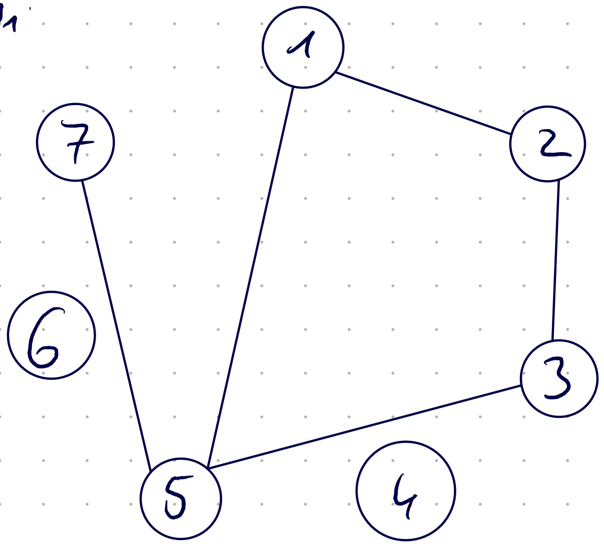
Task 3

We have the following two graphs:

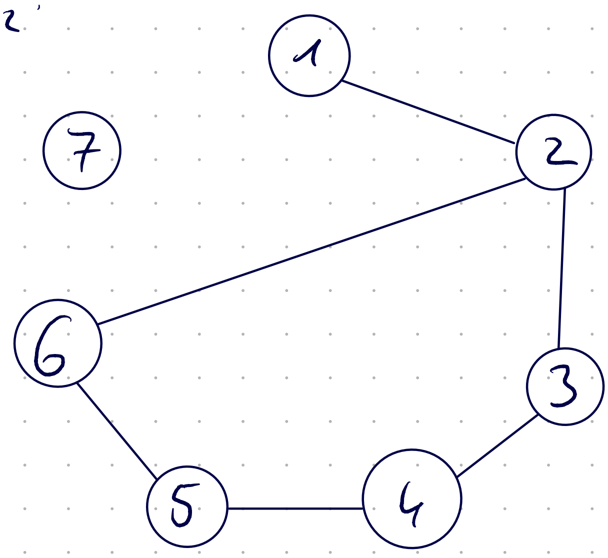
$$G_1 = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \text{ and } G_2 = \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}$$

Using column numbers as labels

G_1:

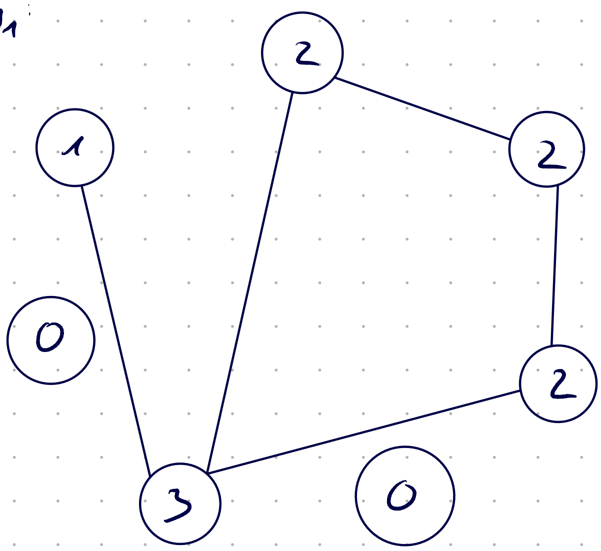


G_2:

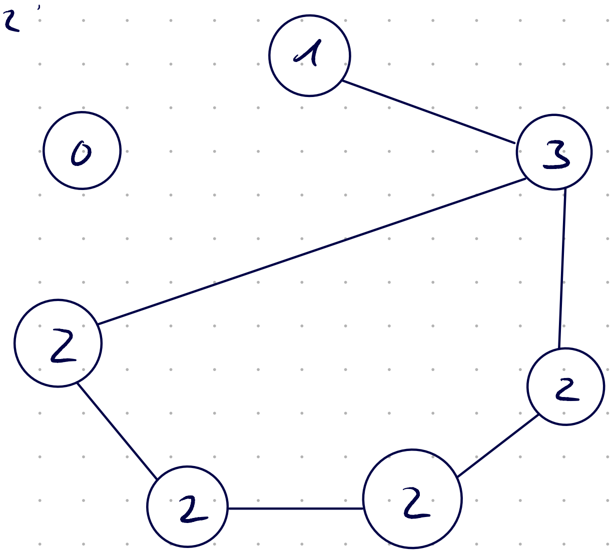


a) **Draw with degrees as labels:**

G_1:

