += v
}

// propagate the lazy
void push(PTreap x){
    if (x && x->lz){ // check x->lz
        if (x->l) update_helper(x->l, 1);
        if (x->r) update_helper(x->r, 1);
        x->lz=0;
    }
}

// updates node with its children information
void pull(PTreap x){
    push(x->l);
    push(x->r);
    x->sz=cnt(x->l)+cnt(x->r)+1;
    x->sum=sum(x->l)+sum(x->r)+x->val;
}

// Updates node value += v
void upd(PTreap x, T v){
    if (!x) return;
    pull(x);
    update_helper(x, v);
}

// O(log(n)) divide the treap in two parts
// [count nodes == left], [the rest of nodes]
pair<PTreap, PTreap> split(PTreap x, int left){
    if (!x) return {nullptr, nullptr};
}

```

```

push(x);
if(cnt(x->l)>=left){
    auto got=split(x->l, left);
    x->l=got.second;
    pull(x);
    return {got.first, x};
} else{
    auto got=split(x->r, left-cnt(x->l)-1);
    x->r=got.first;
    pull(x);
    return {x, got.second};
}
}

// O(log(n)) merge two treap
// [nodes treap x ... nodes treap y]
PTreap merge(PTreap x, PTreap y){
    if(!x) return y;
    if(!y) return x;
    push(x); push(y);
    if(x->prior<=y->prior){
        x->r=merge(x->r, y);
        pull(x);
        return x;
    } else{
        y->l=merge(x, y->l);
        pull(y);
        return y;
    }
}

// O(n) print the treap
void dfs(PTreap x){
    if(!x) return;
    push(x);
    dfs(x->l);
    cout<<x->val<<" ";
    dfs(x->r);
}

```

### 3.10 Implicit Treap Father

```

// Treap => Binary Search Tree + Binary Heap
// 1. create a empty root (PTreap root=nullptr);
// 2. Append the nodes in order (left -> right)
// PTreap tmp=new Treap(x);
// root=merge(root, tmp);

// si se edita un treap, se tiene que hacer un pullAll
// hasta la raiz
// si no se hace esto, el treap queda con informacion
// pasada

// si se va a modificar un treap, hacer un pushAll para
// bajar los lazy

```

```

typedef long long T;
typedef unsigned long long u64;
mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());

T null = 0;
struct Treap{
    Treap *l,*r,*dad; // left child, right child
    u64 prior; // random
    T val,sum; // value, sum subtree
    int sz; // size subtree
    Treap(T v){
        l=r=dad=nullptr;
        prior=rng();
        val=sum=v;
        sz=1;
    }
    ~Treap(){
        delete l;
        delete r;
    }
};

typedef Treap* PTreap;
int cnt(PTreap x){return (!x?0:x->sz);}
T sum(PTreap x){return (!x?0:x->sum);}

// updates node with its children information
void pull(PTreap x){
    x->sz=cnt(x->l)+cnt(x->r)+1;
    x->sum=sum(x->l)+sum(x->r)+x->val;
    if(x->l)x->l->dad=x; //
    if(x->r)x->r->dad=x; //
}

// O(log(n)) divide the treap in two parts
// [count nodes == left], [the rest of nodes]
pair<PTreap, PTreap> split(PTreap x, int left){
    if(!x) return {nullptr, nullptr};
    if(cnt(x->l)>=left){
        auto got=split(x->l, left);
        if(got.first)got.first->dad=nullptr; //
        x->l=got.second;
        x->dad=nullptr; //
        pull(x);
        return {got.first, x};
    } else{
        auto got=split(x->r, left-cnt(x->l)-1);
        if(got.second)got.second->dad=nullptr; //
        x->r=got.first;
        x->dad=nullptr; //
        pull(x);
        return {x, got.second};
    }
}

```

```

// O(log(n)) merge two treap
// [nodes treap x ... nodes treap y]
PTreap merge(PTreap x, PTreap y){
    if(!x) return y;
    if(!y) return x;
    if(x->prior<=y->prior){
        x->r=merge(x->r, y);
        pull(x);
        return x;
    }else{
        y->l=merge(x, y->l);
        pull(y);
        return y;
    }
}

// O(log(n)) propagate the lazy [root->x]
void pushAll(PTreap x){
    if(!x) return;
    pushAll(x->dad);
    push(x);
}

// O(log(n)) update the treap [root->x]
void pullAll(PTreap x){
    if(!x) return;
    pull(x);
    pullAll(x->dad);
}

// O(log(n)) return the root and the position of x (1-
// indexed)
pair<PTreap, int> findRoot(PTreap x){
    pushAll(x);
    int pos=cnt(x->l);
    while(x->dad){
        PTreap f=x->dad;
        if(x==f->r) pos+=cnt(f->l)+1;
        x=f;
    }
    return {x, pos+1};
}

```

### 3.11 Li Chao

```

// inf max abs value that the function may take
typedef long long ty;
struct Line {
    ty m, b;
    Line(){}
    Line(ty m, ty b): m(m), b(b){}
    ty eval(ty x){return m * x + b;}
};

```

```

struct nLiChao{
    // see coments for min
    nLiChao *left = nullptr, *right = nullptr;
    ty l, r;
    Line line;
    nLiChao(ty l, ty r): l(l), r(r){
        line = {0, -inf}; // change to {0, inf};
    }
    // T(Log(Rango)) M(Log(rango))
    void addLine(Line nline){
        ty m = (l + r) >> 1;
        bool lef = nline.eval(l) > line.eval(l);
        // change > to <
        bool mid = nline.eval(m) > line.eval(m);
        // change > to <
        if (mid) swap(nline, line);
        if (r == l) return;
        if (lef != mid){
            if (!left){
                left = new nLiChao(l, m);
                left -> line = nline;
            }
            else left -> addLine(nline);
        }
        else{
            if (!right){
                right = new nLiChao(m +
                    1, r);
                right -> line = nline;
            }
            else right -> addLine(nline);
        }
    }
    // T(Log(Rango))
    ty get(ty x) {
        ty m = (l + r) >> 1;
        ty op1 = -inf, op2 = -inf; // change to
        inf
        if(l == r) return line.eval(x);
        else if(x < m){
            if (left) op1 = left -> get(x);
            return max(line.eval(x), op1); //
            change max to min
        }
        else{
            if (right) op2 = right -> get(x);
            return max(line.eval(x), op2); //

```



```

                                change max to min
        }
    };

    int main() {
        // (rango superior) * (pendiente maxima) puede
        // desbordarse
        // usar double o long double en el eval para
        // estos casos
        // (puede dar problemas de precision)
        nLiChao liChao(0, 1e18);
    }

```

### 3.12 Link Cut Tree

```

// 1-indexed
// All operations are O(log(n))
typedef long long T;
struct SplayTree{
    struct Node{
        int ch[2]={0, 0}, p=0;
        T val=0, path=0; // values for path
        T sub=0, vir=0; // values for subtree
        bool flip=0; // values for lazy
    };
    vector<Node> ns;
    SplayTree(int n):ns(n+1){}
    T path(int u){return (u?ns[u].path:0);}
    T subsum(int u){return (u?ns[u].sub:0);}
    void push(int x){
        if(!x) return;
        int l=ns[x].ch[0], r=ns[x].ch[1];
        if(ns[x].flip){
            ns[l].flip^=1, ns[r].flip^=1;
            swap(ns[x].ch[0], ns[x].ch[1]);
            // if the operation is like a
            // segment tree
            // check swap the values
            ns[x].flip=0;
        }
    }
    void pull(int x){
        int l=ns[x].ch[0], r=ns[x].ch[1];
        push(l); push(r);
        ns[x].path=max({path(l), path(r), ns[x].
            val});
        ns[x].sub=ns[x].vir+subsum(l)+subsum(r)+
            ns[x].val;
    }
    void set(int x, int d, int y){ns[x].ch[d]=y; ns[y]
        .p=x; pull(x);}
    void splay(int x){

```

```

        auto dir=[&](int x){
            int p=ns[x].p; if(!p) return -1;
            return ns[p].ch[0]==x?0:ns[p].ch
                [1]==x?1:-1;
        };
        auto rotate=[&](int x){
            int y=ns[x].p, z=ns[y].p, dx=dir(x)
                , dy=dir(y);
            set(y, dx, ns[x].ch[!dx]);
            set(x, !dx, y);
            if(~dy) set(z, dy, x);
            ns[x].p=z;
        };
        for(push(x); ~dir(x);){
            int y=ns[x].p, z=ns[y].p;
            push(z); push(y); push(x);
            int dx=dir(x), dy=dir(y);
            if(~dy) rotate(dx!=dy?x:y);
            rotate(x);
        }
    };

    struct LinkCut:SplayTree{
        LinkCut(int n):SplayTree(n){}
        // return the root of us tree
        int root(int u){
            access(u); splay(u); push(u);
            while(ns[u].ch[0]){u=ns[u].ch[0]; push(u)
                ;}
            return splay(u), u;
        }
        // return the parent of u
        int parent(int u){
            access(u); splay(u); push(u);
            u=ns[u].ch[0]; push(u);
            while(ns[u].ch[1]){u=ns[u].ch[1]; push(u)
                ;}
            return splay(u), u;
        }
        int access(int x){
            int u=x, v=0;
            for(; u; v=u, u=ns[u].p){
                splay(u);
                int& ov=ns[u].ch[1];
                ns[u].vir+=ns[ov].sub;
                ns[u].vir-=ns[v].sub;
                ov=v; pull(u);
            }
            return splay(x), v;
        }
        // reroot the tree with x as root
        void reroot(int x){
            access(x); ns[x].flip^=1; push(x);
        }
    };

```

```

// create a edge u->v, u is the child of v
void link(int u, int v){
    reroot(u);access(v);
    ns[v].vir+=ns[u].sub;
    ns[u].p=v;pull(v);
}
// delete the edge u->v, u is the child of v
void cut(int u, int v){
    int r=root(u);
    reroot(u);access(v);
    ns[v].ch[0]=ns[u].p=0;pull(v);
    reroot(r);
}
// delete the edge u->parent(u)
void cut(int u){
    access(u);
    ns[ns[u].ch[0]].p=0;
    ns[u].ch[0]=0;pull(u);
}
int lca(int u, int v){
    if(root(u)!=root(v))return -1;
    access(u);return access(v);
}
// return sum of the subtree of u with v as
// father
T subtree(int u, int v){
    int r=root(u);
    reroot(v);access(u);
    T ans=ns[u].vir+ns[u].val;
    return reroot(r),ans;
}
T path(int u, int v){
    int r=root(u);
    reroot(u);access(v);pull(v);
    T ans=ns[v].path;
    return reroot(r),ans;
}
void set(int u, T val){
    access(u);
    ns[u].val=val;
    pull(u);
}
};

```

### 3.13 Link Cut Tree Lazy

```

// 1-indexed
// All operations are O(log(n))
typedef long long T;
struct SplayTree{
    struct Node{
        int ch[2]={0, 0},p=0;
        T val=0,path=0,sz=1; // values for path

```

```

        T sub=0,vir=0,ssz=0,vsz=0; // values for
        subtree
        bool flip=0;T lz=0; // values for lazy
    };
    vector<Node> ns;
    SplayTree(int n):ns(n+1){}
    T path(int u){return (u?ns[u].path:0);}
    T size(int u){return (u?ns[u].sz:0);}
    T subsize(int u){return (u?ns[u].ssz:0);}
    T subsum(int u){return (u?ns[u].sub:0);}
    void push(int x){
        if(!x)return;
        int l=ns[x].ch[0],r=ns[x].ch[1];
        if(ns[x].flip){
            ns[l].flip^=1,ns[r].flip^=1;
            swap(ns[x].ch[0], ns[x].ch[1]);
            // if the operation is like a
            // segment tree
            // check swap the values
            ns[x].flip=0;
        }
        if(ns[x].lz){ // check the lazy
            // propagate the lazy
            ns[x].sub+=ns[x].lz*ns[x].ssz;
            ns[x].vir+=ns[x].lz*ns[x].vsz;
            // ...
        }
    }
    void pull(int x){
        int l=ns[x].ch[0],r=ns[x].ch[1];
        push(l);push(r);
        ns[x].sz=size(l)+size(r)+1;
        ns[x].path=max({path(l), path(r), ns[x].
            val});
        ns[x].sub=ns[x].vir+subsum(l)+subsum(r)+
            ns[x].val;
        ns[x].ssz=ns[x].vsz+subsize(l)+subsize(r)
            +1;
    }
    void set(int x, int d, int y){ns[x].ch[d]=y;ns[y]
        ].p=x;pull(x);}
    void splay(int x){
        auto dir=[&](int x){
            int p=ns[x].p;if(!p)return -1;
            return ns[p].ch[0]==x?0:ns[p].ch
                [1]==x?1:-1;
        };
        auto rotate=[&](int x){
            int y=ns[x].p,z=ns[y].p,dx=dir(x)
                ,dy=dir(y);
            set(y,dx,ns[x].ch[!dx]);
            set(x,!dx,y);

```

```

        if (~dy) set (z, dy, x);
        ns[x].p=z;
    };
    for(push(x); ~dir(x);) {
        int y=ns[x].p, z=ns[y].p;
        push(z); push(y); push(x);
        int dx=dir(x), dy=dir(y);
        if (~dy) rotate(dx!=dy?x:y);
        rotate(x);
    }
};

struct LinkCut:SplayTree{
    LinkCut(int n):SplayTree(n) {}
    // return the root of us tree
    int root(int u){
        access(u); splay(u); push(u);
        while(ns[u].ch[0]) {u=ns[u].ch[0]; push(u);}
        return splay(u), u;
    }
    // return the parent of u
    int parent(int u){
        access(u); splay(u); push(u);
        u=ns[u].ch[0]; push(u);
        while(ns[u].ch[1]) {u=ns[u].ch[1]; push(u);}
        return splay(u), u;
    }
    int access(int x){
        int u=x, v=0;
        for(; u; v=u, u=ns[u].p) {
            splay(u);
            int& ov=ns[u].ch[1];
            ns[u].vir+=ns[ov].sub;
            ns[u].vsz+=ns[ov].ssz;
            ns[u].vir-=ns[v].sub;
            ns[u].vsz-=ns[v].ssz;
            ov=v; pull(u);
        }
        return splay(x), v;
    }
    // reroot the tree with x as root
    void reroot(int x){
        access(x); ns[x].flip^=1; push(x);
    }
    // create a edge u->v, u is the child of v
    void link(int u, int v){
        reroot(u);
        access(v);
        ns[v].vir+=ns[u].sub;

```

```

        ns[v].vsz+=ns[u].ssz;
        ns[u].p=v; pull(v);
    }
    // delete the edge u->v, u is the child of v
    void cut(int u, int v){
        int r=root(u);
        reroot(u);
        access(v);
        ns[v].ch[0]=ns[u].p=0; pull(v);
        reroot(r);
    }
    // delete the edge u->parent(u)
    void cut(int u){
        access(u);
        ns[ns[u].ch[0]].p=0;
        ns[u].ch[0]=0; pull(u);
    }
    int lca(int u, int v){
        if(root(u)!=root(v)) return -1;
        access(u); return access(v);
    }
    int depth(int u){
        int r=root(u);
        reroot(r);
        access(u); splay(u); push(u);
        return ns[u].sz-1;
    }
    T path(int u, int v){
        int r=root(u);
        reroot(u); access(v); pull(v);
        T ans=ns[v].path;
        return reroot(r), ans;
    }
    void set(int u, T val){
        access(u);
        ns[u].val=val;
        pull(u);
    }
    // update the value of the nodes in the path u->v
    // with += val
    void upd(int u, int v, T val){
        int r=root(u);
        reroot(u); access(v); splay(v);
        // change only the lazy
        // ns[v].val+=val;
        reroot(r);
    }
    T comp_size(int u){return ns[root(u)].ssz;}
    T subtree_size(int u){
        int p=parent(u);

```

```

        if(!p) return comp_size(u);
        cut(u); int ans=comp_size(u);
        link(u,p); return ans;
    }
    T subtree_size(int u, int v){ // subtree of u
        with v as father
        int r=root(u);
        reroot(v); access(u);
        T ans=ns[u].vsz+1;
        return reroot(r),ans;
    }
    T comp_sum(int u){ return ns[root(u)].sub; }
    T subtree_sum(int u){
        int p=parent(u);
        if(!p) return comp_sum(u);
        cut(u); T ans=comp_sum(u);
        link(u,p); return ans;
    }
    T subtree_sum(int u, int v){ // subtree of u with
        v as father
        int r=root(u);
        reroot(v); access(u);
        T ans=ns[u].vir+ns[u].val; // por el
        reroot
        return reroot(r),ans;
    }
};

```

### 3.14 Merge Sort Tree

```

// O(n*log(n)) build
// O(log(n)^2) get
typedef long long T;
struct SegTree{
    int size;
    vector<vector<T>> vals;
    void oper(int x){
        merge(all(vals[2*x+1]), all(vals[2*x+2]),
            back_inserter(vals[x]));
    }
    SegTree(vector<T>& a){
        size=1;
        while(size<sz(a)) size*=2;
        vals.resize(2*size);
        build(a, 0, 0, size);
    }
    void build(vector<T>& a, int x, int lx, int rx){
        if(rx-lx==1){
            if(lx<sz(a)) vals[x]={a[lx]};
            return;
        }
        int m=(lx+rx)/2;

```

```

        build(a, 2*x+1, lx, m);
        build(a, 2*x+2, m, rx);
        oper(x);
    }
    int get(int l, int r, int val, int x, int lx, int
        rx){
        if(lx>=r || l>=rx) return 0;
        if(lx>=l && rx<=r){
            return upper_bound(all(vals[x]),
                val)-vals[x].begin();
        }
        int m=(lx+rx)/2;
        int v1=get(l, r, val, 2*x+1, lx, m);
        int v2=get(l, r, val, 2*x+2, m, rx);
        return v1+v2;
    }
    int get(int l, int r, int val){ return get(l, r+1,
        val, 0, 0, size); }
};

```

### 3.15 MOs Algorithm

```

// O((n+q)*sq), sq=n^(1/2)
// 1. fill queries[]
// 2. solve(n);
// 3. print ans[]
int sq;
struct query {int l,r,idx;};
bool cmp(query& a, query& b){
    int x=a.l/sq;
    if(a.l/sq!=b.l/sq) return a.l/sq<b.l/sq;
    return (x&1?a.r<b.r:a.r>b.r);
}

vector<query> queries;
vector<ll> ans;

ll act();
void add(int i); // add a[i]
void remove(int i); // remove a[i]

void solve(int n){
    sq=ceil(sqrt(n));
    sort(all(queries), cmp);
    ans.assign(sz(queries), 0);
    int l=0, r=-1;
    for(auto [li,ri,i]:queries){
        while(r<ri) add(++r);
        while(l>li) add(--l);
        while(r>ri) remove(r--);
        while(l<li) remove(l++);
        ans[i]=act();
    }
}

```

}

### 3.16 MOs Tree

```
// add LCA
struct LCA{};

vector<vector<int>> adj;
const int maxn=1e5+5;
int ver[2*maxn]; // node at position i in euler tour
int st[maxn]; // start time of v
int ft[maxn]; // finish time of v
int pos=0;
LCA tree;

// O((n+q)*sq), sq=n^(1/2)
// 1. build euler tour and lca
// 2. add queries[]
// if(st[a]>st[b])swap(a,b);
// queries.push_back({st[a]+1,st[b],i});
// 3. solve(n);
// 4. print ans[]
int sq;

void dfs(int u=0, int p=-1){
    ver[pos]=u;
    st[u]=pos++;
    for(int v:adj[u]){
        if(v==p) continue;
        dfs(v,u);
    }
    ver[pos]=u;
    ft[u]=pos++;
}

struct query {int l,r,idx;};
bool cmp(query& a, query& b){
    int x=a.l/sq;
    if(a.l/sq!=b.l/sq) return a.l/sq<b.l/sq;
    return (x&1?a.r<b.r:a.r>b.r);
}

vector<query> queries;
vector<ll> ans;
bool vis[maxn];

ll act();
void add(int u); // add node u
void remove(int u); // remove node u
void ask(int u){
    if(!vis[u]) add(u);
    else remove(u);
    vis[u]=!vis[u];
}

void solve(int n){
```

```
sq=ceil(sqrt(n));
sort(all(queries), cmp);
ans.resize(sz(queries));
int l=0,r=-1;
for(auto [li,ri,i]:queries){
    while(r<ri) ask(ver[++r]);
    while(l>li) ask(ver[--l]);
    while(r>ri) ask(ver[r--]);
    while(l<li) ask(ver[l++]);
    int a=ver[l-1],b=ver[r];
    int c=tree.lca(a,b);
    ask(c);
    ans[i]=act();
    ask(c);
}
}
```

### 3.17 MOs Updates

```
// O(q*(s+(n/s)^2) => O(q*(n^(2/3))), s=(2*(n^2))^(1/3) -
// s=n^(2/3)
// 1. fill queries[] and upds[]
// dont confuse index in queries with updates, they are
// different
// the struct upd saves the old value and the new value
// 2. solve(n);
// 3. print ans[]
int sq;
struct upd{int i,old,cur;};
struct query {int l,r,t,idx;};
bool cmp(query& a, query& b){
    int x=a.l/sq;
    if(a.l/sq!=b.l/sq) return a.l/sq<b.l/sq;
    if(a.r/sq!=b.r/sq) return (x&1?a.r<b.r:a.r>b.r);
    return a.t<b.t;
}

vector<query> queries;
vector<upd> upds;
vector<ll> ans;

ll act();
void add(int i); // add a[i]
void remove(int i); // remove a[i]
void update(int i, int v, int l, int r){
    // check if the update is with an active element
    if(l<=i && i<=r){
        remove(i);
        // a[i]=v;
        // ...
        add(i);
    }
    // a[i]=v;
    // ...
}
```

```

}
void solve(int n){
    sq=ceil(pow(n,2.0/3.0));
    sort(all(queries), cmp);
    ans.resize(sz(queries));
    int l=0,r=-1,t=0;
    for(auto [li,ri,ti,i]:queries){
        while(t<ti) update(upds[t].i,upds[t].cur,l,r),++t;
        while(t>ti)--t, update(upds[t].i,upds[t].old,l,r);
        while(r<ri) add(++r);
        while(l>li) add(--l);
        while(r>ri) remove(r--);
        while(l<li) remove(l++);
        ans[i]=act();
    }
}

```

### 3.18 Ordered set

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template<typename T> using ordered_set = tree<T,
    null_type,less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
template<typename T> using ordered_multiset = tree<T,
    null_type,less_equal<T>, rb_tree_tag,
    tree_order_statistics_node_update>;
// ----- CONSTRUCTOR ----- //
// 1. Para ordenar por MAX cambiar less<int> por greater<int>
// 2. Para multiset cambiar less<int> por less_equal<int>
// Para borrar siendo multiset:
// int idx = st.order_of_key(value);
// st.erase(st.find_by_order(idx));
// st.swap(st2);
// ----- METHODS ----- //
st.find_by_order(k) // returns pointer to the k-th
    smallest element
st.order_of_key(x) // returns how many elements are
    smaller than x
st.find_by_order(k) == st.end() // true, if element does
    not exist

```

### 3.19 Persistent Segment Tree

```

// O(n*log(n)) build
// O(log(n)) get, set
// O((n+q)*log(n)) memory

```

```

typedef long long T;
struct Node{
    T val;
    int l,r; // saves the range of the node [l,r]
};
struct SegTree{
    vector<Node> ns;
    vector<int> roots; // roots of the differents
        versions
    T null=0;
    int act=0,size; // act: number of nodes

    T oper(T a, T b){return a+b;}
    SegTree(vector<T>& a, int n){
        size=n;
        roots.push_back(build(a, 0, size));
    }

    void update(int x){
        ns[x].val=oper(ns[ns[x].l].val, ns[ns[x].r].val);
    }

    int newNode(T x){
        Node tmp={x,-1,-1};
        ns.push_back(tmp);
        return act++;
    }

    int newNode(int l, int r){
        Node tmp={null,l,r};
        ns.push_back(tmp);
        update(act);
        return act++;
    }

    int build(vector<T>& a, int l, int r){
        if(r-l==1){return newNode(a[l]);}
        int m=(l+r)/2;
        return newNode(build(a, l, m),build(a, m, r));
    }

    int set(int x, int i, T v, int l, int r){
        if(r-l==1){return newNode(v);}
        int m=(l+r)/2;
        if(i<m){return newNode(set(ns[x].l, i, v, l, m), ns[x].r);}
        else return newNode(ns[x].l, set(ns[x].r, i, v, m, r));
    }

    T get(int x, int lx, int rx, int l, int r){
        if(lx>=r || l>=rx){return null;}
        if(lx>=l && rx<=r){return ns[x].val;}
        int m=(lx+rx)/2;
        T vl=get(ns[x].l, lx, m, l, r);

```

```

        T v2=get(ns[x].r, m, rx, l, r);
        return oper(v1,v2);
    }

T get(int l, int r, int time){return get(roots[
    time], 0, size, l, r+1);}
void set(int i, T v, int time){roots.push_back(
    set(roots[time], i, v, 0, size));}

};

```

### 3.20 Persistent Segment Tree Lazy

```

// O(n*log(n)) build
// O(log(n)) get, upd
// O((n+q)*log(n)) memory
typedef long long T;
struct Node {
    Node* left = nullptr;
    Node* right = nullptr;
    T val = 0, prop = 0;
};
typedef Node* PNode;
struct PerSegTree {
    vector<PNode> roots{};
    vector<T> vec{};
    int n = 0;

    T op(T a, T b){
        return a+b;
    }

    PNode newKid(PNode& curr) {
        PNode newNode = new Node();
        newNode->left = curr->left;
        newNode->right = curr->right;
        newNode->prop = curr->prop;
        newNode->val = curr->val;
        return newNode;
    }

    void lazy(int i, int j, PNode& curr) {
        if (!curr->prop) return;
        curr->val += ((T)(j - i + 1)) * curr->
            prop;
        if (i != j) {
            curr->left = newKid(curr->left);
            curr->right = newKid(curr->right);
            ;
            curr->left->prop += curr->prop;
            curr->right->prop += curr->prop;
        }
        curr->prop = 0;
    }

    PNode build(int i, int j) {
        PNode newNode = new Node();

```

```

        if (i == j) {
            newNode->val = vec[i];
        } else {
            int mid = i + (j - i) / 2;
            PNode leftt = build(i, mid);
            PNode right = build(mid + 1, j);
            newNode->val = op(leftt->val,
                right->val);
            newNode->left = leftt;
            newNode->right = right;
        }
        return newNode;
    }

PNode upd(int i, int j, int l, int r, T value,
    PNode& curr) {
    lazy(i, j, curr);
    if (i >= l && j <= r) {
        PNode newNode = newKid(curr);
        newNode->prop += value;
        lazy(i, j, newNode);
        return newNode;
    }
    if (i > r || j < l) {
        return curr;
    }
    PNode newNode = new Node();
    int mid = i + (j - i) / 2;
    newNode->left = upd(i, mid, l, r, value,
        curr->left);
    newNode->right = upd(mid + 1, j, l, r,
        value, curr->right);
    newNode->val = op(newNode->left->val,
        newNode->right->val);
    return newNode;
}

T get(int i, int j, int l, int r, PNode& curr) {
    lazy(i, j, curr);
    if (j < l || r < i) {
        return 0;
    }
    if (i >= l && j <= r) {
        return curr->val;
    }
    int mid = i + (j - i) / 2;
    return op(get(i, mid, l, r, curr->left),
        get(mid + 1, j, l, r, curr->right));
}

// public methods
void build(vector<T>& vec) {
    if (vec.empty()) return;
    n = vec.size();
    this->vec = vec;

```

```

        auto root = build(0, n - 1);
        roots.push_back(root);
    }
    void upd(int l, int r, T value, int time) {
        roots.push_back(upd(0, n - 1, l, r, value
            , roots[time]));
    }
    T get(int l, int r, int time) {
        return get(0, n - 1, l, r, roots[time]);
    }
    int size() { return roots.size(); }
};

```

### 3.21 Polynomial Updates

```

ll gauss(ll x){return (x*(x+1ll))/2ll;}
struct Node{
    ll sum=0; // the nodes value
    ll acum=0; // count completed levels
    ll cnt=0; // count of updates +1, +2, +3, ...
    void build(ll v){
        acum=cnt=0;
        sum=v;
    }
    void oper(Node& a, Node& b){
        sum=a.sum+b.sum;
        acum=cnt=0;
    }
    void lazy(ll len, ll _acum, ll _cnt){
        sum+=_acum*len+gauss(len)*_cnt;
        acum+=_acum;
        cnt+=_cnt;
    }
};
struct SegTree{
    vector<Node> vals;
    Node null;
    int size;
    SegTree(vector<ll>& a){
        size=1;
        while(size<sz(a))size*=2;
        vals.resize(2*size);
        build(a, 0, 0, size);
    }
    void build(vector<ll>& a, int x, int lx, int rx){
        if(rx-lx==1){
            if(lx<sz(a))vals[x].build(a[lx]);
            return;
        }
        int m=(lx+rx)/2;
        build(a, 2*x+1, lx, m);

```

```

        build(a, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }
    void propagate(int x, int lx, int rx){
        if(rx-lx==1)return;
        if(vals[x].cnt==0)return;
        int m=(rx+lx)/2;
        vals[2*x+1].lazy(m-lx, vals[x].acum, vals
            [x].cnt);
        vals[2*x+2].lazy(rx-m, vals[x].acum+1l(m-
            lx)*vals[x].cnt, vals[x].cnt);
        vals[x].acum=vals[x].cnt=0;
    }
    void upd(int l, int r, ll v, int x, int lx, int
        rx){
        if(rx<=l || r<=lx)return;
        if(l<=lx && rx<=r){
            vals[x].lazy(rx-lx, v*(lx-l), v);
            return;
        }
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        upd(l, r, v, 2*x+1, lx, m);
        upd(l, r, v, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }
    ll get(int l, int r, int x, int lx, int rx){
        if(rx<=l || r<=lx)return null.sum;
        if(l<=lx && rx<=r)return vals[x].sum;
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        ll v1=get(l, r, 2*x+1, lx, m);
        ll v2=get(l, r, 2*x+2, m, rx);
        return v1+v2;
    }
    ll get(int l, int r){return get(l, r+1, 0, 0, size);}
    void upd(int l, int r, ll v){upd(l, r+1, v, 0, 0, size
        );}
    // v es la cantidad de veces que se aplica la
    // operacion +1, +2, +3
};

```

### 3.22 Segment Tree Iterativo

```

struct segtree{
    int n; vl v; ll nulo = 0;
    ll op(ll a, ll b) {return a + b;}
    segtree(int n) : n(n) {v = vl(2*n, nulo);}
    segtree(vl &a) : n(sz(a)), v(2*n){

```



```

        for(int i = 0; i<n; i++) v[n + i] = a[i];
        for (int i = n-1; i>=1; --i) v[i] = op(v[
            i<<1], v[i<<1|1]);
    }
    void upd(int k, ll nv){
        for (v[k += n] = nv; k > 1; k >>= 1) v[k
            >>1] = op(v[k], v[k^1]);
    }
    ll get(int l, int r){
        ll vl = nulo, vr = nulo;
        for (l += n, r += n+1; l < r; l >>= 1, r
            >>= 1){
            if (l&1) vl = op(vl, v[l++]);
            if (r&1) vr = op(v[--r], vr);
        }
        return op(vl, vr);
    }
};

```

### 3.23 Segment Tree Recursivo

```

typedef long long T;
struct SegTree{
    vector<T> vals, lazy;
    T null=0, nolz=0;
    int size;

    T op(T a, T b){return a+b;}
    SegTree(vector<T>& a){
        size=1;
        while(size<sz(a))size*=2;
        vals.resize(2*size);
        lazy.assign(2*size, nolz);
        build(a, 0, 0, size);
    }
    void build(vector<T>& a, int x, int lx, int rx){
        if(rx-lx==1){
            if(lx<sz(a))vals[x]=a[lx];
            return;
        }
        int m=(lx+rx)/2;
        build(a, 2*x+1, lx, m);
        build(a, 2*x+2, m, rx);
        vals[x]=op(vals[2*x+1], vals[2*x+2]);
    }
    void propagate(int x, int lx, int rx){
        if(rx-lx==1)return;
        if(lazy[x]==nolz)return;
        int m=(lx+rx)/2;
        lazy[2*x+1]+=lazy[x];
        vals[2*x+1]+=lazy[x]*((T)(m-lx));
    }
};

```

```

        lazy[2*x+2]+=lazy[x];
        vals[2*x+2]+=lazy[x]*((T)(rx-m));
        lazy[x]=nolz;
    }
    void upd(int l, int r, T v, int x, int lx, int rx)
    {
        if(rx<=l || r<=lx)return;
        if(l<=lx && rx<=r){
            lazy[x]+=v;
            vals[x]+=v*((T)(rx-lx));
            return;
        }
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        upd(l, r, v, 2*x+1, lx, m);
        upd(l, r, v, 2*x+2, m, rx);
        vals[x]=op(vals[2*x+1], vals[2*x+2]);
    }
    void set(int i, T v, int x, int lx, int rx){
        if(rx-lx==1){
            vals[x]=v;
            return;
        }
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        if(i<m)set(i, v, 2*x+1, lx, m);
        else set(i, v, 2*x+2, m, rx);
        vals[x]=op(vals[2*x+1], vals[2*x+2]);
    }
    T get(int l, int r, int x, int lx, int rx){
        if(rx<=l || r<=lx)return null;
        if(l<=lx && rx<=r)return vals[x];
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        T v1=get(l, r, 2*x+1, lx, m);
        T v2=get(l, r, 2*x+2, m, rx);
        return op(v1, v2);
    }
    T get(int l, int r){return get(l, r+1, 0, 0, size);}
    void upd(int l, int r, T v){upd(l, r+1, v, 0, 0, size)
        ;}
    void set(int i, T val){set(i, val, 0, 0, size);}
};

```

### 3.24 Segment Tree 2D

```

// O(n^2*log(n^2)) build
// O(log(n)^2) get, set
const int N=1000+1;
typedef int T;
T st[2*N][2*N];

