TP3: Lenguaje Imperativo Simple Monadico

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EJERCICIO 1 a)

Vamos a demostrar que State es una mónada probando las tres leyes de mónadas para la instancia dada.

Monad.1: $return\ a >>= k = k\ a$

$$return\ x >>= f$$

$$= < \text{return.1} >$$

$$State\ (\lambda s \to (x :!: s)) >>= f$$

$$= < (>>=).1 >$$

$$\begin{array}{l} State\ (\lambda s'' \to let (v :!: s') = runState\ (State\ (\lambda s \to\ (x :!: s)))\ s'' \\ \qquad in\ runState\ (f\ v)\ s') \end{array}$$

$$= < \text{Lema 1: } (runState.State) = (State.runState) = id, \text{ id.1, Def. } (.) >$$

$$\begin{array}{l} State\ (\lambda s'' \to let (v :!: s') = (\lambda s \to\ (x :!: s))\ s'' \\ \qquad in\ runState\ (f\ v)\ s') \end{array}$$

$$= < \text{Aplicación} >$$

$$\begin{array}{l} State\ (\lambda s'' \to let\ (v :!: s') = (x :!: s'') \\ \qquad in\ runState\ (f\ v)\ s') \end{array}$$

$$= < \text{Def. let} >$$

$$State\ (\lambda s'' \to runState\ (f\ x)\ s'')$$

$$= < \eta\text{-reducción: } (\lambda x \to\ f\ x) = f >$$

$$State\ (runState\ (f\ x))$$

$$= < \text{Lema 1, id.1, Def. } (.) >$$

$$f\ x$$

Monad.2: $m >>= return = m$

$$(State\ h) >>= return$$

$$= <\ (>>=).1\ >$$

$$\begin{aligned}&State(\lambda s \to let(v\ \ :!:\ \ s') = runState(State\ h)s\\&\quad in\ runState(return\ v)s')\end{aligned}$$

$$= <\text{ Lema 1: runState.State = State.runState = id }>$$

$$\begin{aligned}&State(\lambda s \to let\ (v\ \ :!:\ \ s') = h\ s\\&\quad in\ runState\ (return\ v)\ s')\end{aligned}$$

$$= <\text{ def return }>$$

$$\begin{aligned}&State(\lambda s \to let\ (v\ \ :!:\ \ s') = h\ s\\&\quad in\ runState\ (State(\lambda s'' \to\ (v\ \ :!:\ \ s'')))\ s')\end{aligned}$$

$$= <\text{ Lema 1 }>$$

$$\begin{aligned}&State(\lambda s \to let\ (v\ \ :!:\ \ s') = h\ s\\&\quad in\ (\lambda s'' \to\ (v\ \ :!:\ \ s''))s')\end{aligned}$$

$$= <\text{ App }>$$

$$\begin{aligned}&State(\lambda s \to let\ (v\ \ :!:\ \ s') = h\ s\\&\quad in\ (v\ \ :!:\ \ s'))\end{aligned}$$

$$= <\text{ def let }>$$

$$State(\lambda s \to\ h\ s)$$

$$= <\ \eta\text{-reducción }>$$

(State h)

Monad.3: $m >>= (\lambda x \;\rightarrow\; k\ x >>= h) = (m >>= k) >>= h$