CH No. 5 Pg:- 127

⇒ Integration:-

⇒ Area under the curve.

Newton cote's formula's:-

ⓐ Trapezodial rule:- Pg:- 133

$$I_T = \frac{h}{2}\left[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i\right]$$

Ex: 1:-

$$\int_0^{0.6} f(x)$$ (equal interval)

fp

$$I_T = \frac{0.1}{2}\Big[(0.0000 + 0.5646) + 2\,(0.00998 + 0.1987 + 0.2955$$

$$0.3894 + 0.4794)\Big]$$

$$= \frac{0.1}{2} \times 3.4902$$

$$\boxed{I_T = 0.1745}$$

ⓑ Simpson's 1/3 rd:-

$$I_{SR} = \frac{h}{3}\left[f_0 + 4(f_1 + f_3 + f_5) + 2(f_2 + f_4) + f_6\right]$$

ⓒ Simspon's 3/8 th:-

$$I_{SR} = \frac{3h}{8}\left[f_0 + 3(f_1 + f_2) + 2f_3 + 3(f_4 + f_5) + f_6\right]$$

(d) Boole's Rule:-

$$I_B = \frac{2h}{45}\left[7(f_0+f_6) + 32(f_1+f_3+f_5) + 12(f_2+f_4)\right]$$

(e) Weddle's Rule:-

$$I_W = \frac{3h}{10}\left[f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + f_6\right]$$

Ex 1:-

Simpson's 1/3rd:-

$$I_S = \frac{h}{3}\left[f_0 + 4(f_1+f_3+f_5) + 2(f_2+f_4) + f_6\right]$$

$$= \frac{(0.1)}{3}\Big[(0.0000) + 4\big((0.0998) + (0.2955) + (0.4794)\big) + 2\big((0.1987) + (0.3894)\big) + (0.5646)\Big]$$

$$= \frac{(0.1)}{3} \times 5.2396$$

$$= 0.1747$$

Romberg Integration:-

$$\int_1^{2.6} \frac{dx}{x}, \quad \text{Let } n = 8$$

$a = 1, \; b = 2.6$

$$h = \frac{b-a}{n} = \frac{2.6-1}{8} = 0.2$$

$$I_{11} = \frac{1}{2}(f_0 + f_8)$$

$$= \frac{1}{2}(1.00 + 0.385) = \boxed{0.6925}$$

$T_{11} = h' \, I_{11}$

$T_{11} = 1.6 \times 0.6925$

$$\boxed{T_{11} = 1.2080}$$

fp

$$I_{12} = \left[I_{11} + f\left(a + \frac{h'}{2}\right)\right]$$

$$= \left[I_{11} + f_4\right]$$

$$= \left[0.6925 + 0.556\right] = \boxed{1.2485}$$

$$T_{12} = \frac{h'}{2} I_{12}$$

$$= \frac{1.6}{2}(1.2485)$$

$$\boxed{T_{12} = 0.9988}$$

Example 5

$I_{11}, I_{12}, I_{13}, I_{14} = ?$

$T_{11}, T_{12}, T_{13}, T_{14} = ?$

$$h' = \frac{(a-b)}{1}$$

$= 2.6 - 1 = 1.6$

$f\left(a + \frac{h'}{2}\right) = 1.8 = f_4$

$$I_{23} = \left[I_{22} + f\left(a + \frac{h'}{4}\right) + f\left(a + \frac{3h'}{4}\right) \right]$$

$$= \left[I_{22} + f_2 + f_6 \right]$$

$$= \left((1.2485) + (0.714) + (0.455) \right)$$

$$\boxed{I_{13} = 2.4175}$$

$$T_{13} = \frac{h'}{4} I_{13}$$

$$= \frac{1.6}{4} \times (2.4175)$$

$$\boxed{T_{13} = 0.9670}$$

$$I_{24} = \left(I_{23} + f\left(a + \frac{h'}{8}\right) + f\left(a + \frac{3h'}{8}\right) + f\left(a + \frac{5h'}{8}\right) + f\left(a + \frac{7h'}{8}\right) \right]$$