```

```

struct SegTree{
    int n,m,neutro=0;
    T op(T a, T b){return a+b;}
    SegTree(int n, int m): n(n), m(m){
        for(int i=0;i<2*n;++i)for(int j=0;j<2*m;
            ++j)st[i][j]=neutro;
    }
    SegTree(vector<vector<T>>& a): n(sz(a)), m(n ? sz
        (a[0]) : 0){ build(a); }

    void build(vector<vector<T>>& a){
        for(int i=0;i<n;++i)for(int j=0;j<m;++j)
            st[i+n][j+m]=a[i][j];
        for(int i=0;i<n;++i)for(int j=m-1;j>=1;--
            j)st[i+n][j]=op(st[i+n][j<<1], st[i+n
            ][j<<1|1]);
        for(int i=n-1;i>=1;--i)for(int j=0;j<2*m
            ;++j)st[i][j]=op(st[i<<1][j], st[i
            <<1|1][j]);
    }

    void set(int x, int y, T v){
        st[x+n][y+m]=v;
        for(int j=y+m;j>1;j>>=1)st[x+n][j>>1]=op(
            st[x+n][j], st[x+n][j^1]);
        for(int i=x+n;i>1;i>>=1)for(int j=y+m;j;j
            >>=1)st[i>>1][j]=op(st[i][j], st[i^1][
            j]);
    }

    T get(int x0, int y0, int x1, int y1){
        T r=neutro;
        for(int i0=x0+n,i1=x1+n+1;i0<i1;i0>>=1,i1
            >>=1){
            int t[4],q=0;
            if(i0&1)t[q++]=i0++;
            if(i1&1)t[q++]--i1;
            for(int k=0;k<q;++k)for(int j0=y0
                +m,j1=y1+m+1;j0<j1;j0>>=1,j1
                >>=1){
                if(j0&1)r=op(r,st[t[k]][
                    j0++]);
                if(j1&1)r=op(r,st[t[k
                    ]][--j1]);
            }
        }
        return r;
    }
};

```

### 3.25 Segment Tree Beats

```

// O(n*log(n)) build
// O(log(n)) get, upd

```

```

// updMax[l,r] -> ai = max(ai, v)
// updMin[l,r] -> ai = min(ai, v)
// updAdd[l,r] -> ai = ai + v
// get[l,r] -> return sum of the range [l,r]
typedef long long T;
T null=0,noVal=0;
T INF=1e18;
struct Node{
    T sum,lazy;
    T max1,max2,maxc;
    T min1,min2,minc;

    void build(T x){
        sum=max1=min1=x;
        maxc=minc=1;
        lazy=noVal;
        max2=-INF;
        min2=INF;
    }

    void oper(Node& a, Node& b){
        sum=a.sum+b.sum;

        if(a.max1>b.max1){
            max1=a.max1;
            maxc=a.maxc;
            max2=max(a.max2, b.max1);
        }else if(a.max1<b.max1){
            max1=b.max1;
            maxc=b.maxc;
            max2=max(b.max2, a.max1);
        }else{
            max1=a.max1;
            maxc=a.maxc+b.maxc;
            max2=max(a.max2, b.max2);
        }

        if(a.min1<b.min1){
            min1=a.min1;
            minc=a.minc;
            min2=min(a.min2, b.min1);
        }else if(a.min1>b.min1){
            min1=b.min1;
            minc=b.minc;
            min2=min(b.min2, a.min1);
        }else{
            min1=a.min1;
            minc=a.minc+b.minc;
            min2=min(a.min2, b.min2);
        }
    }
};

struct SegTree{
    vector<Node> vals;
    int size;

    SegTree(vector<T>& a){

```

```

        size=1;
        while(size<sz(a)) size*=2;
        vals.resize(2*size);
        build(a, 0, 0, size);
    }

    void build(vector<T>& a, int x, int lx, int rx){
        if(rx-lx==1){
            if(lx<sz(a)) vals[x].build(a[lx]);
            return;
        }
        int m=(lx+rx)/2;
        build(a, 2*x+1, lx, m);
        build(a, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }

    void propagateMax(T v, int x, int lx, int rx){
        if(vals[x].min1>=v) return;
        vals[x].sum-=vals[x].min1*vals[x].minc;
        vals[x].min1=v;
        vals[x].sum+=vals[x].min1*vals[x].minc;
        if(rx-lx==1){
            vals[x].max1=v;
        }else{
            if(v>=vals[x].max1){
                vals[x].max1=v;
            }else if(v>vals[x].max2){
                vals[x].max2=v;
            }
        }
    }

    void propagateMin(T v, int x, int lx, int rx){
        if(vals[x].max1<=v) return;
        vals[x].sum-=vals[x].max1*vals[x].maxc;
        vals[x].max1=v;
        vals[x].sum+=vals[x].max1*vals[x].maxc;
        if(rx-lx==1){
            vals[x].min1=v;
        }else{
            if(v<=vals[x].min1){
                vals[x].min1=v;
            }else if(v<vals[x].min2){
                vals[x].min2=v;
            }
        }
    }

    void propagateAdd(T v, int x, int lx, int rx){
        vals[x].sum+=v*((T)(rx-lx));
        vals[x].lazy+=v;
        vals[x].max1+=v;
        vals[x].min1+=v;
        if(vals[x].max2!=-INF) vals[x].max2+=v;
        if(vals[x].min2!=-INF) vals[x].min2+=v;
    }

```

```

    }

    void propagate(int x, int lx, int rx){
        if(rx-lx==1) return;
        int m=(lx+rx)/2;
        if(vals[x].lazy!=noVal){
            propagateAdd(vals[x].lazy, 2*x+1,
                lx, m);
            propagateAdd(vals[x].lazy, 2*x+2,
                m, rx);
            vals[x].lazy=noVal;
        }

        propagateMin(vals[x].max1, 2*x+1, lx, m);
        propagateMin(vals[x].max1, 2*x+2, m, rx);

        propagateMax(vals[x].min1, 2*x+1, lx, m);
        propagateMax(vals[x].min1, 2*x+2, m, rx);
    }

    void updAdd(int l, int r, T v, int x, int lx, int
        rx){
        if(lx>=r || l>=rx) return;
        if(lx>=l && rx<=r){
            propagateAdd(v, x, lx, rx);
            return;
        }
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        updAdd(l, r, v, 2*x+1, lx, m);
        updAdd(l, r, v, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }

    void updMax(int l, int r, T v, int x, int lx, int
        rx){
        if(lx>=r || l>=rx || vals[x].min1>v)
            return;
        if(lx>=l && rx<=r && vals[x].min2>v){
            propagateMax(v, x, lx, rx);
            return;
        }
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        updMax(l, r, v, 2*x+1, lx, m);
        updMax(l, r, v, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }

    void updMin(int l, int r, T v, int x, int lx, int
        rx){
        if(lx>=r || l>=rx || vals[x].max1<v)
            return;
        if(lx>=l && rx<=r && vals[x].max2<v){
            propagateMin(v, x, lx, rx);
            return;
        }
    }

```

```

        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        updMin(l, r, v, 2*x+1, lx, m);
        updMin(l, r, v, 2*x+2, m, rx);
        vals[x].oper(vals[2*x+1], vals[2*x+2]);
    }

    T get(int l, int r, int x, int lx, int rx){
        if(lx>=r || l>=rx) return null;
        if(lx>=l && rx<=r) return vals[x].sum;
        propagate(x, lx, rx);
        int m=(lx+rx)/2;
        T v1=get(l, r, 2*x+1, lx, m);
        T v2=get(l, r, 2*x+2, m, rx);
        return v1+v2;
    }

    T get(int l, int r){return get(l, r+1, 0, 0, size);}
    void updAdd(int l, int r, T v){updAdd(l, r+1, v, 0, 0, size);}
    void updMin(int l, int r, T v){updMin(l, r+1, v, 0, 0, size);}
    void updMax(int l, int r, T v){updMax(l, r+1, v, 0, 0, size);}
};

```

### 3.26 Sparse Table

```

// O(n*log(n)) build
// O(1) get
typedef long long T;
T op(T a, T b); // max, min, gcd ...
struct Table{
    vector<vector<T>> st;
    Table(vector<T>& v){
        st.clear();
        int n=v.size();
        st.push_back(v);
        for(int j=1; (1<<j)<=n;++j){
            st.push_back(vector<T>(n));
            for(int i=0; i+(1<<(j-1))<n;++i){
                st[j][i]=op(st[j-1][i], st[j-1][i+(1<<(j-1))]);
            }
        }
    }
    T get(int l, int r){
        int j=31-__builtin_clz(r-l+1);
        return op(st[j][l], st[j][r-(1<<j)+1]);
    }
};

```

### 3.27 Sparse Table 2D

```

// O(n*m*log(n)*log(m)) build
// O(1) get
typedef int T;
const int maxn = 1000, logn = 10;
T st[logn][maxn][logn][maxn];
int lg2[maxn+1];
T op(T a, T b); // min, max, gcd...

void build(int n, int m, vector<vector<T>>& a){
    for(int i=2; i<=max(n, m); ++i) lg2[i]=lg2[i/2]+1;
    for(int i=0; i<n; ++i){
        for(int j=0; j<m; ++j){
            st[0][i][0][j]=a[i][j];
            for(int k2=1; k2<logn; ++k2)
                for(int j=0; j+(1<<(k2-1))<m; ++j)
                    st[0][i][k2][j]=op(st[0][i][k2-1][j], st[0][i][k2-1][j+(1<<(k2-1))]);
        }
        for(int k1=1; k1<logn; ++k1)
            for(int i=0; i<n; ++i)
                for(int k2=0; k2<logn; ++k2)
                    for(int j=0; j<m; ++j)
                        st[k1][i][k2][j]=
                            op(st[k1-1][i][k2][j],
                               op(st[k1-1][i][k2-1][j],
                                  st[k1-1][i+(1<<(k1-1))][k2][j]));
    }
}

T get(int x1, int y1, int x2, int y2){
    x2++; y2++;
    int a=lg2[x2-x1];
    int b=lg2[y2-y1];
    return op(
        op(st[a][x1][b][y1],
           st[a][x2-(1<<a)][b][y1]),
        op(st[a][x1][b][y2-(1<<b)],
           st[a][x2-(1<<a)][b][y2-(1<<b)]));
}

```

### 3.28 Sqrt Decomposition

```

// O(n) build
// O(n/b+b) get, set
typedef long long T;
struct SQRT{
    int b; // check b
    vector<T> a, bls;
};

```

```

SQR T(vector<T>& arr, int n){
    b=ceil(sqrt(n));a=arr;
    bls.assign(b, 0);
    for(int i=0;i<n;++i){
        bls[i/b]+=a[i];
    }
}

void set(int x, int v){
    bls[x/b]-=a[x];
    a[x]=v;
    bls[x/b]+=a[x];
}

T get(int r){
    T res=0;
    for(int i=0;i<r/b;++i){res+=bls[i];}
    for(int i=(r/b)*b;i<r;++i){res+=a[i];}
    return res;
}

T get(int l, int r){
    return get(r+1)-get(l);
}

};

```

### 3.29 Treap

```

// Treap => Binary Search Tree + Binary Heap
// 1. create a empty root (PTreap root=nullptr;)
// 2. Append the nodes in asc order
// PTreap tmp=new Treap(x);
// root=merge(root, tmp);

typedef long long T;
typedef unsigned long long u64;
mt19937_64 rng (chrono::steady_clock::now().
    time_since_epoch().count());

T null = 0;
struct Treap{
    Treap *l,*r,*dad; // left child, right child
    u64 prior; // random
    T val; // value
    int sz; // size subtree
    Treap(T v){
        l=r=nullptr;
        prior=rng();
        val=v;sz=1;
    }
    ~Treap(){
        delete l;
        delete r;
    }
};

typedef Treap* PTreap;
int cnt(PTreap x){return (!x?0:x->sz);}

```

```

// updates node with its children information
void pull(PTreap x){
    x->sz=cnt(x->l)+cnt(x->r)+1;
    if(x->l)x->l->dad=x;
    if(x->r)x->r->dad=x;
}

// O(log(n)) divide the treap in two parts
// [nodes value <= key], [nodes value > key]
pair<PTreap, PTreap> split(PTreap x, T key){
    if(!x)return {nullptr, nullptr};
    if(x->val>key){
        auto got=split(x->l, key);
        x->l=got.second;
        pull(x);
        return {got.first, x};
    }else{
        auto got=split(x->r, key);
        x->r=got.first;
        pull(x);
        return {x, got.second};
    }
}

// O(log(n)) merge two treap
// if all values in treap x < all values in treap y
PTreap merge(PTreap x, PTreap y){
    if(!x)return y;
    if(!y)return x;
    if(x->prior<=y->prior){
        x->r=merge(x->r, y);
        pull(x);
        return x;
    }else{
        y->l=merge(x, y->l);
        pull(y);
        return y;
    }
}

// O(n*log(n))
// Combine two treap into one
PTreap combine(PTreap x, PTreap y){
    if(!x)return y;
    if(!y)return x;
    if(x->prior<y->prior)swap(x, y);
    auto z=split(y, x->val);
    x->r=combine(x->r, z.second);
    x->l=combine(z.first, x->l);
    return x;
}

// O(log(n))
// return kth element - indexed 0
T kth(PTreap& x, int k){

```

```

    if(!x) return null;
    if(k==cnt(x->l)) return x->val;
    if(k<cnt(x->l)) return kth(x->l, k);
    return kth(x->r, k-cnt(x->l)-1);
}

// O(log(n))
// return {index, val}
pair<int, T> lower_bound(PTreap x, T key){
    if(!x) return {0, null};
    if(x->val<key){
        auto y=lower_bound(x->r, key);
        y.first+=cnt(x->l)+1;
        return y;
    }
    auto y=lower_bound(x->l, key);
    if(y.first==cnt(x->l)) y.second=x->val;
    return y;
}

// O(n) print the treap
void dfs(PTreap x){
    if(!x) return;
    dfs(x->l);
    cout<<x->val<<" ";
    dfs(x->r);
}

```

### 3.30 Trie Bit

```

struct node{
    int childs[2]{-1, -1};
};

struct TrieBit{
    vector<node> nds;
    vi passNums;

    TrieBit(){
        nds.pb(node());
        passNums.pb(0);
    }

    void insert(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;
            if(nds[cur].childs[bit] == -1){
                nds[cur].childs[bit] =
                    nds.size();
                nds.pb(node());
                passNums.pb(0);
            }
            cur = nds[cur].childs[bit];
        }
        passNums[cur]++;
    }
}

```

```

        cur = nds[cur].childs[bit];
    }
    passNums[cur]++;

    void remove(int num){
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;
            passNums[cur]--;
            cur = nds[cur].childs[bit];
        }
        passNums[cur]--;

    }

    int maxXor(int num){
        int ans = 0;
        int cur = 0;
        for(int i = 30; i >= 0; i--){
            bool bit = (num >> i) & 1;

            int n1 = nds[cur].childs[!bit];
            if (n1 != -1 && passNums[n1]){
                ans += (1 << i);
                bit = !bit;
            }

            cur = nds[cur].childs[bit];
        }

        return ans;
    }
};

```

### 3.31 Two Stacks

```

// O(1) push, pop, get
typedef long long T;
struct Node{ T val, acum; };
struct TwoStacks{
    stack<Node> s1, s2;

    void push(T x){
        Node tmp={x, x};
        if(!s2.empty()){
            // tmp.acum + s2.top().acum
        }
        s2.push(tmp);
    }

    void pop(){
        if(s1.empty()){
            while(!s2.empty()){
                Node tmp=s2.top();
                if(s1.empty()){

```

```

        // tmp.acum = tmp
        .val
    }else{
        // tmp.acum + s1.
        top().acum
    }
    s1.push(tmp);
    s2.pop();
}
s1.pop();
}
bool get(){
    if(s1.empty() && s2.empty())return false;
    else if(!s1.empty() && s2.empty()){
        return true; // eval s1.top();
    }else if(s1.empty() && !s2.empty()){
        return true; // eval s2.top();
    }else{
        return true; // eval s1.top() +
        s2.top()
    }
}
};

```

### 3.32 Wavelet Tree

```

const int maxn = 1e5+5;
const int maxv = 1e9;
const int minv = -1e9;

// O(n*log(n)) build
// O(log(n)) kth, lte, cnt, sum
// 1. int a[maxn];
// 2. WaveletTree wt;
// 3. fill a[1;n]
// 4. wt.build(a+1, a+n+1, minv, maxv);
struct WaveletTree { // indexed 1
    int lo, hi;
    WaveletTree *l, *r;
    int *b, bsz, csz;
    ll *c;

    WaveletTree() {
        hi=bsz=csz=0;
        l=r=NULL;
        lo=1;
    }

    void build(int *from, int *to, int x, int y){
        lo=x, hi=y;
        if(from>=to) return;
        int mid=lo+(hi-lo)/2;
        auto f=[mid](int x){return x<=mid;};
    }
}

```

```

b=(int*)malloc((to-from+2)*sizeof(int));
bsz=0;
b[bsz++]=0;
c=(ll*)malloc((to-from+2)*sizeof(ll));
csz=0;
c[csz++]=0;
for(auto it=from; it!=to; ++it){
    b[bsz]=(b[bsz-1]+f(*it));
    c[csz]=(c[csz-1]+(*it));
    bsz++;csz++;
}
if(hi==lo) return;
auto pivot=stable_partition(from, to, f);
l=new WaveletTree();
l->build(from, pivot, lo, mid);
r=new WaveletTree();
r->build(pivot, to, mid+1, hi);
}

//kth smallest element in [l, r]
int kth(int l, int r, int k){
    if(l>r) return 0;
    if(lo==hi) return lo;
    int inLeft=b[r]-b[l-1], lb=b[l-1], rb=b[r];
    if(k<=inLeft) return this->l->kth(lb+1, rb, k);
    return this->r->kth(l-lb, r-rb, k-inLeft);
}

//count of numbers in [l, r] Less than or equal
to k
int lte(int l, int r, int k){
    if(l>r || k<lo) return 0;
    if(hi<=k) return r-l+1;
    int lb=b[l-1], rb=b[r];
    return this->l->lte(lb+1, rb, k)+this->r->lte(l-lb, r-rb, k);
}

//count of numbers in [l, r] equal to k
int count(int l, int r, int k){
    if(l>r || k<lo || k>hi) return 0;
    if(lo==hi) return r-l+1;
    int lb=b[l-1], rb=b[r];
    int mid=(lo+hi)>>1;
    if(k<=mid) return this->l->count(lb+1, rb, k);
    return this->r->count(l-lb, r-rb, k);
}

//sum of numbers in [l, r] less than or equal to
k
ll sum(int l, int r, int k){
    if(l>r || k<lo) return 0;
}

```

```

        if(hi<=k) return c[r]-c[l-1];
        int lb=b[l-1], rb=b[r];
        return this->l->sum(lb+1, rb, k)+this->r
            ->sum(l-lb, r-rb, k);
    }
    ~WaveletTree() {
        delete l;
        delete r;
    }
};

```

## 4 Flujos

### 4.1 Blossom

```

// O(|E||V|^2)
struct network {
    struct struct_edge { int v; struct_edge * n; };
    typedef struct_edge* edge;
    int n;
    struct_edge pool[MAXE]; ///2*n*n;
    edge top;
    vector<edge> adj;
    queue<int> q;
    vector<int> f, base, inq, inb, inp, match;
    vector<vector<int>> ed;
    network(int n) : n(n), match(n, -1), adj(n), top(pool),
        f(n), base(n),
            inq(n), inb(n), inp(n), ed(n, vector<
                int>(n)) {}
    void add_edge(int u, int v) {
        if(ed[u][v]) return;
        ed[u][v] = 1;
        top->v = v, top->n = adj[u], adj[u] = top++;
        top->v = u, top->n = adj[v], adj[v] = top++;
    }
    int get_lca(int root, int u, int v) {
        fill(inp.begin(), inp.end(), 0);
        while(1) {
            inp[u = base[u]] = 1;
            if(u == root) break;
            u = f[ match[u] ];
        }
        while(1) {
            if(inp[v = base[v]]) return v;
            else v = f[ match[v] ];
        }
    }
    void mark(int lca, int u) {
        while(base[u] != lca) {
            int v = match[u];
            inb[ base[u] ] = 1;

```

```

            inb[ base[v] ] = 1;
            u = f[v];
            if(base[u] != lca) f[u] = v;
        }
    }
    void blossom_contraction(int s, int u, int v) {
        int lca = get_lca(s, u, v);
        fill(inb.begin(), inb.end(), 0);
        mark(lca, u); mark(lca, v);
        if(base[u] != lca) f[u] = v;
        if(base[v] != lca) f[v] = u;
        for(int u = 0; u < n; u++)
            if(inb[base[u]]) {
                base[u] = lca;
                if(!inq[u]) {
                    inq[u] = 1;
                    q.push(u);
                }
            }
    }
    int bfs(int s) {
        fill(inq.begin(), inq.end(), 0);
        fill(f.begin(), f.end(), -1);
        for(int i = 0; i < n; i++) base[i] = i;
        q = queue<int>();
        q.push(s);
        inq[s] = 1;
        while(q.size()) {
            int u = q.front(); q.pop();
            for(edge e = adj[u]; e; e = e->n) {
                int v = e->v;
                if(base[u] != base[v] && match[u] != v) {
                    if((v == s) || (match[v] != -1 && f[match[v]]
                        != -1))
                        blossom_contraction(s, u, v);
                    else if(f[v] == -1) {
                        f[v] = u;
                        if(match[v] == -1) return v;
                        else if(!inq[match[v]]) {
                            inq[match[v]] = 1;
                            q.push(match[v]);
                        }
                    }
                }
            }
        }
        return -1;
    }
    int doit(int u) {
        if(u == -1) return 0;
        int v = f[u];
        doit(match[v]);
        match[v] = u; match[u] = v;
        return u != -1;
    }
}

```



```

/// (i < net.match[i]) => means match
int maximum_matching() {
    int ans = 0;
    for(int u = 0; u < n; u++)
        ans += (match[u] == -1) && doit(bfs(u));
    return ans;
}
};

```

## 4.2 Dinic

```

// O(|E|*|V|^2)
struct edge { ll v, cap, inv, flow, ori; };
struct network {
    ll n, s, t;
    vector<ll> lvl;
    vector<vector<edge>> g;
    network(ll n) : n(n), lvl(n), g(n) {}
    void add_edge(int u, int v, ll c) {
        g[u].push_back({v, c, sz(g[v]), 0, 1});
        g[v].push_back({u, 0, sz(g[u])-1, c, 0});
    }
    bool bfs() {
        fill(lvl.begin(), lvl.end(), -1);
        queue<ll> q;
        lvl[s] = 0;
        for(q.push(s); q.size(); q.pop()) {
            ll u = q.front();
            for(auto &e : g[u]) {
                if(e.cap > 0 && lvl[e.v]
                    == -1) {
                    lvl[e.v] = lvl[u]
                        +1;
                    q.push(e.v);
                }
            }
        }
        return lvl[t] != -1;
    }
    ll dfs(ll u, ll nf) {
        if(u == t) return nf;
        ll res = 0;
        for(auto &e : g[u]) {
            if(e.cap > 0 && lvl[e.v] == lvl[u]
                +1) {
                ll tf = dfs(e.v, min(nf,
                    e.cap));
                res += tf; nf -= tf; e.
                    cap -= tf;
                g[e.v][e.inv].cap += tf;
                g[e.v][e.inv].flow -= tf;
                e.flow += tf;
                if(nf == 0) return res;
            }
        }
    }
}

```

```

}
if(!res) lvl[u] = -1;
return res;
}
ll max_flow(ll so, ll si, ll res = 0) {
    s = so; t = si;
    while(bfs()) res += dfs(s, LONG_LONG_MAX);
    return res;
}
void min_cut() {
    queue<ll> q;
    vector<bool> vis(n, 0);
    vis[s] = 1;
    for(q.push(s); q.size(); q.pop()) {
        ll u = q.front();
        for(auto &e : g[u]) {
            if(e.cap > 0 && !vis[e.v]
                ) {
                q.push(e.v);
                vis[e.v] = 1;
            }
        }
    }
    vii ans;
    for (int i = 0; i < n; i++) {
        for (auto &e : g[i]) {
            if (vis[i] && !vis[e.v]
                && e.ori) {
                ans.push_back({i
                    +1, e.v+1});
            }
        }
    }
    for (auto [x, y] : ans) cout << x << ' '
        << y << ln;
}
bool dfs2(vi &path, vector<bool> &vis, int u) {
    vis[u] = 1;
    for (auto &e : g[u]) {
        if (e.flow > 0 && e.ori && !vis[e.v]) {
            if (e.v == t || dfs2(path,
                vis, e.v)) {
                path.push_back(e.v);
                e.flow = 0;
                return 1;
            }
        }
    }
    return 0;
}
void disjoint_paths() {
    vi path;
}

```

```

        vector<bool> vis(n, 0);
        while (dfs2(path, vis, s)) {
            path.push_back(s);
            reverse(all(path));
            cout << sz(path) << ln;
            for (int v : path) cout << v+1 <<
                " ";
            cout << ln;
            path.clear(); vis.assign(n, 0);
        }
    };
};

```

### 4.3 L-R Flow

```

const long long inf = 1LL << 61;
struct Dinic {
    struct edge {
        int to, rev;
        long long flow, w;
        int id;
    };
    int n, s, t, mxid;
    vector<int> d, flow_through;
    vector<int> done;
    vector<vector<edge>> g;
    Dinic() {}
    Dinic(int _n) {
        n = _n + 10;
        mxid = 0;
        g.resize(n);
    }
    void add_edge(int u, int v, long long w, int id = -1) {
        edge a = {v, (int)g[v].size(), 0, w, id};
        edge b = {u, (int)g[u].size(), 0, 0, -1}; //for
        bidirectional edges cap(b) = w
        g[u].emplace_back(a);
        g[v].emplace_back(b);
        mxid = max(mxid, id);
    }
    bool bfs() {
        d.assign(n, -1);
        d[s] = 0;
        queue<int> q;
        q.push(s);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            for (auto &e : g[u]) {
                int v = e.to;
                if (d[v] == -1 && e.flow < e.w) d[v] = d[u] + 1,
                    q.push(v);
            }
        }
    }
};

```

```

        return d[t] != -1;
    }
    long long dfs(int u, long long flow) {
        if (u == t) return flow;
        for (int &i = done[u]; i < (int)g[u].size(); i++) {
            edge &e = g[u][i];
            if (e.w <= e.flow) continue;
            int v = e.to;
            if (d[v] == d[u] + 1) {
                long long nw = dfs(v, min(flow, e.w - e.flow));
                if (nw > 0) {
                    e.flow += nw;
                    g[v][e.rev].flow -= nw;
                    return nw;
                }
            }
        }
        return 0;
    }
    long long max_flow(int _s, int _t) {
        s = _s;
        t = _t;
        long long flow = 0;
        while (bfs()) {
            done.assign(n, 0);
            while (long long nw = dfs(s, inf)) flow += nw;
        }
        flow_through.assign(mxid + 10, 0);
        for (int i = 0; i < n; i++) for (auto e : g[i]) if (e.id
            >= 0) flow_through[e.id] = e.flow;
        return flow;
    }
};
//flow_through[i] = extra flow beyond 'low' sent through
edge i
struct LR_Flow {
    Dinic F;
    int n, s, t;
    struct edge {
        int u, v, l, r, id;
    };
    vector<edge> edges;
    LR_Flow() {}
    LR_Flow(int _n) {
        n = _n + 10;
        s = n - 2, t = n - 1;
        edges.clear();
    }
    void add_edge(int u, int v, int l, int r, int id = -1) {
        assert(0 <= l && l <= r);
        edges.push_back({u, v, l, r, id});
    }
    bool feasible(int _s = -1, int _t = -1, int L = -1, int
        R = -1) {

```

```

if (L != -1) edges.push_back({_t, _s, L, R, -1});
F = Dinic(n);
long long target = 0;
for (auto e : edges) {
    int u = e.u, v = e.v, l = e.l, r = e.r, id = e.id;
    if (l != 0) {
        F.add_edge(s, v, l);
        F.add_edge(u, t, l);
        target += l;
    }
    F.add_edge(u, v, r - l, id);
}
auto ans = F.max_flow(s, t);
if (L != -1) edges.pop_back();
if (ans < target) return 0; //not feasible
return 1;
}
int max_flow(int _s, int _t) { //-1 means flow is not
feasible
    int mx = 1e5 + 9;
    if (!feasible(_s, _t, 0, mx)) return -1;
    return F.max_flow(_s, _t);
}
int min_flow(int _s, int _t) { //-1 means flow is not
feasible
    int mx = 1e9;
    int ans = -1, l = 0, r = mx;
    while (l <= r) {
        int mid = l + r >> 1;
        if (feasible(_s, _t, 0, mid)) ans = mid, r = mid -
1;
        else l = mid + 1;
    }
    return ans;
}
};

```

#### 4.4 Edmonds Karp

```

// O(V * E^2)
ll bfs(vector<vi> &adj, vector<vl> &capacity, int s, int
t, vi& parent) {
    fill(parent.begin(), parent.end(), -1);
    parent[s] = -2;
    queue<pll> q;
    q.push({s, INFL});
    while (!q.empty()) {
        int cur = q.front().first;
        ll flow = q.front().second;
        q.pop();
        for (int next : adj[cur]) {
            if (parent[next] == -1LL &&

```

```

capacity[cur][next]) {
        parent[next] = cur;
        ll new_flow = min(flow,
            capacity[cur][next]);
        if (next == t)
            return new_flow;
        q.push({next, new_flow});
    }
}
return 0;
}
}
ll maxflow(vector<vi> &adj, vector<vl> &capacity, int s,
int t, int n) {
    ll flow = 0;
    vi parent(n);
    ll new_flow;
    while ((new_flow = bfs(adj, capacity, s, t,
parent))) {
        flow += new_flow;
        int cur = t;
        while (cur != s) {
            int prev = parent[cur];
            capacity[prev][cur] -= new_flow;
            capacity[cur][prev] += new_flow;
            cur = prev;
        }
    }
    return flow;
}
}

```

#### 4.5 Hopcroft Karp

```

// O(E*sqrt(V))
struct mbm {
    vector<vector<int>> g;
    vector<int> d, match;
    int nil, l, r;
    // u -> 0 to l, v -> 0 to r
    mbm(int l, int r) : g(l+r), d(l+l+r, INF), match(l+r, l
+r),
        nil(l+r), l(l), r(r) {}
    void add_edge(int a, int b) {
        g[a].push_back(l+b);
        g[l+b].push_back(a);
    }
    bool bfs() {
        queue<int> q;
        for(int u = 0; u < l; u++) {
            if(match[u] == nil) {
                d[u] = 0;

```

```

    q.push(u);
  } else d[u] = INF;
}
d[nil] = INF;
while(q.size()) {
  int u = q.front(); q.pop();
  if(u == nil) continue;
  for(auto v : g[u]) {
    if(d[match[v]] == INF) {
      d[match[v]] = d[u] + 1;
      q.push(match[v]);
    }
  }
}
return d[nil] != INF;
}
bool dfs(int u) {
  if(u == nil) return true;
  for(int v : g[u]) {
    if(d[match[v]] == d[u] + 1 && dfs(match[v])) {
      match[v] = u; match[u] = v;
      return true;
    }
  }
  d[u] = INF;
  return false;
}
int max_matching() {
  int ans = 0;
  while(bfs()) {
    for(int u = 0; u < l; u++) {
      ans += (match[u] == nil && dfs(u));
    }
  }
  return ans;
}
void matchs() {
  for(int i = 0; i < l; i++) {
    if(match[i] == l + r) continue;
    cout << i + 1 << " " << match[i] + 1 - l << "\n";
  }
}
};

```

## 4.6 Hungarian

```

#define rep(i, a, b) for(int i = a; i < (b); ++i)
typedef double type;
const type INF_TYPE = LLONG_MAX;
pair<type, vi> hungarian(const vector<vector<type>>> &a) {
  if(a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vector<type> u(n), v(m); vi p(m), ans(n - 1);
  rep(i, 1, n) {

```

```

    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vector<type> dist(m, INF_TYPE); vi pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1; type delta = INF_TYPE;
      rep(j, 1, m) if(!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if(cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if(dist[j] < delta) delta = dist[j], j1 = j;
      }
      rep(j, 0, m) {
        if(done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while(p[j0]);
    while(j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
    rep(j, 1, m) if(p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
  }
}

```

## 4.7 Maximum Bipartite Matching

```

// O(|E|*|V|)
struct mbm {
  int l, r;
  vector<vector<int>>> g;
  vector<int> match, seen;
  mbm(int l, int r) : l(l), r(r), g(l), match(r), seen(r) {}
  void add_edge(int l, int r) { g[l].push_back(r); }
  bool dfs(int u) {
    for(auto v : g[u]) {
      if(seen[v]++) continue;
      if(match[v] == -1 || dfs(match[v])) {
        match[v] = u;
        return true;
      }
    }
  }
}

```

```

    }
    return false;
}
int max_matching() {
    int ans = 0;
    fill(match.begin(), match.end(), -1);
    for(int u = 0; u < l; ++u) {
        fill(seen.begin(), seen.end(), 0);
        ans += dfs(u);
    }
    return ans;
}
void matchs() {
    for (int i = 0; i < r; i++) {
        if (match[i] == -1) continue;
        cout << match[i]+1 << ' ' << i+1
              << ln;
    }
}
};

```

## 4.8 Minimum Cost Maximum Flow

```

// O(|V|*|E|^2*log(|E|))
template <class type>
struct mcmf {
    struct edge { int u, v, cap, flow; type cost; };
    int n;
    vector<edge> ed;
    vector<vector<int>> g;
    vector<int> p;
    vector<type> d, phi;
    mcmf(int n) : n(n), g(n), p(n), d(n), phi(n) {}
    void add_edge(int u, int v, int cap, type cost) {
        g[u].push_back(ed.size());
        ed.push_back({u, v, cap, 0, cost});
        g[v].push_back(ed.size());
        ed.push_back({v, u, 0, 0, -cost});
    }
    bool dijkstra(int s, int t) {
        fill(d.begin(), d.end(), INF_TYPE);
        fill(p.begin(), p.end(), -1);
        set<pair<type, int>> q;
        d[s] = 0;
        for(q.insert({d[s], s}); q.size();) {
            int u = (*q.begin()).second; q.erase(q.begin());
            for(auto v : g[u]) {
                auto &e = ed[v];
                type nd = d[e.u]+e.cost+
                    phi[e.u]-phi[e.v];
                if(0 < (e.cap-e.flow) &&
                    nd < d[e.v]) {

