Problem 3

December 16, 2021

1 Problem 3

[1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

First we will define the $W^{(i)}$ from the assignment sheet.

[2]:
```
W1 = np.loadtxt("W1.txt")
W2 = np.loadtxt("W2.txt")
k = 2
```

We define W and later also P and S to be 4D tensors, where the first dimension is the lower index, the second is the upper index, and dimensions three and four are the matrix. So e.g. $(W_i^{(j)})_{kl}$ would be `W[i, j, k, l]`.

[3]:
```
W = np.array([[W1, W2]])
print(W)
```

```
[[[[1.  0.5 0.3 0.1 0.1]
   [0.5 1.  0.4 0.1 0.1]
   [0.3 0.4 1.  0.3 0.3]
   [0.1 0.1 0.3 1.  0.5]
   [0.1 0.1 0.3 0.5 1. ]]

  [[1.  0.2 0.5 0.1 0.1]
   [0.2 1.  0.3 0.1 0.1]
   [0.5 0.3 1.  0.3 0.3]
   [0.1 0.1 0.3 1.  0.5]
   [0.1 0.1 0.3 0.5 1. ]]]]
```

[4]:
```
# 4D tesor
def calc_P_from_W(W):
    P = np.zeros_like(W)
    for i, Wstep in enumerate(W):
        for j, Windex in enumerate(Wstep):
            P[i, j] = _calc_P_from_W(Windex) # calculate all matricies
    return P
# 2D matrix
```

```
def _calc_P_from_W(W):
    # np.diag(np.diag(X)) gives a Matrix that has just the diagonal of X and 0␣
↪everywhere else
    W_zero_diag = W - np.diag(np.diag(W))
    denominator = 2 * W_zero_diag.sum(axis=-1, keepdims=True)

    W_zero_diag_normalized = W_zero_diag / denominator
    return W_zero_diag_normalized + np.diag(0.5 * np.ones(W.shape[0]))

# 4D tesor
def calc_S_from_W(W, k=k):
    S = np.zeros_like(W)
    for i, Wstep in enumerate(W):
        for j, Windex in enumerate(Wstep):
            S[i, j] = _calc_S_from_W(Windex, k) # calculate all matricies
    return S
# 2D matrix
def _calc_S_from_W(W, k):
    # get neighbour indexes, sorted by similarity and select the k largest
    neighbours = np.argsort(W)[:, -k:]
    # select corresponding neighbour values
    kNN = np.take_along_axis(W, neighbours, axis=-1)
    # calculate the sum (denominatior)
    kNN_sum = kNN.sum(axis=-1)
    S = np.zeros_like(W)

    for i, row in enumerate(W):
        for j, x in enumerate(row):
            if j in neighbours[i]:
                S[i,j] = x / kNN_sum[i]
    return S
```

1.1 a)

Now we can easily calculate P and S:

[5]:
```
P = calc_P_from_W(W)
S = calc_S_from_W(W)

print("P-matricies: ")
print("P^(1):")
print(P[0, 0])
print()
print("P^(2):")
print(P[0, 1])
print()
print()
```

```
print("S-matricies: ")
print("S^(1):")
print(S[0, 0])
print()
print("S^(2):")
print(S[0, 1])
```

```
P-matricies:
P^(1):
[[0.5        0.25       0.15       0.05       0.05      ]
 [0.22727273 0.5        0.18181818 0.04545455 0.04545455]
 [0.11538462 0.15384615 0.5        0.11538462 0.11538462]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]

P^(2):
[[0.5        0.11111111 0.27777778 0.05555556 0.05555556]
 [0.14285714 0.5        0.21428571 0.07142857 0.07142857]
 [0.17857143 0.10714286 0.5        0.10714286 0.10714286]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]


S-matricies:
S^(1):
[[0.66666667 0.33333333 0.         0.         0.        ]
 [0.33333333 0.66666667 0.         0.         0.        ]
 [0.         0.28571429 0.71428571 0.         0.        ]
 [0.         0.         0.         0.66666667 0.33333333]
 [0.         0.         0.         0.33333333 0.66666667]]

S^(2):
[[0.66666667 0.         0.33333333 0.         0.        ]
 [0.         0.76923077 0.23076923 0.         0.        ]
 [0.33333333 0.         0.66666667 0.         0.        ]
 [0.         0.         0.         0.66666667 0.33333333]
 [0.         0.         0.         0.33333333 0.66666667]]
```