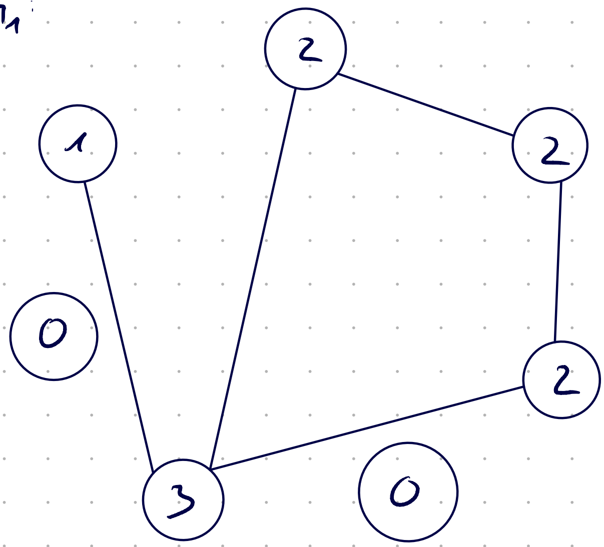


G_2:

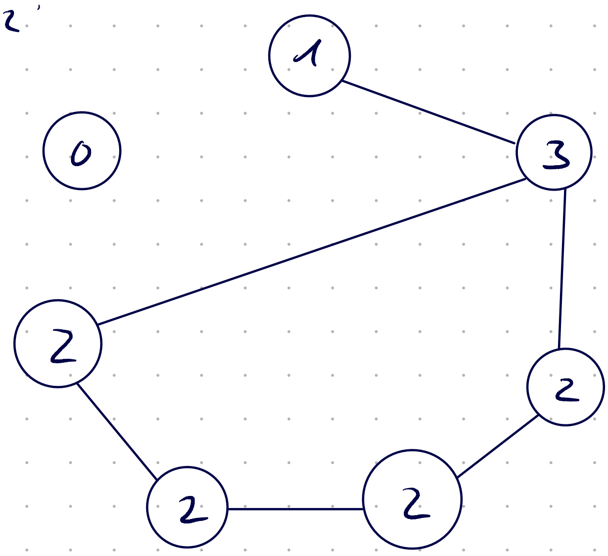


b) Weisfeiler - Lehmann

G_1:

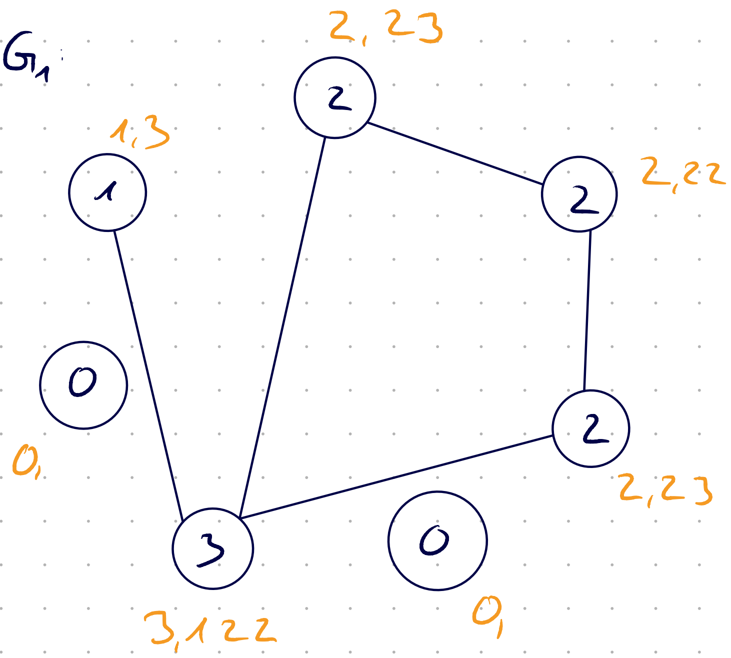


G_2:

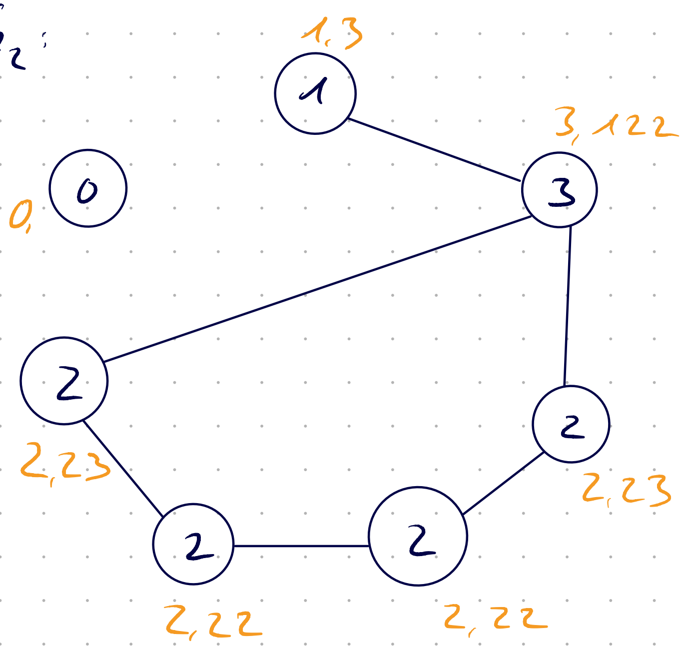


first iteration:

G_1:



G_2:

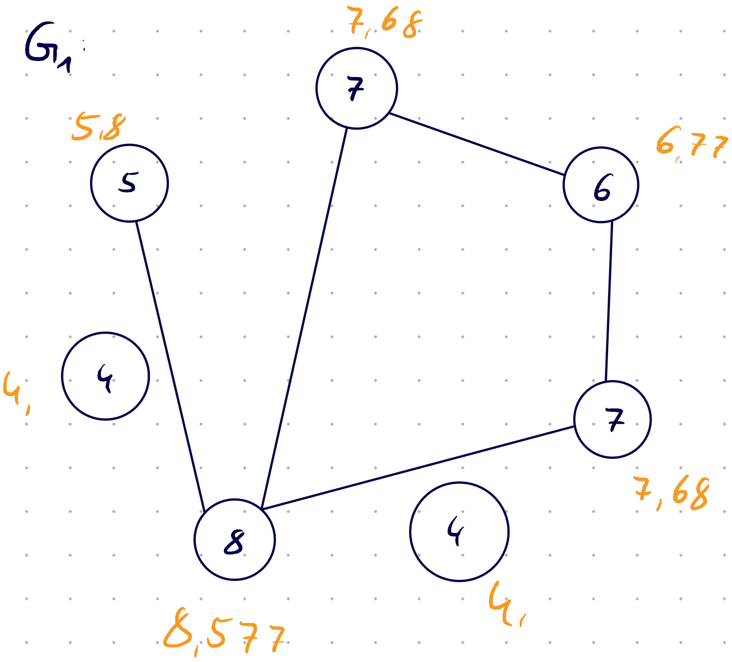


rename labels:

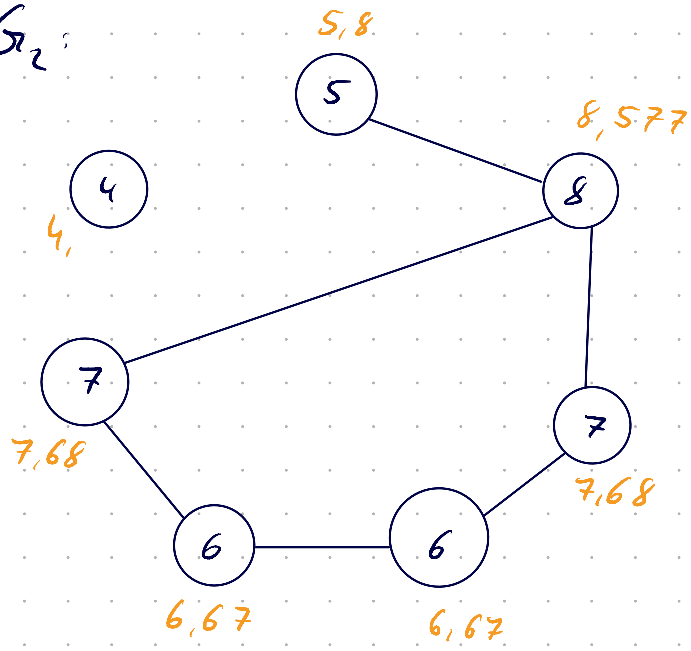
old	0,	1, 3	2, 22	2, 23	3, 122
new	4	5	6	7	8