```

```

                    q.erase({d[e.v], e.v});
                    d[e.v] = nd; p[e.v] = v;
                    q.insert({d[e.v], e.v});
                }
            }
        }
        for(int i = 0; i < n; i++) phi[i] = min(
            INF_TYPE, phi[i]+d[i]);
        return d[t] != INF_TYPE;
    }
    pair<int, type> max_flow(int s, int t) {
        type mc = 0;
        int mf = 0;
        fill(phi.begin(), phi.end(), 0);
        while(dijkstra(s, t)) {
            int flow = INF;
            for(int v = p[t]; v != -1; v = p[ed[v].u])
                flow = min(flow, ed[v].cap-ed[v].flow);
            for(int v = p[t]; v != -1; v = p[ed[v].u]) {
                edge &e1 = ed[v];
                edge &e2 = ed[v^1];
                mc += e1.cost*flow;
                e1.flow += flow;
                e2.flow -= flow;
            }
            mf += flow;
        }
        return {mf, mc};
    }
};

```

## 4.9 MCMF Vasito

```

// O(|E|*|F|*log(|V|))
typedef int tf;
typedef int tc;
const tf INFFLOW=1e9;
const tc INFCOST=1e9;
struct MCF{
    int n;
    vector<tc> prio, pot; vector<tf> curflow; vector<int>
        prevedge, prevnode;
    priority_queue<pair<tc, int>, vector<pair<tc, int>>,
        greater<pair<tc, int>>> q;
    struct edge{int to, rev; tf f, cap; tc cost;};
    vector<vector<edge>> g;
    MCF(int n) : n(n), prio(n), curflow(n), prevedge(n), prevnode
        (n), pot(n), g(n) {}

```

```

void add_edge(int s, int t, tf cap, tc cost) {
    g[s].push_back((edge){t, sz(g[t]), 0, cap, cost});
    g[t].push_back((edge){s, sz(g[s])-1, 0, 0, -cost});
}
pair<tf,tc> get_flow(int s, int t) {
    tf flow=0; tc flowcost=0;
    while(1){
        q.push({0, s});
        fill(all(prio), INFCOST);
        prio[s]=0; curflow[s]=INFFLOW;
        while(!q.empty()) {
            auto cur=q.top();
            tc d=cur.first;
            int u=cur.second;
            q.pop();
            if(d!=prio[u]) continue;
            for(int i=0; i<sz(g[u]); ++i) {
                edge &e=g[u][i];
                int v=e.to;
                if(e.cap<=e.f) continue;
                tc nprio=prio[u]+e.cost+pot[u]-pot[v];
                if(prio[v]>nprio) {
                    prio[v]=nprio;
                    q.push({nprio, v});
                    prevnode[v]=u; prevedge[v]=i;
                    curflow[v]=min(curflow[u], e.cap-e.f);
                }
            }
        }
        if(prio[t]==INFCOST) break;
        for(int i=0; i<n; i++) pot[i]+=prio[i];
        tf df=min(curflow[t], INFFLOW-flow);
        flow+=df;
        for(int v=t; v!=s; v=prevnode[v]) {
            edge &e=g[prevnode[v]][prevedge[v]];
            e.f+=df; g[v][e.rev].f-=df;
            flowcost+=df*e.cost;
        }
    }
    return {flow, flowcost};
}
};

```

## 4.10 L-R MCMF

```

//Works for both directed, undirected and with negative
//cost too
//doesn't work for negative cycles
//for undirected edges just make the directed flag false
//Complexity:  $O(\min(E^2 * V \log V, E \log V * \text{flow}))$ 
using T = long long;
const T inf = 1LL << 61;
struct MCMF {
    struct edge {

```

```

        int u, v;
        T cap, cost; int id;
        edge(int _u, int _v, T _cap, T _cost, int _id){
            u = _u; v = _v; cap = _cap; cost = _cost; id
            = _id;
        }
    };
    int n, s, t, mxid; T flow, cost;
    vector<vector<int>>> g; vector<edge> e;
    vector<T> d, potential, flow_through;
    vector<int> par; bool neg;
    MCMF() {}
    MCMF(int _n) { // 0-based indexing
        n = _n + 10;
        g.assign(n, vector<int> ());
        neg = false; mxid = 0;
    }
    void add_edge(int u, int v, T cap, T cost, int id =
        -1, bool directed = true) {
        if(cost < 0) neg = true;
        g[u].push_back(e.size());
        e.push_back(edge(u, v, cap, cost, id));
        g[v].push_back(e.size());
        e.push_back(edge(v, u, 0, -cost, -1));
        mxid = max(mxid, id);
        if(!directed) add_edge(v, u, cap, cost, -1, true)
        ;
    }
    bool dijkstra() {
        par.assign(n, -1);
        d.assign(n, inf);
        priority_queue<pair<T, T>, vector<pair<T, T>>,
            greater<pair<T, T>>> > q;
        d[s] = 0;
        q.push(pair<T, T>(0, s));
        while (!q.empty()) {
            int u = q.top().second;
            T nw = q.top().first;
            q.pop();
            if(nw != d[u]) continue;
            for (int i = 0; i < (int)g[u].size(); i++) {
                int id = g[u][i];
                int v = e[id].v; T cap = e[id].cap;
                T w = e[id].cost + potential[u] -
                    potential[v];
                if (d[u] + w < d[v] && cap > 0) {
                    d[v] = d[u] + w;
                    par[v] = id;
                    q.push(pair<T, T>(d[v], v));
                }
            }
        }
        for (int i = 0; i < n; i++) {
            if (d[i] < inf) d[i] += (potential[i] -
                potential[s]);

```

```

    }
    for (int i = 0; i < n; i++) {
        if (d[i] < inf) potential[i] = d[i];
    }
    // return d[t] != inf; // for max flow min cost
    return d[t] <= 0; // for min cost flow
}
T send_flow(int v, T cur) {
    if(par[v] == -1) return cur;
    int id = par[v];
    int u = e[id].u; T w = e[id].cost;
    T f = send_flow(u, min(cur, e[id].cap));
    cost += f * w;
    e[id].cap -= f;
    e[id^1].cap += f;
    return f;
}
//returns {maxflow, mincost}
pair<T, T> solve(int _s, int _t, T goal = inf) {
    s = _s; t = _t;
    flow = 0, cost = 0;
    potential.assign(n, 0);
    if (neg) {
        // run Bellman-Ford to find starting
        // potential
        d.assign(n, inf);
        d[s] = 0;
        bool relax = true;
        for (int i = 0; i < n && relax; i++) {
            relax = false;
            for (int u = 0; u < n; u++) {
                for (int k = 0; k < (int)g[u].size();
                    k++) {
                    int id = g[u][k];
                    int v = e[id].v; T cap = e[id].
                    cap, w = e[id].cost;
                    if (d[v] > d[u] + w && cap > 0) {
                        d[v] = d[u] + w;
                        relax = true;
                    }
                }
            }
        }
        for(int i = 0; i < n; i++) if(d[i] < inf)
            potential[i] = d[i];
    }
    while (flow < goal && dijkstra()) flow +=
        send_flow(t, goal - flow);
    flow_through.assign(mxid + 10, 0);
    for (int u = 0; u < n; u++) {
        for (auto v: g[u]) {
            if (e[v].id >= 0) flow_through[e[v].id] =
                e[v ^ 1].cap;
        }
    }
}

```

```

        return make_pair(flow, cost);
    }
};
//flow_through[i] = extra flow beyond 'low' sent through
//edge i
//it finds the feasible solution with minimum cost
struct LR_Flow{
    MCMF F;
    static const T INF = 1e12;
    // sum of cost should be < INF / 2
    // flow * INF must not overflow in data type
    int n;
    T target;
    LR_Flow() {}
    LR_Flow(int _n) {
        n = _n + 10; target = 0;
        F = MCMF(n);
    }
    void add_edge(int u, int v, T l, T r, T cost = 0, int
        id = -1, bool directed = true) {
        assert(0 <= l && l <= r);
        target += l;
        F.add_edge(u, v, l, -INF + cost, -1, directed);
        // will try to take this edge first
        F.add_edge(u, v, r - l, cost, id, directed);
    }
    pair<T, T> solve(int s, int t, T goal = inf) {
        auto ans = F.solve(s, t, goal);
        ans.second += INF * target;
        if (abs(ans.second) >= INF / 2) return {-1, -1};
        // not feasible
        return ans;
    }
};

```

## 4.11 Scaling Algorithm

```

// O(|E|^2*log(C)) C = maximum edge weight of the graph
struct MaxFlow {
    static const ll INF = 1e18;
    struct Edge {int u,v;ll w;};
    int n, s, t;
    vector<vector<int>>> g;
    vector<Edge> ed;
    vector<bool> vis;
    ll flow = 0;
    MaxFlow(int n, int s, int t) : n(n), s(s), t(t), g(n)
        {}
    int add_edge(int u, int v, ll forward, ll backward =
        0) {
        const int id = (int)ed.size();
        g[u].emplace_back(id);
        ed.push_back({u, v, forward});
    }
}

```

```

    g[v].emplace_back(id + 1);
    ed.push_back({v, u, backward});
    return id;
}
bool dfs(int node, ll lim) {
    if (node == t) return true;
    if (vis[node]) return false;
    vis[node] = true;
    for (int i : g[node]) {
        auto &e = ed[i];
        auto &back = ed[i ^ 1];
        if (e.w >= lim) {
            if (dfs(e.v, lim)) {
                e.w -= lim;
                back.w += lim;
                return true;
            }
        }
    }
    return false;
}
ll max_flow() {
    for (ll bit = 1ll << 62; bit > 0; bit /= 2) {
        bool found = false;
        do {
            vis.assign(n, false);
            found = dfs(s, bit);
            flow += bit * found;
        } while (found);
    }
    return flow;
}
};

```

## 4.12 Weighted Matching

```

// O(|V|^3)
typedef int type;
struct matching_weighted {
    int l, r;
    vector<vector<type>> c;
    matching_weighted(int l, int r) : l(l), r(r), c(l,
        vector<type>(r)) {
        assert(l <= r);
    }
    void add_edge(int a, int b, type cost) { c[a][b] = cost; }
}
type matching() {
    vector<type> v(r), d(r); // v: potential
    vector<int> ml(l, -1), mr(r, -1); // matching pairs
    vector<int> idx(r), prev(r);
    iota(idx.begin(), idx.end(), 0);
    auto residue = [&](int i, int j) { return c[i][j] - v[j]

```

```

    ]; };
    for(int f = 0; f < l; ++f) {
        for(int j = 0; j < r; ++j) {
            d[j] = residue(f, j);
            prev[j] = f;
        }
    }
    type w;
    int j, l;
    for (int s = 0, t = 0;;) {
        if (s == t) {
            l = s;
            w = d[ idx[t++] ];
            for(int k = t; k < r; ++k) {
                j = idx[k];
                type h = d[j];
                if (h <= w) {
                    if (h < w) t = s, w = h;
                    idx[k] = idx[t];
                    idx[t++] = j;
                }
            }
            for (int k = s; k < t; ++k) {
                j = idx[k];
                if (mr[j] < 0) goto aug;
            }
        }
        int q = idx[s++], i = mr[q];
        for (int k = t; k < r; ++k) {
            j = idx[k];
            type h = residue(i, j) - residue(i, q) + w;
            if (h < d[j]) {
                d[j] = h;
                prev[j] = i;
                if (h == w) {
                    if (mr[j] < 0) goto aug;
                    idx[k] = idx[t];
                    idx[t++] = j;
                }
            }
        }
        aug: for (int k = 0; k < l; ++k)
            v[ idx[k] ] += d[ idx[k] ] - w;
        int i;
        do {
            mr[j] = i = prev[j];
            swap(j, ml[i]);
        } while (i != f);
    }
    type opt = 0;
    for (int i = 0; i < l; ++i)
        opt += c[i][ml[i]]; // (i, ml[i]) is a solution
    return opt;
}
};

```



## 5 Geometria

### 5.1 2D Tree

```
// given a set of points, answer queries of nearest point
// in O(log(n))
bool onx(pt a, pt b){return a.x < b.x;}
bool ony(pt a, pt b){return a.y < b.y;}
struct Node {
    pt pp;
    lf x0 = inf, x1 = -inf, y0 = inf, y1 = -inf;
    Node *first = 0, *second = 0;

    lf distance(pt p){
        lf x = min(max(x0, p.x), x1);
        lf y = min(max(y0, p.y), y1);
        return norm2(pt(x, y) - p);
    }

    Node(vector<pt>&& vp) : pp(vp[0]){
        for(pt p : vp){
            x0 = min(x0, p.x);
            x1 = max(x1, p.x);
            y0 = min(y0, p.y);
            y1 = max(y1, p.y);
        }
        if(vp.size() > 1){
            sort(all(vp), x1 - x0 >= y1 - y0
                ? onx : ony);
            int m = vp.size() / 2;
            first = new Node({vp.begin(), vp.
                begin() + m});
            second = new Node({vp.begin() + m
                , vp.end()});
        }
    }
};

struct KDTree {
    Node* root;
    KDTree(const vector<pt>& vp): root(new Node({all(
        vp)})){}
    pair<lf, pt> search(pt p, Node *node){
        if(!node->first){
            // avoid query point as answer
            // if(p.x == node->pp.x && p.y ==
            // node->pp.y) return {inf, pt()};
            return {norm2(p-node->pp), node->
                pp};
        }
        Node *f = node->first, *s = node->second;
        lf bf = f->distance(p), bs = s->
            distance(p);
```

```
        if(bf > bs) swap(bf, bs), swap(f, s);
        auto best = search(p, f);
        if(bs < best.ff) best = min(best, search(
            p, s));
        return best;
    }
    pair<lf, pt> nearest(pt p){ return search(p, root
    ); }
};
```

### 5.2 3D

```
typedef double lf;
struct p3 {
    lf x, y, z;
    p3(){}
    p3(lf x, lf y, lf z): x(x), y(y), z(z){}
    p3 operator + (p3 p){ return {x + p.x, y + p.y, z + p
        .z}; }
    p3 operator - (p3 p){ return {x - p.x, y - p.y, z - p
        .z}; }
    p3 operator * (lf d){ return {x * d, y * d, z * d}; }
    p3 operator / (lf d){ return {x / d, y / d, z / d}; }
    // only for floating point
    // Some comparators
    bool operator == (p3 p){ return tie(x, y, z) == tie(p
        .x, p.y, p.z); }
    bool operator != (p3 p){ return !operator == (p); }
    void print(){ cout << x << " " << y << " " << z
        << "\n"; }
    // scale: (newnorm / norm) * p3
};

lf dot(p3 v, p3 w){ return v.x * w.x + v.y * w.y + v.z *
    w.z; }

p3 cross(p3 v, p3 w){
    return { v.y * w.z - v.z * w.y, v.z * w.x - v.x * w.z
        , v.x * w.y - v.y * w.x };
}

lf norm2(p3 v){ return dot(v, v); }
lf norm(p3 v){ return sqrt(norm2(v)); }
p3 unit(p3 v){ return v / norm(v); }

// ang(RAD)
double angle(p3 v, p3 w){
    double cos_theta = dot(v, w) / norm(v) / norm(w);
    return acos(max(-1.0, min(1.0, cos_theta)));
}

// orient s, pqr form a triangle pos: 'up', zero = on,
// neg = 'dow'
lf orient(p3 p, p3 q, p3 r, p3 s){
    return dot(cross((q - p), (r - p)), (s - p));
}
```

```

// same as 2D but in n-normal direction
lf orient_by_normal(p3 p, p3 q, p3 r, p3 n){
    return dot(cross((q - p), (r - p)), n);
}

struct plane {
    p3 n; lf d; // n: normal d: dist to zero
    // From normal n and offset d
    plane(p3 n, lf d): n(n), d(d) {}
    // From normal n and point P
    plane(p3 n, p3 p): n(n), d(dot(n, p)) {}
    // From three non-collinear points P,Q,R
    plane(p3 p, p3 q, p3 r): plane(cross((q - p), (r - p)
    ), p) {}
    // - these work with lf = int
    lf side(p3 p) { return dot(n, p) - d; }
    double dist(p3 p) { return abs(side(p)) / norm(n); }
    plane translate(p3 t) { return {n, d + dot(n, t)}; }
    /// - these require lf = double
    plane shift_up(double dist) { return {n, d + dist *
    norm(n)}; }
    p3 proj(p3 p) { return p - n * side(p) / norm2(n); }
    p3 refl(p3 p) { return p - n * 2 * side(p) / norm2(n); }
};

struct line3d {
    p3 d, o; // d: dir o: point on line
    // From two points P, Q
    line3d(p3 p, p3 q): d(q - p), o(p) {}
    // From two planes p1, p2 (requires lf = double)
    line3d(plane p1, plane p2) {
        d = cross(p1.n, p2.n);
        o = cross((p2.n * p1.d - p1.n * p2.d), d)
        / norm2(d);
    }
    // - these work with lf = int
    double dist2(p3 p) { return norm2(cross(d, (p - o)
    )) / norm2(d); }
    double dist(p3 p) { return sqrt(dist2(p)); }
    bool cmp_proj(p3 p, p3 q) { return dot(d, p) < dot
    (d, q); }
    // - these require lf = double
    p3 proj(p3 p) { return o + d * dot(d, (p - o)) /
    norm2(d); }
    p3 refl(p3 p) { return proj(p) * 2 - p; }
    p3 inter(plane p) { return o - d * p.side(o) / dot
    (p.n, d); }
    // get other point: pl.o + pl.d * t;
};

double dist(line3d l1, line3d l2) {
    p3 n = cross(l1.d, l2.d);
    if(n == p3(0, 0, 0)) return l1.dist(l2.o); //
    parallel

```

```

        return abs(dot((l2.o - l1.o), n)) / norm(n);
    }
    // closest point on l1 to l2
    p3 closest_on_line1(line3d l1, line3d l2) {
        p3 n2 = cross(l2.d, cross(l1.d, l2.d));
        return l1.o + l1.d * (dot((l2.o - l1.o), n2)) /
        dot(l1.d, n2);
    }
    double small_angle(p3 v, p3 w) { return acos(min(abs(dot(v
    , w)) / norm(v) / norm(w), 1.0)); } // 0 90
    double angle(plane p1, plane p2) { return small_angle(p1.n
    , p2.n); }
    bool is_parallel(plane p1, plane p2) { return cross(p1.n,
    p2.n) == p3(0, 0, 0); }
    bool is_perpendicular(plane p1, plane p2) { return dot(p1.
    n, p2.n) == 0; }
    double angle(line3d l1, line3d l2) { return small_angle(l1
    .d, l2.d); }
    bool is_parallel(line3d l1, line3d l2) { return cross(l1.d
    , l2.d) == p3(0, 0, 0); }
    bool is_perpendicular(line3d l1, line3d l2) { return dot(
    l1.d, l2.d) == 0; }
    double angle(plane p, line3d l) { return M_PI / 2 -
    small_angle(p.n, l.d); }
    bool is_parallel(plane p, line3d l) { return dot(p.n, l.d)
    == 0; }
    bool is_perpendicular(plane p, line3d l) { return cross(p.
    n, l.d) == p3(0, 0, 0); }
    line3d perp_through(plane p, p3 o) { return line3d(o, o +
    p.n); }
    plane perp_through(line3d l, p3 o) { return plane(l.d, o);
    }
    pair<p3, lf> smallest_enclosing_sphere(vector<p3> p) {
        int n = p.size();
        p3 c(0, 0, 0);
        for(int i = 0; i < n; i++) c = c + p[i];
        c = c / n;
        double ratio = 0.1;
        int pos = 0;
        int it = 100000;
        while (it--) {
            pos = 0;
            for (int i = 1; i < n; i++) {
                if(norm2(c - p[i]) > norm2(c - p[pos])) pos =
                i;
            }
            c = c + (p[pos] - c) * ratio;
            ratio *= 0.998;
        }
        return {c, sqrt(norm2(c - p[pos]))};
    }
}

```

## 5.3 Circulos

```

// add Lines Points
enum {OUT, IN, ON};
struct circle {
    pt center; lf r;
    // (x - xo)^2 + (y - yo)^2 = r^2
    circle(pt c, lf r): center(c), r(r){};
    // circle that passes through abc
    circle(pt a, pt b, pt c) {
        b = b - a, c = c - a;
        assert(cross(b, c) != 0); // no
        // circumcircle if A, B, C aligned
        pt cen = a + rot90(b * norm2(c) - c *
            norm2(b)) / cross(b, c) / 2;
        center = cen;
        r = norm(a - cen);
    }
    // diameter = segment pq
    circle(pt p, pt q) {
        center = (p + q) * 0.5L;
        r = dis(p, q) * 0.5L;
    }
    int contains(pt &p) {
        lf det = r * r - dis2(center, p);
        if(fabs1(det) <= EPS) return ON;
        return (det > EPS ? IN : OUT);
    }
    bool in(circle c){ return norm(center - c.center)
        + r <= c.r + EPS; } // non strict
};

// centers of the circles that pass through ab and has
// radius r
vector<pt> centers(pt a, pt b, lf r) {
    if (norm(a - b) > 2 * r + EPS) return {};
    pt m = (a + b) / 2;
    double f = sqrt(r * r / norm2(a - m) - 1);
    pt c = rot90(a - m) * f;
    return {m - c, m + c};
}

vector<pt> inter_cl(circle c, line l){
    vector<pt> s;
    pt p = l.proj(c.center);
    lf d = norm(p - c.center);
    if(d - EPS > c.r) return s;
    if(abs(d - c.r) <= EPS){ s.push_back(p); return s;
        ; }
    d=sqrt(c.r * c.r - d * d);
    s.push_back(p + normalize(l.v) * d);

```

```

        s.push_back(p - normalize(l.v) * d);
        return s;
    }
    vector<pt> inter_cc(circle c1, circle c2) {
        pt dir = c2.center - c1.center;
        lf d2 = dis2(c1.center, c2.center);
        if(d2 <= E0) {
            //assert( fabs1( c1.r - c2.r ) > E0 );
            return {};
        }
        lf td = 0.5L * ( d2 + c1.r * c1.r - c2.r * c2.r );
        lf h2 = c1.r * c1.r - td / d2 * td;
        pt p = c1.center + dir * (td / d2);
        if(fabs1(h2) < EPS) return {p};
        if(h2 < 0.0L) return {};
        pt dir_h = rot90(dir) * sqrt1(h2 / d2);
        return {p + dir_h, p - dir_h};
    }
    //compute intersection of line through points a and b
    //with
    //circle centered at c with radius r > 0
    vector<pt> circle_line_intersection(pt c, lf r, pt a, pt
        b) {
        vector<pt> ret;
        b = b - a; a = a - c;
        lf A = dot(b, b), B = dot(a, b);
        lf C = dot(a, a) - r * r, D = B * B - A * C;
        if (D < -EPS) return ret;
        ret.push_back(c + a + b * (-B + sqrt(D + EPS)) / A);
        if (D > EPS) ret.push_back(c + a + b * (-B - sqrt(D))
            / A);
        return ret;
    }
    // circle-line inter = 1, inner: 1 = o x o 0 = o = o
    vector<pair<pt, pt>> tangents(circle c1, circle c2, bool
        inner){
        vector<pair<pt, pt>> out;
        if (inner) c2.r = -c2.r; // inner tangent
        pt d = c2.center - c1.center;
        double dr = c1.r - c2.r, d2 = norm2(d), h2 = d2 -
            dr * dr;
        if (d2 == 0 || h2 < 0) { assert(h2 != 0); return
            {}; } // (identical)
        for (double s : {-1, 1}) {
            pt v = (d * dr + rot90(d) * sqrt(h2) * s)
                / d2;
            out.push_back({c1.center + v * c1.r, c2.
                center + v * c2.r});
        }
    }

```

```

        return out; // if size 1: circle are tangent
    }
    // circle tangent passing through pt p
    pair<pt, pt> tangent_through_pt(circle c, pt p){
        pair<pt, pt> out;
        double d = norm2(p - c.center);
        if (d < c.r) return {};
        pt base = c.center - p;
        double w = sqrt(norm2(base) - c.r * c.r);
        pt a = {w, c.r}, b = {w, -c.r};
        pt s = p + base * a / norm2(base) * w;
        pt t = p + base * b / norm2(base) * w;
        out = {s, t};
        return out;
    }
    if safeAcos(lf x) {
        if (x < -1.0) x = -1.0;
        if (x > 1.0) x = 1.0;
        return acos(x);
    }
    if areaOfIntersectionOfTwoCircles(circle c1, circle c2){
        lf r1 = c1.r, r2 = c2.r, d = dis(c1.center, c2.
            center);
        if(d >= r1 + r2) return 0.0L;
        if(d <= fabs(r2 - r1)) return PI * (r1 < r2 ? r1
            * r1 : r2 * r2);
        lf alpha = safeAcos((r1 * r1 - r2 * r2 + d * d) /
            (2.0L * d * r1));
        lf betha = safeAcos((r2 * r2 - r1 * r1 + d * d) /
            (2.0L * d * r2));
        lf a1 = r1 * r1 * (alpha - sinl(alpha) * cosl(
            alpha));
        lf a2 = r2 * r2 * (betha - sinl(betha) * cosl(
            betha));
        return a1 + a2;
    };
    if intertriangle(circle& c, pt a, pt b){ // area of
        intersection with oab
        if(abs(cross((c.center - a), (c.center - b))) <=
            EPS) return 0.;
        vector<pt> q = {a}, w = inter_cl(c, line(a, b));
        if(w.size() == 2) for(auto p: w) if(dot((a - p),
            (b - p)) < -EPS) q.push_back(p);
        q.push_back(b);
        if(q.size() == 4 && dot((q[0] - q[1]), (q[2] - q
            [1])) > EPS) swap(q[1], q[2]);
        lf s = 0;
        for(int i = 0; i < q.size() - 1; ++i){
            if(!c.contains(q[i]) || !c.contains(q[i +
                1])) s += c.r * c.r * min_angle((q[i]
                - c.center), q[i+1] - c.center) / 2;
            else s += abs(cross((q[i] - c.center), (q

```

```

                [i + 1] - c.center)) / 2);
            }
            return s;
        }
        bool circumcircle_contains(vector<pt> tr, pt D) { //
            triange CCW
            pt A = tr[0] - D, B = tr[1] - D, C = tr[2] - D;
            lf norm_a = norm2(tr[0]) - norm2(D);
            lf norm_b = norm2(tr[1]) - norm2(D);
            lf norm_c = norm2(tr[2]) - norm2(D);
            lf det1 = A.x * (B.y * norm_c - norm_b * C.y);
            lf det2 = B.x * (C.y * norm_a - norm_c * A.y);
            lf det3 = C.x * (A.y * norm_b - norm_a * B.y);
            return det1 + det2 + det3 > E0;
        }
        // r[k]: area covered by at least k circles
        // O(n^2 log n) (high constant)
        vector<lf> intercircles(vector<circle> c){
            vector<lf> r(c.size() + 1);
            for(int i = 0; i < c.size(); ++i){
                int k = 1; pt O = c[i].center;
                vector<pair<pt, int>> p = {
                    {c[i].center + pt(1,0) * c[i].r,
                        0},
                    {c[i].center - pt(1,0) * c[i].r,
                        0}};
                for(int j = 0; j < c.size(); ++j) if(j !=
                    i){
                    bool b0 = c[i].in(c[j]), b1 = c[j]
                        .in(c[i]);
                    if(b0 && (!b1 || i < j)) ++k;
                    else if(!b0 && !b1){
                        auto v = inter_cc(c[i], c
                            [j]);
                        if(v.size() == 2){
                            swap(v[0], v[1]);
                            p.push_back({v
                                [0], 1});
                            p.push_back({v
                                [1], -1});
                            if(polar_cmp(v[1]
                                - O, v[0] - O
                                    )) ++k;
                        }
                    }
                }
            }
            sort(all(p), [&](auto& a, auto& b){
                return polar_cmp(a.first - O, b.first
                    - O); });
            for(int j = 0; j < p.size(); ++j){

```

```

        pt p0 = p[j ? j - 1 : p.size()
        -1].first, p1 = p[j].first;
        lf a = min_angle((p0 - c[i].
        center), (p1 - c[i].center));
        r[k] += (p0.x - p1.x) * (p0.y +
        p1.y) / 2 + c[i].r * c[i].r *
        (a - sin(a)) / 2;
        k += p[j].second;
    }
    return r;
}

```

## 5.4 Closest Points

```

// O(nlogn)
pair<pt, pt> closest_points(vector<pt> v){
    sort(v.begin(), v.end());
    pair<pt, pt> ans;
    lf d2 = INF;

    function<void( int, int )> solve = [&](int l, int
    r) {
        if(l == r) return;
        int mid = (l + r) / 2;
        lf x_mid = v[mid].x;
        solve(l, mid);
        solve(mid + 1, r);

        vector<pt> aux;
        int p1 = l, p2 = mid + 1;
        while (p1 <= mid && p2 <= r) {
            if(v[p1].y < v[p2].y) aux.
                push_back(v[p1++]);
            else aux.push_back(v[p2++]);
        }
        while(p1 <= mid) aux.push_back(v[p1++]);
        while(p2 <= r) aux.push_back(v[p2++]);

        vector<pt> nb;
        for(int i = l; i <= r; ++i){
            v[i] = aux[i - 1];
            lf dx = (x_mid - v[i].x);
            if(dx * dx < d2)
                nb.push_back(v[i]);
        }

        for(int i = 0; i < (int) nb.size(); ++i){
            for(int k = i + 1; k < (int) nb.size();
            ++k){
                lf dy = (nb[k].y - nb[i].y);
                if(dy * dy > d2) break;
                lf nd2 = dis2(nb[i], nb[k]);
                if(nd2 < d2) d2 = nd2, ans = {nb[

```

```

                i], nb[k]}};
            }
        };
        solve(0, v.size() - 1);
        return ans;
    }
}

```

## 5.5 Convex Hull

```

// CCW order
// if colinear are needed, use > in orient and remove
// repeated points
vector<pt> chull(vector<pt>& p){
    if(p.size() < 3) return p;

    vector<pt> r; //r.reserve(p.size());
    sort(p.begin(), p.end()); // first x, then y

    for(int i = 0; i < p.size(); i++){ // lower hull
        while(r.size() >= 2 && orient(r[r.size()
        - 2], p[i], r.back()) >= 0) r.pop_back
        ();
        r.pb(p[i]);
    }
    r.pop_back();

    int k = r.size();
    for(int i = p.size() - 1; i >= 0; --i){ // upper
        hull
        while(r.size() >= k + 2 && orient(r[r.
        size() - 2], p[i], r.back()) >= 0) r.
        pop_back();
        r.pb(p[i]);
    }
    r.pop_back();
    return r;
}

```

## 5.6 Dynamic Convex Hull

```

bool ccw(pt p, pt q, pt r) { // counter-clockwise
    return cross((q - p), (r - q)) > 0; // >= if
    colinear are needed
}

struct upper {
    set<pt> se;
    set<pt>::iterator it;

    int is_under(pt p) { // 1 -> inside ; 2 -> border
        it = se.lower_bound(p);
        if (it == se.end()) return 0;
    }
}

```

```

        if (it == se.begin()) return p == *it ? 2
        : 0;
        if (ccw(p, *it, *prev(it))) return 1;
        return ccw(p, *prev(it), *it) ? 0 : 2;
    }
    void insert(pt p) {
        if (is_under(p)) return;

        if (it != se.end()) while (next(it) != se
            .end() and !ccw(*next(it), *it, p))
            it = se.erase(it);
        if (it != se.begin()) while (--it != se.
            begin() and !ccw(p, *it, *prev(it)))
            it = se.erase(it);

        se.insert(p);
    }
};

struct dyn_hull { // 1 -> inside ; 2 -> border
    upper U, L;

    int is_inside(pt p) {
        int u = U.is_under(p), l = L.is_under({-p
            .x, -p.y});
        if (!u or !l) return 0;
        return max(u, l);
    }

    void insert(pt p) {
        U.insert(p);
        L.insert({-p.x, -p.y});
    }

    int size() {
        int ans = U.se.size() + L.se.size();
        return ans <= 2 ? ans/2 : ans-2;
    }
};

// farthest_dynamic
vector<vector<pt>> pols;

// log^2(n) amortized (binary mergin)
void add(pt pi) {
    vector<pt> pa = {pi};
    while (sz(pols) && sz(pols.back()) < 2 * sz(pa)) {
        for (pt pi : pols.back()) pa.pb(pi);
        pols.pop_back();
    }
    vector<pt> ch = hull(pa);
    pols.pb(ch);
}

// log^2(n)
lf query(pt &dir) {
    lf maxi = LONG_LONG_MIN;

```

```

        for (vector<pt> &pol : pols) {
            lf ans = dot(farthest(pol, dir), dir);
            maxi = max(maxi, ans);
        }
        return maxi;
    }
}

```

## 5.7 Delaunay

```

// Returns planar graph representing Delaunay's
// triangulation.
// Edges for each vertex are in ccw order.
// Voronoi vertices = the circumcenters of the Delaunay
// triangles.
// O(nlogn)
typedef struct QuadEdge* Q;
struct QuadEdge {
    int id, used;
    pt o;
    Q rot, nxt;
    QuadEdge(int id=-1, pt o=pt(INF, INF)) : id(id), used
        (0), o(o), rot(0), nxt(0) {}
    Q rev() { return rot->rot; }
    Q next() { return nxt; }
    Q prev() { return rot->next()->rot; }
    pt dest() { return rev()->o; }
};

Q edge(pt a, pt b, int ida, int idb) {
    Q e1=new QuadEdge(ida, a);
    Q e2=new QuadEdge(idb, b);
    Q e3=new QuadEdge;
    Q e4=new QuadEdge;
    tie(e1->rot, e2->rot, e3->rot, e4->rot) = {e3, e4, e2, e1};
    tie(e1->nxt, e2->nxt, e3->nxt, e4->nxt) = {e1, e2, e4, e3};
    return e1;
}

void splice(Q a, Q b) {
    swap(a->nxt->rot->nxt, b->nxt->rot->nxt);
    swap(a->nxt, b->nxt);
}

void del_edge(Q& e, Q ne) {
    splice(e, e->prev()); splice(e->rev(), e->rev()->
        prev());
    delete e->rev()->rot; delete e->rev();
    delete e->rot; delete e;
    e=ne;
}

Q conn(Q a, Q b) {
    Q e=edge(a->dest(), b->o, a->rev()->id, b->id);