1.2 b)

Now we will define a function, that does one step:

[6]:
```
def do_step(P, S, normalize = False): # This is the version for more than two␣
↪matricies, not sure if that was required
    Pt = P[-1]
    St = S[-1]

    Ptplus1 = []
```

```
    for i, Si in enumerate(St):
        Ptplus1.append(
            Si @ (
                np.concatenate([ Pt[:i], Pt[i+1:] ]).mean(axis=0) # leave out i␣
↪and average
            ) @ Si.T
        )

    # If requested, normalize as described in the paper
    if normalize:
        for i, p in Ptplus1:
            Ptplus1[i] = _calc_P_from_W(p)

    return np.array(Ptplus1)
```

So now we can do the steps and print the matricies:

[7]:
```
P = calc_P_from_W(W)
S = calc_S_from_W(W)
Pcs = [ #initialize P^(c)
    (P[0, 0] + P[0, 1]) / 2
]
for i in range(2):
    P1, P2 = do_step(P, S)
    P = np.concatenate([P, [[P1, P2]]])
    Pcs.append(
        (P1 + P2) / 2
    )
Pcs = np.array(Pcs)
```

[8]:
```
for i, (Ps, Pc) in enumerate(zip(P, Pcs)):
    print("#"*100)
    print()
    print(f"Matricies #{i}:")
    print("-"*100)
    print(f"P_{i}^(1): ")
    print(Ps[0])

    print("-"*100)
    print(f"P_{i}^(2): ")
    print(Ps[1])

    print("-"*100)
    print(f"P_{i}^(c): ")
    print(Pc)
    print("-"*100)
```

```python
    print()
print("#"*100)
```

```
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Matricies #0:
--------------------------------------------------------------------------------
--------------------
P_0^(1):
[[0.5        0.25       0.15       0.05       0.05      ]
 [0.22727273 0.5        0.18181818 0.04545455 0.04545455]
 [0.11538462 0.15384615 0.5        0.11538462 0.11538462]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]
--------------------------------------------------------------------------------
--------------------
P_0^(2):
[[0.5        0.11111111 0.27777778 0.05555556 0.05555556]
 [0.14285714 0.5        0.21428571 0.07142857 0.07142857]
 [0.17857143 0.10714286 0.5        0.10714286 0.10714286]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]
--------------------------------------------------------------------------------
--------------------
P_0^(c):
[[0.5        0.18055556 0.21388889 0.05277778 0.05277778]
 [0.18506494 0.5        0.19805195 0.05844156 0.05844156]
 [0.14697802 0.13049451 0.5        0.11126374 0.11126374]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]
--------------------------------------------------------------------------------
--------------------

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Matricies #1:
--------------------------------------------------------------------------------
--------------------
P_1^(1):
[[0.33421517 0.28747795 0.25207861 0.06084656 0.06084656]
 [0.29805996 0.33421517 0.27399849 0.06613757 0.06613757]
 [0.18537415 0.20238095 0.36151603 0.09693878 0.09693878]
 [0.05       0.05       0.12142857 0.38888889 0.36111111]
 [0.05       0.05       0.12142857 0.36111111 0.38888889]]
```

```
--------------------------------------------------------------------------------
--------------------
P_1^(2):
[[0.33675214 0.22919132 0.3017094  0.07179487 0.07179487]
 [0.21938318 0.38207059 0.23731397 0.06159225 0.06159225]
 [0.29017094 0.23145957 0.33675214 0.09358974 0.09358974]
 [0.08333333 0.07307692 0.11666667 0.38888889 0.36111111]
 [0.08333333 0.07307692 0.11666667 0.36111111 0.38888889]]
--------------------------------------------------------------------------------
--------------------
P_1^(c):
[[0.33548365 0.25833464 0.27689401 0.06632072 0.06632072]
 [0.25872157 0.35814288 0.25565623 0.06386491 0.06386491]
 [0.23777254 0.21692026 0.34913409 0.09526426 0.09526426]
 [0.06666667 0.06153846 0.11904762 0.38888889 0.36111111]
 [0.06666667 0.06153846 0.11904762 0.36111111 0.38888889]]
--------------------------------------------------------------------------------
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Matricies #2:
--------------------------------------------------------------------------------
--------------------
P_2^(1):
[[0.29180313 0.2859771  0.28021763 0.068394   0.068394  ]
 [0.28270772 0.30690928 0.27944532 0.06499313 0.06499313]
 [0.27146101 0.27297615 0.29866982 0.0844476  0.0844476 ]
 [0.07991453 0.07649573 0.10421245 0.37654321 0.37345679]
 [0.07991453 0.07649573 0.10421245 0.37345679 0.37654321]]
--------------------------------------------------------------------------------
--------------------
P_2^(2):
[[0.28592025 0.26590739 0.287239   0.0728773  0.0728773 ]
 [0.27943531 0.30157719 0.28681508 0.07324554 0.07324554]
 [0.26500418 0.25250591 0.29502054 0.08490804 0.08490804]
 [0.07380952 0.06648352 0.09761905 0.37654321 0.37345679]
 [0.07380952 0.06648352 0.09761905 0.37345679 0.37654321]]
--------------------------------------------------------------------------------
--------------------
P_2^(c):
[[0.28886169 0.27594225 0.28372832 0.07063565 0.07063565]
 [0.28107152 0.30424323 0.2831302  0.06911933 0.06911933]
 [0.26823259 0.26274103 0.29684518 0.08467782 0.08467782]
 [0.07686203 0.07148962 0.10091575 0.37654321 0.37345679]
 [0.07686203 0.07148962 0.10091575 0.37345679 0.37654321]]
--------------------------------------------------------------------------------
```