$$= \left[I_{23} + f_2 + f_3 + f_5 + f_7 \right]$$

$$= \left[(2.4175) + (0.833) + (0.625) + (0.500) + (0.417) \right]$$

$$\boxed{I_{14} = 4.7925}$$

$$T_{14} = \frac{h'}{8} I_{14}$$

$$= \frac{1.6}{8} \times (4.7925)$$

$$\boxed{T_{14} = 0.9585}$$

Pg No. 169
Ch No. 6:-

Picard's Method:-

$$y^{(0)} = y_0 + \int_{x_0}^{x} f(x, y) \, dx$$

Fp

$$y^{(1)} = y_0 + \int_{x_0}^{x_0+h} f(x, y^{(0)}) \, dx$$

$$y^{(2)} = y_0 + \int_{x_0}^{x_1+h} f(x, y^{(1)}) \, dx$$

$$y^{(3)} = y_0 + \int_{x_0}^{x_2+h} f(x, y^{(2)}) \, dx$$

Pg No. 170 Example No. 1

$y' = x + y^2$

Sometimes ↓

$\Rightarrow y = y_0$ at $x_0 = 0 \rightarrow y_0 = 0 \rightarrow y_0 = 0 = x_0$

Sol:- $y^{(0)} = y_0 = 0$

First App:-

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y^{(0)}) \, dx$$

$$= 0 + \int_{x_0}^{x} (x+y)^2 \, dx$$

$$= 0 + \int_{x_0}^{x} (x + (y^{(0)})^2) \, dx$$

$$= 0 + \int_{0}^{x} x \, dx$$

$$= \frac{x^2}{2}$$

Second App:-

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)}) \, dx$$

$$= 0 + \int_{0}^{x} f(x, y^{(1)}) \, dx$$

$$= \int_{0}^{x} (x + (y^{(1)})^2) \, dx$$

$$= \int_{0}^{x} x + \left(\frac{x^2}{2}\right)^2 dx$$

$$= \int_{0}^{x} x + \frac{x^4}{4} \, dx$$

$$= \frac{x^2}{2} + \frac{x^5}{20}$$

$$\int \frac{x^4}{4} dx$$

$$= \frac{1}{4} \int x^4 \, dx$$

$$\frac{1}{4} \cdot \frac{x^{4+1}}{4+1} dx$$

$$\frac{x^5}{20}$$

Third App:-

$$y^{(3)} = y_0 \int_{x_0}^{x} f(x, y^{(2)}) \, dx$$

$$= y_0 \int_{0}^{x} \left(x + (y^{(2)})^2\right) dx$$

$$= y_0 + \int_{0}^{x} x + \left(\frac{x^2}{2} + \frac{x^5}{20}\right)^2 dx$$

$$= 0 + \int_0^x x + \frac{x^4}{4} + \frac{x^7}{20} + \frac{x^{10}}{400} \, dx$$

$$= \frac{x^2}{2} + \frac{x^5}{20} + \frac{x^6}{160} + \frac{x^{11}}{4400}$$

- 20 160 4400

Taylor's Series

$$y = y_0 + (x - x_0) y_0 + \frac{(x-x_0)^2}{2!} y_0'' + \frac{(x-x_0)^3}{3!} y_0''' + \frac{(x-x_0)^4}{4!} y_0^4$$

$$y = y_0 + hf(x_0, y_0) + \frac{h^2}{2!} f_1(x_0, y_0) + \frac{h^3}{3!} f_2(x_0, y_0) + \cdots$$

$$\boxed{h = x - x_0}$$

Example 2:-

$y' = 0.1(x^3 + y^2)$, $y(0) = 1$ $\quad x_0 = 0$, $y_0 = 1$

$y_0' = 0.1(x_0 + y_0)$

$= (0.1)(0 + 1) = 0.1$ $\quad \boxed{y_0' = 0.1}$

By differentiating:-

$y'' = 0.1(3x^2 + 2y\,y')$

$y_0'' = f(x_0, y_0)$

$= 0.1(3x_0^2 + 2y_0 y_0')$

$= 0.1(3(0) + 2(1)(0.1))$

$\boxed{y_0'' = 0.02}$

$y''' = 0.1(6x + 2yy'' + 2(y')^2)$

$y_0''' = 0.1(6x_0 + 2y_0 y_0'' + 2(y_0')^2)$

$y_0''' = (0.1)(6(0) + 2(1)(0.02) + 2(0.1)^2)$

$\boxed{y_0''' = 0.006}$

Euler's Method

$y_{n+1} = y_n + hf\left[x_n + \frac{h}{2},\ y_n + \frac{h}{2} f(x_n, y_n)\right]$ Pg No. 175

