Simulating quantum circuits by classical circuits

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Abstract

In a recent breakthrough, Bravyi, Gosset, and König (BGK) [1] proved that shallow quantum circuits cannot be "simulated" by shallow classical circuits. Indeed, they show a lower bound of log depth. **Our work addresses the upper bound question.**

We first explicitly define their implicit notion of simulation which we call "possibilistic simulation" (see right). Essentially, we say a classical circuit simulates a quantum circuit if, over all inputs, the output of the classical circuit is a possible output of the quantum circuit.

In this sense, classically simulating the BGK quantum circuits that solve their Hidden Linear Function (HLF) problem is equivalent to classically solving HLF [1].

We then show the following two incomparable results.

Result 1. Any quantum circuit of depth ***D*** with Clifford gates and ***t*** T gates can be simulated in depth

$$O(D+t)$$

where *O* conceals a constant independent of the quantum circuit, in particular, its number of qubits.

Result 2. The BGK quantum circuits can be simulated in complexity class NC^2, i.e. by classical circuits of **log squared** depth and polynomial size (in number of qubits).

Discussion

Result 1 follows from the construction to the right. While Result 1 is inspired by Gottesman-Knill (GK) and an extension by Bravyi and Gosset [2], it is not directly implied by them. For example, GK does not directly imply Result 1 with $t=0$ because a usual GK simulator would "measure" n bits sequentially using depth $O(n)$. In fact, one way to interpret Result 1 is that it parallelises classical simulators in the context of possibilistic simulation.

Result 2 follows because HLF can be solved by finding a matrix kernel and then solving a linear equation [1]. But these tasks are in complexity class NC^2, e.g. [3].

Correction. The current (to-be-updated) arXiv paper costs the depth of a "switchboard circuit" in its "Construction B" as $O(\log t)$. This should be $O(t)$ as pointed out to me by Luke Schaeffer. The main consequence is that "Result 4" in the paper, saying log depth separation is the maximum achievable, is revised to be Result 1 above.

Ongoing work for v3. Extending Result 1 to parallelise other simulation techniques, e.g. tensor network based.

Possibilistic simulation definition

Definition 1. *We make the following definitions for circuits with n variable input bits and m output bits.*

- *A relation on Cartesian product $\{0,1\}^n \times \{0,1\}^m$ is a subset $\mathcal{R} \subset \{0,1\}^n \times \{0,1\}^m$.*
- *A quantum circuit Q on n input qubit lines and measured on m qubit lines in the computational basis defines a relation $\mathcal{R}(Q) \subset \{0,1\}^n \times \{0,1\}^m$ by:*