##
####################

1.3 c)

Now all that is left is to implement the stopping criterion and put everything together.

[9]:
```python
def should_stop(Pcs, epsilon=1E-6):
    Pc_t = Pcs[-2]
    Pc_tplus1 = Pcs[-1]

    E = np.linalg.norm(Pc_tplus1 - Pc_t) / np.linalg.norm(Pc_t)
    return E < epsilon, E
```

[10]:
```python
def SNF(W, k, epsilon=1e-6, normalize=False, max_iter=100):
    # calculate P and S

    P = calc_P_from_W(W)
    S = calc_S_from_W(W, k=k)

    # initialize Pc
    Pcs = [
        (P[0, 0] + P[0, 1]) / 2
    ]
    Es = []

    #update
    for i in range(max_iter):
        Pis = do_step(P, S, normalize=normalize)
        P = np.concatenate([P, [Pis]])
        Pcs.append(
            Pis.mean(axis=0)
        )

        # check for convergence
        converged, E = should_stop(Pcs, epsilon=epsilon)
        # save E
        Es.append(E)
        # and stop if converged
        if converged:
            break
    Pcs = np.array(Pcs)
    Es = np.array(Es)

    # return everything, that could be wanted
    return Pcs, Es, P, S
```

[11]:
```
output, errors, _, _ = SNF(W, 2) # I have also tried with normalization as described in the paper, but it is not required in the task and did not help anyways.
```

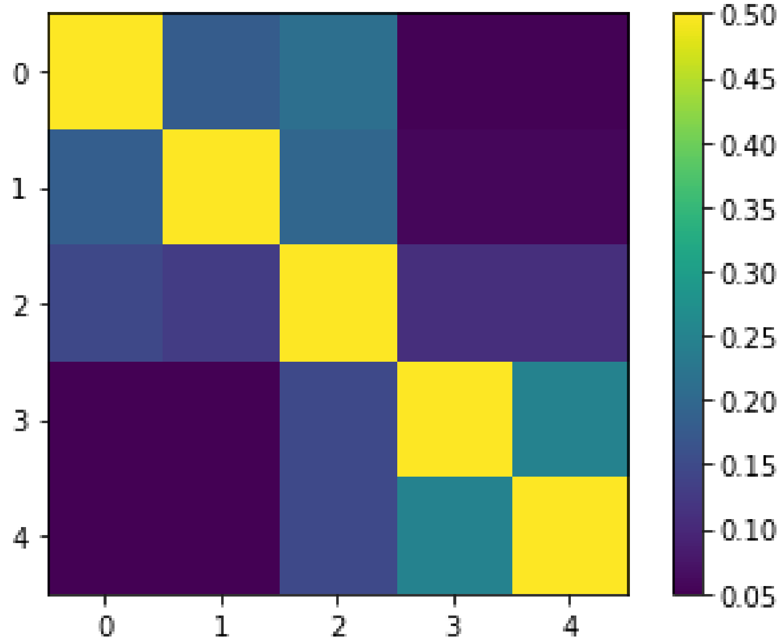
[12]:
```
for i, (pc, e) in enumerate(zip(output, [np.inf] + list(errors))):
    print("#"*100)
    print()
    print(f"\033[1m Iteration {i} \033[0m")
    print()
    print("-"*100)
    print(f"P_{i}^(c): ")
    print(pc)
    print("-"*100)
    plt.imshow(pc)
    plt.colorbar()
    plt.show()
    print("-"*100)
    print(f"Error: E = {e}")
    print("-"*100)

    print()
print("#"*100)
```

```
################################################################################
####################

 Iteration 0

--------------------------------------------------------------------------------
--------------------
P_0^(c):
[[0.5        0.18055556 0.21388889 0.05277778 0.05277778]
 [0.18506494 0.5        0.19805195 0.05844156 0.05844156]
 [0.14697802 0.13049451 0.5        0.11126374 0.11126374]
 [0.05       0.05       0.15       0.5        0.25      ]
 [0.05       0.05       0.15       0.25       0.5       ]]
--------------------------------------------------------------------------------
--------------------
```