```

```

    splice(e, a->rev()->prev());
    splice(e->rev(), b);
    return e;
}

auto area(pt p, pt q, pt r){ return cross((q-p), (r-q)); }

bool circumcircle_contains(vector<pt> tr, pt D){
    if (orient(tr[0], tr[1], tr[2]) < 0) reverse(all(
        tr));

    pt A = tr[0] - D, B = tr[1] - D, C = tr[2] - D;
    lf norm_a = norm2(tr[0]) - norm2(D);
    lf norm_b = norm2(tr[1]) - norm2(D);
    lf norm_c = norm2(tr[2]) - norm2(D);

    lf det1 = A.x * (B.y * norm_c - norm_b * C.y);
    lf det2 = B.x * (C.y * norm_a - norm_c * A.y);
    lf det3 = C.x * (A.y * norm_b - norm_a * B.y);

    return det1 + det2 + det3 > 0;
}

pair<Q,Q> build_tr(vector<pt>& p, int l, int r){
    if(r-l+1<=3){
        Q a=edge(p[l], p[l+1], l, l+1), b=edge(p[l+1], p[r], l+1, r);
        if(r-l+1==2) return {a, a->rev()};
        splice(a->rev(), b);
        auto ar=area(p[l], p[l+1], p[r]);
        Q c=abs(ar)>EPS?conn(b, a):0;
        if(ar>=-EPS) return {a, b->rev()};
        return {c->rev(), c};
    }
    int m=(l+r)/2;
    auto [la, ra]=build_tr(p, l, m);
    auto [lb, rb]=build_tr(p, m+1, r);
    while(1){
        if(orient(lb->o, ra->o, ra->dest()) > 0)
            ra=ra->rev()->prev();
        else if(orient(lb->o, ra->o, lb->dest()) > 0) lb=lb->rev()->next();
        else break;
    }
    Q b=conn(lb->rev(), ra);
    auto valid=[&](Q e){ return orient(e->dest(), b->dest(), b->o) > 0; };
    if(ra->o==la->o) la=b->rev();
    if(lb->o==rb->o) rb=b;
    while(1){
        Q L=b->rev()->next();
        if(valid(L)) while(circumcircle_contains(
            ({b->dest(), b->o, L->dest(), L->next()->dest()}), del_edge(L, L->next()));
        Q R=b->prev();
        if(valid(R)) while(circumcircle_contains

```

```

        ({b->dest(), b->o, R->dest(), R->prev()->dest()}), del_edge(R, R->prev()));
        if(!valid(L)&&!valid(R)) break;
        if(!valid(L) || (valid(R)&&
            circumcircle_contains({L->dest(), L->o, R->o, R->dest()})) b=conn(R, b->rev()));
        else b=conn(b->rev(), L->rev());
    }
    return {la, rb};
}

vector<vector<int>> delaunay(vector<pt> v){
    int n=v.size(); auto tmp=v;
    vector<int> id(n); iota(all(id), 0);
    sort(all(id), [&](int l, int r){ return v[l]<v[r]; });
    for(int i = 0; i < n; ++i) v[i]=tmp[id[i]];
    assert(unique(all(v))==v.end());
    vector<vector<int>> g(n);
    int col=1;
    for(int i = 2; i < n; ++i) col &= abs(area(v[i], v[i-1], v[i-2])) <= EPS;
    if(col){
        for(int i = 1; i < n; i++) g[id[i-1]].pb(id[i]), g[id[i]].pb(id[i-1]);
    }
    else{
        Q e=build_tr(v, 0, n-1).first;
        vector<Q> edg={e};
        for(int i=0; i<edg.size(); i=e=edg[i++]){
            for(Q at=e; !at->used; at=at->next()){
                at->used=1;
                g[id[at->id]].pb(id[at->rev()->id]);
                edg.pb(at->rev());
            }
        }
    }
    return g;
}

```

## 5.8 Halfplanes

```

const lf INF = 1e100;
struct Halfplane {
    pt p, pq; // p: point on line, pq: dir, take left
    lf angle;
    Halfplane(){}
    Halfplane(pt& a, pt& b): p(a), pq(b - a){
        angle = atan2l(pq.y, pq.x);
    }
    bool out(const pt& r){ return cross(pq, r - p) <

```



```

        -EPS;} // checks if p is inside the half plane
bool operator < (const Halfplane& e) const {
    return angle < e.angle; }

};

// intersection pt of the lines of 2 halfplanes
pt inter(const Halfplane& s, const Halfplane& t){
    if (abs(cross(s.pq, t.pq)) <= EPS) return {INF,
        INF};
    lf alpha = cross((t.p - s.p), t.pq) / cross(s.pq,
        t.pq);
    return s.p + (s.pq * alpha);
}

// O(nlogn) return CCW polygon
vector<pt> hp_intersect(vector<Halfplane>& H) {
    pt box[4] = {pt(INF, INF), pt(-INF, INF), pt(-INF,
        -INF), pt(INF, -INF)};

    for(int i = 0; i < 4; ++i) {
        Halfplane aux(box[i], box[(i + 1) % 4]);
        H.push_back(aux);
    }

    sort(H.begin(), H.end());
    deque<Halfplane> dq;
    int len = 0;
    for(int i = 0; i < int(H.size()); ++i){
        while (len > 1 && H[i].out(inter(dq[len -
            1], dq[len - 2]))) {
            dq.pop_back();
            --len;
        }

        while (len > 1 && H[i].out(inter(dq[0],
            dq[1]))) {
            dq.pop_front();
            --len;
        }

        if (len > 0 && fabs1(cross(H[i].pq, dq[
            len - 1].pq)) < EPS) {
            if (dot(H[i].pq, dq[len - 1].pq)
                < 0.0) return vector<pt>();

            if (H[i].out(dq[len - 1].p)) {
                dq.pop_back();
                --len;
            } else continue;
        }

        dq.push_back(H[i]);
        ++len;
    }

    while (len > 2 && dq[0].out(inter(dq[len - 1], dq
        [len - 2]))) {

```

```

        dq.pop_back();
        --len;
    }

    while (len > 2 && dq[len - 1].out(inter(dq[0], dq
        [1]))) {
        dq.pop_front();
        --len;
    }

    if (len < 3) return vector<pt>();

    vector<pt> ret(len);
    for(int i = 0; i + 1 < len; ++i) ret[i] = inter(
        dq[i], dq[i + 1]);

    ret.back() = inter(dq[len - 1], dq[0]);
    // remove repeated points if needed
    return ret;
}

// -----
// intersection of halfplanes
vector<pt> hp_intersect(vector<halfplane>& b){
    vector<pt> box = {{inf, inf}, {-inf, inf}, {-inf,
        -inf}, {inf, -inf}};
    for(int i = 0; i < 4; i++){
        b.push_back({box[i], box[(i + 1) % 4]});
    }

    sort(b.begin(), b.end());
    int n = b.size(), q = 1, h = 0;
    vector<halfplane> c(n + 10);
    for(int i = 0; i < n; i++){
        while(q < h && b[i].out(inter(c[h], c[h -
            1]))) h--;
        while(q < h && b[i].out(inter(c[q], c[q +
            1]))) q++;
        c[++h] = b[i];
        if(q < h && abs(cross(c[h].pq, c[h-1].pq)
            ) < EPS) {
            if(dot(c[h].pq, c[h - 1].pq) <=
                0) return {};
            h--;
            if(b[i].out(c[h].p)) c[h] = b[i];
        }
    }

    while(q < h - 1 && c[q].out(inter(c[h], c[h - 1])
        )) h--;
    while(q < h - 1 && c[h].out(inter(c[q], c[q + 1])
        )) q++;
    if(h - q <= 1) return {};
    c[h + 1] = c[q];
    vector<pt> s;
    for(int i = q; i < h + 1; i++) s.pb(inter(c[i], c
        [i + 1]));
    return s;
}

```



## 5.9 KD Tree

```

}

const ll INF = 2e18;
const int D = 2; // dimension
struct ptd{
    int p[D];
    bool operator !=(const ptd &a) const {
        bool ok = 1;
        for(int i = 0; i < D; i++) ok &= (p[i] == a.p[i]);
        return !ok;
    }
};

struct kd_node{
    ptd p;
    int axis;
    kd_node *left, *right;
};

struct cmp_points {
    int axis;
    cmp_points(){}
    cmp_points(int x): axis(x){}

    bool operator ()(const ptd &a, const ptd &b) const {
        return a.p[axis] < b.p[axis];
    }
};

ll dis2(ptd a, ptd b) {
    ll ans = 0;
    for(int i = 0; i < D; i++) ans += (a.p[i] - b.p[i]) * 1ll * (a.p[i] - b.p[i]);
    return ans;
}

struct KDTree{
    vector<ptd> arr;
    kd_node* root;

    KDTree(vector<ptd> &vptd): arr(vptd){
        build(root, 0, sz(vptd) - 1);
    }

    // O(nlogn)
    void build(kd_node* &node, int l, int r){
        if(l > r) {
            node = nullptr;
            return;
        }
        node = new kd_node();

```

```

        if(l == r) {
            node->p = arr[l];
            node->left = nullptr;
            node->right = nullptr;
            return;
        }

        ll bAxis = 0;
        ll mRange = 0;
        for (int axis = 0; axis < D; ++axis) {
            ll minVal = INF, maxVal = -INF;
            for (int i = l; i <= r; ++i){
                minVal = min(minVal, (ll) arr[i].p[axis]);
                maxVal = max(maxVal, (ll) arr[i].p[axis]);
            }

            if (maxVal - minVal > mRange) {
                mRange = maxVal - minVal;
                bAxis = axis;
            }
        }

        int mid = (l + r) / 2;
        nth_element(arr.begin() + l, arr.begin() + mid, arr.begin() + r + 1, cmp_points(bAxis));
        node->p = arr[mid];
        node->axis = bAxis;
        build(node->left, l, mid);
        build(node->right, mid + 1, r);
    }

    void nearest(kd_node* node, ptd q, pair<ll, ptd> &ans){
        if(node == NULL) return;
        if(node->left == NULL && node->right == NULL) {
            if(!(q != node->p)) return; // avoid query point as answer
            if (ans.first > dis2(node->p, q))
                ans = {dis2(node->p, q), node->p};
            return;
        }

        int axis = node->axis;
        int value = node->p.p[axis];
        if(q.p[axis] <= value){
            nearest(node->left, q, ans);
            ll diff = value - q.p[axis];

```

```

        if(diff * diff <= ans.ff) nearest
            (node->right, q, ans);
    }else{
        nearest(node->right, q, ans);
        ll diff = q.p[axis] - value;
        if(diff * diff <= ans.ff) nearest
            (node->left, q, ans);
    }
}

// O(logn) Returns {squared distance, nearest point}
pair<ll, ptd> nearest(ptd q){
    pair<ll, ptd> ans = {INF, ptd()};
    nearest(root, q, ans);
    return ans;
}

};

```

## 5.10 Lineas

```

// add points operators
struct line {
    pt v; lf c; // v: dir, c: mov y, cross(v, p) = c
    line(pt v, lf c) : v(v), c(c) {}
    line(lf a, lf b, lf c) : v({b, -a}), c(c) {} //
        ax + by = c
    line(pt p, pt q) : v(q - p), c(cross(v, p)) {}
    line(lf m, lf b) : v({1, m}), c(b) {}

    bool operator < (line l){ return cross(v, l.v) >
        0; }
    bool operator == (line l){ return (abs(cross(v, l
        .v)) <= E0) && c == l.c; } // abs(c) == abs(l
        c)

    lf side(pt p){ return cross(v, p) - c; }
    lf dist(pt p){ return abs(side(p)) / norm(v); }
    lf dist2(pt p){ return side(p) * side(p) / (lf)
        norm2(v); }
    line perp_through(pt p){ return {p, p + rot90(v)
        }; } // line perp to v passing through p
    bool cmp_proj(pt p, pt q){ return dot(v, p) < dot
        (v, q); } // order for points over the line
    // use: auto fsort = [&ll](const pt &a, const pt
        &b){ return ll.cmp_proj(a, b); };
    line translate(pt t){ return {v, c + cross(v, t)
        }; }
    line shift_left(lf d){ return {v, c + d*norm(v)};
        }

    pt proj(pt p){ return p - rot90(v) * side(p) /
        norm2(v); } // pt proyected on the line
    pt refl(pt p){ return p - rot90(v) * 2 * side(p)
        / norm2(v); } // pt reflected on the other
        side of the line

```

```

    bool has(pt p){ return abs(cross(v, p) - c) <= E0
        ; }; // pt on line

    lf evalx(lf x){
        assert(fabssl(v.x) > EPS);
        return (c + v.y * x) / v.x;
    }

};

pt inter_ll(line l1, line l2) {
    if (abs(cross(l1.v, l2.v)) <= EPS) return {INF,
        INF}; // parallel
    return (l2.v * l1.c - l1.v * l2.c) / cross(l1.v,
        l2.v); // floating points
}

// bisector divides the angle in 2 equal angles
// interior line goes on the same direction as l1 and l2
line bisector(line l1, line l2, bool interior) {
    // assert(cross(l1.v, l2.v) != 0); // l1 and l2
        cannot be parallel
    lf sign = interior ? 1 : -1;
    return {l2.v / norm(l2.v) + l1.v / norm(l1.v) *
        sign,
        l2.c / norm(l2.v) + l1.c / norm(
        l1.v) * sign};
}

```

## 5.11 Manhattan

```

struct pt {
    ll x, y;
};

// Returns a list of edges in the format (weight, u, v).
// Passing this list to Kruskal algorithm will give the
    Manhattan MST.
vector<tuple<ll, ll, ll>> manhattan_mst_edges(vector<pt>
    ps){
    vl ids(sz(ps));
    forx(i, sz(ps)) ids[i] = i;
    vector<tuple<ll, ll, ll>> edges;

    for (ll rot = 0; rot < 4; rot++) {
        sort(ids.begin(), ids.end(), [&](ll i, ll
            j){
                return (ps[i].x + ps[i].y) < (ps[j].x + ps[j
                    ].y);
            });
        map<ll, ll, greater<ll>> active; // (xs, id)
        for(auto i : ids){
            for(auto it = active.lower_bound(
                ps[i].x); it != active.end();

```

```

        active.erase(it++){
            ll j = it->second;
            if (ps[i].x - ps[j].x > ps[j].y - ps[j].y)
                break;
            assert(ps[i].x >= ps[j].x && ps[i].y >=
                ps[j].y);
            edges.push_back((ps[i].x - ps[j].x) + (
                ps[i].y - ps[j].y), i, j));
        }
        active[ps[i].x] = i;
    }
    for (auto &p : ps){ // rotate
        if (rot & 1) p.x *= -1;
        else swap(p.x, p.y);
    }
    return edges;
}

```

## 5.12 Min Circle

```

// minimo circulo que encierra todos los puntos
// Promedio: O(n), Peor: O(n^2)
Circle min_circle(vector<pt> v){
    random_shuffle(v.begin(), v.end()); // shuffle(
        all(vec), rng);
    auto f2 = [&](int a, int b){
        Circle ans(v[a], v[b]);
        for(int i = 0; i < a; ++i)
            if(ans.contains(v[i]) == OUT) ans =
                Circle(v[i], v[a], v[b]);
        return ans;
    };
    auto f1 = [&](int a){
        Circle ans(v[a], 0.0L);
        for(int i = 0; i < a; ++i)
            if(ans.contains(v[i]) == OUT) ans = f2(i, a);
        return ans;
    };
    Circle ans(v[0], 0.0L);
    for(int i = 1; i < (int) v.size(); ++i)
        if(ans.contains(v[i]) == OUT) ans = f1(i);
    return ans;
}

```

## 5.13 Puntos

```

typedef long double lf;
const lf EPS = 1e-9;
const lf E0 = 0.0L; //Keep = 0 for integer coordinates,
    otherwise = EPS
const lf PI = acos(-1);
struct pt {
    lf x, y;
    pt(){}
    pt(lf a, lf b): x(a), y(b){}
    pt(lf ang): x(cos(ang)), y(sin(ang)){} // Polar
        unit point: ang(RAD)
    pt operator - (const pt &q) const { return {x - q
        .x, y - q.y}; }
    pt operator + (const pt &q) const { return {x + q
        .x, y + q.y}; }
    // pt operator * (pt p){ return {x * p.x - y * p.
        y, x * p.y + y * p.x}; }
    pt operator * (const lf &t) const { return {x * t
        , y * t}; }
    pt operator / (const lf &t) const { return {x / t
        , y / t}; }
    bool operator == (pt p){ return abs(x - p.x) <=
        EPS && abs(y - p.y) <= EPS; }
    bool operator != (pt p){ return !operator==(p); }
    bool operator < (const pt &q) const { // set /
        sort
        if(fabsl(x - q.x) > E0) return x < q.x;
        return y < q.y;
    }
    void print(){ cout << x << " " << y << "\n"; }
};

pt normalize(pt p){
    lf norm = hypotl(p.x, p.y);
    if(fabsl(norm) > EPS) return {p.x /= norm, p.y /=
        norm};
    else return p;
}

int cmp(lf a, lf b){ return (a + EPS < b ? -1 : (b + EPS <
    a ? 1 : 0)); } // float comparator

// rota ccw
pt rot90(pt p){ return {-p.y, p.x}; }
// w(RAD)
pt rot(pt p, lf w){ return {cosl(w) * p.x - sinl(w) * p.y
    , sinl(w) * p.x + cosl(w) * p.y}; }

lf norm2(pt p){ return p.x * p.x + p.y * p.y; }
lf norm(pt p){ return hypotl(p.x, p.y); }

lf dis2(pt p, pt q){ return norm2(p - q); }
lf dis(pt p, pt q){ return norm(p - q); }

lf arg(pt a){return atan2(a.y, a.x); } // ang(RAD) a x-
    pos
lf dot(pt a, pt b){ return a.x * b.x + a.y * b.y; } // x

```

```

    = 90 -> cos = 0
    if cross(pt a, pt b){ return a.x * b.y - a.y * b.x; } //
    x = 180 -> sin = 0
    if orient(pt a, pt b, pt c){ return cross(b - a, c - a);
    } // AB clockwise = -
    int sign(1f x){ return (EPS < x) - (x < -EPS); }
    // p inside angle abc (center in a)
    bool in_angle(pt a, pt b, pt c, pt p) {
        //assert(fabs(orient(a, b, c)) > E0);
        if(orient(a, b, c) < -E0)
            return orient(a, b, p) >= -E0 || orient(a
            , c, p) <= E0;
        return orient(a, b, p) >= -E0 && orient(a, c, p)
            <= E0;
    }
    if min_angle(pt a, pt b){ return acos(max((1f)-1.0, min((
    1f)1.0, dot(a, b)/norm(a)/norm(b)))); } // ang(RAD)
    if angle(pt a, pt b){ return atan2(cross(a, b), dot(a, b
    )); } // ang(RAD)
    if angle(pt a, pt b, pt c){ // ang(RAD) AB AC ccw
        1f ang = angle(b - a, c - a);
        if (ang < 0) ang += 2 * PI;
        return ang;
    }
    bool half(pt p){ // true if is in (0, 180] (line is x
    axis)
        // assert(p.x != 0 || p.y != 0); // the argument
        of (0, 0) is undefined
        return p.y > 0 || (p.y == 0 && p.x < 0);
    }
    bool half_from(pt p, pt v = {1, 0}) {
        return cross(v,p) < 0 || (cross(v,p) == 0 && dot(
        v,p) < 0);
    }
    // polar sort
    bool polar_cmp(const pt &a, const pt &b){
        return make_tuple(half(a), 0) < make_tuple(half(b
        ), cross(a,b));
    }
    void polar_sort(vector<pt> &v, pt o){ // sort points in
    counterclockwise with respect to point o
        sort(v.begin(), v.end(), [&](pt a,pt b) {
            return make_tuple(half(a - o), 0.0, norm2
            ((a - o))) < make_tuple(half(b - o),
            cross(a - o, b - o), norm2((b - o)));
        });
    }
    int cuad(pt p){ // REVISAR
        if(p.x > 0 && p.y >= 0) return 0;
        if(p.x <= 0 && p.y > 0) return 1;

```

```

        if(p.x < 0 && p.y <= 0) return 2;
        if(p.x >= 0 && p.y < 0) return 3;
        return -1; // x == 0 && y == 0
    }
    bool cmp(pt p1, pt p2){
        int c1 = cuad(p1), c2 = cuad(p2);
        return c1 == c2 ? p1.y * p2.x < p1.x * p2.y : c1
            < c2;
    }
    // O(n*2^d*d)
    // Return the max manhattan distance between points with
    d-dimension.
    ll max_distance_manhattan(vector<vi> p, int d){
        long long ans = 0;
        for (int msk = 0; msk < (1 << d); msk++) {
            long long mx = LLONG_MIN, mn = LLONG_MAX;
            for (int i = 0; i < n; i++) {
                long long cur = 0;
                for (int j = 0; j < d; j++) {
                    if (msk & (1 << j)) cur
                        += p[i][j];
                    else cur -= p[i][j];
                }
                mx = max(mx, cur);
                mn = min(mn, cur);
            }
            ans = max(ans, mx - mn);
        }
        return ans;
    }
    ll sd_to_ll(string num, int canDec = 6){
        string nnum = "";
        bool ok = 0;
        for(int i = 0; i < sz(num); i++){
            if (num[i] == '.' || '0') {
                ok = 1;
                continue;
            }
            if (ok) canDec--;
            nnum.pb(num[i]);
        }
        while(canDec-- > 0) nnum.pb('0');
        return stoll(nnum);
    }

```

## 5.14 Poligonos

// add Points Lines Segments Circles

```

// points in polygon(vector<pt>) ccw or cw
enum {OUT, IN, ON};

lf area(vector<pt>& p){
    lf r = 0.;
    for(int i = 0, n = p.size(); i < n; ++i){
        r += cross(p[i], p[(i + 1) % n]);
    }
    return r / 2; // negative if CW, positive if CCW
}

lf perimeter(vector<pt>& p) {
    lf per = 0;
    for (int i = 0, n = p.size(); i < n; ++i){
        per += norm(p[i] - p[(i + 1) % n]);
    }
    return per;
}

bool is_convex(vector<pt>& p) {
    bool pos = 0, neg = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int o = orient(p[i], p[(i + 1) % n], p[(i + 2) % n]);
        if (o > 0) pos = 1;
        if (o < 0) neg = 1;
    }
    return !(pos && neg);
}

int point_in_polygon(vector<pt>& pol, pt& p){
    int wn = 0;
    for(int i = 0, n = pol.size(); i < n; ++i) {
        lf c = orient(p, pol[i], pol[(i + 1) % n]);
        if(fabs(c) <= E0 && dot(pol[i] - p, pol[(i + 1) % n] - p) <= E0) return ON;
        // on segment

        if(c > 0 && pol[i].y <= p.y + E0 && pol[(i + 1) % n].y - p.y > E0) ++wn;
        if(c < 0 && pol[(i + 1) % n].y <= p.y + E0 && pol[i].y - p.y > E0) --wn;
    }
    return wn ? IN : OUT;
}

// O(logn) polygon CCW, remove collinear
int point_in_convex_polygon(const vector<pt> &pol, const pt &p){
    int low = 1, high = pol.size() - 1;
    while(high - low > 1){
        int mid = (low + high) / 2;
        if(orient(pol[0], pol[mid], p) >= -E0)
            low = mid;
        else high = mid;
    }
}

```

```

if(orient(pol[0], pol[low], p) < -E0) return OUT;
if(orient(pol[low], pol[high], p) < -E0) return OUT;
if(orient(pol[high], pol[0], p) < -E0) return OUT;

if(low == 1 && orient(pol[0], pol[low], p) <= E0)
    return ON;
if(orient(pol[low], pol[high], p) <= E0) return ON;
if(high == (int) pol.size() - 1 && orient(pol[high], pol[0], p) <= E0) return ON;
return IN;
}

// convex polygons in some order (CCW, CW)
vector<pt> minkowski(vector<pt> P, vector<pt> Q) {
    rotate(P.begin(), min_element(P.begin(), P.end()), P.end());
    rotate(Q.begin(), min_element(Q.begin(), Q.end()), Q.end());

    P.push_back(P[0]), P.push_back(P[1]);
    Q.push_back(Q[0]), Q.push_back(Q[1]);

    vector<pt> ans;
    size_t i = 0, j = 0;
    while(i < P.size() - 2 || j < Q.size() - 2){
        ans.push_back(P[i] + Q[j]);
        lf dt = cross(P[i + 1] - P[i], Q[j + 1] - Q[j]);
        if(dt >= E0 && i < P.size() - 2) ++i;
        if(dt <= E0 && j < Q.size() - 2) ++j;
    }
    return ans;
}

pt centroid(vector<pt>& p){
    pt c{0, 0};
    lf scale = 6. * area(p);
    for (int i = 0, n = p.size(); i < n; ++i){
        c = c + (p[i] + p[(i + 1) % n]) * cross(p[i], p[(i + 1) % n]);
    }
    return c / scale;
}

void normalize(vector<pt>& p) { // polygon CCW
    int bottom = min_element(p.begin(), p.end()) - p.begin();
    vector<pt> tmp(p.begin() + bottom, p.end());
    tmp.insert(tmp.end(), p.begin(), p.begin() + bottom);
    p.swap(tmp);
    bottom = 0;
}

void remove_col(vector<pt>& p){

```

```

vector<pt> s;
for(int i = 0, n = p.size(); i < n; i++){
    if(!on_segment(p[(i - 1 + n) % n], p[(i + 1) % n], p[i])) s.push_back(p[i]);
}
p.swap(s);
}

void delete_repetead(vector<pt>& p){
    vector<pt> aux;
    sort(p.begin(), p.end());
    for (pt &pi : p){
        if (aux.empty() || aux.back() != pi) aux.push_back(pi);
    }
    p.swap(aux);
}

pt farthest(vector<pt>& p, pt v){ // O(log(n)) only
    CONVEX, v: dir
    int n = p.size();
    if(n < 10){
        int k = 0;
        for(int i = 1; i < n; i++) if(dot(v, (p[i] - p[k])) > EPS) k = i;
        return p[k];
    }
    pt a = p[1] - p[0];
    int s = 0, e = n, ua = dot(v, a) > EPS;
    if(!ua && dot(v, (p[n - 1] - p[0])) <= EPS)
        return p[0];
    while(1){
        int m = (s + e) / 2;
        pt c = p[(m + 1) % n] - p[m];
        int uc = dot(v, c) > EPS;
        if(!uc && dot(v, (p[(m - 1 + n) % n] - p[m])) <= EPS) return p[m];
        if(ua && (!uc || dot(v, (p[s] - p[m])) > EPS)) e = m;
        else if(ua || uc || dot(v, (p[s] - p[m])) >= -EPS) s = m, a = c, ua = uc;
        else e = m;
        assert(e > s + 1);
    }
}

vector<pt> cut(vector<pt>& p, line l){
    // cut CONVEX polygon by line l
    // returns part at left of l.pq
    vector<pt> q;
    for(int i = 0, n = p.size(); i < n; i++) {
        int d0 = sign(l.side(p[i]));
        int d1 = sign(l.side(p[(i + 1) % n]));
        if(d0 >= 0) q.push_back(p[i]);
        line m(p[i], p[(i + 1) % n]);
    }
}

```

```

        if(d0 * d1 < 0 && !(abs(cross(l.v, m.v)) <= EPS)){
            q.push_back((inter_ll(l, m)));
        }
    }
    return q;
}

// O(n)
vector<pair<int, int>> antipodal(vector<pt>& p){
    vector<pair<int, int>> ans;
    int n = p.size();
    if (n == 2) ans.push_back({0, 1});
    if (n < 3) return ans;
    auto nxt = [&](int x){ return (x + 1 == n ? 0 : x + 1); };
    auto area2 = [&](pt a, pt b, pt c){ return cross(b - a, c - a); };
    int b0 = 0;
    while (abs(area2(p[n - 1], p[0], p[nxt(b0)])) > abs(area2(p[n - 1], p[0], p[b0]))) ++b0;
    for (int b = b0, a = 0; b != 0 && a <= b0; ++a) {
        ans.push_back({a, b});
        while (abs(area2(p[a], p[nxt(a)], p[nxt(b)])) > abs(area2(p[a], p[nxt(a)], p[b]))) {
            b = nxt(b);
            if (a != b0 || b != 0) ans.push_back({a, b});
            else return ans;
        }
        if (abs(area2(p[a], p[nxt(a)], p[nxt(b)]) == abs(area2(p[a], p[nxt(a)], p[b]))) {
            if (a != b0 || b != n - 1) ans.push_back({a, nxt(b)});
            else ans.push_back({nxt(a), b});
        }
    }
    return ans;
}

// O(n)
// square distance of most distant points, prereq: convex, ccw, NO COLLINEAR POINTS
lf callipers(vector<pt>& p){
    int n = p.size();
    lf r = 0;
    for(int i = 0, j = n < 2 ? 0 : 1; i < j; ++i){
        for(;; j = (j + 1) % n){
            r = max(r, norm2(p[i] - p[j]));
            if(cross((p[(i + 1) % n] - p[i]), (p[(j + 1) % n] - p[j])) <= EPS) break;
        }
    }
}

```

```

    return r;
}
// O(n + m) max_dist between 2 points (pa, pb) of 2
// Convex polygons (a, b)
lf rotating_callipers(vector<pt>& a, vector<pt>& b){ //
    REVISAR
    if (a.size() > b.size()) swap(a, b); // <- del or
    add
    pair<ll, int> start = {-1, -1};
    if(a.size() == 1) swap(a, b);
    for(int i = 0; i < a.size(); i++) start = max(
        start, {norm2(b[0] - a[i]), i});
    if(b.size() == 1) return start.first;

    lf r = 0;
    for(int i = 0, j = start.second; i < b.size(); ++
        i){
        for(;; j = (j + 1) % a.size()){
            r = max(r, norm2(b[i] - a[j]));
            if(cross((b[(i + 1) % b.size()] -
                b[i]), (a[(j + 1) % a.size()]
                - a[j])) <= EPS) break;
        }
    }
    return r;
}

lf intercircle(vector<pt>& p, circle c){ // area of
    intersection with circle
    lf r=0.;
    for(int i = 0, n = p.size(); i < n; i++){
        int j = (i + 1) % n;
        lf w = intertriangle(c, p[i], p[j]);
        if(cross((p[j] - c.center), (p[i] - c.
            center)) > 0) r += w;
        else r -= w;
    }
    return abs(r);
}

ll pick(vector<pt>& p){
    ll boundary = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        int j = (i + 1 == n ? 0 : i + 1);
        boundary += __gcd((ll)abs(p[i].x - p[j].x
            ), (ll)abs(p[i].y - p[j].y));
    }
    return abs(area(p)) + 1 - boundary / 2;
}

// minimum distance between two parallel lines (non
// necessarily axis parallel)
// such that the polygon can be put between the lines
// O(n) CCW polygon
lf width(vector<pt> &p) {

```

```

    int n = (int)p.size();
    if (n <= 2) return 0;
    lf ans = inf;
    int i = 0, j = 1;
    while (i < n){
        while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n
            ] - p[j]) >= 0) j = (j + 1) % n;
        line ll(p[i], p[(i + 1) % n]);
        ans = min(ans, ll.dist(p[j]));
        i++;
    }
    return ans;
}

// O(n) {minimum perimeter, minimum area} CCW polygon
pair<ld, ld> minimum_enclosing_rectangle(vector<pt> &p) {
    int n = p.size();
    if (n <= 2) return {perimeter(p), 0};
    int mndot = 0;
    lf tmp = dot(p[1] - p[0], p[0]);
    for (int i = 1; i < n; i++) {
        if (dot(p[1] - p[0], p[i]) <= tmp) {
            tmp = dot(p[1] - p[0], p[i]);
            mndot = i;
        }
    }
    ld ansP = inf;
    ld ansA = inf;
    int i = 0, j = 1, mxdot = 1;
    while (i < n) {
        pt cur = p[(i + 1) % n] - p[i];
        while (cross(cur, p[(j + 1) % n] - p[j]) >= 0) j
            = (j + 1) % n;
        while (dot(p[(mxdot + 1) % n], cur) >= dot(p[
            mxdot], cur)) mxdot = (mxdot + 1) % n;
        while (dot(p[(mndot + 1) % n], cur) <= dot(p[
            mndot], cur)) mndot = (mndot + 1) % n;
        line ll(p[i], p[(i + 1) % n]);

        // minimum perimeter
        ansP = min(ansP, 2.0 * ((dot(p[mxdot], cur) /
            norm(cur) - dot(p[mndot], cur) / norm(cur)) +
            ll.dist(p[j])));
        // minimum area
        ansA = min(ansA, (dot(p[mxdot], cur) / norm(cur)
            - dot(p[mndot], cur) / norm(cur)) * ll.dist(p[
            j]));
        i++;
    }
    return {ansP, ansA};
}

// maximum distance from a convex polygon to another
// convex polygon
lf maximum_dist_from_polygon_to_polygon(vector<pt> &u,