second iteration

G_1:



G_2:

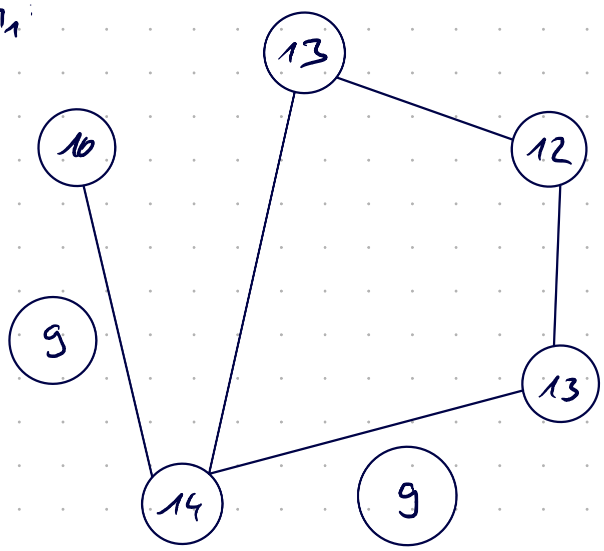


rename labels:

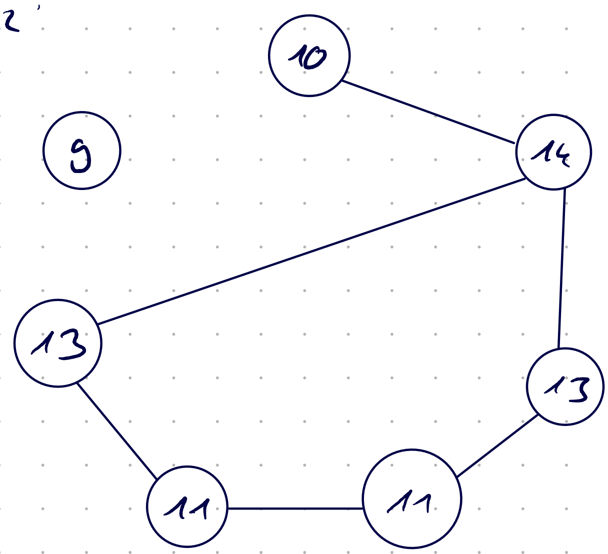
old	4,	5,8	6,67	6,77	7,68	8,577
new	9	10	11	12	13	14

Redraw graph

G_1:



G_2:



Now we can stop since

$$L_1 := \{ \ell_2(v) \mid v \in V_1 \} \neq \{ \ell_2(v) \mid v \in V_2 \} =: L_2$$

where V_1 is the set of edges of G_1 and V_2 the set of edges of G_2.

This is because

$12 \in L_1$ $\qquad$ $12 \in L_2$

$11 \notin L_1$ $\qquad$ $11 \in L_2$

Therefore the algorithm terminates and G_1 and G_2 are <u>not</u> isomorpic.

c)

These graphs are obviously not isomorphic. However the algorithm will not detect that. This is because there are only two types of nodes: Those with degree 0 and those with 1.
And those with degree 0 are (obviously) unconnected, and those with 1 have one connection to another node with

degree 1. So the 0 - nodes will all be renamed the same and the 1 - nodes will all be renamed the same. Since both graphs have both types of nodes, the labelsets will always be the same.
So the algorithm will stay in an infinite loop despite the fact, that the graphs are not isomorphic.