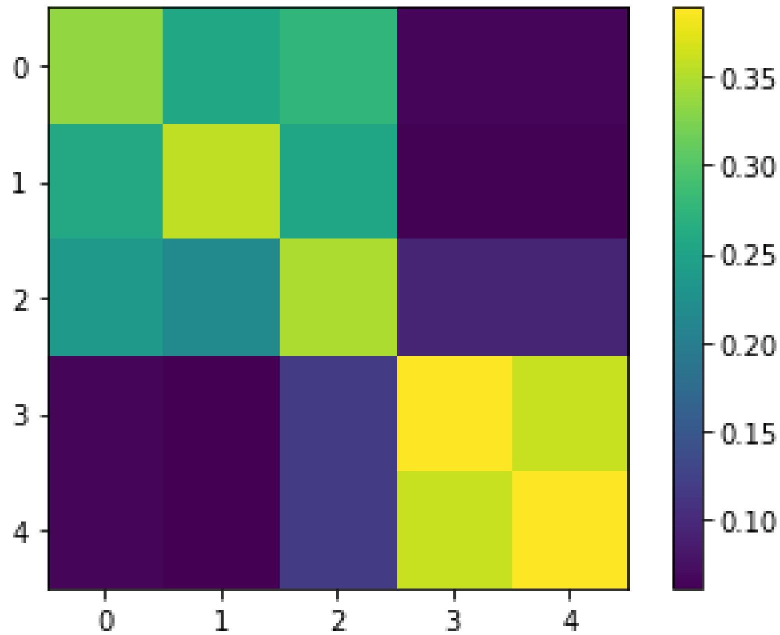



```
--------------------------------------------------------------------------------
--------------------
Error: E = inf
--------------------------------------------------------------------------------
--------------------

################################################################################
####################

 Iteration 1

--------------------------------------------------------------------------------
--------------------
P_1^(c):
[[0.33548365 0.25833464 0.27689401 0.06632072 0.06632072]
 [0.25872157 0.35814288 0.25565623 0.06386491 0.06386491]
 [0.23777254 0.21692026 0.34913409 0.09526426 0.09526426]
 [0.06666667 0.06153846 0.11904762 0.38888889 0.36111111]
 [0.06666667 0.06153846 0.11904762 0.36111111 0.38888889]]
--------------------------------------------------------------------------------
--------------------
```