```



```

vector<pt> &v){ //O(n)
    int n = (int)u.size(), m = (int)v.size();
    if ans = 0;
    if (n < 3 || m < 3) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) ans = max(ans,
                dis2(u[i], v[j]));
        }
        return sqrt(ans);
    }
    if (u[0].x > v[0].x) swap(n, m), swap(u, v);
    int i = 0, j = 0, step = n + m + 10;
    while (j + 1 < m && v[j].x < v[j + 1].x) j++;
    while (step-- > 0) {
        if (cross(u[(i + 1) % n] - u[i], v[(j + 1) % m] -
            v[j]) >= 0) j = (j + 1) % m;
        else i = (i + 1) % n;
        ans = max(ans, dis2(u[i], v[j]));
    }
    return sqrt(ans);
}

// -----

pt project_from_point_to_seg(pt a, pt b, pt c) {
    double r = dis2(a, b);
    if (sign(r) == 0) return a;
    r = dot(c - a, b - a) / r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b - a) * r;
}

// minimum distance from point c to segment ab
lf pt_to_seg(pt a, pt b, pt c) {
    return dis(c, project_from_point_to_seg(a, b, c));
}

pair<pt, int> point_poly_tangent(vector<pt> &p, pt Q, int
    dir, int l, int r) {
    while (r - l > 1) {
        int mid = (l + r) >> 1;
        bool pvs = sign(orient(Q, p[mid], p[mid - 1])) !=
            -dir;
        bool nxt = sign(orient(Q, p[mid], p[mid + 1])) !=
            -dir;
        if (pvs && nxt) return {p[mid], mid};
        if (!(pvs || nxt)) {
            auto p1 = point_poly_tangent(p, Q, dir, mid +
                1, r);
            auto p2 = point_poly_tangent(p, Q, dir, l,
                mid - 1);
            return sign(orient(Q, p1.first, p2.first)) ==
                dir ? p1 : p2;
        }
    }
}

```

```

    if (!pvs) {
        if (sign(orient(Q, p[mid], p[l])) == dir) r
            = mid - 1;
        else if (sign(orient(Q, p[l], p[r])) == dir)
            r = mid - 1;
        else l = mid + 1;
    }
    if (!nxt) {
        if (sign(orient(Q, p[mid], p[l])) == dir) l
            = mid + 1;
        else if (sign(orient(Q, p[l], p[r])) == dir)
            r = mid - 1;
        else l = mid + 1;
    }
}
pair<pt, int> ret = {p[l], l};
for (int i = l + 1; i <= r; i++) ret = sign(orient(Q,
    ret.first, p[i])) != dir ? make_pair(p[i], i) :
    ret;
return ret;
}

// (ccw, cw) tangents from a point that is outside this
// convex polygon
// returns indexes of the points
// ccw means the tangent from Q to that point is in the
// same direction as the polygon ccw direction
pair<int, int> tangents_from_point_to_polygon(vector<pt>
    &p, pt Q) {
    int ccw = point_poly_tangent(p, Q, 1, 0, (int)p.size()
        - 1).second;
    int cw = point_poly_tangent(p, Q, -1, 0, (int)p.size()
        - 1).second;
    return make_pair(ccw, cw);
}

// minimum distance from a point to a convex polygon
// it assumes point lie strictly outside the polygon
lf dist_from_point_to_polygon(vector<pt> &p, pt z) {
    lf ans = inf;
    int n = p.size();
    if (n <= 3) {
        for (int i = 0; i < n; i++) ans = min(ans,
            pt_to_seg(p[i], p[(i + 1) % n], z));
        return ans;
    }
    pair<int, int> dum = tangents_from_point_to_polygon(p,
        z);
    int r = dum.first;
    int l = dum.second;
    if (l > r) r += n;
    while (l < r) {
        int mid = (l + r) >> 1;
        lf left = dis2(p[mid % n], z), right = dis2(p[(mid
            + 1) % n], z);
        ans = min({ans, left, right});
    }
}

```



```

        if(left < right) r = mid;
        else l = mid + 1;
    }
    ans = sqrt(ans);
    ans = min(ans, pt_to_seg(p[l % n], p[(l + 1) % n], z)
    );
    ans = min(ans, pt_to_seg(p[l % n], p[(l - 1 + n) % n
    ], z));
    return ans;
}

// minimum distance from a convex polygon to another
// convex polygon
// the polygon doesnot overlap or touch
lf dist_from_polygon_to_polygon(vector<pt> &p1, vector<pt>
> &p2) { // O(n log n)
    lf ans = inf;
    for (int i = 0; i < p1.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p2, p1[
        i]));
    }
    for (int i = 0; i < p2.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p1, p2[
        i]));
    }
    return ans;
}

// it returns a point such that the sum of distances
// from that point to all points in p is minimum
// O(n log^2 MX)
PT geometric_median(vector<PT> p) {
    auto tot_dist = [&](PT z) {
        double res = 0;
        for (int i = 0; i < p.size(); i++) res +=
            dist(p[i], z);
        return res;
    };
    auto findY = [&](double x) {
        double yl = -1e5, yr = 1e5;
        for (int i = 0; i < 60; i++) {
            double yml = yl + (yr - yl) / 3;
            double ym2 = yr - (yr - yl) / 3;
            double d1 = tot_dist(PT(x, yml));
            double d2 = tot_dist(PT(x, ym2));
            if (d1 < d2) yr = ym2;
            else yl = yml;
        }
        return pair<double, double> (yl, tot_dist(PT(
        x, yl)));
    };
    double xl = -1e5, xr = 1e5;
    for (int i = 0; i < 60; i++) {
        double xml = xl + (xr - xl) / 3;
        double xm2 = xr - (xr - xl) / 3;

```

```

        double yl, d1, y2, d2;
        auto z = findY(xml); yl = z.first; d1 = z.second;
        z = findY(xm2); y2 = z.first; d2 = z.second;
        if (d1 < d2) xr = xm2;
        else xl = xml;
    }
    return {xl, findY(xl).first };
}

// ear decomposition, O(n^3) but faster
vector<vector<pt>> triangulate(vector<pt> p) {
    vector<vector<pt>> v;

    while (p.size() >= 3){
        for (int i = 0, n = p.size(); i < n; i++)
        {
            int pre = i == 0 ? n - 1 : i -
            1;;
            int nxt = i == n - 1 ? 0 : i +
            1;;
            lf ori = orient(p[i], p[pre], p[
            nxt]);
            if (ori < 0){
                int ok = 1;
                for (int j = 0; j < n; j
                ++){
                    if (j == i || j
                    == pre || j ==
                    nxt) continue;
                    vector<pt> tr = {
                    p[i], p[pre],
                    p[nxt]};
                    if (
                    point_in_polygon
                    (tr, p[j]) !=
                    OUT){
                        ok = 0;
                        break;
                    }
                }
                if (ok){
                    v.push_back({p[
                    pre], p[i], p[
                    nxt]});
                    p.erase(p.begin()
                    + i);
                    break;
                }
            }
        }
    }
    return v;
}

// CH DP

```

```

// vecs contains all vectors u -> v that lie within the
// original polygon
sort(vecs.begin(), vecs.end(), cmp);
for (auto [u, v] : vecs)
    for (int s = 0; s < n; s++)
        dp[s][v] += dp[s][u];

// Suma del area de todos los triangulos de un poligono
int main() {
    int n; cin >> n;
    vector<pt> pts(n);
    for (int i = 0; i < n; i++) cin >> pts[i].x >> pts[i].y;
    vector<pt> pref(n + 1);
    for (int i = 1; i < n + 1; i++) {
        pref[i] = pref[i - 1] + pts[i - 1];
    }
    ld totalT = 0;
    for (int i = 0; i < n; i++) {
        pt p1 = (pts[i] * (ld)i) - pref[i];
        pt p2 = (pref[n] - pref[i + 1]) - (pts[i] * (ld)(n - 1 - i));
        totalT += cross(p1, p2);
    }
    // area = totalT / 2
}

```

## 5.15 Segmentos

```

// add Lines Points
bool in_disk(pt a, pt b, pt p) { // pt p inside ab disk
    return dot(a - p, b - p) <= E0;
}

bool on_segment(pt a, pt b, pt p) { // p on ab
    return orient(a, b, p) == 0 && in_disk(a, b, p);
}

// ab crossing cd
bool proper_inter(pt a, pt b, pt c, pt d, pt& out) {
    if (oa = orient(c, d, a);
        ob = orient(c, d, b);
        oc = orient(a, b, c);
        od = orient(a, b, d);
        // Proper intersection exists iff opposite signs
        if (oa * ob < 0 && oc * od < 0) {
            out = (a * ob - b * oa) / (ob - oa);
            return true;
        }
    }
}

```

```

    return false;
}

// intersection bwn segments
set<pt> inter_ss(pt a, pt b, pt c, pt d) {
    pt out;
    if (proper_inter(a, b, c, d, out)) return {out};
    set<pt> s;
    if (on_segment(c, d, a)) s.insert(a); // a in cd
    if (on_segment(c, d, b)) s.insert(b); // b in cd
    if (on_segment(a, b, c)) s.insert(c); // c in ab
    if (on_segment(a, b, d)) s.insert(d); // d in ab
    return s;
}

lf pt_to_seg(pt a, pt b, pt p) { // p to ab
    if (a != b) {
        line l(a, b);
        if (l.cmp_proj(a, p) && l.cmp_proj(p, b))
            // if closest to projection = (a, p, b)
            return l.dist(p); // output distance to line
    }
    return min(norm(p - a), norm(p - b)); // otherwise distance to A or B
}

lf seg_to_seg(pt a, pt b, pt c, pt d) {
    pt dummy;
    if (proper_inter(a, b, c, d, dummy)) return 0; // ab intersects cd
    return min({pt_to_seg(a, b, c), pt_to_seg(a, b, d),
                pt_to_seg(c, d, a), pt_to_seg(c, d, b)});
    // try the 4 pts
}

int length_union(vector<pt>& a) { // REVISAR
    int n = a.size();
    vector<pair<int, bool>> x(n * 2);
    for (int i = 0; i < n; i++) {
        x[i * 2] = {a[i].x, false};
        x[i * 2 + 1] = {a[i].y, true};
    }
    sort(x.begin(), x.end());
    int result = 0;
    int c = 0;
    for (int i = 0; i < n * 2; i++) {
        if (i > 0 && x[i].first > x[i - 1].first
            && c > 0) result += x[i].first - x[i - 1].first;
        if (x[i].second) c--;
        else c++;
    }
    return result;
}

```

## 5.16 Triangle Union

```

// Area of the union of a set of n triangles
// T(n^2 logn) M(n)

typedef double dbl;
const dbl eps = 1e-9;

inline bool eq(dbl x, dbl y){
    return fabs(x - y) < eps;
}

inline bool lt(dbl x, dbl y){
    return x < y - eps;
}

inline bool gt(dbl x, dbl y){
    return x > y + eps;
}

inline bool le(dbl x, dbl y){
    return x < y + eps;
}

inline bool ge(dbl x, dbl y){
    return x > y - eps;
}

struct ptT{
    dbl x, y;
    ptT(){}
    ptT(dbl x, dbl y): x(x), y(y){}
    inline ptT operator - (const ptT & p) const{
        return ptT{x - p.x, y - p.y};
    }
    inline ptT operator + (const ptT & p) const{
        return ptT{x + p.x, y + p.y};
    }
    inline ptT operator * (dbl a) const{
        return ptT{x * a, y * a};
    }
    inline dbl cross(const ptT & p) const{
        return x * p.y - y * p.x;
    }
    inline dbl dot(const ptT & p) const{
        return x * p.x + y * p.y;
    }
    inline bool operator == (const ptT & p) const{
        return eq(x, p.x) && eq(y, p.y);
    }
};

struct LineT{
    ptT p[2];
    LineT(){}
    LineT(ptT a, ptT b):p{a, b}{}

```

```

    ptT vec() const{
        return p[1] - p[0];
    }
    ptT& operator [] (size_t i){
        return p[i];
    }
};

inline bool lexComp(const ptT & l, const ptT & r){
    if(fabs(l.x - r.x) > eps){
        return l.x < r.x;
    }
    else return l.y < r.y;
}

vector<ptT> interSegSeg(LineT l1, LineT l2){
    if(eq(l1.vec().cross(l2.vec()), 0)){
        if(!eq(l1.vec().cross(l2[0] - l1[0]), 0))
            return {};
        if(!lexComp(l1[0], l1[1]))
            swap(l1[0], l1[1]);
        if(!lexComp(l2[0], l2[1]))
            swap(l2[0], l2[1]);
        ptT l = lexComp(l1[0], l2[0]) ? l2[0] : l1[0];
        ptT r = lexComp(l1[1], l2[1]) ? l1[1] : l2[1];
        if(l == r)
            return {l};
        else return lexComp(l, r) ? vector<ptT>{l, r} :
            vector<ptT>();
    }
    else{
        dbl s = (l2[0] - l1[0]).cross(l2.vec()) / l1.vec()
            .cross(l2.vec());
        ptT inter = l1[0] + l1.vec() * s;
        if(ge(s, 0) && le(s, 1) && le((l2[0] - inter).dot
            (l2[1] - inter), 0))
            return {inter};
        else
            return {};
    }
}

inline char get_segtype(LineT segment, ptT other_point){
    if(eq(segment[0].x, segment[1].x))
        return 0;
    if(!lexComp(segment[0], segment[1]))
        swap(segment[0], segment[1]);
    return (segment[1] - segment[0]).cross(other_point -
        segment[0]) > 0 ? 1 : -1;
}

dbl union_area(vector<tuple<ptT, ptT, ptT> > triangles){
    vector<LineT> segments(3 * triangles.size());
    vector<char> segtype(segments.size());
    for(size_t i = 0; i < triangles.size(); i++){
        ptT a, b, c;

```

```

tie(a, b, c) = triangles[i];
segments[3 * i] = lexComp(a, b) ? LineT(a, b) :
    LineT(b, a);
segtype[3 * i] = get_segtype(segments[3 * i], c);
segments[3 * i + 1] = lexComp(b, c) ? LineT(b, c)
    : LineT(c, b);
segtype[3 * i + 1] = get_segtype(segments[3 * i +
    1], a);
segments[3 * i + 2] = lexComp(c, a) ? LineT(c, a)
    : LineT(a, c);
segtype[3 * i + 2] = get_segtype(segments[3 * i +
    2], b);
}
vector<dbl> k(segments.size()), b(segments.size());
for(size_t i = 0; i < segments.size(); i++){
    if(segtype[i]){
        k[i] = (segments[i][1].y - segments[i][0].y)
            / (segments[i][1].x - segments[i][0].x);
        b[i] = segments[i][0].y - k[i] * segments[i]
            [0].x;
    }
}
dbl ans = 0;
for(size_t i = 0; i < segments.size(); i++){
    if(!segtype[i])
        continue;
    dbl l = segments[i][0].x, r = segments[i][1].x;
    vector<pair<dbl, int>> evts;
    for(size_t j = 0; j < segments.size(); j++){
        if(!segtype[j] || i == j)
            continue;
        dbl l1 = segments[j][0].x, r1 = segments[j]
            [1].x;
        if(ge(l1, r) || ge(l, r1))
            continue;
        dbl common_l = max(l, l1), common_r = min(r,
            r1);
        auto pts = interSegSeg(segments[i], segments[
            j]);
        if(pts.empty()){
            dbl y11 = k[j] * common_l + b[j];
            dbl y1 = k[i] * common_l + b[i];
            if(lt(y11, y1) == (segtype[i] == 1)){
                int evt_type = -segtype[i] * segtype[
                    j];
                evts.emplace_back(common_l, evt_type)
                    ;
                evts.emplace_back(common_r, -evt_type
                    );
            }
        }
        else if(pts.size() == 1u){
            dbl y1 = k[i] * common_l + b[i], y11 = k[
                j] * common_l + b[j];

```

```

                int evt_type = -segtype[i] * segtype[j];
                if(lt(y11, y1) == (segtype[i] == 1)){
                    evts.emplace_back(common_l, evt_type)
                        ;
                    evts.emplace_back(pts[0].x, -evt_type
                        );
                }
                y1 = k[i] * common_r + b[i], y11 = k[j] *
                    common_r + b[j];
                if(lt(y11, y1) == (segtype[i] == 1)){
                    evts.emplace_back(pts[0].x, evt_type)
                        ;
                    evts.emplace_back(common_r, -evt_type
                        );
                }
            }
        }
        else{
            if(segtype[j] != segtype[i] || j > i){
                evts.emplace_back(common_l, -2);
                evts.emplace_back(common_r, 2);
            }
        }
    }
    evts.emplace_back(1, 0);
    sort(evts.begin(), evts.end());
    size_t j = 0;
    int balance = 0;
    while(j < evts.size()){
        size_t ptr = j;
        while(ptr < evts.size() && eq(evts[j].first,
            evts[ptr].first)){
            balance += evts[ptr].second;
            ++ptr;
        }
        if(!balance && !eq(evts[j].first, r)){
            dbl next_x = ptr == evts.size() ? r :
                evts[ptr].first;
            ans -= segtype[i] * (k[i] * (next_x +
                evts[j].first) + 2 * b[i]) * (next_x -
                evts[j].first);
        }
        j = ptr;
    }
}
return ans/2;
}

```

## 6 Grafos

### 6.1 2sat

```

// O(n+m)
// (x1 or y1) and (x2 or y2) and ... and (xn or yn)

```

```

struct sat2{
    vector<vector<vi>> g;
    vector<bool> vis, val;
    stack<int> st;
    vi comp;
    int n;

    sat2(int n):n(n),g(2, vector<vi>(2*n)),vis(2*n),
        val(2*n),comp(2*n){}

    int neg(int x){return 2*n-x-1;} // get not x
    void make_true(int u){add_edge(neg(u), u);}
    void make_false(int u){make_true(neg(u));}
    void add_or(int u, int v){implication(neg(u),v);}
    // (u or v)
    void diff(int u, int v){eq(u, neg(v));} // u != v
    void eq(int u, int v){
        implication(u, v);
        implication(v, u);
    }

    void implication(int u,int v){
        add_edge(u, v);
        add_edge(neg(v),neg(u));
    }

    void add_edge(int u, int v){
        g[0][u].push_back(v);
        g[1][v].push_back(u);
    }

    void dfs(int id, int u, int t=0){
        vis[u]=true;
        for(auto &v:g[id][u])
            if(!vis[v])dfs(id, v, t);
        if(id)comp[u]=t;
        else st.push(u);
    }

    void kosaraju() {
        for(int u=0;u<n;++u){
            if(!vis[u])dfs(0, u);
            if(!vis[neg(u)])dfs(0, neg(u));
        }
        vis.assign(2*n, false);
        int t=0;
        while(!st.empty()){
            int u=st.top();st.pop();
            if(!vis[u])dfs(1, u, t++);
        }

        // return true if satisfiable, fills val[]
        bool check() {
            kosaraju();
            for(int i=0;i<n;++i){

```

```

                if(comp[i]==comp[neg(i)]) return
                    false;
                val[i]=comp[i]>comp[neg(i)];
            }
            return true;
        }
    };

    int m,n;cin>>m>>n;
    sat2 s(n);
    char c1,c2;
    for(int a,b,i=0;i<m;++i){
        cin>>c1>>a>>c2>>b;
        a--;b--;
        if(c1=='-')a=s.neg(a);
        if(c2=='-')b=s.neg(b);
        s.add_or(a,b);
    }
    if(s.check()){
        for(int i=0;i<n;++i)cout<<(s.val[i]?'+':'-')<<" "
            ;
        cout<<"\n";
    }else cout<<"IMPOSSIBLE\n";
}

```

## 6.2 Bellman Ford

```

// O(V*E)
vi bellman_ford(vector<vii> &adj, int s, int n){
    vi dist(n, INF); dist[s] = 0;
    for (int i = 0; i<n-1; i++){
        bool modified = false;
        for (int u = 0; u<n; u++){
            if (dist[u] != INF)
                for (auto &[v, w] : adj[u]){
                    if (dist[v] <=
                        dist[u] + w)
                        continue;
                    dist[v] = dist[u]
                        + w;
                    modified = true;
                }
            if (!modified) break;
        }

        bool negativeCicle = false;
        for (int u = 0; u<n; u++){
            if (dist[u] != INF)
                for (auto &[v, w] : adj[u]){
                    if (dist[v] > dist[u] + w)
                        negativeCicle = true;
                }
        }

        return dist;
    }
}

```

### 6.3 Block Cut Tree

```

// O(n) build
// bi_connected save the edges
const int maxn = 1e5+5;
int lowLink[maxn] , dfn[maxn];
vector<vector<ii>> bi_connected;
stack<ii> comps;
int ndfn;

void tarjan(vector<vi>& adj, int u=0, int par=-1){
    dfn[u] = lowLink[u] = ndfn++;
    for(auto &v : adj[u]){
        if (v != par && dfn[v] < dfn[u])
            comps.push({u, v});
        if(dfn[v] == -1){
            tarjan(adj, v, u);
            lowLink[u] = min(lowLink[u] ,
                lowLink[v]);
            if (lowLink[v] >= dfn[u]){
                bi_connected.emplace_back
                    (vector<ii>());
                ii edge;
                do{
                    edge = comps.top
                        ();
                    comps.pop();
                    bi_connected.back
                        ().
                        emplace_back(
                            edge);
                }while(edge.first != u ||
                    edge.second != v);
                reverse(all(bi_connected.
                    back()));
            }
        }else if(v != par){
            lowLink[u] = min(lowLink[u] , dfn
                [v]);
        }
    }
}

void init(vector<vi>& adj){
    for(int i=0;i<sz(adj);++i)
        dfn[i]=-1;
    bi_connected.clear();
    comps=stack<ii>();
    ndfn=0;
    tarjan(adj);
}

```

### 6.4 Bridges Online

```

vector<int> par, dsu_2ecc, dsu_cc, dsu_cc_size;
int bridges;
int lca_iteration;
vector<int> last_visit;

void init(int n) {
    par.resize(n);
    dsu_2ecc.resize(n);
    dsu_cc.resize(n);
    dsu_cc_size.resize(n);
    lca_iteration = 0;
    last_visit.assign(n, 0);
    for (int i=0; i<n; ++i) {
        dsu_2ecc[i] = i;
        dsu_cc[i] = i;
        dsu_cc_size[i] = 1;
        par[i] = -1;
    }
    bridges = 0;
}

int find_2ecc(int v) {
    if (v == -1)
        return -1;
    return dsu_2ecc[v] == v ? v : dsu_2ecc[v] =
        find_2ecc(dsu_2ecc[v]);
}

int find_cc(int v) {
    v = find_2ecc(v);
    return dsu_cc[v] == v ? v : dsu_cc[v] = find_cc(
        dsu_cc[v]);
}

void make_root(int v) {
    int root = v;
    int child = -1;
    while (v != -1) {
        int p = find_2ecc(par[v]);
        par[v] = child;
        dsu_cc[v] = root;
        child = v;
        v = p;
    }
    dsu_cc_size[root] = dsu_cc_size[child];
}

void merge_path (int a, int b) {
    ++lca_iteration;
    vector<int> path_a, path_b;
    int lca = -1;
    while (lca == -1) {
        if (a != -1) {
            a = find_2ecc(a);

```

```

        path_a.push_back(a);
        if (last_visit[a] ==
            lca_iteration){
            lca = a;
            break;
        }
        last_visit[a] = lca_iteration;
        a = par[a];
    }
    if (b != -1) {
        b = find_2ecc(b);
        path_b.push_back(b);
        if (last_visit[b] ==
            lca_iteration){
            lca = b;
            break;
        }
        last_visit[b] = lca_iteration;
        b = par[b];
    }
}

for (int v : path_a) {
    dsu_2ecc[v] = lca;
    if (v == lca)
        break;
    --bridges;
}
for (int v : path_b) {
    dsu_2ecc[v] = lca;
    if (v == lca)
        break;
    --bridges;
}

}

void add_edge(int a, int b) {
    a = find_2ecc(a);
    b = find_2ecc(b);
    if (a == b)
        return;

    int ca = find_cc(a);
    int cb = find_cc(b);
    if (ca != cb) {
        ++bridges;
        if (dsu_cc_size[ca] > dsu_cc_size[cb]) {
            swap(a, b);
            swap(ca, cb);
        }
        make_root(a);
        par[a] = dsu_cc[a] = b;
        dsu_cc_size[cb] += dsu_cc_size[a];
    } else {
        merge_path(a, b);
    }
}

```

```

    }
}

```

## 6.5 Camino Mas Corto De Longitud Fija

```

/*
  Modificar operacion * de matrix de esta forma:
  En la exponenciacion binaria inicializar matrix ans = b
*/
const ll INFL = 2e18;
matrix operator * (const matrix &b){
    matrix ans(this->r, b.c, vector<vl>(this->r, vl(b
        .c, INFL)));
    for (int i = 0; i<this->r; i++) {
        for (int k = 0; k<b.r; k++){
            for (int j = 0; j<b.c; j++){
                ans.m[i][j] = min(ans.m[i
                    ][j], m[i][k] + b.m[k
                        ][j]);
            }
        }
    }
    return ans;
}

int main() {
    int n, m, k; cin >> n >> m >> k;
    vector<vl> adj(n, vl(n, INFL));

    for (int i = 0; i<m; i++){
        ll a, b, c; cin >> a >> b >> c; a--; b--;
        adj[a][b] = min(adj[a][b], c);
    }

    matrix graph(n, n, adj);
    graph = pow(graph, k-1);

    cout << (graph.m[0][n-1]==INFL ? -1 : graph.m[0][
        n-1]) << "\n";

    return 0;
}

```

## 6.6 Clique

```

/**
 * Credit: kactl
 * Given a graph as a symmetric bitset matrix (without
   any self edges)
 * Finds the maximum clique
 * Can be used to find the maximum independent set by
   finding a clique of the complement graph.

```

```

* Runs in about 1s for n=155, and faster for sparse graphs
* 0 indexed
*/
const int N = 40;
typedef vector<bitset<N>> graph;
struct Maxclique {
    double limit = 0.025, pk = 0;
    struct Vertex {
        int i, d = 0;
    };
    typedef vector<Vertex> vv;
    graph e;
    vv V;
    vector<vector<int>> C;
    vector<int> qmax, q, S, old;
    void init(vv& r) {
        for (auto& v : r) v.d = 0;
        for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
        sort(r.begin(), r.end(), [](auto a, auto b) {
            return a.d > b.d;
        });
        int mxD = r[0].d;
        for (int i = 0; i < sz(r); i++) r[i].d = min(i, mxD) + 1;
    }
    void expand(vv& R, int lev = 1) {
        S[lev] += S[lev - 1] - old[lev];
        old[lev] = S[lev - 1];
        while (sz(R)) {
            if (sz(q) + R.back().d <= sz(qmax)) return;
            q.push_back(R.back().i);
            vv T;
            for(auto v : R) if (e[R.back().i][v.i]) T.push_back({v.i});
            if (sz(T)) {
                if (S[lev]++ / ++pk < limit) init(T);
                int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
                C[1].clear(), C[2].clear();
                for (auto v : T) {
                    int k = 1;
                    auto f = [&](int i) {
                        return e[v.i][i];
                    };
                    while (any_of(C[k].begin(), C[k].end(), f)) k++;
                    if (k > mxk) mxk = k, C[mxk + 1].clear();
                    if (k < mnk) T[j++].i = v.i;
                    C[k].push_back(v.i);
                }
                if (j > 0) T[j - 1].d = 0;
                for (int k = mnk; k <= mxk; k++) for (int i : C[k])

```

```

                T[j].i = i, T[j++].d = k;
                expand(T, lev + 1);
            } else if (sz(q) > sz(qmax)) qmax = q;
            q.pop_back(), R.pop_back();
        }
    }
    Maxclique(graph g) : e(g), C(sz(e) + 1), S(sz(C)), old(S) {
        for (int i = 0; i < sz(e); i++) V.push_back({i});
    }
    vector<int> solve() { // returns the clique
        init(V), expand(V);
        return qmax;
    }
};

```

## 6.7 Cycle Directed

```

vector<vi> adj;
vi parent, color;
int cy0, cy1;

bool dfs(int v) {
    color[v] = 1;
    for(int u : adj[v]) {
        if (color[u] == 0) {
            parent[u] = v;
            if (dfs(u)) return true;
        } else if (color[u] == 1) {
            cy1 = v;
            cy0 = u;
            return true;
        }
    }
    color[v] = 2;
    return false;
}

// O(m)
void find_cycle(int n) {
    color.assign(n, 0);
    parent.assign(n, -1);
    cy0 = -1;
    for(int v = 0; v < n; ++v) {
        if (color[v] == 0) {
            if (dfs(v)) break;
        }
    }
    if (cy0 == -1) {
        cout << "IMPOSSIBLE\n";
        return;
    }
    vi cycle;
    cycle.push_back(cy0);

```



```

        for(int v=cy1;v!=cy0;v=parent[v]) cycle.push_back(
            v);
        cycle.push_back(cy0);
        reverse(cycle.begin(), cycle.end());
        print(cycle);
    }

```

## 6.8 Cycle Undirected

```

vector<vi> adj;
vector<bool> visited;
int cy0,cy1;
vi parent;

bool dfs(int v, int par){
    visited[v]=true;
    for(int u:adj[v]){
        if(u==par) continue;
        if(visited[u]){
            cy1=v;
            cy0=u;
            return true;
        }
        parent[u]=v;
        if(dfs(u,parent[u])) return true;
    }
    return false;
}

// O(m)
void find_cycle(int n){
    visited.assign(n, false);
    parent.assign(n, -1);
    cy0=-1;
    for(int v=0;v<n;++v){
        if(!visited[v]){
            if(dfs(v, parent[v])) break;
        }
    }
    if(cy0===-1){
        cout<<"IMPOSSIBLE\n";
        return;
    }
    vi cycle;
    cycle.push_back(cy0);
    for(int v=cy1;v!=cy0;v=parent[v]) cycle.push_back(
        v);
    cycle.push_back(cy0);
    print(cycle);
}

```

## 6.9 Dial Algorithm

```

const int maxn = 2e5+5;
vector<ii> adj[maxn];

// O(n*k+m)
// bfs for edge weights in the range [0, k]
void bfs(int s, int n, int k){
    vector<queue<int>> qs(k+1, queue<int>());
    vector<bool> vis(n, false);
    vector<int> dist(n, 1e9);
    qs[0].push(s);
    dist[s]=0;
    int pos=0,num=1;
    while(num){
        while(qs[pos%(k+1)].empty()) pos++;
        int u=qs[pos%(k+1)].front();
        qs[pos%(k+1)].pop();
        num--;
        if(vis[u]) continue;
        vis[u]=true;
        for(auto [w,v]:adj[u]){
            if(dist[v]>dist[u]+w){
                dist[v]=dist[u]+w;
                qs[dist[v]%(k+1)].push(v);
                num++;
            }
        }
    }
}

```

## 6.10 Dijkstra

```

// O ((V+E)*log V)
vl dijkstra(vector<vector<pll>> &adj, int s, int n){
    vl dist(n, INFL); dist[s] = 0;
    priority_queue<pll, vector<pll>, greater<pll> >
        pq; pq.push(pll(0, s));
    while(!pq.empty()){
        pll front = pq.top(); pq.pop();
        ll d = front.first, u = front.second;
        if (d > dist[u]) continue;
        for (auto &[v, w] : adj[u]){
            if (dist[u] + w < dist[v]){
                dist[v] = dist[u] + w;
                pq.push(pll(dist[v], v));
            }
        }
    }
    return dist;
}

```

## 6.11 Dijkstra Sparse Graphs

```
// O(E*log(V))
vl dijkstra(vector<vector<pll>> &adj, int s, int n){
    vl dist(n, INFL); dist[s] = 0;
    set<pll> pq;
    pq.insert(pll(0, s));
    while(!pq.empty()){
        pll front = *pq.begin(); pq.erase(pq.
            begin());
        ll d = front.first, u = front.second;
        for (auto &[v, w] : adj[u]){
            if (dist[u] + w < dist[v]){
                pq.erase(pll(dist[v], v));
                dist[v] = dist[u] + w;
                pq.insert(pll(dist[v], v));
            }
        }
    }
    return dist;
}
```

## 6.12 Eulerian Path Directed

```
const int maxn = 1e5+5;
vector<int> adj[maxn], path;
int out[maxn], in[maxn]; // remember
void dfs(int v){
    while(!adj[v].empty()){
        int u=adj[v].back();
        adj[v].pop_back();
        dfs(u);
    }
    path.push_back(v);
}

// n -> nodes, m -> edges, s -> start, e -> end
void eulerian_path(int n, int m, int s, int e){
    for(int i=0; i<n; ++i){
        if(i==s || i==e) continue;
        if(in[i]!=out[i]){
            cout<<"IMPOSSIBLE\n";
            return;
        }
    }
    if(out[s]-in[s]!=1 || in[e]-out[e]!=1){
        cout<<"IMPOSSIBLE\n";
        return;
    }
    dfs(s);
    reverse(path.begin(), path.end());
}
```

```
if(sz(path)!=m+1 || path.back()!=e) cout<<"
    IMPOSSIBLE\n";
else print(path);
}
```

## 6.13 Eulerian Path Undirected

```
const int maxn = 1e5+5;
const int maxm = 2e5+5;
vector<ii> adj[maxn]; // adj[a].push_back({b, i});
vector<int> path;
int grade[maxn]; // remember
bool vis[maxn];
void dfs(int v){
    while(!adj[v].empty()){
        ii x=adj[v].back();
        adj[v].pop_back();
        if(vis[x.second]) continue;
        vis[x.second]=true;
        dfs(x.first);
    }
    path.push_back(v+1);
}

// check if end is equal to start
void eulerian_path(int n, int m, int s){
    for(int i=0; i<n; ++i){
        if(grade[i]%2!=0){
            cout<<"IMPOSSIBLE\n";
            return;
        }
    }
    dfs(s);
    if(sz(path)!=m+1) cout<<"IMPOSSIBLE\n";
    else print(path);
}
```

## 6.14 Floyd Warshall

```
// O(n^3)
vector<vi> adjMat(n+1, vi(n+1));
//Condicion previa: adjMat[i][j] contiene peso de la
//arista (i, j)
//o INF si no existe esa arista y adjMat[i][i] = 0
for (int k = 0; k < n; ++k) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (adjMat[i][k] < INF && adjMat[k][j] < INF)
                adjMat[i][j] = min(adjMat[i][j], adjMat[i][k] + adjMat[k][j]);
        }
    }
}
```

```

    }
}