Example No. 3:-

$y' = x+y$

$y(0) = 1 \qquad y_0 = 1,\ x_0 = 0$

$h = 0.1$

$[0,\ 0.5] \rightarrow$ interval

	x_n	y_n	$K_1 = x_n + y_n$	$y_{n+1} = y_n + hK_1$
$=x_0$	0	$1.0 = y_0$	$0 + 1.0 = 1.0$	1.1
$=x_1$	0.1	1.1	$0.1 + 1.1 = 1.2$	1.22
$=x_2$	0.2	1.22	$0.2 + 1.22 = 1.42$	1.362
$=x_3$	0.3	1.362	$0.3 + 1.362 = 1.662$	1.5282
$=x_4$	0.4	1.5282	$0.4 + 1.5282 = 1.9282$	1.72102
$=x_5$	0.5	1.72102	2.22102	1.943221

$y_{n+1} = y_n + hK_1$

$K_1 = f(x_n, y_n)$

$y_n = y_{n-1} + hK_1$

$y_{n+1} = y_n + hK_1$

$y_{0+1} = y_0 + hK_1(0)$

$y_1 = 1 + (0.1)(1.0)$

$y_1 = 1.1$

$K_1 = x + y$

$= 0 + 0.1 = 0.1$

$y_n = y_{n-1} + hK_1$

$y_1 = y_{1-1} + hK_1(0)$

$y_1 = y_0 + hK_1$

$= 1 + (0.1)(1)$

$y_1 = 1.1$

for 1:-

$y_n = y_{n-1} + hK_1$

$y_n = 1.0 + (0.1)(1.0)$

$\boxed{y_n = 1.1}$

$y_{n+1} = y_n + hK_1$

$y_{1+1} = y_1 + (0.1)(1.2)$

$y_2 = 1.1 + (0.1)(1.2)$

$\boxed{y_2 = 1.22}$

for 3:-

$y_n = y_{n-1} + hK_1$

$y_n = 1.22 + (0.1)(1.42)$

2:-

$y_n = y_{n-1} + hK_1$

$y_n = 1.1 + (0.1)(1.2)$

$= 1.22$

$y_{n+1} = y_n + hK_1$

$y_{2+1} = y_2 + (0.1)(1.42)$

$y_3 = (1.22) + 0.142$

$\boxed{y_3 = 1.362}$

$y_{n+1} = y_n + hK_1$

$y_{3+1} = y_3 + hK_1$

$y_4 = (1.362) + (0.1)(1.662)$

$\boxed{y_4 = 1.5282}$

For 4:-

$y_n = y_{n-1} + hK_1$

$y_n = 1.362 + (0.1)(1.662)$

$\boxed{y_n = 1.5282}$

$y_{n+1} = y_n + hK_1$

$y_{4+1} = y_4 + (0.1)(1.9282)$

$y_5 = (1.5282) + (0.1)(1.9282)$

$\boxed{y_5 = 1.72102}$

For 5:-

$y_n = y_{n-1} + hK_1$

$y_n = (1.5282) + (0.1)(1.9282)$

$\boxed{y_n = 2.22102}$

$y_{n+1} = y_n + hK_1$

$y_{5+1} = y_5 + (0.1)(2.221$

$y_6 = 1.72102 + 0.22102$

$\boxed{y_6 = 1.943122}$

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Runge - Kutta :-

Exa

Example 4:-

$$y' = \frac{y-x}{y+x} \qquad x_0 = 0, \; y_0 = 1$$

$$y(0) = 1$$

$h = 0.1$, $\quad 0 \leq x \leq 0.5$

n	x_n	y_n	K_1	K_2	K_3	K_4	$y_{n+1} = y_n + h$
0	0.0	1.0	0.1	0.0909	0.0909	0.0832	1.0911
1	0.1	1.0911					
2	0.2						
3	0.3						
4	0.4						
5	0.5						