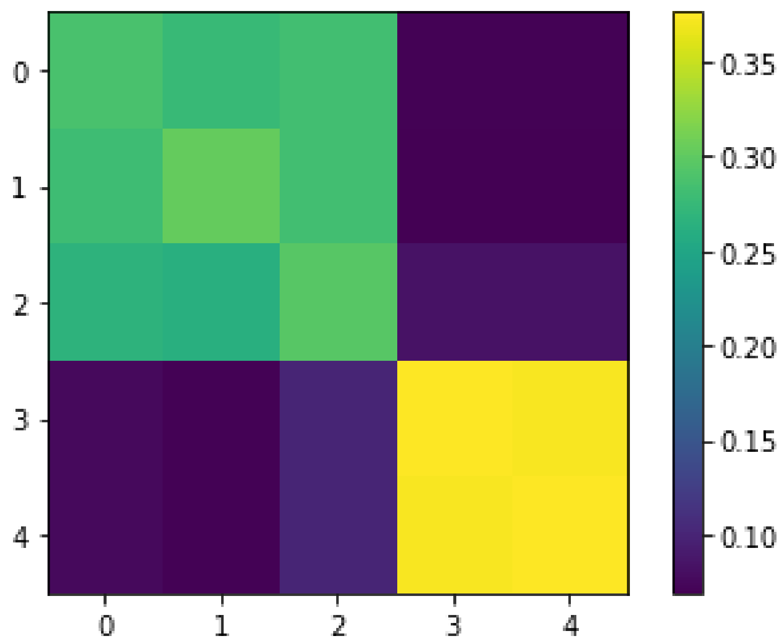



```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.30823076923503195
--------------------------------------------------------------------------------
--------------------

################################################################################
####################
```

Iteration 2

```
--------------------------------------------------------------------------------
--------------------
P_2^(c):
[[0.28886169 0.27594225 0.28372832 0.07063565 0.07063565]
 [0.28107152 0.30424323 0.2831302  0.06911933 0.06911933]
 [0.26823259 0.26274103 0.29684518 0.08467782 0.08467782]
 [0.07686203 0.07148962 0.10091575 0.37654321 0.37345679]
 [0.07686203 0.07148962 0.10091575 0.37345679 0.37654321]]
--------------------------------------------------------------------------------
--------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.10210461245409906
--------------------------------------------------------------------------------
--------------------

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 Iteration 3

--------------------------------------------------------------------------------
--------------------
P_3^(c):
[[0.28362129 0.28125912 0.28518008 0.07337262 0.07337262]
 [0.28283126 0.29128104 0.28518095 0.07130271 0.07130271]
 [0.27563059 0.27410647 0.28648235 0.08033615 0.08033615]
 [0.07969068 0.07590871 0.09241816 0.37517147 0.37482853]
 [0.07969068 0.07590871 0.09241816 0.37482853 0.37517147]]
--------------------------------------------------------------------------------
--------------------
```
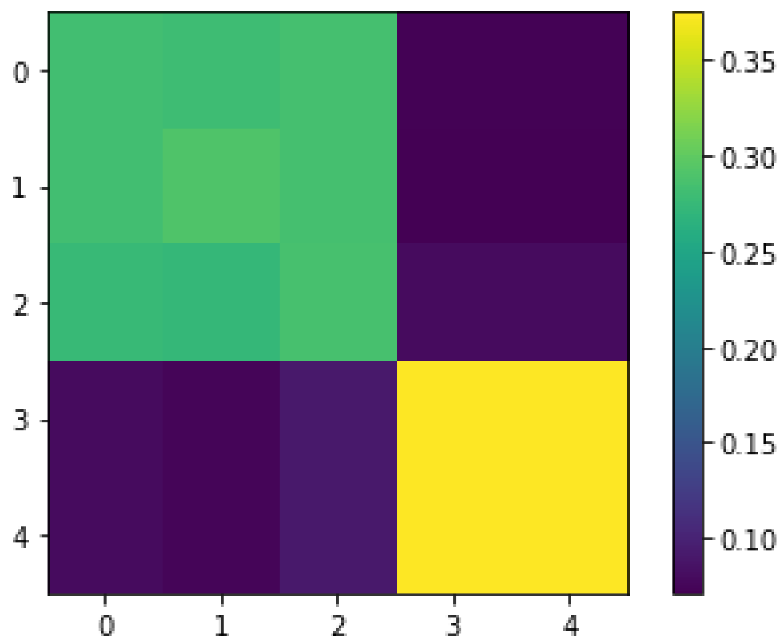
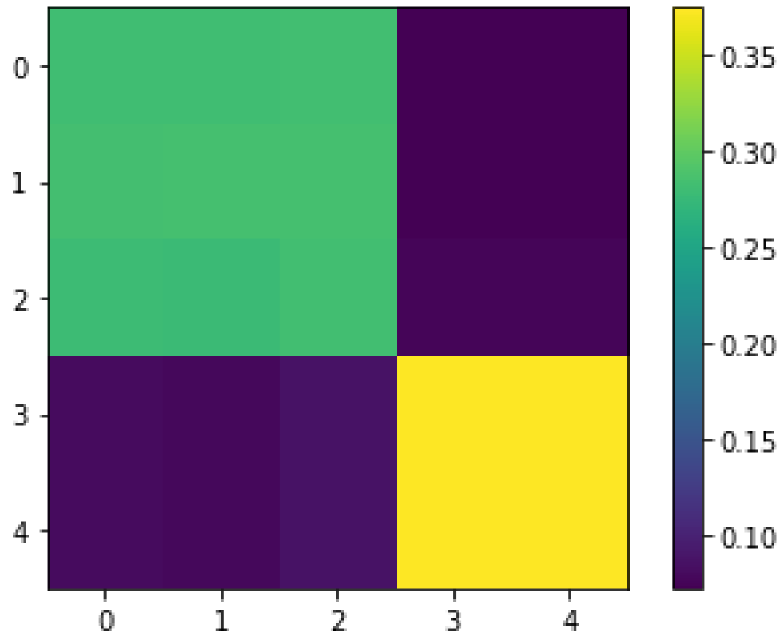