```

## 6.15 Kosaraju

```

const int maxn = 1e5+5;
// construir el grafo inverso
// remember adj[a]->b, adj_rev[b]->a
vi adj[maxn], adj_rev[maxn];
bool used[maxn];
int idx[maxn]; // componente de cada nodo
vi order, comp;

// O(n+m)
void dfs1(int v) {
    used[v]=true;
    for(int u:adj[v])
        if(!used[u]) dfs1(u);
    order.push_back(v);
}

void dfs2(int v) {
    used[v]=true;
    comp.push_back(v);
    for(int u:adj_rev[v])
        if(!used[u]) dfs2(u);
}

// retorna el numero de componentes
int init(int n) {
    for(int i=0; i<n; ++i) if(!used[i]) dfs1(i);
    for(int i=0; i<n; ++i) used[i]=false;
    reverse(all(order));
    int j=0;
    for(int v:order) {
        if(!used[v]) {
            dfs2(v);
            for(int u:comp) idx[u]=j;
            comp.clear();
            j++;
        }
    }
    return j;
}

```

## 6.16 kruskal

```

// peso, nodo a, node b
vector<tuple<int,int,int>> edges;
struct DSU{};
// O(m*log(m))

```

```

void kruskal(int n) {
    sort(all(edges));
    DSU dsu(n);
    ll ans=0;
    for(auto& [w,u,v]:edges) {
        if(dsu.get(u)!=dsu.get(v)) {
            dsu.unite(u, v);
            ans+=w;
        }
    }
    if(dsu.sets!=1) cout<<"IMPOSSIBLE\n";
    else cout<<ans<<"\n";
}

```

## 6.17 Prim

```

// O(E * log V)
// check: primer parametro de prim
// check: cuando no hay mst
vector<vii> adj;
vi tomado;
priority_queue<ii> pq;
void process(int u) {
    tomado[u] = 1;
    for (auto &[v, w] : adj[u]) {
        if (!tomado[v]) pq.emplace(-w, -v);
    }
}

int prim(int v, int n) {
    tomado.assign(n, 0);
    process(v);
    int mst_costo = 0, tomados = 0;
    while (!pq.empty()) {
        auto [w, u] = pq.top(); pq.pop();
        w = -w; u = -u;
        if (tomado[u]) continue;
        mst_costo += w;
        process(u);
        tomados++;
        if (tomados == n-1) break;
    }
    return mst_costo;
}

```

## 6.18 Puentes y Puntos

```

const int maxn = 1e5+5;
vector<bool> vis;
vi adj[maxn]; // undirected
vi tin, low;
int timer;

void dfs(int u, int p=-1) {

```



```

if (k == 1) return {0};
vi seq, aux(n + 1);
int cont = 0;
function<void(int,int)> gen = [&](int t, int p) {
    if (t > n){
        if (n % p == 0) for(int i = 1; i
            < p + 1; i++){
            if (cont >= lim) return;
            seq.push_back(aux[i]);
            cont++;
        }
    } else {
        aux[t] = aux[t - p];
        gen(t + 1, p);
        while (++aux[t] < k){
            if (cont >= lim) return;
            gen(t + 1, t);
        }
    }
};
gen(1, 1);
// for (int i = 0; i<n-1; i++) seq.push_back(0);
return seq;
}

```

## 7.2 Convoluciones

```

// c[k] = sumatoria (i&j = k, += a[i]*b[j]) AND
convolution
// c[k] = sumatoria (i|j = k, += a[i]*b[j]) OR
convolution
// c[k] = sumatoria (i^j = k, += a[i]*b[j]) XOR
convolution
// c[k] = sumatoria (gcd(i,j) = k, += a[i]*b[j]) GCD
convolution
// c[k] = sumatoria (lcm(i,j) = k, += a[i]*b[j]) LCM
convolution
// todas las funciones tienen operaciones con modulo
// si es indexando en 1 entonces se pone un cero al
principio y listo

template<int MOD> struct mint {
    static const int mod = MOD;
    int v;
    explicit operator int() const { return v; }
    mint():v(0) {}
    mint(ll _v):v(int(_v%MOD)) { v += (v<0)*MOD; }
    void build(ll _v) { v=int(_v%MOD), v+=(v<0)*MOD; }
    mint& operator+=(mint o) {
        if ((v += o.v) >= MOD) v -= MOD;
        return *this; }
    mint& operator-=(mint o) {
        if ((v -= o.v) < 0) v += MOD;

```

```

        return *this; }
    mint& operator*=(mint o) {
        v = int((ll)v*o.v%MOD); return *this; }
    friend mint pow(mint a, ll p) { assert(p >= 0);
        return p==0?1:pow(a*a,p/2)*(p&1?a:1); }
    friend mint inv(mint a) { assert(a.v != 0);
        return pow(a,MOD-2); }
    friend mint operator+(mint a, mint b) { return a
        += b; }
    friend mint operator-(mint a, mint b) { return a
        -= b; }
    friend mint operator*(mint a, mint b) { return a
        *= b; }
};
using mi = mint<998244353>;

template<typename T>
void SubsetZetaTransform(vector<T>& v) {
    const int n = v.size(); // n must be a power of 2
    for (int j = 1; j < n; j <= 1) {
        for (int i = 0; i < n; i++)
            if (i & j) v[i] += v[i ^ j];
    }
}

template<typename T>
void SubsetMobiusTransform(vector<T>& v) {
    const int n = v.size(); // n must be a power of 2
    for (int j = 1; j < n; j <= 1) {
        for (int i = 0; i < n; i++)
            if (i & j) v[i] -= v[i ^ j];
    }
}

template<typename T>
void SupersetZetaTransform(vector<T>& v) {
    const int n = v.size(); // n must be a power of 2
    for (int j = 1; j < n; j <= 1) {
        for (int i = 0; i < n; i++)
            if (i & j) v[i ^ j] += v[i];
    }
}

template<typename T>
void SupersetMobiusTransform(vector<T>& v) {
    const int n = v.size(); // n must be a power of 2
    for (int j = 1; j < n; j <= 1) {
        for (int i = 0; i < n; i++)
            if (i & j) v[i ^ j] -= v[i];
    }
}

vector<int> PrimeEnumerate(int n) {
    vector<int> P; vector<bool> B(n + 1, 1);
    for (int i = 2; i <= n; i++) {
        if (B[i]) P.push_back(i);

```

```

        for (int j : P) { if (i * j > n) break; B
          [i * j] = 0; if (i % j == 0) break; }
    }
    return P;
}

template<typename T>
void DivisorZetaTransform(vector<T>& v) {
    const int n = sz(v) - 1;
    for (int p : PrimeEnumerate(n)) {
        for (int i = 1; i * p <= n; i++)
            v[i * p] += v[i];
    }
}

template<typename T>
void DivisorMobiusTransform(vector<T>& v) {
    const int n = sz(v) - 1;
    for (int p : PrimeEnumerate(n)) {
        for (int i = n / p; i; i--)
            v[i * p] -= v[i];
    }
}

template<typename T>
void MultipleZetaTransform(vector<T>& v) {
    const int n = sz(v) - 1;
    for (int p : PrimeEnumerate(n)) {
        for (int i = n / p; i; i--)
            v[i] += v[i * p];
    }
}

template<typename T>
void MultipleMobiusTransform(vector<T>& v) {
    const int n = sz(v) - 1;
    for (int p : PrimeEnumerate(n)) {
        for (int i = 1; i * p <= n; i++)
            v[i] -= v[i * p];
    }
}

template<typename T>
vector<T> AndConvolution(vector<T> A, vector<T> B) {
    SupersetZetaTransform(A);
    SupersetZetaTransform(B);
    for (int i = 0; i < sz(A); i++) A[i] *= B[i];
    SupersetMobiusTransform(A);
    return A;
}

template<typename T>
vector<T> OrConvolution(vector<T> A, vector<T> B) {
    SubsetZetaTransform(A);
    SubsetZetaTransform(B);
    for (int i = 0; i < sz(A); i++) A[i] *= B[i];
    SubsetMobiusTransform(A);

```

```

    return A;
}

template<typename T>
vector<T> GCDConvolution(vector<T> A, vector<T> B) {
    MultipleZetaTransform(A);
    MultipleZetaTransform(B);
    for (int i = 0; i < sz(A); i++) A[i] *= B[i];
    MultipleMobiusTransform(A);
    return A;
}

template<typename T>
vector<T> LCMConvolution(vector<T> A, vector<T> B) {
    DivisorZetaTransform(A);
    DivisorZetaTransform(B);
    for (int i = 0; i < sz(A); i++) A[i] *= B[i];
    DivisorMobiusTransform(A);
    return A;
}

template<typename T>
vector<T> XORConvolution(vector<T> A, vector<T> B) {
    const int n = sz(A);
    auto FWT = [&](vector<T>& v) {
        for (int len = 1; len < n; len <= 1) {
            for (int i = 0; i < n; i += (len
                < 1)) {
                for (int j = 0; j < len;
                    j++) {
                    T u(v[i + j]);
                    T w(v[i + j + len
                        ]);
                    v[i + j] = u + w;
                    v[i + j + len
                        ] = u - w;
                }
            }
        }
    };
    FWT(A); FWT(B);
    for (int i = 0; i < n; i++) A[i] *= B[i];
    FWT(A);
    T inv_n(inv(T(n)));
    for (int i = 0; i < n; i++) A[i] *= inv_n;
    return A;
}

void main2() {
    int n;
    cin >> n;
    vector<mi> a(1<<n), b(1<<n);
    for (int x, i = 0; i < sz(a); ++i) { cin >> x; a[i].build(x); }
    for (int x, i = 0; i < sz(b); ++i) { cin >> x; b[i].build(x); }
    vector<mi> ans = XORConvolution(a, b);
    for (int i = 0; i < sz(ans); ++i) cout << ans[i].v << " ";
}

```

}

### 7.3 Criba

```
// O(n*log(log(n)))
vector<ll> primes;
vector<bool> is_prime;
void criba(ll n){
    is_prime.assign(n+1,true);
    for(ll i=2;i<=n;++i){
        if(!is_prime[i])continue;
        for(ll j=i*i;j<=n;j+=i)is_prime[j]=false;
        primes.push_back(i);
    }
}

// O(sqrt(n)/log(sqrt(n)))
void fact(ll n, map<ll, int>& f){
    for(int i=0;i<sz(primes) && primes[i]*primes[i]<=
        n;++i)
        while(n%primes[i]==0ll)f[primes[i]]++,n/=
            primes[i];
    if(n>1)f[n]++;
}

// O((R-L+1)log(log(R))+sqrt(R)log(log(sqrt(R))))
// R-L+1 <= 1e7, R <= 1e14
void segmentedSieve(long long L, long long R) {
    // generate all primes up to sqrt(R)
    long long lim = sqrt(R)+3;
    vector<bool> mark(lim + 1, false);
    vector<long long> primes;
    for (long long i = 2; i <= lim; ++i) {
        if (!mark[i]) {
            primes.emplace_back(i);
            for (long long j = i * i; j <= lim; j += i)
                mark[j] = true;
        }
    }

    vector<bool> isPrime(R - L + 1, true);
    for (long long i : primes)
        for (long long j = max(i * i, (L + i - 1) / i * i);
            j <= R; j += i)
            isPrime[j - L] = false;
    if (L == 1)
        isPrime[0] = false;
}
```

### 7.4 Chinese Remainder Theorem

```
/// Complexity: |N|*log(|N|)
```

```
/// Tested: Not yet.
/// finds a suitable x that meets: x is congruent to a_i
/// mod n_i
/** Works for non-coprime moduli.
    Returns {-1,-1} if solution does not exist or input is
    invalid.
    Otherwise, returns {x,L}, where x is the solution unique
    to mod L = LCM of mods
    */

pll crt( vl A, vl M ) {
    ll n = A.size(), a1 = A[0], m1 = M[0];
    for(ll i = 1; i < n; i++) {
        ll a2 = A[i], m2 = M[i];
        ll g = __gcd(m1, m2);
        if( a1 % g != a2 % g ) return {-1,-1};
        ll p, q;
        extended_euclid(m1/g, m2/g, p, q);
        ll mod = m1 / g * m2;
        q %= mod; p %= mod;
        ll x = ((1ll*(a1%mod)*(m2/g))%mod*q + (1
            ll*(a2%mod)*(m1/g))%mod*p) % mod; //
            if WA there is overflow
        a1 = x;
        if (a1 < 0) a1 += mod;
        m1 = mod;
    }
    return {a1, m1};
}
```

### 7.5 Divisors

```
// d(n) = (a1+1)*(a2+1)*...*(ak+1)
ll numDiv(map<ll, ll>& f){
    ll ans=1;
    for(auto [_ ,pot]:f)ans=mul(ans, (pot+1ll));
    return ans;
}

// sigma(n) = (p1^(a1+1)-1)/(p1-1) * (p2^(a2+1)-1)/(p2-1)
// ... * (pk^(ak+1)-1)/(pk-1)
// suma divisores a la xth potencia
ll sumDiv(map<ll, ll>& f){
    ll ans=1,potencia=1;
    for(auto [num, pot]:f){
        ll p=binpow(num, (pot+1ll)*potencia)-1ll;
        ans=mul(ans, mul(p, inv(num-1ll)));
    }
    return ans;
}

ll productDiv(map<ll, ll>& f){
    ll pi=1,res=1;
    for(auto [num, pot]:f){
        ll p=binpow(num, pot*(pot+1ll)/2ll);
    }
```

```

        res=mul(binpow(res, pot+1ll),binpow(p, pi
        ));
        pi=mul(pi, pot+1ll, MOD-1ll);
    }
    return res;
}
// si a y b son coprimos, entonces:
// sigma(a*b) = sigma(a)*sigma(b)
// d(a*b) = d(a)*d(b)

```

## 7.6 Ecuaciones Diofanticas

```

// O(log(n))
ll extended_euclid(ll a, ll b, ll &x, ll &y) {
    ll xx = y = 0;
    ll yy = x = 1;
    while (b) {
        ll q = a / b;
        ll t = b; b = a % b; a = t;
        t = xx; xx = x - q * xx; x = t;
        t = yy; yy = y - q * yy; y = t;
    }
    return a;
}
// a*x+b*y=c. returns valid x and y if possible.
// all solutions are of the form (x0 + k * b / g, y0 - k
// * b / g)
bool find_any_solution (ll a, ll b, ll c, ll &x0, ll &y0,
ll &g) {
    if (a == 0 and b == 0) {
        if (c) return false;
        x0 = y0 = g = 0;
        return true;
    }
    g = extended_euclid (abs(a), abs(b), x0, y0);
    if (c % g != 0) return false;
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 *= -1;
    if (b < 0) y0 *= -1;
    return true;
}
void shift_solution(ll &x, ll &y, ll a, ll b, ll cnt) {
    x += cnt * b;
    y -= cnt * a;
}
// returns the number of solutions where x is in the
// range[minx, maxx] and y is in the range[miny, maxy]
ll find_all_solutions(ll a, ll b, ll c, ll minx, ll maxx,
ll miny, ll maxy) {
    ll x, y, g;
    if (find_any_solution(a, b, c, x, y, g) == 0)
        return 0;
    if (a == 0 and b == 0) {

```

```

        assert(c == 0);
        return 1LL * (maxx - minx + 1) * (maxy -
        miny + 1);
    }
    if (a == 0) {
        return (maxx - minx + 1) * (miny <= c / b
        and c / b <= maxy);
    }
    if (b == 0) {
        return (maxy - miny + 1) * (minx <= c / a
        and c / a <= maxx);
    }
    a /= g, b /= g;
    ll sign_a = a > 0 ? +1 : -1;
    ll sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx) shift_solution(x, y, a, b, sign_b);
    if (x > maxx) return 0;
    ll lx1 = x;
    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx) shift_solution(x, y, a, b, -sign_b
    );
    ll rx1 = x;
    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny) shift_solution(x, y, a, b, -sign_a
    );
    if (y > maxy) return 0;
    ll lx2 = x;
    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy) shift_solution(x, y, a, b, sign_a);
    ll rx2 = x;
    if (lx2 > rx2) swap (lx2, rx2);
    ll lx = max(lx1, lx2);
    ll rx = min(rx1, rx2);
    if (lx > rx) return 0;
    return (rx - lx) / abs(b) + 1;
}
///finds the first k | x + b * k / gcd(a, b) >= val
ll greater_or_equal_than(ll a, ll b, ll x, ll val, ll g)
{
    ld got = 1.0 * (val - x) * g / b;
    return b > 0 ? ceil(got) : floor(got);
}

```

## 7.7 Exponenciacion binaria

```

ll binpow(ll b, ll n, ll m) {
    b %= m;
    ll res = 1;
    while (n > 0) {
        if (n & 1)
            res = res * b % m;
        b = b * b % m;

```



```

        n >= 1;
    }
    return res % m;
}

```

## 7.8 Exponenciación matricial

```

struct matrix {
    int r, c; vector<vl> m;
    matrix(int r, int c, const vector<vl> &m) : r(r),
        c(c), m(m) {}

    matrix operator * (const matrix &b) {
        matrix ans(this->r, b.c, vector<vl>(this
            ->r, vl(b.c, 0)));

        for (int i = 0; i < this->r; i++) {
            for (int k = 0; k < b.r; k++) {
                if (m[i][k] == 0)
                    continue;
                for (int j = 0; j < b.c; j
                    ++){
                    ans.m[i][j] +=
                        mod(m[i][k],
                            MOD) * mod(b.m
                                [k][j], MOD);
                    ans.m[i][j] = mod
                        (ans.m[i][j],
                            MOD);
                }
            }
        }
        return ans;
    }
};

matrix pow(matrix &b, ll p) {
    matrix ans(b.r, b.c, vector<vl>(b.r, vl(b.c, 0)));
    for (int i = 0; i < b.r; i++) ans.m[i][i] = 1;
    while (p) {
        if (p & 1) {
            ans = ans * b;
        }
        b = b * b;
        p >>= 1;
    }
    return ans;
}

```

## 7.9 Fast Fourier Transform

///Complexity:  $O(N \log N)$

```

//tested: https://codeforces.com/gym/104373/problem/E
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define sz(v) ((int)v.size())
#define trav(a, x) for(auto& a : x)
#define all(v) v.begin(), v.end()
typedef vector<ll> vl;
typedef vector<int> vi;
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C> &a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if
        double)
    for (static int k = 2; k < n; k *= 2) {
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i, k, 2*k) rt[i] = R[i] = i & 1 ? R[i/2]
            * x : R[i/2];
    }
    vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) /
        2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j
            , 0, k) {
            // C z = rt[j+k] * a[i+j+k]; //
            (25% faster if hand-rolled)
            /// include-line
            auto x = (double *)&rt[j+k], y =
                (double *)&a[i+j+k]; //
            / exclude-line
            C z(x[0]*y[0] - x[1]*y[1], x[0]*y
                [1] + x[1]*y[0]); //
            / exclude-line
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
}

vl conv(const vl& a, const vl& b) {
    if (a.empty() || b.empty()) return {};
    vd res(sz(a) + sz(b) - 1);
    int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
    vector<C> in(n), out(n);
    copy(all(a), begin(in));
    rep(i, 0, sz(b)) in[i].imag(b[i]);
    fft(in);
    trav(x, in) x *= x;
    rep(i, 0, n) out[i] = in[-i & (n - 1)] - conj(in[i
        ]);
    fft(out);
    vector<ll> resp(sz(res));
    rep(i, 0, sz(res)) resp[i] = round(imag(out[i]) /

```

```

        (4.0 * n));
    return resp;
}

```

## 7.10 Fibonacci Fast Doubling

```

// O(log n) muy rapido
// F(2n+1) = F(n)^2 + F(n+1)^2
// F(2n) = F(n+1)^2 - F(n-1)^2
pair<int, int> fib (int n) {
    if (n == 0)
        return {0, 1};

    auto p = fib(n >> 1);
    int c = p.first * (2 * p.second - p.first);
    int d = p.first * p.first + p.second * p.second;
    if (n & 1)
        return {d, c + d};
    else
        return {c, d};
}

```

## 7.11 Fraction

```

typedef __int128 T;
struct Fraction{
    T num,den;
    Fraction():num(0),den(1){}
    Fraction(T n):num(n),den(1){}
    Fraction(T n,T d):num(n),den(d){reduce();}
    void reduce(){
        // assert(den!=0);
        T gcd=__gcd(num,den); // <-
        num/=gcd,den/=gcd;
        if(den<0) num=-num,den=-den;
    }
    Fraction fractional_part() const { // x - floor(x)
        Fraction fp=Fraction(num%den,den);
        if(fp<Fraction(0)) fp+=Fraction(1);
        return fp;
    }
    T compare(Fraction f) const {return num*f.den-den*f.num;}
    Fraction operator + (const Fraction& f){return
        Fraction(num*f.den+den*f.num,den*f.den);}
    Fraction operator - (const Fraction& f){return
        Fraction(num*f.den-den*f.num,den*f.den);}
    Fraction operator * (const Fraction& f){
        Fraction a=Fraction(num,f.den);
        Fraction b=Fraction(f.num,den);
        return Fraction(a.num*b.num,a.den*b.den);
    }
}

```

```

Fraction operator / (const Fraction& f){return *
    this*Fraction(f.den,f.num);}
Fraction operator += (const Fraction& f){return *
    this=*this+f;}
Fraction operator -= (const Fraction& f){return *
    this=*this-f;}
Fraction operator *= (const Fraction& f){return *
    this=*this*f;}
Fraction operator /= (const Fraction& f){return *
    this=*this/f;}
bool operator == (const Fraction& f) const {return
    compare(f)==0;}
bool operator != (const Fraction& f) const {return
    compare(f)!=0;}
bool operator >= (const Fraction& f) const {return
    compare(f)>=0;}
bool operator <= (const Fraction& f) const {return
    compare(f)<=0;}
bool operator > (const Fraction& f) const {return
    compare(f)>0;}
bool operator < (const Fraction& f) const {return
    compare(f)<0;}
};
Fraction operator - (const Fraction& f){return Fraction(-
    f.num,f.den);}
ostream& operator << (ostream& os, const Fraction& f){
    return os<<"("<<(ll)f.num<<"/"<<(ll)f.den<<")";}

```

## 7.12 Freivalds algorithm

```

mt19937 rnd(chrono::steady_clock::now().time_since_epoch
    ().count());
// check if two n*n matrix a*b=c within complexity (
    iteration*n^2)
// probability of error 2^(-iteration)
// O(iter*n^2)
int Freivalds(matrix &a, matrix &b, matrix &c) {
    int n = a.r, iteration = 20;
    matrix zero(n, 1), r(n, 1);
    while (iteration--){
        for(int i = 0; i < n; i++) r.m[i][0] =
            rnd() % 2;
        matrix ans = (a * (b * r)) - (c * r);
        if(ans.m != zero.m) return 0;
    }
    return 1;
}

```

## 7.13 Gauss Jordan

```

const int EPS = 1;

```

```

int gauss (vector<vector<int>> a, vector<int> &ans) {
    int n = a.size(), m = a[0].size()-1;
    vector<int> where(m, -1);
    for(int col = 0, row = 0; col < m && row < n; ++col) {
        int sel = row;
        for(int i = row; i < n; ++i)
            if(abs(a[i][col]) > abs(a[sel][col])) sel = i;
        if(abs(a[sel][col]) < EPS) continue;
        swap(a[sel], a[row]);
        where[col] = row;
        for(int i = 0; i < n; ++i)
            if(i != row) {
                int c = divide(a[i][col], a[row][col]); //
                precalc inverses
                for(int j = col; j <= m; ++j)
                    a[i][j] = sbt(a[i][j], mul(a[row][j], c));
            }
        ++row;
    }
    ans.assign(m, 0);
    for(int i = 0; i < m; ++i)
        if(where[i] != -1) ans[i] = divide(a[where[i]][m], a[
            where[i]][i]);
    for(int i = 0; i < n; ++i) {
        int sum = 0;
        for(int j = 0; j < m; ++j)
            sum = add(sum, mul(ans[j], a[i][j]));
        if(sum != a[i][m]) return 0;
    }
    for(int i = 0; i < m; ++i)
        if(where[i] == -1) return -1; // infinite solutions
    return 1;
}

```

## 7.14 Gauss Jordan mod 2

```

// O(min(n, m) * n * m)
int gauss (vector<bitset<N>> &a, int n, int m, bitset<
    N> &ans) {
    vector<int> where(m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        for (int i=row; i<n; ++i)
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        if (! a[row][col])
            continue;
        where[col] = row;
        for (int i=0; i<n; ++i)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        ++row;
    }
}

```

```

}
for (int i=0; i<m; ++i)
    if (where[i] != -1)
        ans[i] = a[where[i]][m] / a[where
            [i]][i];
for (int i=0; i<n; ++i) {
    double sum = 0;
    for (int j=0; j<m; ++j)
        sum += ans[j] * a[i][j];
    if (abs (sum - a[i][m]) > EPS)
        return 0;
}
for (int i=0; i<m; ++i)
    if (where[i] == -1)
        return INF;
return 1;
}
}

```

## 7.15 GCD y LCM

```

//O(log10 n) n == max(a, b)
int gcd(int a, int b) { return b == 0 ? a : gcd(b, a%b);
}
int lcm(int a, int b) { return a / gcd(a, b) * b; }
//gcd(a, b, c) = gcd(a, gcd(b, c))
//gcd(a, b) = gcd(a, b-a)
// O(log(min(a, b)) - a*x+b*y=gcd(a,b)
ll gcd(ll a, ll b, ll &x, ll &y){
    x=1,y=0;
    ll x1=0,y1=1,a1=a,b1=b;
    while(b1){
        ll q=a1/b1;
        tie(x, x1)=make_tuple(x1, x-q*x1);
        tie(y, y1)=make_tuple(y1, y-q*y1);
        tie(a1, b1)=make_tuple(b1, a1-q*b1);
    }return a1;
}
}

```

## 7.16 Integral Definida

```

const int steps = 1e6; // %2==0
double f(double x);
double simpson(double a, double b){
    double h=(b-a)/steps;
    double s=f(a)+f(b);
    for(int i=1;i<=steps-1;i++){
        double x=a+h*i;
        s+=f(x)*((i&1)?4:2);
    }
}

```

```

    s*=h/3;
    return s;
}

```

## 7.17 Inverso modular

```

ll mod(ll a, ll m){
    return ((a%m) + m) % m;
}

ll modInverse(ll b, ll m){
    ll x, y;
    ll d = extEuclid(b, m, x, y); //obtiene b*x + m*
    y == d
    if (d != 1) return -1; //indica error
    // b*x + m*y == 1, ahora aplicamos (mod m) para
    // obtener b*x == 1 (mod m)
    return mod(x, m);
}

// Otra forma
// O(log MOD)
ll inv (ll a){
    return binpow(a, MOD-2, MOD);
}

//Modulo constante
inv[1] = 1;
for(int i = 2; i < p; ++i)
    inv[i] = (p - (p / i) * inv[p % i] % p) % p;

```

## 7.18 Lagrange

```

const int N = 1e6;
int f[N], fr[N];
void initC(){
    f[0] = 1;
    for(int i=1; i<N; i++) f[i] = mul(f[i-1], i);
    fr[N-1] = inv(f[N-1]);
    for(int i=N-1; i>=1; --i) fr[i-1] = mul(fr[i], i);
}
// mint C(int n, int k) { return k<0 || k>n ? 0 : f[n] *
// fr[k] * fr[n-k]; }
struct LagrangePol {
    int n;
    vi y, den, l, r;
    LagrangePol(vector<int> f): n(sz(f)), y(f), den(n), l(n),
    r(n){ // f[i] := f(i)
        // Calcula interpol. pol P in O(n) := deg(P) = sz(v)
        -1
    }
    initC();
    for (int i = 0; i<n; i++) {
        den[i] = mul(fr[n-1-i], fr[i]);
    }
}

```

```

        if((n-1-i) & 1) den[i] = mod(-den[i]);
    }
}

int eval(int x){ // Evaluate LagrangePoly P(x) in O(n)
    l[0] = r[n-1] = 1;
    for (int i = 1; i<n; i++) l[i] = mul(l[i-1], mod(x -
        i + 1));
    for(int i=n-2; i>=0; --i) r[i] = mul(r[i+1], mod(x -
        i - 1));
    int ans = 0;
    for (int i = 0; i<n; i++) ans = add(ans, mul(mul(l[i],
        r[i]), mul(y[i], den[i]))));
    return ans;
}

}

////////////////////////////////////
// Para Xs que no sean de [0, N]
// Complexity: O(|N|^2)
// Tested: https://tinyurl.com/y23sh38k
vector<lf> X, F;
lf f(lf x) {
    lf answer = 0;
    for(int i = 0; i < sz(X); i++) {
        lf prod = F[i];
        for(int j = 0; j < sz(X); j++) {
            if(i == j) continue;
            prod = mul(prod, divide(sbt(x, X[j]), sbt(X[i], X[j]
                ))));
        }
        answer = add(answer, prod);
    }
    return answer;
}

//given y=f(x) for x [0,degree]
vi interpolation( vi &y ) {
    int n = sz(y);
    vi u = y, ans( n ), sum( n );
    ans[0] = u[0], sum[0] = 1;
    for( int i = 1; i < n; ++i )
    {
        int inv = binpow( i, MOD - 2 );
        for( int j = n - 1; j >= i; --j )
            u[j] = 1LL * (u[j] - u[j - 1] + MOD) * inv % MOD;

        for( int j = i; j > 0; --j )
        {
            sum[j] = (sum[j - 1] - 1LL * (i - 1) * sum[j] % MOD
                + MOD) % MOD;
            ans[j] = (ans[j] + 1LL * sum[j] * u[i]) % MOD;
        }
        sum[0] = 1LL * (i - 1) * (MOD - sum[0]) % MOD;
        ans[0] = (ans[0] + 1LL * sum[0] * u[i]) % MOD;
    }
    return ans;
}

```

}

## 7.19 Logaritmo Discreto

```
// O(sqrt(m))
// Returns minimum x for which a ^ x % m = b % m.
int solve(int a, int b, int m) {
    // if (a == 0) return b == 0 ? 1 : -1; Casos 0^x
    // = b
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 111 * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
    for (int i = 0; i < n; ++i)
        an = (an * 111 * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 111 * a) % m;
    }

    for (int p = 1, cur = k; p <= n; ++p) {
        cur = (cur * 111 * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur] + add;
            return ans;
        }
    }
    return -1;
}
```

## 7.20 Miller Rabin

```
ll mul (ll a, ll b, ll mod) {
    ll ret = 0;
    for(a %= mod, b %= mod; b != 0;
        b >>= 1, a <=<= 1, a = a >= mod ? a - mod
        : a) {
        if (b & 1) {
            ret += a;
            if (ret >= mod) ret -= mod;
        }
    }
}
```

```
return ret;
}
ll fpow (ll a, ll b, ll mod) {
    ll ans = 1;
    for (; b; b >>= 1, a = mul(a, a, mod))
        if (b & 1)
            ans = mul(ans, a, mod);

    return ans;
}
bool witness (ll a, ll s, ll d, ll n) {
    ll x = fpow(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int i = 0; i < s - 1; ++i) {
        x = mul(x, x, n);
        if (x == 1) return true;
        if (x == n - 1) return false;
    }
    return true;
}
ll test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 0};
bool is_prime (ll n) {
    if (n < 2) return false;
    if (n == 2) return true;
    if (n % 2 == 0) return false;
    ll d = n - 1, s = 0;
    while (d % 2 == 0) ++s, d /= 2;
    for (int i = 0; test[i] && test[i] < n; ++i)
        if (!witness(test[i], s, d, n))
            return false;
    return true;
}
```

## 7.21 Miller Rabin Probabilístico

```
using u64 = uint64_t;
using ul28 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1)
            result = (ul28)result * base %
                mod;
        base = (ul28)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1)
        return false;
}
```

```

    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1)
            return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if
    n is probably prime, else returns false.
    if (n < 4)
        return n == 2 || n == 3;

    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

```

## 7.22 Mobius

```

// 1 if n is 1
// 0 if n has a squared prime factor
// (-1)^k if n is a product of k distinct prime factors
const int N = 1e6+1;
int mob[N];
void mobius() {
    mob[1] = 1;
    for (int i = 2; i < N; i++){
        mob[i]--;
        for (int j = i + i; j < N; j += i) {
            mob[j] -= mob[i];
        }
    }

    // to count coprime pairs
    // ans=n*(n-1)/2
    // for(int x:a){
    //     for(int y:divisors[a])cnt[y]++;
    // }
    // ans+=(mobius[v]*cnt[v]*(cnt[v]-1))/2

```

## 7.23 Number Theoretic Transform

```

const int N = 1 << 20;
const int mod = 469762049; //998244353
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {
    for (; y >>= 1; x = (long long) x * x % mod)
        if (y & 1) ans = (long long) ans * x % mod;
    return ans;
}
void precompute(int len) {
    lim = wn[0] = 1; int s = -1;
    while (lim < len) lim <<= 1, ++s;
    for (int i = 0; i < lim; ++i) rev[i] = rev[i >>
        1] >> 1 | (i & 1) << s;
    const int g = POW(root, (mod - 1) / lim);
    inv_lim = POW(lim, mod - 2);
    for (int i = 1; i < lim; ++i) wn[i] = (long long)
        wn[i - 1] * g % mod;
}
void ntt(vector<int> &a, int typ) {
    for (int i = 0; i < lim; ++i) if (i < rev[i])
        swap(a[i], a[rev[i]]);
    for (int i = 1; i < lim; i <<= 1) {
        for (int j = 0, t = lim / i / 2; j < i;
            ++j) w[j] = wn[j * t];
        for (int j = 0; j < lim; j += i << 1) {
            for (int k = 0; k < i; ++k) {
                const int x = a[k + j], y
                    = (long long) a[k + j
                        + i] * w[k] % mod;
                reduce(a[k + j] += y -
                    mod), reduce(a[k + j +
                        i] = x - y);
            }
        }
    }
    if (!typ) {
        reverse(a.begin() + 1, a.begin() + lim);
        for (int i = 0; i < lim; ++i) a[i] = (
            long long) a[i] * inv_lim % mod;
    }
}
vector<int> multiply(vector<int> &f, vector<int> &g) {
    int n=(int)f.size() + (int)g.size() - 1;
    precompute(n);
    vector<int> a = f, b = g;
    a.resize(lim); b.resize(lim);
    ntt(a, 1), ntt(b, 1);
    for (int i = 0; i < lim; ++i) a[i] = (long long)
        a[i] * b[i] % mod;
    ntt(a, 0);
    a.resize(n + 1);
    return a;
}