$$K = \frac{1}{6}\left(K_1 + 2(K_2 + K_3) + K_4\right)$$

$$K_1 = hf\,(x_n, y_n)$$

$$K_2 = hf\left[x_n + \frac{h}{2}, \; y_n + \frac{K_1}{2}\right]$$

$$K_3 = hf\left[x_n + \frac{h}{2}, \; y_n + \frac{K_2}{2}\right]$$

$$K_4 = hf\left[x_n + h, \; y_n + K_3\right]$$

$$K_1 = hf(x_0, y_0) = h\left[\frac{y_0 - x_0}{x_0 + y_0}\right]$$

$$= (0.1)\left[\frac{1-0}{\cancel{0+1}}\right] = 0.1$$

$$\boxed{K_1 = 0.1}$$

$$K_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right]$$

$$= hf\left(0 + \frac{(0.1)}{2}, 1 + \frac{(0.1)}{2}\right)$$

$$= hf(0.05, 1.05) \rightarrow hf(x_0, y_0)$$

$$= hf\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$$

$$= (0.1)\left(\frac{1.05 - 0.05}{1.05 + 0.05}\right) = \frac{0.1 \times 1}{1.1} = 0.0909$$

$$\boxed{K_2 = 0.0909}$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= hf\left(0 + \frac{(0.1)}{2}, 1 + \frac{0.0909}{2}\right)$$

$$= hf(0.05, 1.0455) \rightarrow hf(x_0, y_0)$$

$$= hf\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$$

$$= (0.1)\left(\frac{1.0455 - 0.05}{1.0455 + 0.05}\right)$$

$$\boxed{K_3 = 0.0909}$$

$$\frac{2 + 0.0909}{2}$$

$$\frac{2.0909}{2}$$

$$1.0455$$

Exa

$K_4 = hf(x_0 + h, y_0 + K_3)$
$= hf(0 + 0.1, 1 + 0.0909)$
$= hf(0.1, 1.0909)$

$= hf\left(\frac{y_0 - x_0}{y_0 + x_0}\right)$
$= (0.1)\left(\frac{1.0909 - 0.1}{1.0909 + 0.1}\right)$

$\boxed{K_4 = 0.0832}$

$K = \frac{1}{6}\left[K_1 + 2(K_2 + K_3) + K_4\right]$

$K = \frac{1}{6}\left[0.1 + 2(0.0909 + 0.0909) + (0.0832)\right]$

$\boxed{K = 0.0911}$

$y_1 = y_0 + K$
$= 1.0 + 0.0911 = 1.0911$
$\boxed{y_1 = 1.0911}$

$y_2 = y_1 + k$

$y_4 = y_3 + k$

Pg No. 184

⇒ Predictor and Corrector:- (6.8)

Milne Simpson:-

Pg No. 188

Example 5:-

$y' = x - 1$ at $(0, 1)$ $x_0 = 0$, $y_0 = 1$, $h = 0.1$

x	$0.0 = x_0$	0.1	0.2	0.3	0.4
y	$1.0000 = y_0$	0.9097	0.8375	0.7816	$\boxed{0.7407}$

find $y(0.4)$ $y(0.4) = ?$

$f = x - y$

$-1.0000 = f_0$ (Predictor) ↓

$-0.8097 = f_1$

$-0.6375 = f_2$

$-0.4816 = f_3$

$-0.3407 = f_4$

(Milne Predictor)

$$y(0.4) = y_0 + \frac{4h}{3}\left[2f_1 - f_2 + 2f_3\right]$$

$$\boxed{y(0.4) = 0.7407}$$

(Simpsons rule)

$$y(0.4) = y_2 + \frac{h}{3}\left(f_2 + 4f_3 + f_4\right)$$

$$\boxed{y(0.4) = 0.7407}$$

Corrector Method:-

$$y(0.4) = 2e^{-x} + x - 1$$

$$= 2e^{-0.4} + 0.4 - 1$$

$$\boxed{y(0.4) = 0.7406}$$