Logistic Regression:

Conditional Likelihood

$$\mathcal{L}(\underline{\beta}) = \prod_{n=1}^{N} \underbrace{\left[f_1(\underline{x}_n,\underline{\beta})\right]}_{\substack{f_1(\underline{x}_n,\underline{\beta}) \\ = f(H_1|\underline{x}_n)}}^{y_n} \underbrace{\left[1-f_1(\underline{x}_n,\underline{\beta})\right]}_{\substack{1-f_1(\underline{x}_n,\underline{\beta}) \\ = 1-f(H_1|\underline{x}_n) = f(H_0|\underline{x}_n)}}^{1-y_n}$$

Conditional Log-Likelihood

$$\ell(\underline{\beta}) = \ln\mathcal{L}(\underline{\beta}) = \sum_{n=1}^{N}\left\{ y_n \ln f_1(\underline{x}_n,\underline{\beta}) + (1-y_n)\ln\left[1-f_1(\underline{x}_n,\underline{\beta})\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n \ln f_1(\underline{x}_n,\underline{\beta}) + \ln\left[1-f_1(\underline{x}_n,\underline{\beta})\right] - y_n\ln\left[1-f_1(\underline{x}_n,\underline{\beta})\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n\left(\ln f_1(\underline{x}_n,\underline{\beta}) - \ln\left[1-f_1(\underline{x}_n,\underline{\beta})\right]\right) + \ln\left[1-f_1(\underline{x}_n,\underline{\beta})\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n \underbrace{\ln\left[\frac{f_1(\underline{x}_n,\underline{\beta})}{1-f_1(\underline{x}_n,\underline{\beta})}\right]}_{\text{Recall: } \ln\left[\frac{f_1(\underline{x}_n,\underline{\beta})}{1-f_1(\underline{x}_n,\underline{\beta})}\right] = \underline{\beta}^T\underline{x}_n} + \ln\left[1-\underbrace{f_1(\underline{x}_n,\underline{\beta})}_{}\right]\right\}$$

Recall: $f_1(\underline{x}_n,\underline{\beta}) = \dfrac{e^{\underline{\beta}^T\underline{x}_n}}{1+e^{\underline{\beta}^T\underline{x}_n}}$

$$= \sum_{n=1}^{N}\left\{ y_n(\underline{\beta}^T\underline{x}_n) + \ln\left[1-\frac{e^{\underline{\beta}^T\underline{x}_n}}{1+e^{\underline{\beta}^T\underline{x}_n}}\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n(\underline{\beta}^T\underline{x}_n) + \ln\left[\frac{(1+e^{\underline{\beta}^T\underline{x}_n}) - (e^{\underline{\beta}^T\underline{x}_n})}{1+e^{\underline{\beta}^T\underline{x}_n}}\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n(\underline{\beta}^T\underline{x}_n) + \ln\left[\frac{1}{1+e^{\underline{\beta}^T\underline{x}_n}}\right]\right\}$$

$$= \sum_{n=1}^{N}\left\{ y_n(\underline{\beta}^T\underline{x}_n) - \ln\left[1+e^{\underline{\beta}^T\underline{x}_n}\right]\right\}$$