```

}

## 7.24 Pollard Rho

```
//O(n^(1/4)) (?)
ll pollard_rho(ll n, ll c) {
    ll x = 2, y = 2, i = 1, k = 2, d;
    while (true) {
        x = (mul(x, x, n) + c);
        if (x >= n) x -= n;
        d = __gcd(x - y, n);
        if (d > 1) return d;
        if (++i == k) y = x, k <= 1;
    }
    return n;
}
void factorize(ll n, vector<ll> &f) {
    if (n == 1) return;
    if (is_prime(n)) {
        f.push_back(n);
        return;
    }
    ll d = n;
    for (int i = 2; d == n; i++)
        d = pollard_rho(n, i);
    factorize(d, f);
    factorize(n/d, f);
}
```

## 7.25 Simplex

```
// Maximizar c1*x1 + c2*x2 + c3*x3 ...
// Restricciones a11*x1 + a12*x2 <= b1, a22*x2 + a23*x3
// <= b2 ...
// Retorna valor optimo y valores de las variables
// O(c^2*b), O(c*b) - variables c, restricciones b
typedef double lf;
const lf EPS = 1e-9;
struct Simplex{
    vector<vector<lf>> A;
    vector<lf> B,C;
    vector<int> X,Y;
    lf z;
    int n,m;

    Simplex(vector<vector<lf>> _a, vector<lf> _b,
            vector<lf> _c){
        A=_a;B=_b;C=_c;
        n=B.size();m=C.size();z=0.;
        X=vector<int>(m);Y=vector<int>(n);
        for(int i=0;i<m;++i)X[i]=i;
        for(int i=0;i<n;++i)Y[i]=i+m;
    }
};
```

}

```
void pivot(int x,int y){
    swap(X[Y],Y[X]);
    B[X]/=A[X][Y];
    for(int i=0;i<m;++i)if(i!=y)A[X][i]/=A[X][Y];
    A[X][Y]=1/A[X][Y];
    for(int i=0;i<n;++i)if(i!=x&&abs(A[i][Y])>EPS){
        B[i]-=A[i][Y]*B[X];
        for(int j=0;j<m;++j)if(j!=y)A[i][j]-=A[i][Y]*A[X][j];
        A[i][Y]=-A[i][Y]*A[X][Y];
    }
    z+=C[Y]*B[X];
    for(int i=0;i<m;++i)if(i!=y)C[i]-=C[Y]*A[X][i];
    C[Y]=-C[Y]*A[X][Y];
}

pair<lf, vector<lf>> maximize(){
    while(1){
        int x=-1,y=-1;
        lf mn=-EPS;
        for(int i=0;i<n;++i)if(B[i]<mn)mn=B[i],x=i;
        if(x<0)break;
        for(int i=0;i<m;++i)if(A[X][i]<-EPS){y=i;break;}
        // assert(y>=0) -> y<0, no solution to Ax<=B
        pivot(x,y);
    }
    while(1){
        lf mx=EPS;
        int x=-1,y=-1;
        for(int i=0;i<m;++i)if(C[i]>mx)mx=C[i],y=i;
        if(y<0)break;
        lf mn=1e200;
        for(int i=0;i<n;++i)if(A[i][Y]>EPS&&B[i]/A[i][Y]<mn)mn=B[i]/A[i][Y],x=i;
        // assert(x>=0) -> x<0, unbounded
        pivot(x,y);
    }
    vector<lf> r(m);
    for(int i=0;i<n;++i)if(Y[i]<m)r[Y[i]]=B[i];
    return {z,r};
}
```

};

## 7.26 Simplex Int

```

// Maximizar  $c_1x_1 + c_2x_2 + c_3x_3 \dots$ 
// Restricciones  $a_{11}x_1 + a_{12}x_2 \leq b_1, a_{22}x_2 + a_{23}x_3 \leq b_2 \dots$ 
// Retorna valor optimo y valores de las variables
//  $O(c^2 \cdot b)$ ,  $O(c \cdot b)$  - variables  $c$ , restricciones  $b$  (tle)
struct Fraction{};
typedef Fraction lf;
const lf ZERO(0), INF(1e18);
struct Simplex{
    vector<vector<lf>> A;
    vector<lf> B,C;
    vector<int> X,Y;
    lf z;
    int n,m;

    Simplex(vector<vector<lf>> _a, vector<lf> _b,
            vector<lf> _c){
        A=_a;B=_b;C=_c;
        n=B.size();m=C.size();z=ZERO;
        X=vector<int>(m);Y=vector<int>(n);
        for(int i=0;i<m;++i)X[i]=i;
        for(int i=0;i<n;++i)Y[i]=i+m;
    }

    void pivot(int x,int y){
        swap(X[Y],Y[X]);
        B[X]/=A[X][Y];
        for(int i=0;i<m;++i)if(i!=y)A[X][i]/=A[X][Y];
        A[X][Y]=Fraction(1)/A[X][Y];
        for(int i=0;i<n;++i)if(i!=x && A[i][Y]!=ZERO){
            B[i]-=A[i][Y]*B[X];
            for(int j=0;j<m;++j)if(j!=y)A[i][j]-=A[i][Y]*A[X][j];
            A[i][Y]=-A[i][Y]*A[X][Y];
        }
        z+=C[Y]*B[X];
        for(int i=0;i<m;++i)if(i!=y)C[i]-=C[Y]*A[X][i];
        C[Y]=-C[Y]*A[X][Y];
    }

    pair<lf, vector<lf>> maximize(){
        while(1){
            int x=-1,y=-1;
            lf mn=ZERO;
            for(int i=0;i<n;++i)if(B[i]<mn)mn=B[i],x=i;
            if(x<0)break;
            for(int i=0;i<m;++i)if(A[X][i]<ZERO){y=i;break;}
            // assert(y>=0) -> y<0, no

```

```

            // solution to  $Ax \leq B$ 
            pivot(x,y);
        }
        while(1){
            lf mx=ZERO;
            int x=-1,y=-1;
            for(int i=0;i<m;++i)if(C[i]>mx)mx=C[i],y=i;
            if(y<0)break;
            lf mn=INF;
            for(int i=0;i<n;++i)if(A[i][Y]>ZERO && B[i]/A[i][Y]<mn)mn=B[i]/A[i][Y],x=i;
            // assert(x>=0) -> x<0, unbounded
            pivot(x,y);
        }
        vector<lf> r(m);
        for(int i=0;i<n;++i)if(Y[i]<m)r[Y[i]]=B[i];
        return {z,r};
    }

    pair<Fraction, vector<Fraction>> maximize_int(){
        while(1){
            auto sol=maximize();
            bool all_int=true;
            for(auto &x:sol.second)all_int&=x.fractional_part()==ZERO;
            if(all_int)return sol;
            Fraction nw_b=ZERO;
            int id=-1;
            for(int i=0;i<n;++i){
                Fraction fp=B[i].fractional_part();
                if(fp>nw_b)nw_b=fp,id=i;
            }
            vector<Fraction> nw_a;
            for(auto &x:A[id])nw_a.push_back(-x.fractional_part());
            A.push_back(nw_a);
            B.push_back(-nw_b);
            Y.push_back(n+m);n++;
        }
    }
};

```

## 7.27 Totient y Divisores

```

vector<int> count_divisors_sieve() {
    bitset<mx> is_prime; is_prime.set();
    vector<int> cnt(mx, 1);
    is_prime[0] = is_prime[1] = 0;
    for(int i = 2; i < mx; i++) {

```



```

        if(!is_prime[i]) continue;
        cnt[i]++;
        for(int j = i+i; j < mx; j += i) {
            int n = j, c = 1;
            while( n%i == 0 ) n /= i, c++;
            cnt[j] *= c;
            is_prime[j] = 0;
        }
    }
    return cnt;
}
vector<int> euler_phi_sieve() {
    bitset<mx> is_prime; is_prime.set();
    vector<int> phi(mx);
    iota(phi.begin(), phi.end(), 0);
    is_prime[0] = is_prime[1] = 0;
    for(int i = 2; i < mx; i++) {
        if(!is_prime[i]) continue;
        for(int j = i; j < mx; j += i) {
            phi[j] -= phi[j]/i;
            is_prime[j] = 0;
        }
    }
    return phi;
}
ll euler_phi(ll n) {
    ll ans = n;
    for(ll i = 2; i * i <= n; ++i) {
        if(n % i == 0) {
            ans -= ans / i;
            while(n % i == 0) n /= i;
        }
    }
    if(n > 1) ans -= ans / n;
    return ans;
}

```

## 7.28 Xor Basis

```

template<typename T = int, int B = 31>
struct Basis {
    T a[B];
    Basis() {
        memset(a, 0, sizeof a);
    }
    void insert(T x){
        for (int i = B - 1; i >= 0; i--) {
            if (x >> i & 1) {
                if (a[i]) x ^= a[i];
                else {
                    a[i] = x;
                    break;
                }
            }
        }
    }
}

```

```

    }
    bool can(T x) {
        for(int i = B - 1; i >= 0; i--) {
            x = min(x, x ^ a[i]);
        }
        return x == 0;
    }
    T max_xor(T ans = 0) {
        for(int i = B - 1; i >= 0; i--) {
            ans = max(ans, ans ^ a[i]);
        }
        return ans;
    }
};

// Basis<long long, 63> B;
// Cantidad de xor diferentes es 2^sz(base)
// Cantidad de subsets xor = 0 es 2^(n-sz(base))

```

## 8 Programacion dinamica

### 8.1 Aliens Trick

```

// We binary search for A and find the highest A such
// that c(A) >= K.
// Let the optimal value be Aopt. Then our answer is v(
// Aopt) + AoptK

int main() {
    int n, k;
    cin >> n >> k;

    int a[n];
    for (int &i : a) { cin >> i; }

    /**
     * the maximum sum along with the number of
     * subarrays used
     * if creating a subarray penalizes the sum by "
     * lmb" and
     * there is no limit to the number of subarrays
     * you can create
     */
    auto solve_lambda = [&](ll lmb) {
        pair<ll, ll> dp[n][2];
        dp[0][0] = {0, 0};
        dp[0][1] = {a[0] - lmb, 1};
        for (int i = 1; i < n; i++) {
            dp[i][0] = max(dp[i - 1][0], dp[i - 1][1]);
            dp[i][1] =

```

```

        max(make_pair(dp[i - 1][0].
            first + a[i] - lmb, dp[i -
            1][0].second + 1),
            make_pair(dp[i - 1][1].
                first + a[i], dp[i -
                1][1].second));
    }
    return max(dp[n - 1][0], dp[n - 1][1]);
};

ll lo = 0;
ll hi = 1e18;
while (lo < hi) {
    ll mid = (lo + hi + 1) / 2;
    solve_lambda(mid).second >= k ? lo = mid
        : hi = mid - 1;
}

cout << solve_lambda(lo).first + lo * k << endl;
}

```

## 8.2 Bin Packing

```

int main() {
    ll n, capacidad;
    cin >> n >> capacidad;
    vl pesos(n, 0);
    forx(i, n) cin >> pesos[i];

    vector<pll> dp((1 << n));
    dp[0] = {1, 0};
    // dp[X] = {#numero de paquetes, peso de min
    // paquete}

    // La idea es probar todos los subset y en cada
    // uno preguntarnos
    // quien es mejor para subirse de ultimo buscando
    // minimizar
    // primero el numero de paquetes
    for (int subset = 1; subset < (1 << n); subset++)
    {
        dp[subset] = {21, 0};
        for (int iPer = 0; iPer < n; iPer++) {
            if ((subset >> iPer) & 1) {
                pll ant = dp[subset ^ (1
                    << iPer)];
                ll k = ant.ff;
                ll w = ant.ss;

                if (w + pesos[iPer] >
                    capacidad) {
                    k++;
                    w = min(pesos[
                        iPer], w);
                }
            }
        }
    }
}

```

```

    } else {
        w += pesos[iPer];
    }

    dp[subset] = min(dp[
        subset], {k, w});
}

}

cout << dp[(1 << n) - 1].ff << ln;
}

```

## 8.3 Convex Hull Trick

```

// - Me dan las pendientes ordenadas
// Caso 1: Me hacen las queries ordenadas
// O(N + Q)
// Caso 2: Me hacen queries arbitrarias
// O(N + QlogN)

struct CHT {
    // funciona tanto para min como para max, depende del
    // orden en que pasamos las lineas

    struct Line {
        int slope, yIntercept;

        Line(int slope, int yIntercept) : slope(slope),
            yIntercept(yIntercept) {}
        int val(int x) { return slope * x + yIntercept; }
        int intersect(Line y) {
            return (y.yIntercept - yIntercept + slope - y
                .slope - 1) / (slope - y.slope);
        }
    };

    deque<pair<Line, int>> dq;

    void insert(int slope, int yIntercept) {
        // lower hull si m1 < m2 < m3
        // upper hull si si m1 > m2 > m3
        Line newLine(slope, yIntercept);
        while (!dq.empty() && dq.back().second >= dq.back
            ().first.intersect(newLine)) dq.pop_back();
        if (dq.empty()) {
            dq.emplace_back(newLine, 0);
            return;
        }
        dq.emplace_back(newLine, dq.back().first.
            intersect(newLine));
    }

    int query(int x) { // cuando las consultas son
        // crecientes
        while (dq.size() > 1) {

```

```

        if (dq[1].second <= x) dq.pop_front();
        else break;
    }
    return dq[0].first.val(x);
}

int query2(int x) { // cuando son arbitrarias
    auto qry = *lower_bound(dq.rbegin(), dq.rend(),
                           make_pair(Line(0, 0), x),
                           [&](const pair<Line, int>
                               &a, const pair<Line,
                               int> &b) {
                               return a.second > b.
                                   second;
                           });
    return qry.first.val(x);
}
};

```

## 8.4 CHT Dynamic

```

// O((N+Q) log N) <- usando set para add y bs para q
// lineas de la forma mx + b
#pragma once
struct Line {
    mutable ll m, b, p;
    bool operator<(const Line& o) const { return m <
        o.m; }
    bool operator<(ll x) const { return p < x; }
};

struct CHT : multiset<Line, less<>> {
    // (for doubles, use inf = 1/.0, div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    static const bool mini = 0; // <---- 1 FOR MIN
    ll div(ll a, ll b){ // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y){
        if (y == end()) return x->p = inf, 0;
        if (x->m == y->m) x->p = x->b > y->b ?
            inf : -inf;
        else x->p = div(y->b - x->b, x->m - y->m)
            ;
        return x->p >= y->p;
    }
    void add(ll m, ll b){
        if (mini){ m *= -1, b *= -1; }
        auto z = insert({m, b, 0}), y = z++, x =
            y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(
            x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >=
            y->p)

```

```

            isect(x, erase(y));
        }
        ll query(ll x) {
            assert(!empty());
            auto l = *lower_bound(x);
            if (mini) return -l.m * x + -l.b;
            else return l.m * x + l.b;
        }
    };

```

## 8.5 Digit DP

```

// dp[pos][count of d][limit]
ll dp[20][20][2];
int k, d;

// count numbers <= c with k occurrences of d
ll dfs(string& c, int x=0, int y=0, bool z=0){
    if(dp[x][y][z] != -1) return dp[x][y][z];
    dp[x][y][z] = (y==k);
    if(x==(int)c.size()){
        return dp[x][y][z];
    }
    int limit=9;
    if(!z){
        limit=c[x]-'0';
    }
    dp[x][y][z]=0;
    for(int i=0; i<=limit; ++i){
        if(z) dp[x][y][z] += dfs(c, x+1, y+(i==d), z
            );
        else dp[x][y][z] += dfs(c, x+1, y+(i==d), i
            < limit);
    }
    return dp[x][y][z];
}

// count(0,m) - count(0,n-1) = count(n,m)
ll query(ll n, ll m){
    string s1=to_string(m);
    string s2=to_string(n-1ll);
    memset(dp, -1, sizeof(dp));
    ll ans=dfs(s1);
    if(n<=0ll) return ans; // check
    memset(dp, -1, sizeof(dp));
    return ans-dfs(s2);
}

```

## 8.6 Divide Conquer

```

// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c
< d.
int m, n;

```

```

vector<long long> dp_before(n), dp_cur(n);
long long C(int i, int j);
// compute dp_cur[l], ... dp_cur[r] (inclusive)
void compute(int l, int r, int optl, int optr) {
    if (l > r)
        return;

    int mid = (l + r) >> 1;
    pair<long long, int> best = {LLONG_MAX, -1};
    for (int k = optl; k <= min(mid, optr); k++) {
        best = min(best, {(k ? dp_before[k - 1] :
            0) + C(k, mid), k});
    }
    dp_cur[mid] = best.first;
    int opt = best.second;
    compute(l, mid - 1, optl, opt);
    compute(mid + 1, r, opt, optr);
}

int solve() {
    for (int i = 0; i < n; i++)
        dp_before[i] = C(0, i);

    for (int i = 1; i < m; i++) {
        compute(0, n - 1, 0, n - 1);
        dp_before = dp_cur;
    }

    return dp_before[n - 1];
}

```

## 8.7 Edit Distances

```

int editDistances(string& wor1, string& wor2) {
    // O(tam1*tam2)
    // minimo de letras que debemos insertar, eliminar
    // o reemplazar
    // de wor1 para obtener wor2
    ll tam1=wor1.size();
    ll tam2=wor2.size();
    vector<vl> dp(tam2+1, vl(tam1+1, 0));
    for (int i=0; i<=tam1; i++) dp[0][i]=i;
    for (int i=0; i<=tam2; i++) dp[i][0]=i;
    dp[0][0]=0;
    for (int i=1; i<=tam2; i++) {
        for (int j=1; j<=tam1; j++) {
            ll op1 = min(dp[i-1][j], dp[i][j-1]) + 1;
            // el minimo entre eliminar o
            // insertar
            ll op2 = dp[i-1][j-1]; //
            reemplazarlo

```

```

        if (wor1[j-1] != wor2[i-1]) op2++;
        // si el reemplazo tiene efecto o
        // quedo igual
        dp[i][j] = min(op1, op2);
    }
}

return dp[tam2][tam1];
}

```

## 8.8 Kadane 2D

```

int main() {
    ll fil, col; cin >> fil >> col;
    vector<vl> grid(fil, vl(col, 0));

    // Algoritmo de Kadane/DP para suma maxima de una matriz
    // 2D en O(n^3)
    for (int i=0; i<fil; i++) {
        for (int e=0; e<col; e++) {
            ll num; cin >> num;
            if (e>0) grid[i][e] = num + grid[i][e-1];
            else grid[i][e] = num;
        }
    }

    ll maxGlobal = LONG_LONG_MIN;
    for (int l=0; l<col; l++) {
        for (int r=l; r<col; r++) {
            ll maxLoc=0;
            for (int row=0; row<fil; row++) {
                if (l>0) maxLoc += grid[row][r] - grid[row][l-1];
                else maxLoc += grid[row][r];
                if (maxLoc<0) maxLoc=0;
                maxGlobal = max(maxGlobal, maxLoc);
            }
        }
    }
}

```

## 8.9 Knuth

```

// C[b][c] <= C[a][d]
// C[a][c] + C[b][d] <= C[a][d] + C[b][c] where a < b < c
// < d.
int solve() {
    int N;
    ... // read N and input
    int dp[N][N], opt[N][N];

```

```

auto C = [&](int i, int j) {
    ... // Implement cost function C.
};

for (int i = 0; i < N; i++) {
    opt[i][i] = i;
    ... // Initialize dp[i][i] according to
         the problem
}

for (int i = N-2; i >= 0; i--) {
    for (int j = i+1; j < N; j++) {
        int mn = INT_MAX;
        int cost = C(i, j);
        for (int k = opt[i][j-1]; k <=
            min(j-1, opt[i+1][j]); k++) {
            if (mn >= dp[i][k] + dp[k
                +1][j] + cost) {
                opt[i][j] = k;
                mn = dp[i][k] +
                    dp[k+1][j] +
                    cost;
            }
        }
        dp[i][j] = mn;
    }
}

cout << dp[0][N-1] << endl;
}

```

## 8.10 LIS

```

// O(n*log(n))
// retorna los indices de un lis
// cambiar el tipo y revisar si permite iguales
typedef int T;
vi lis(vector<T>& a, bool equal){
    vi prev(sz(a));
    typedef pair<T, int> p;
    vector<p> res;
    for(int i=0; i<sz(a); ++i){
        auto it=lower_bound(all(res), p{a[i], (
            equal?i:0)});
        if(it==res.end()) res.emplace_back(), it=
            res.end()-1;
        *it={a[i], i};
        prev[i]=(it==res.begin())?0:(it-1)->
            second;
    }
    int l=sz(res), act=res.back().second;
    vi ans(l);
    while(l-->0) ans[l]=act, act=prev[act];
    return ans;
}

```

## 8.11 SOS

```

const int bits = 23;
int dp[1<<bits];

// O(n*2^n)
void SOS(){
    for(int i = 0; i < (1 << bits); ++i) dp[i] = A[i];

    // top - down (informacion de las submascaras)
    for(int i = 0; i < bits; ++i){
        for(int s = 0; s < (1 << bits); ++s){
            if(s & (1 << i)){
                dp[s] += dp[s ^ (1 << i)];
            }
        }
    }

    // bottom - up (informacion de las supermascaras)
    for(int i = 0; i < bits; ++i){
        for(int s = (1 << bits) - 1; s >= 0; --s){
            if(s & (1 << i)){
                dp[s ^ (1 << i)] += dp[s];
            }
        }
    }
}

int dp2[1<<bits][bits+1];

// O(2^n*n^2)
void cnt(){
    vector<int> a;
    for(int x:a) dp2[x][0]++;

    // dp[s][c] = number of s^ai with c bits
    for(int i=0; i<bits; ++i){
        for(int c=i; c>=0; --c){
            s=0; s<(1<<bits); ++s){
                dp2[s^(1<<i)][c+1] += dp2[s][c];
            }
        }
    }
}

```

## 9 Strings

### 9.1 Aho Corasick

```
// 1) init() trie and add() strings
// 2) build() aho-corasick
// 3) process the text
// 4) dfs to calculate dp

// suf: longest proper suffix that's also in the trie
// dad: closest suffix link that is terminal
// cnt: number of strings that end exactly at node v
const int maxn = 2e5+5;
const int alpha = 26;
vector<int> adj[maxn];
int to[maxn][alpha], cnt[maxn], dad[maxn], suf[maxn], act; //
    not to change
int conv(char ch){return ((ch>='a' && ch<='z')?ch-'a':ch-
    'A'+26);}
void init(){
    for(int i=0;i<=act;++i){
        suf[i]=cnt[i]=dad[i]=0;
        adj[i].clear();
        memset(to[i], 0, sizeof(to[i]));
    }
    act=0;
}
int add(string& s){
    int u=0;
    for(char ch:s){
        int c=conv(ch);
        if(!to[u][c])to[u][c]=++act;
        u=to[u][c];
    }
    cnt[u]++;
    return u;
}
// O(sum(|s|)*alpha)
void build(){
    queue<int> q{{0}};
    while(!q.empty()){
        int u=q.front();q.pop();
        for(int i=0;i<alpha;++i){
            int v=to[u][i];
            if(!v)to[u][i]=to[suf[u]][i];
            else q.push(v);
            if(!u || !v)continue;
            suf[v]=to[suf[u]][i];
            dad[v]=cnt[suf[v]]?suf[v]:dad[suf
                [v]];
        }
    }
}
```

```
for(int i=1;i<=act;++i){
    adj[i].push_back(dad[i]);
    adj[dad[i]].push_back(i);
}
}
```

### 9.2 Hashing

```
// O(n) build - O(1) get
// 1. prepare() in the main
// 2. hashing<string> hs("hello");
// 3. hs.get(l,r);

// Chars are in [1, BASE)
// BASE is prime or random(lim, MOD-lim)
// If chars are in [0, BASE) then compare the hashes for
length
// 1000234999, 1000567999, 1000111997, 1000777121,
// 1001265673, 1001864327, 999727999, 1070777777
// if hash(multiset 1)==hash(multiset 2) then (r+a1)*(r+
a2)...(r+an)==(r+b1)*(r+b2)...(r+bn) // (Collision n/
MOD)

const ii BASE(257, 367);
const int MOD[2] = { 1001864327, 1001265673 };

int add(int a, int b, int m){return a+b>=m?a+b-m:a+b;}
int sbt(int a, int b, int m){return a-b<0?a-b+m:a-b;}
int mul(int a, int b, int m){return ll(a)*b%m;}
ll operator ! (const ii a) { return (ll(a.first) << 32) |
    a.second; }
ii operator + (const ii& a, const ii& b){return {add(a.
first, b.first, MOD[0]), add(a.second, b.second, MOD
[1])};}
ii operator - (const ii& a, const ii& b){return {sbt(a.
first, b.first, MOD[0]), sbt(a.second, b.second, MOD
[1])};}
ii operator * (const ii& a, const ii& b){return {mul(a.
first, b.first, MOD[0]), mul(a.second, b.second, MOD
[1])};}

const int maxn = 1e6+5;
ii pot[maxn];
void prepare(){ // remember!!!
    pot[0] = ii{1,1};
    rep(i,1,maxn) pot[i] = pot[i-1] * BASE;
}

template <class type>
struct Hashing{
    vector<ii> h;
    Hashing(type& t){
        h.assign(sz(t)+1, ii{0,0});
        rep(i,1,sz(h)) h[i] = h[i-1] * BASE + ii{
            t[i-1], t[i-1]};
    }
}
```

```

    }
    ii get(int l, int r){
        return h[r+1] - h[l] * pot[r-l+1];
    }
};
ii combine(ii a, ii b, int lenb){
    return a * pot[lenb] + b;
}

```

### 9.3 Hashing 2D

```

// Revisar primero los comentarios en hashing!!!
// 1-indexed

const ii BX(3731, 3731), BY(2999, 2999);
const int MOD[2] = { 998244353, 1001265673 };

int add(int a, int b, int m){return a+b>=m?a+b-m:a+b;}
int sbt(int a, int b, int m){return a-b<0?a-b+m:a-b;}
int mul(int a, int b, int m){return ll(a)*b%m;}
ll operator ! (const ii a) { return (ll(a.first) << 32) |
    a.second; }
ii operator + (const ii& a, const ii& b){return {add(a.
    first, b.first, MOD[0]), add(a.second, b.second, MOD
    [1])};}
ii operator - (const ii& a, const ii& b){return {sbt(a.
    first, b.first, MOD[0]), sbt(a.second, b.second, MOD
    [1])};}
ii operator * (const ii& a, const ii& b){return {mul(a.
    first, b.first, MOD[0]), mul(a.second, b.second, MOD
    [1])};}

const int maxn = 1e6+5;
ii PX[maxn], PY[maxn];
void prepare(){ // remember!!!
    PX[0] = PY[0] = ii{1,1};
    rep(i,1,maxn) {
        PX[i] = PX[i-1] * BX;
        PY[i] = PY[i-1] * BY;
    }
}

template <class type>
struct Hashing2D { // 1-indexed
    vector<vector<ii>> hs;
    int n, m;
    Hashing2D(vector<type>& s) {
        n = sz(s), m = sz(s[0]);
        hs.assign(n + 1, vector<ii>(m + 1, {0,0})
        );
        rep(i, 0, n) rep(j, 0, m)
            hs[i + 1][j + 1] = {s[i][j], s[i
            ][j]};
    }
}

```

```

    rep(i, 0, n+1) rep(j, 0, m)
        hs[i][j + 1] = hs[i][j + 1] + hs[
        i][j] * BY;
    rep(i, 0, n) rep(j, 0, m+1)
        hs[i + 1][j] = hs[i + 1][j] + hs[
        i][j] * BX;
}
ii get(int x1, int y1, int x2, int y2) {
    assert(1 <= x1 && x1 <= x2 && x2 <= n);
    assert(1 <= y1 && y1 <= y2 && y2 <= m);
    x1--;y1--;
    int dx = x2 - x1, dy = y2 - y1;
    return (hs[x2][y2] - hs[x2][y1] * PY[dy])
        (hs[x1][y2] - hs[x1][y1] * PY[dy
        ]) * PX[dx];
}
};

```

### 9.4 KMP

```

// O(n)
vector<int> phi(string& s){
    int n=sz(s);
    vector<int> tmp(n);
    for(int i=1,j=0;i<n;++i){
        while(j>0 && s[j]!=s[i])j=tmp[j-1];
        if(s[i]==s[j])j++;
        tmp[i]=j;
    }
    return tmp;
}

// O(n+m)
int kmp(string& s, string& p){
    int n=sz(s),m=sz(p),cnt=0;
    vector<int> pi=phi(p);
    for(int i=0,j=0;i<n;++i){
        while(j && s[i]!=p[j])j=pi[j-1];
        if(s[i]==p[j])j++;
        if(j==m){
            cnt++;
            j=pi[j-1];
        }
    }
    return cnt;
}

```

### 9.5 KMP Automaton

```

const int maxn = 1e5+5;
const int alpha = 26;
int to[maxn][alpha];

```

```

int conv(char ch){return ((ch>='a' && ch<='z')?ch-'a':ch-
'A'+26);}
// O(n*alpha)
void build(string& s){
    to[0][conv(s[0])]=1;
    int n=sz(s);
    for(int i=1,p=0;i<n+1;++i){
        for(int j=0;j<alpha;++j)to[i][j]=to[p][j];
        if(i<n){
            to[i][conv(s[i])]=i+1;
            p=to[p][conv(s[i])];
        }
    }
}

```

## 9.6 Lyndon Factorization

```

// A string is called simple if it is strictly smaller
// than all its nontrivial cyclic shifts.
// The Lyndon factorization of the string is s = w1 w2
// ... wk
// where all strings wi are simple, and they are in non-
// increasing order
// w1 >= w2 >= ... >= wk
// this factorization exists and it is unique
// O(n)
vector<string> duval(string& s){
    vector<string> factorization;
    int n=sz(s),i=0;
    while(i<n){
        int j=i+1,k=i;
        while(j<n && s[k]<=s[j]){
            if(s[k]<s[j])k=i;
            else k++;
            j++;
        }
        while(i<=k){
            factorization.push_back(s.substr(
                i, j-k));
            i+=j-k;
        }
    }
    return factorization;
}

```

## 9.7 Manacher

```

// O(n), par (raiz, izq, der) 1 - impar 0
vi manacher(string& s, int par){

```

```

int l=0,r=-1,n=sz(s);
vi m(n,0);
for(int i=0;i<n;++i){
    int k=(i>r?(l-par):min(m[l+r-i+par], r-i+par))+par;
    while(i+k-par<n && i-k>=0 && s[i+k-par]==s[i-k])++k;
    m[i]=k-par;--k;
    if(i+k-par>r)l=i-k,r=i+k-par;
}
for(int i=0;i<n;++i)m[i]=(m[i]-l+par)*2+l-par;
return m;
}

```

## 9.8 Minimum Expression

```

// O(n)
int minimum_expression(string s){
    s=s+s;int n=sz(s),i=0,j=1,k=0;
    while(i+k<n && j+k<n){
        if(s[i+k]==s[j+k])k++;
        else if(s[i+k]>s[j+k])i=i+k+1,k=0; // <
        else j=j+k+1,k=0;
        if(i==j)j++;
    }
    return min(i, j);
}

```

## 9.9 Next Permutation

```

// O(n)
// 1) find the last i such that ai<ai+1
// 2) find the last j such that ai<aj
// 3) swap i and j, then reverse the segment [i+1, n-1]
string nextPermutation(string& s){
    string ans(s);
    int n=sz(s);
    int i=n-2;
    while(i>=0 && ans[i]>=ans[i+1])i--;
    if(i<0)return "no permutation";
    int j=n-1;
    while(ans[i]>=ans[j])j--;
    swap(ans[i], ans[j]);
    int l=i+1,r=n-1;
    while(r>l)swap(ans[r--], ans[l++]);
    return ans;
}

```

## 9.10 Palindromic Tree



```

const int alpha = 26;
const char mini = 'a';

// tree.suf: the longest suffix-palindrome link
// tree.dad - tree.to: the parent palindrome by removing
// the first and last character
// node 0 = root with len -1 for odd
// node 1 = root with len 0 for even
struct Node {
    int to[alpha], suf, len, cnt, dad;
    Node(int x, int l = 0, int c = 1): len(x), suf(l),
        cnt(c) {
        memset(to, 0, sizeof(to));
    }
    int& operator [] (int i) { return to[i]; }
};

struct PalindromicTree {
    vector<Node> tree;
    vector<int> palo; // longest suffix-palindrome in
        the position i
    string s;
    int n, last; // max suffix palindrome

    PalindromicTree(string t = "") {
        n = last = 0;
        tree.push_back(Node(-1));
        tree.push_back(Node(0));
        for(char& c:t) add_char(c);
        // Propagate counts up the suffix links
        for(int i=sz(tree)-1; i>=2; i--){
            tree[tree[i].suf].cnt+=tree[i].
                cnt;
        }
    }

    int getsuf(int p) {
        while (n - tree[p].len - 1 < 0 || s[n - tree[p].
            len - 1] != s[n])
            p = tree[p].suf;
        return p;
    }

    void add_char(char ch) {
        s.push_back(ch);
        int p = getsuf(last), c = ch - mini;
        if (!tree[p][c]) {
            int suf = getsuf(tree[p].suf);
            suf = max(1, tree[suf][c]);
            tree[p][c] = sz(tree);
            tree.push_back(Node(tree[p].len + 2, suf, 0))
        }
        last = tree[p][c];
        tree[last].dad = p;
        tree[last].cnt++; n++;
        palo.push_back(tree[last].len);
    }
};

```

```

    }
};

