Numerical Methods for Partial Differential Equations
A.Y. 2023/2024

Project topics and general instructions

The present document contains the instructions for delivering the project results and a list of possible project topics. Students who want to propose alternative topics are welcome, but these topics should be discussed with the course teachers.

General instructions

1. Students should work in groups of 2 or 3 people. Groups of 3 are strongly recommended due to the complexity of the projects. Students are free to form the groups as they choose.
2. Students should communicate via e-mail to michele.bucelli@polimi.it their choice of group and project topic, as soon as possible.
3. All the projects require the implementation of finite element solvers for problems that were not discussed during the course.
4. Project descriptions are deliberately lacking in detail. Students are encouraged to look for additional information in the scientific literature (including, but not limited to, the works cited in the project descriptions).
5. Students are encouraged to take parallel computing aspects into account when working on the project. Upon request, the teachers will grant access to the HPC resources of the mathematics department for parallel scalability tests.
6. The source code of a project should be uploaded to a GitHub repository obtained by forking the repository at the following URL: https://github.com/michelebucelli/nmpde-projects. Please read carefully the `README.md` file of that repository.
7. The delivery of the code should be done by creating a pull request into the original repository. The pull request page will be used by the course teachers to review the code and suggest changes or improvements.
8. The project report and presentations should demonstrate a clear understanding of the mathematical problem, of its weak and discrete formulation, and of the algorithmic and computational aspects of the solver.
9. The report should be delivered via e-mail to michele.bucelli@polimi.it. Students are not required to deliver the slides prior to the day of the presentation.

Project 1. Preconditioning heterogeneous diffusion problems

Consider the following 3D Poisson problem (1), in which the diffusion coefficient varies significantly (of orders of magnitude) over the domain:

$$\begin{cases} -\boldsymbol{\nabla} \cdot (\mu \boldsymbol{\nabla} u) = f & \text{in } \Omega \ , \\ \mu \boldsymbol{\nabla} u \mathbf{n} = 0 & \text{on } \partial\Omega \ , \end{cases} \tag{1}$$

$$\mu(\mathbf{x}) = \begin{cases} 10^p & \text{if } \mathbf{x} \in B \ , \\ 1 & \text{otherwise} \ , \end{cases} \tag{2}$$

where $p > 0$ and $B = \bigcup_{i=1}^{N} B_i$, with B_i a sphere of center $\mathbf{x}_i$ and radius r_i.

Implement a solver for the problem, and discuss the effectiveness of different preconditioning strategies depending on the heterogeneity. Consider the preconditioners that are offered by `deal.II` within the `TrilinosWrappers` namespace.

Discuss the computational efficiency, optimality and parallel scalability of the different preconditioners under consideration.

Difficulty: easy.