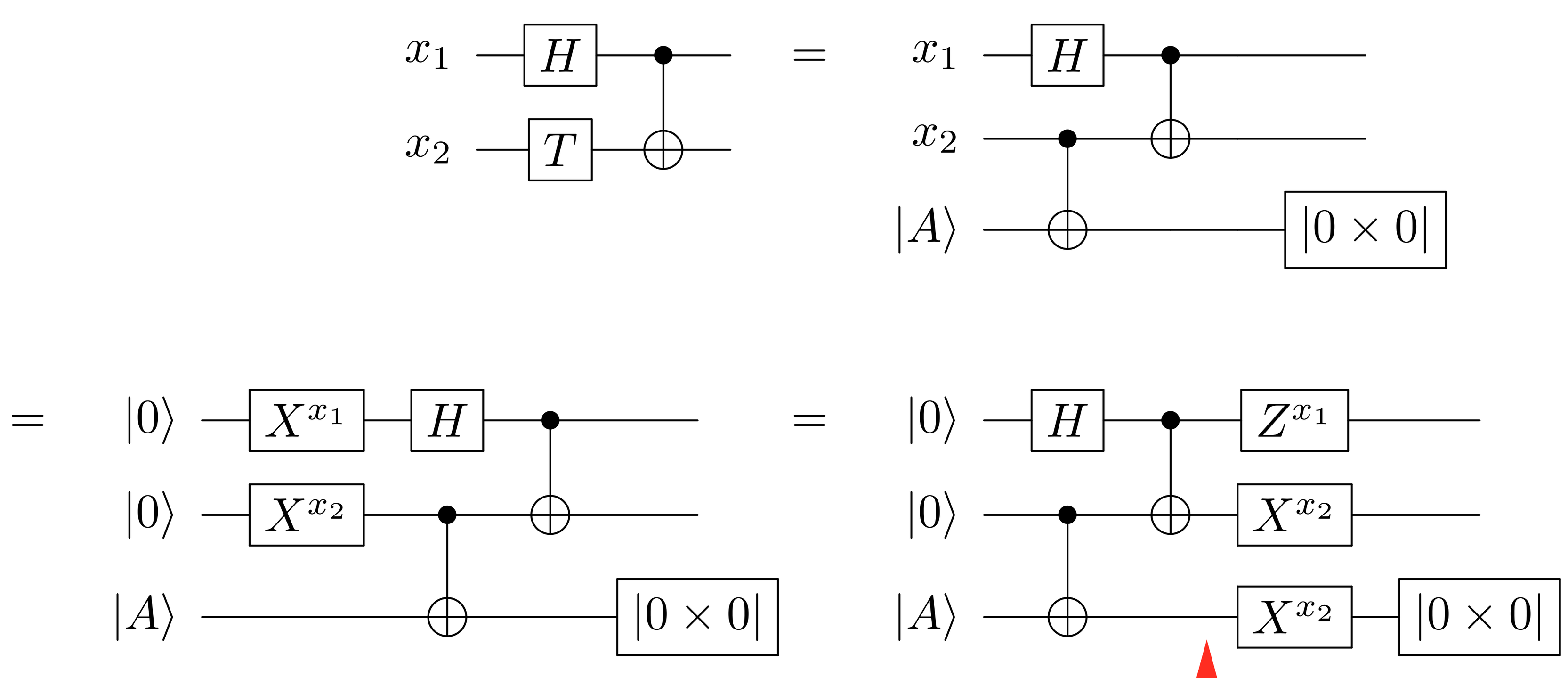
$$(x,y) \in \mathcal{R}(Q) \iff \langle y|\, Q\, |x\rangle \neq 0. \tag{1}$$

- *Let $C : \{0,1\}^n \to \{0,1\}^m$ be a classical circuit, and $\mathcal{R}$ a relation on $\{0,1\}^n \times \{0,1\}^m$. We say C possibilistically simulates $\mathcal{R}$ if:*

$$(x, C(x)) \in \mathcal{R}, \text{ for all } x \in \{0,1\}^n. \tag{2}$$

Construction for Result 1

Idea: use circuit identities like those in the example below together with free pre-computation to construct classical Clifford+*T* simulator.



with $|A\rangle = \frac{1}{\sqrt{2}}(|0\rangle + \omega\,|1\rangle)$ where $\omega = e^{i\pi/4}$

Now we can *precompute* the 3-qubit state at the red arrow:

$$|\psi\rangle = \frac{1}{2}(|000\rangle + |110\rangle + w\,|001\rangle + w\,|111\rangle)$$

which is independent of the input bits x_i.

To simulate the circuit, we may first use x_2 to determine which of the two 2-qubit "sectors" of ψ to project to. Suppose $x_2 = 0$, then we should project to the sector

$$|\psi_0\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

Let s be a bit-string in the *support* of ψ_0, say *00*, which can be precomputed. We can then apply the "classical version" of $Z_1^{x_1} X_2^{x_2}$ to s to classically output a possible output of the quantum circuit. This completes our construction.

The t in Result 1 comes from a "switchboard circuit" that carries out projections onto 2^t sectors in general. The D is because at most 2^D bits x_i can appear in the exponent of each Pauli operator on the right-hand-side of the last circuit above. The 2^t (resp. 2^D) becomes linear in t (resp. D) using binary trees of various kinds.

References
[1] S. Bravyi, D. Gosset, and R. König, Science **362**, 208 (2018).
[2] S. Bravyi and D. Gosset, Phys. Rev. Lett. **116**, 250501 (2016).
[3] K. Mulmuley, Combinatorica **7**, 101 (1987).

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