```

## 9.11 Suffix Array

```

// O(n*log(n)) - char in [1, lim)
// sa: is the starting position of the i-th lex smallest
// suffix
// rnk: is the rank (position in SA) of the suffix
// starting at i
// lcp: is the longest common prefix between sa[i] and sa
// [i+1]
auto SuffixArray(string s, int lim=256) {
    s.push_back(0); int n = sz(s), k = 0, a, b;
    vi sa, lcp, rnk(all(s)), y(n), ws(max(n, lim));
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2),
        lim = p) {
        p = j, iota(all(y), n - j);
        rep(i, 0, n) if (sa[i] >= j) y[p++] = sa[i]
            - j;
        fill(all(ws), 0);
        rep(i, 0, n) ws[rnk[i]]++;
        rep(i, 1, lim) ws[i] += ws[i - 1];
        for (int i = n; i--;) sa[--ws[rnk[y[i]]]]
            = y[i];
        swap(rnk, y), p = 1, rnk[sa[0]] = 0;
        rep(i, 1, n) a = sa[i - 1], b = sa[i], rnk[
            b] =
            (y[a] == y[b] && y[a + j] == y[b
                + j]) ? p - 1 : p++;
    }
    for (int i = 0, j; i < n - 1; lcp[rnk[i++]] = k)
        for (k && k--, j = sa[rnk[i] - 1]; s[i +
            k] == s[j + k]; k++);
    reverse(all(lcp)); lcp.pop_back(); reverse(all(lcp)
        );
    return tuple{sa, rnk, lcp};
}

```

## 9.12 Suffix Automaton

```

// O(n*log(alpha))
// suf: suffix link (like aho if not match)
// len: length of the longest string in this state
// minlen: smallest string of node v = (v.suf==-1?0:v.suf
// .len) + 1
// end: if this state is terminal
// count different strings [i.suf.len+1, i.len]
// para saber cuantos substrings itnee a en b, ir
// procesando los

```

```

// prefijos y al marcarlos procesar la cantidad visitando
// los sufijos de los nodos
// contribucion es u.len - u.suf.len, tener en cuenta con
// que len se llevo
// puede ser un len2 para manejar eso min(u.len, u.len2)-
// u.suf.len

// a->b->c->b->c
// b->c
template<int alpha = 26>
struct SuffixAutomaton {
    struct Node {
        // array<int, alpha> to; TLE, add -> int
        // conv(char ch)
        map<char, int> to;
        int len = 0, suf = 0;
        bool end = false;
    };
    vector<Node> sa;
    int last = 0;
    ll substrs = 0;

    SuffixAutomaton(string &s) {
        sa.reserve(sz(s)*2);
        last = add_node();
        sa[0].suf = -1;
        for (char &c : s) add_char(c);
        for (int p = last; p; p = sa[p].suf) sa[p]
            ].end = 1;
    }

    int add_node() { sa.push_back({}); return sz(sa)
        -1; }

    void add_char(char c) {
        int u = add_node(), p = last;
        sa[u].len = sa[last].len + 1;
        while (p != -1 && !sa[p].to.count(c)) {
            sa[p].to[c] = u;
            substrs += p != 0 ? sa[p].len -
                sa[sa[p].suf].len : 1;
            p = sa[p].suf;
        }
        if (p != -1) {
            int q = sa[p].to[c];
            if (sa[p].len + 1 != sa[q].len) {
                int clone = add_node();
                sa[clone] = sa[q];
                sa[clone].len = sa[p].len
                    + 1;
                sa[q].suf = sa[u].suf =
                    clone;
                while (p != -1 && sa[p].
                    to[c] == q) {
                    sa[p].to[c] =
                        clone;

```

```

                p = sa[p].suf;
            } else sa[u].suf = q;
        }
        last = u;
    }

    // Aplicaciones
    int dfs(int u) { // count
        if (sa[u].cnt != -1) return sa[u].cnt;
        sa[u].cnt = sa[u].end;
        for (auto [_, v] : sa[u].to) {
            sa[u].cnt += dfs(v);
        }
        return sa[u].cnt;
    }

    void dfs2(int u) { // grade primero
        sa[u].pre--;
        if (sa[u].pre > 0) return;
        for (auto [_, v] : sa[u].to) {
            sa[v].cnt2 += sa[u].cnt2;
            dfs2(v);
        }
    }

    void dfs2() {
        vector<int> order(sz(sa)-1);
        for (int i=1; i<sz(sa); ++i) order[i-1]=i;
        sort(order.begin(), order.end(), [&](int
            a, int b) { return sa[a].len > sa[b].
                len; });
        for (auto &i : order) {
            // suf.cnt += i.cnt
        }
    }

    int lcs(string& t) {
        int u=0, l=0, ans=0;
        for (char c:t) {
            while (u && !sa[u].to.count(c)) {
                u=sa[u].suf;
                l=sa[u].len;
            }
            if (sa[u].to.count(c)) {
                u=sa[u].to[c];
                l++;
            }
            ans = max(ans, l);
        }
        return ans;
    }

    bool query(string& t) {
        int u=0;
        for (char c:t) {

```

```

        if(!sa[u].to.count(c)) return
        false;
        u=sa[u].to[c];
    }
    return true;
}

void cyclic(string& t){ // dfs(0) primero
    int u=0,l=0;
    int m=sz(t);
    t+=t;
    unordered_set<int> s; // vector<bool>
    for(char ch:t){
        int c=conv(ch);
        while(u && !sa[u].to[c]){
            u=sa[u].suf;
            l=sa[u].len;
        }
        if(sa[u].to[c]){
            u=sa[u].to[c];
            l++;
        }
        if(l==m){
            s.insert(u);
            if(sa[u].minlen==m){
                u=sa[u].suf;
                l=sa[u].len;
            }else{
                l--;
            }
        }
    }
    ll ans=0;
    for(int u:s) ans+=sa[u].cnt;
    cout<<ans<<"\n";
}
};

```

### 9.13 Suffix Tree

```

// O(n)
// pos: start of the edge
// len: edge length
// link: suffix link
struct SuffixTree{
    vector<map<char,int>> to;
    vector<int> pos,len,link;
    int size=0,inf=1e9;
    string s;

    int make(int _pos, int _len){
        to.push_back(map<char,int>());
        pos.push_back(_pos);
        len.push_back(_len);
        link.push_back(-1);
    }
};

```

```

        return size++;
    }

    void add(int& p, int& lef, char c){
        s+=c; ++lef; int lst=0;
        for(; lef; p=link[p]:lef--){
            while(lef>1 && lef>len[to[p][s[sz(s)-lef]]]){
                p=to[p][s[sz(s)-lef]], lef
                -=len[p];
            }
            char e=s[sz(s)-lef];
            int& q=to[p][e];
            if(!q){
                q=make(sz(s)-lef,inf),
                link[lst]=p,lst=0;
            }else{
                char t=s[pos[q]+lef-1];
                if(t==c){link[lst]=p;
                    return;}
                int u=make(pos[q],lef-1);
                to[u][c]=make(sz(s)-1,inf);
                to[u][t]=q;
                pos[q]+=lef-1;
                if(len[q]!=inf) len[q]-=
                lef-1;
                q=u,link[lst]=u,lst=u;
            }
        }
    }

    SuffixTree(string& _s){
        make(-1,0); int p=0,lef=0;
        for(char c:_s) add(p,lef,c);
        add(p,lef,'$'); // smallest char
        s.pop_back();
    }

    int query(string& p){
        for(int i=0,u=0,n=sz(p);i<n;i++){
            if(i==n || !to[u].count(p[i]))
                return i;
            u=to[u][p[i]];
            for(int j=0;j<len[u];j++){
                if(i==n || s[pos[u]+j]!=p[i]) return i;
                i++;
            }
        }
    }

    vector<int> sa;
    void genSA(int x=0, int Len=0){
        if(!sz(to[x])) sa.push_back(pos[x]-Len);
        else for(auto t:to[x]) genSA(t.second,Len+

```

```

        len[x]);
    }
};

```

## 9.14 Trie

```

const int maxn = 2e6+5;
const int alpha = 26;

int to[maxn][alpha]; // to[u][c]: node u edge with the
                    // letter c
int cnt[maxn]; // count of word ending in this node
int act; // trie node count
int conv(char ch){return ((ch>='a' && ch<='z')?ch-'a':ch-
'A'+26);}

void init(){
    for(int i=0;i<=act;++i){
        memset(to[i],0,sizeof(to[i]));
        cnt[i]=0;
    }
    act=0;
}

void add(string& s){
    int u=0;
    for(char ch:s){
        int c=conv(ch);
        if(!to[u][c])to[u][c]=++act;
        u=to[u][c];
    }
    cnt[u]++;
}

```

## 9.15 Trie Bit

```

const int maxn = 5e5+5;
const int bits = 30;
const int alpha = 2;

int to[maxn*bits][alpha]; // to[u][c]: node u edge with
                        // the letter c
int cnt[maxn*bits]; // count of word ending in this node
int act; // trie node count
int conv(int x, int i){return ((x&(1<<i))?1:0);}

void init(){
    for(int i=0;i<=act;++i){
        memset(to[i],0,sizeof(to[i]));
        cnt[i]=0;
    }
    act=0;
}

```

```

void add(int x){
    int u=0;
    for(int i=bits;i>=0;--i){
        int c=conv(x,i);
        if(!to[u][c])to[u][c]=++act;
        cnt[u]++;
        u=to[u][c];
    }
    cnt[u]++;
}

int mini(int x){
    int u=0,ans=0;
    for(int i=bits;i>=0;--i){
        int c=conv(x,i);
        if(!to[u][c] || cnt[to[u][c]]==0){
            u=to[u][!c];
            ans+=(1<<i);
        }else{
            u=to[u][c];
        }
    }
    return ans;
}

```

## 9.16 Z Algorithm

```

// O(n)
vector<int> z_function(string& s){
    int n=sz(s),l=0,r=0;
    vector<int> z(n);
    for(int i=1;i<n;i++){
        if(i<r)z[i]=min(r-i, z[i-l]);
        while(i+z[i]<n && s[z[i]]==s[i+z[i]])z[i]
            ++;
        if(i+z[i]>r){
            l=i;
            r=i+z[i];
        }
    }
    return z;
}

```

## 9.17 El especial

```

#include<bits/stdc++.h>
using namespace std;
const int N = 1e5 + 9;

struct ST {
    #define lc (n << 1)
    #define rc ((n << 1) | 1)
}

```

```

long long t[4 * N], lazy[4 * N];
ST() {
    memset(t, 0, sizeof t);
    memset(lazy, 0, sizeof lazy);
}
inline void push(int n, int b, int e) {
    if (lazy[n] == 0) return;
    t[n] = t[n] + lazy[n] * (e - b + 1);
    if (b != e) {
        lazy[lc] = lazy[lc] + lazy[n];
        lazy[rc] = lazy[rc] + lazy[n];
    }
    lazy[n] = 0;
}
inline long long combine(long long a, long long b) {
    return a + b;
}
inline void pull(int n) {
    t[n] = t[lc] + t[rc];
}
void upd(int n, int b, int e, int i, int j, int v) {
    push(n, b, e);
    if (j < b || e < i) return;
    if (i <= b && e <= j) {
        lazy[n] = v; //set lazy
        return;
    }
    int mid = (b + e) >> 1;
    upd(lc, b, mid, i, j, v);
    upd(rc, mid + 1, e, i, j, v);
    pull(n);
}
long long query(int n, int b, int e, int i, int j) {
    push(n, b, e);
    if (i > e || b > j) return 0; //return null
    if (i <= b && e <= j) return t[n];
    int mid = (b + e) >> 1;
    return combine(query(lc, b, mid, i, j), query(rc, mid
        + 1, e, i, j));
}
}st;

struct node {
    int len, link, firstpos;
    map<char, int> nxt;
};
vector<node> t;
struct SuffixAutomaton {
    int sz, last;
    vector<int> terminal;
    vector<int> dp;
    vector<vector<int>> g;
    SuffixAutomaton() {}
    SuffixAutomaton(int n) {

```

```

        t.clear(); t.resize(2 * n);
        terminal.resize(2 * n, 0);
        dp.resize(2 * n, -1); sz = 1; last = 0;
        g.resize(2 * n);
        t[0].len = 0; t[0].link = -1; t[0].firstpos = 0;
    }
    void extend(char c) {
        int p = last;
        int cur = sz++;
        t[cur].len = t[last].len + 1;
        t[cur].firstpos = t[cur].len;
        p = last;
        while (p != -1 && !t[p].nxt.count(c)) {
            t[p].nxt[c] = cur;
            p = t[p].link;
        }
        if (p == -1) t[cur].link = 0;
        else {
            int q = t[p].nxt[c];
            if (t[p].len + 1 == t[q].len) t[cur].link = q;
            else {
                int clone = sz++;
                t[clone] = t[q];
                t[clone].len = t[p].len + 1;
                while (p != -1 && t[p].nxt[c] == q) {
                    t[p].nxt[c] = clone;
                    p = t[p].link;
                }
                t[q].link = t[cur].link = clone;
            }
        }
        last = cur;
    }
};

pair<int, int> modifies[N * 2];
int cnt;
namespace lct {
    int par[N * 2], lazy[N * 2], last[N * 2], c[N * 2][2];
    void mark(int x, int v) {
        lazy[x] = last[x] = v;
    }
    void push(int x) {
        if (lazy[x]) {
            if (c[x][0]) {
                mark(c[x][0], lazy[x]);
            }
            if (c[x][1]) {
                mark(c[x][1], lazy[x]);
            }
            lazy[x] = 0;
        }
    }
    bool is_root(int x) {
        return c[par[x]][0] != x && c[par[x]][1] != x;
    }

```

```

}
void rotate(int x) {
    int y = par[x], z = par[y], k = c[y][1] == x;
    if (!is_root(y)) {
        c[z][c[z][1] == y] = x;
    }
    par[c[y][k] = c[x][!k]] = y;
    par[par[c[x][!k] = y] = x] = z;
}
void splay(int x) {
    static int st[N];
    int top = 0;
    st[++top] = x;
    for (int i = x; !is_root(i); i = par[i]) {
        st[++top] = par[i];
    }
    while (top) {
        push(st[top--]);
    }
    while (!is_root(x)) {
        int y = par[x], z = par[y];
        if (!is_root(y)) {
            rotate((c[y][1] == x) == (c[z][1] == y) ? y : x);
        }
        rotate(x);
    }
}
void access(int x, int v) {
    int z = 0;
    cnt = 0;
    while (x) {
        splay(x);
        modifies[++cnt] = make_pair(t[x - 1].len, last[x]);
        c[x][1] = z;
        mark(x, v);
        z = x;
        x = par[x];
    }
}

int pos[N];
vector<pair<int, int>> Q[N];
long long ans[N];
int32_t main() {
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, q; cin >> n >> q;
    string s; cin >> s;
    SuffixAutomaton sa(n);
    for (int i = 1; i <= q; i++) {
        int l, r; cin >> l >> r;
        ++l; ++r;
        Q[r].push_back({l, i});
    }
    s = "." + s;

```

```

pos[0] = 1;
for (int i = 1; i <= n; ++i) {
    sa.extend(s[i]);
    pos[i] = sa.last + 1;
}
for (int i = 1; i <= sa.sz; ++i) {
    lct::par[i] = t[i - 1].link + 1;
}
for (int i = 1; i <= n; ++i) {
    st.upd(1, 1, n, 1, i, 1);
    lct::access(pos[i], i);
    int last = 0;
    for (int j = cnt; j > 1; --j) {
        pair<int, int> p = modifies[j];
        if (p.first) {
            if (p.second) {
                st.upd(1, 1, n, p.second - p.first + 1, p.
                    second - last, -1);
            }
            last = p.first;
        }
    }
    // st.query(1, 1) = number of distinct substrings
    // which lastly occured in starting position 1 for
    // prefix [1, i]
    for (auto [l, id]: Q[i]) {
        ans[id] = st.query(1, 1, n, l, i);
    }
}
for (int i = 1; i <= q; i++) {
    cout << ans[i] << '\n';
}
return 0;
}

```

## 10 Misc

### 10.1 Counting Sort

```

// O(n+k)
void counting_sort(vi& a) {
    int n=sz(a);
    int maxi=*max_element(all(a));
    int mini=*min_element(all(a));
    int k=maxi-mini+1;
    vi cnt(k, 0);
    for (int i=0; i<n; ++i) ++cnt[a[i]-mini];
    for (int i=0, j=0; i<k; ++i)
        while (cnt[i]--) a[j++] = i+mini;
}

```

## 10.2 Dates

```

int dateToInt(int y, int m, int d){
    return 1461*(y+4800+(m-14)/12)/4+367*(m-2-(m-14)
        /12*12)/12-
        3*((y+4900+(m-14)/12)/100)/4+d-32075;
}
void intToDate(int jd, int& y, int& m, int& d){
    int x,n,i,j;x=jd+68569;
    n=4*x/146097;x=(146097*n+3)/4;
    i=(4000*(x+1))/1461001;x-=1461*i/4-31;
    j=80*x/2447;d=x-2447*j/80;
    x=j/11;m=j+2-12*x;y=100*(n-49)+i+x;
}
int DayOfWeek(int d, int m, int y){    //starting on
    Sunday
    static int ttt[]={0, 3, 2, 5, 0, 3, 5, 1, 4, 6,
        2, 4};
    y-=m<3;
    return (y+y/4-y/100+y/400+ttt[m-1]+d)%7;
}

```

## 10.3 Expression Parsing

```

// O(n) - eval() de python
bool delim(char c){return c=='(';}
bool is_op(char c){return c=='+' || c=='-' || c=='*' || c
    =='/';}
bool is_unary(char c){return c=='+' || c=='-';}

int priority(char op){
    if(op<0) return 3;
    if(op=='+' || op=='-') return 1;
    if(op=='*' || op=='/') return 2;
    return -1;
}

void process_op(stack<int>& st, char op){
    if(op<0){
        int l=st.top();st.pop();
        switch(-op){
            case '+':st.push(l);break;
            case '-':st.push(-l);break;
        }
    }else{
        int r=st.top();st.pop();
        int l=st.top();st.pop();
        switch(op){
            case '+':st.push(l+r);break;
            case '-':st.push(l-r);break;
            case '*':st.push(l*r);break;
            case '/':st.push(l/r);break;
        }
    }
}

```

```

}

int evaluate(string& s){
    stack<int> st;
    stack<char> op;
    bool may_be_unary=true;
    for(int i=0;i<sz(s);++i){
        if(delim(s[i]))continue;
        if(s[i] == '('){
            op.push('(');
            may_be_unary=true;
        }else if(s[i]==')'){
            while(op.top()!='('){
                process_op(st, op.top());
                op.pop();
            }
            op.pop();
            may_be_unary=false;
        }else if(is_op(s[i])){
            char cur_op=s[i];
            if(may_be_unary && is_unary(
                cur_op)) cur_op=-cur_op;
            while(!op.empty() && ((cur_op >=
                0 && priority(op.top()) >=
                priority(cur_op)) || (cur_op <
                0 && priority(op.top()) >
                priority(cur_op)))){
                process_op(st, op.top());
                op.pop();
            }
            op.push(cur_op);
            may_be_unary=true;
        }else{
            int number=0;
            while(i<sz(s) && isalnum(s[i]))
                number=number*10+s[i++]-'0';
            --i;
            st.push(number);
            may_be_unary=false;
        }
    }

    while(!op.empty()){
        process_op(st, op.top());
        op.pop();
    }
    return st.top();
}

```

## 10.4 Hanoi

```

// hanoi(n) = 2 * hanoi(n-1) + 1
// hanoi(n, 1, 3)

```

```
vector<int> ans;
void hanoi(int x, int start, int end){
    if(!x) return;
    hanoi(x-1, start, 6-start-end);
    ans.push_back({start, end});
    hanoi(x-1, 6-start-end, end);
}
```

## 10.5 K mas frecuentes

```
// los k numeros mas frecuentes
// el cero es un valor neutral dentro del vector
// no usarlo en el array original (a[i] > 0, i e [0,n-1])
// el vector guarda {valor, contador}
// pero contador es para el algo, no es la cantidad real

// algoritmo de misra-gries O(k^2)
vector<ii> null(k, {0,0});
vector<ii> init(int v){
    vector<ii> a=null;
    a[0]={v,1};
    return a;
}
vector<ii> oper(vector<ii> a, vector<ii> b, int k) {
    for (int i = 0; i < k; ++i) if (b[i].first) {
        int p = -1, q = -1;
        for (int j = 0; j < k; ++j) {
            if (b[i].first == a[j].first) p = j;
            if (!a[j].first) q = j;
        }
        if (p != -1) {
            a[p].second += b[i].second;
        } else if (q != -1) {
            a[q] = b[i];
        } else {
            int mn = b[i].second;
            for (int j = 0; j < k; ++j) mn = min(mn, a[j].second);
            for (int j = 0; j < k; ++j) a[j].second -= mn;
            b[i].second -= mn;
            for (int j = 0; j < k; ++j) if (!a[j].second) {
                if (b[i].second > 0) {
                    a[j] = b[i], b[i].second = 0;
                } else {
                    a[j].first = 0;
                }
            }
        }
    }
    return a;
}
```

```
}
```

## 10.6 Prefix3D

```
const int N = 100;
int A[N][N][N];
int prefix[N + 1][N + 1][N + 1];

void build(int n){
    for (int x = 1; x <= n; x++){
        for (int y = 1; y <= n; y++){
            for (int z = 1; z <= n; z++){
                prefix[x][y][z] = A[x - 1][y - 1][z - 1]
                    + prefix[x - 1][y][z]
                    + prefix[x][y - 1][z]
                    + prefix[x][y][z - 1]
                    - prefix[x - 1][y - 1][z]
                    - prefix[x - 1][y][z - 1]
                    - prefix[x][y - 1][z - 1]
                    + prefix[x - 1][y - 1][z - 1];
            }
        }
    }

    ll query(int lx, int rx, int ly, int ry, int lz, int rz){
        ll ans = prefix[rx][ry][rz]
            - prefix[lx - 1][ry][rz] - prefix[rx][ly - 1][rz]
            - prefix[lx - 1][ly - 1][rz] + prefix[lx - 1][ry][lz - 1]
            + prefix[lx - 1][ly - 1][lz - 1] + prefix[rx][ly - 1][lz - 1]
            - prefix[lx - 1][ly - 1][lz - 1];
        return ans;
    }
}
```

## 10.7 Ternary Search

```
// O(log((r-l)/eps))
// retorna el maximo valor de f(x) en [l,r]
const double eps = 1e-9;
double f(double x);
double ternary(){
    // ...
}
```



```

double l, r;
while (r-l > eps) {
    double m1 = l + (r-l)/3.0;
    double m2 = r - (r-l)/3.0;
    if (f(m1) < f(m2)) l = m1;
    else r = m2;
} return max(f(l), f(r));
}

// ternary search para enteros
// O(log((r-l)/eps))
// retorna el maximo valor de f(x) en [l, r]
int f(int x);
int ternary() {
    int l, r;
    while (r-l > 6) {
        int m1 = l + (r-l)/3;

```

```

        int m2 = r - (r-l)/3;
        if (f(m1) < f(m2)) l = m1; // revisar desempate
        else r = m2;
    }
    int ans = l, val = f(l);
    for (int i = l+1; i <= r; ++i) {
        int val2 = f(i);
        if (val2 > val) {
            val = val2;
            ans = i;
        }
    }
    return val;
}

```

## 11 Teoría y miscelánea

### 11.1 Sumatorias

- $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{i=1}^n i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{i=1}^n i^5 = \frac{(n(n+1))^2(2n^2+2n-1)}{12}$
- $\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{i=0}^n x^i = \frac{x^{n+1}-1}{x-1}$  para  $x \neq 1$

### 11.2 Teoría de Grafos

#### 11.2.1 Teorema de Euler

En un grafo conectado planar, se cumple que  $V - E + F = 2$ , donde  $V$  es el número de vértices,  $E$  es el número de aristas y  $F$  es el número de caras. Para varios componentes la formula es:  $V - E + F = 1 + C$ , siendo  $C$  el número de componentes.

#### 11.2.2 Planaridad de Grafos

Un grafo es planar si y solo si no contiene un subgrafo homeomorfo a  $K_5$  (grafo completo con 5 vértices) ni a  $K_{3,3}$  (grafo bipartito completo con 3 vértices en cada conjunto).

#### 11.2.3 Truco del Cow Game

Dadas restricciones de la forma:

$$x_a - x_b \leq d$$

podemos transformar cada desigualdad en una arista dirigida:

$$b \rightarrow a \quad \text{con peso } d$$

Luego, ejecutando un algoritmo de camino más corto desde un nodo inicial  $s$ , obtenemos:

$$\text{dist}[i] = \max(x_i - x_s)$$

**Nota:** Pueden aparecer pesos negativos, por lo que se debe usar Bellman-Ford o SPFA, no Dijkstra.

### 11.3 Teoría de Números

#### 11.3.1 Ecuaciones Diofánticas Lineales

Una ecuación diofántica lineal es una ecuación en la que se buscan soluciones enteras  $x$  e  $y$  que satisfagan la relación lineal  $ax + by = c$ , donde  $a$ ,  $b$  y  $c$  son constantes dadas.

Para encontrar soluciones enteras positivas en una ecuación diofántica lineal, podemos seguir el siguiente proceso:

1. Encontrar una solución particular: Encuentra una solución particular  $(x_0, y_0)$  de la ecuación. Esto puede hacerse utilizando el algoritmo de Euclides extendido.

2. Encontrar la solución general: Una vez que tengas una solución particular, puedes obtener la solución general utilizando la fórmula:

$$x = x_0 + \frac{b}{\text{mcd}(a, b)} \cdot t$$

$$y = y_0 - \frac{a}{\text{mcd}(a, b)} \cdot t$$

donde  $t$  es un parámetro entero.

3. Restringir a soluciones positivas: Si deseas soluciones positivas, asegúrate de que las soluciones generales satisfagan  $x \geq 0$  y  $y \geq 0$ . Puedes ajustar el valor de  $t$  para cumplir con estas restricciones.

### 11.3.2 Pequeño Teorema de Fermat

Si  $p$  es un número primo y  $a$  es un entero no divisible por  $p$ , entonces  $a^{p-1} \equiv 1 \pmod{p}$ .

### 11.3.3 Teorema de Euler

Para cualquier número entero positivo  $n$  y un entero  $a$  coprimo con  $n$ , se cumple que  $a^{\phi(n)} \equiv 1 \pmod{n}$ , donde  $\phi(n)$  es la función phi de Euler, que representa la cantidad de enteros positivos menores que  $n$  y coprimos con  $n$ .

## 11.4 Geometría

### 11.4.1 Teorema de Pick

Sea un polígono simple cuyos vertices tienen coordenadas enteras. Si  $B$  es el número de puntos enteros en el borde,  $I$  el número de puntos enteros en el interior del polígono, entonces el área  $A$  del polígono se puede calcular con la formula:

$$A = I + \frac{B}{2} - 1$$

### 11.4.2 Fórmula de Herón

Si los lados del triángulo tienen longitudes  $a$ ,  $b$  y  $c$ , y  $s$  es el semiperímetro (es decir,  $s = \frac{a+b+c}{2}$ ), entonces el área  $A$  del triángulo está dada por:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

### 11.4.3 Relación de Existencia Triangular

Para un triángulo con lados de longitud  $a$ ,  $b$ , y  $c$ , la relación de existencia triangular se expresa como:

$$b - c < a < b + c, \quad a - c < b < a + c, \quad a - b < c < a + b$$

## 11.5 Combinatoria

### 11.5.1 Permutaciones

El número de permutaciones de  $n$  objetos distintos tomados de a  $r$  a la vez (sin repetición) se denota como  $P(n, r)$  y se calcula mediante:

$$P(n, r) = \frac{n!}{(n-r)!}$$

### 11.5.2 Combinaciones

El número de combinaciones de  $n$  objetos distintos tomados de a  $r$  a la vez (sin repetición) se denota como  $C(n, r)$  o  $\binom{n}{r}$  y se calcula mediante:

$$C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

### 11.5.3 Permutaciones con Repetición

El número de permutaciones de  $n$  objetos tomando en cuenta repeticiones se denota como  $P_{\text{rep}}(n; n_1, n_2, \dots, n_k)$  y se calcula mediante:

$$P_{\text{rep}}(n; n_1, n_2, \dots, n_k) = \frac{n!}{n_1!n_2! \dots n_k!}$$

### 11.5.4 Combinaciones con Repetición

El número de combinaciones de  $n$  objetos tomando en cuenta repeticiones se denota como  $C_{\text{rep}}(n; n_1, n_2, \dots, n_k)$  y se calcula mediante:

$$C_{\text{rep}}(n; n_1, n_2, \dots, n_k) = \binom{n+k-1}{n} = \binom{n+k-1}{k-1}$$

### 11.5.5 Números de Catalan

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

Los números de Catalan también pueden calcularse utilizando la siguiente fórmula recursiva:

$$C_0 = 1$$

$$C_{n+1} = \frac{4n+2}{n+2} C_n$$

Usos:

- $\text{Cat}(n)$  cuenta el número de árboles binarios distintos con  $n$  vértices.

- $\text{Cat}(n)$  cuenta el número de expresiones que contienen  $n$  pares de paréntesis correctamente emparejados.
- $\text{Cat}(n)$  cuenta el número de formas diferentes en que se pueden colocar  $n + 1$  factores entre paréntesis, por ejemplo, para  $n = 3$  y  $3 + 1 = 4$  factores:  $a, b, c, d$ , tenemos:  $(ab)(cd), a(b(cd)), ((ab)c)d$  y  $a((bc)d)$ .
- Los números de Catalan cuentan la cantidad de caminos no cruzados en una rejilla  $n \times n$  que se pueden trazar desde una esquina de un cuadrado o rectángulo a la esquina opuesta, moviéndose solo hacia arriba y hacia la derecha.
- Los números de Catalan representan el número de árboles binarios completos con  $n + 1$  hojas.
- $\text{Cat}(n)$  cuenta el número de formas en que se puede triangular un polígono convexo de  $n + 2$  lados. Otra forma de decirlo es como la cantidad de formas de dividir un polígono convexo en triángulos utilizando diagonales no cruzadas.

### 11.5.6 Estrellas y barras

Número de soluciones de la ecuación  $x_1 + x_2 + \cdots + x_k = n$ .

- Con  $x_i \geq 0$ :  $\binom{n+k-1}{n}$
- Con  $x_i \geq 1$ :  $\binom{n-1}{k-1}$

Número de sumas de enteros con límite inferior:

Esto se puede extender fácilmente a sumas de enteros con diferentes límites inferiores. Es decir, queremos contar el número de soluciones para la ecuación:

$$x_1 + x_2 + \cdots + x_k = n$$

con  $x_i \geq a_i$ .

Después de sustituir  $x'_i := x_i - a_i$  recibimos la ecuación modificada:

$$(x'_1 + a_1) + (x'_2 + a_2) + \cdots + (x'_k + a_k) = n$$

$$\Leftrightarrow x'_1 + x'_2 + \cdots + x'_k = n - a_1 - a_2 - \cdots - a_k$$

con  $x'_i \geq 0$ . Así que hemos reducido el problema al caso más simple con  $x'_i \geq 0$  y nuevamente podemos aplicar el teorema de estrellas y barras.

## 11.6 DP Optimization Theory

Name	Original Recurrence	Sufficient Condition	From	To
CH 1	$dp[i] = \min_{j < i} \{dp[j] + b[j] * a[i]\}$	$b[j] \geq b[j+1]$ Optionally $a[i] \leq a[i+1]$	$O(n^2)$	$O(n)$
CH 2	$dp[i][j] = \min_{k < j} \{dp[i-1][k] + b[k] * a[j]\}$	$b[k] \geq b[k+1]$ Optionally $a[j] \leq a[j+1]$	$O(kn^2)$	$O(kn)$
D&Q	$dp[i][j] = \min_{k < j} \{dp[i-1][k] + C[k][j]\}$	$A[i][j] \leq A[i][j+1]$	$O(kn^2)$	$O(kn \log n)$
Knuth	$dp[i][j] = \min_{i < k < j} \{dp[i][k] + dp[k][j] + C[i][j]\}$	$A[i, j-1] \leq A[i, j] \leq A[i+1, j]$	$O(n^3)$	$O(n^2)$

Notes:

- $A[i][j]$  - the smallest  $k$  that gives the optimal answer, for example in  $dp[i][j] = dp[i-1][k] + C[k][j]$
- $C[i][j]$  - some given cost function
- We can generalize a bit in the following way  $dp[i] = \min_{j < i} \{F[j] + b[j] * a[i]\}$ , where  $F[j]$  is computed from  $dp[j]$  in constant time

## 11.7 Lista de números con mayor cantidad de divisores hasta $10^n$

$n$	Número	# divisores
1	6	4
2	60	12
3	840	32
4	7560	64
5	83160	128
6	720720	240
7	8648640	448
8	73513440	768
9	735134400	1344
10	6983776800	2304
11	97772875200	4032
12	963761198400	6720
13	9316358251200	10752
14	97821761637600	17280
15	866421317361600	26880
16	8086598962041600	41472
17	74801040398884800	64512
18	897612484786617600	103680

## 11.8 Games

**Grundy numbers.** For a two-player, normal-play (last to move wins) game on a graph  $(V, E)$ :  $G(x) = \text{mex}(\{G(y) : (x, y) \in E\})$ , where  $\text{mex}(S) = \min\{n \geq 0 : n \notin S\}$ .  $x$  is losing iff  $G(x) = 0$ .

**Sums of games.**

- *Player chooses a game and makes a move in it.* Grundy number of a position is xor of grundy numbers of positions in summed games.
- *Player chooses a non-empty subset of games (possibly, all) and makes moves in all of them.* A position is losing iff each game is in a losing position.

- *Player chooses a proper subset of games (not empty and not all), and makes moves in all chosen ones.* A position is losing iff grundy numbers of all games are equal.

- *Player must move in all games, and loses if can't move in some game.* A position is losing if any of the games is in a losing position.

**Misère Nim.** A position with pile sizes  $a_1, a_2, \dots, a_n \geq 1$ , not all equal to 1, is losing iff  $a_1 \oplus a_2 \oplus \dots \oplus a_n = 0$  (like in normal nim.) A position with  $n$  piles of size 1 is losing iff  $n$  is *odd*.