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.02416007977648423
--------------------------------------------------------------------------------
--------------------

################################################################################
####################
```

Iteration 4

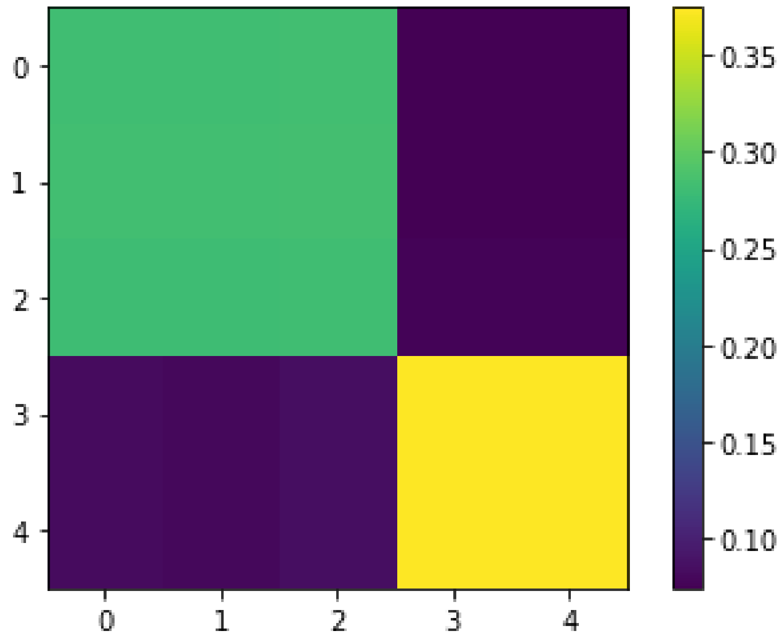
```
--------------------------------------------------------------------------------
--------------------
P_4^(c):
[[0.28240852 0.28189187 0.28383356 0.0740915  0.0740915 ]
 [0.28418409 0.28599468 0.28542605 0.07298849 0.07298849]
 [0.28009694 0.27941161 0.28341068 0.07753344 0.07753344]
 [0.08172963 0.07904671 0.0876368  0.37501905 0.37498095]
 [0.08172963 0.07904671 0.0876368  0.37498095 0.37501905]]
--------------------------------------------------------------------------------
--------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.011744649087641394
--------------------------------------------------------------------------------
--------------------

################################################################################
####################
```

Iteration 5

```
--------------------------------------------------------------------------------
--------------------
P_5^(c):
[[0.28263458 0.2827536  0.28369413 0.07463152 0.07463152]
 [0.28353285 0.28438268 0.28448865 0.07374775 0.07374775]
 [0.28131186 0.28133    0.28290926 0.07634203 0.07634203]
 [0.08212473 0.08054913 0.08528284 0.37500212 0.37499788]
 [0.08212473 0.08054913 0.08528284 0.37499788 0.37500212]]
--------------------------------------------------------------------------------
--------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.004747451072213179
--------------------------------------------------------------------------------
--------------------

################################################################################
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 Iteration 6

--------------------------------------------------------------------------------
--------------------
P_6^(c):
[[0.28282901 0.28273859 0.28326392 0.07472709 0.07472709]
 [0.28357053 0.28364779 0.28395632 0.07426883 0.07426883]
 [0.28233883 0.28220018 0.28288688 0.07558869 0.07558869]
 [0.08252054 0.0815087  0.08400203 0.37500024 0.37499976]
 [0.08252054 0.0815087  0.08400203 0.37499976 0.37500024]]
--------------------------------------------------------------------------------
--------------------
```
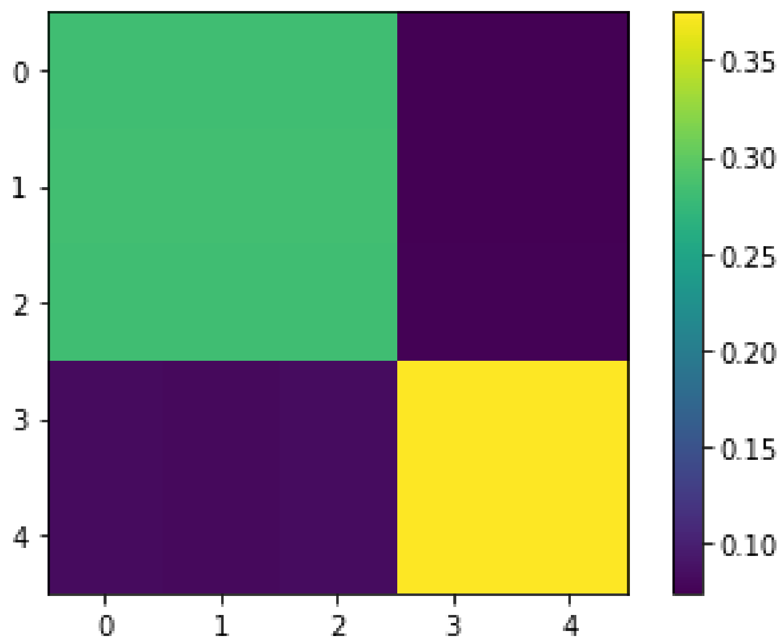
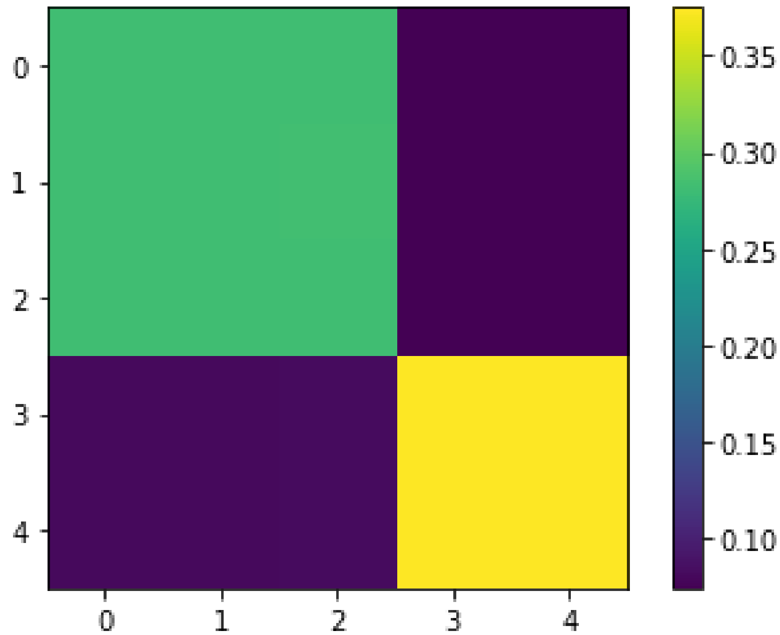




```
----------------------------------------------------------------------------------------------------
Error: E = 0.002712593804803818
----------------------------------------------------------------------------------------------------

####################################################################################################

 Iteration 7

----------------------------------------------------------------------------------------------------
P_7^(c):
[[0.28294498 0.28297979 0.28322979 0.07482968 0.07482968]
 [0.28327404 0.28336235 0.28354285 0.0745147  0.0745147 ]
 [0.282623   0.28264198 0.28294092 0.07525877 0.07525877]
 [0.08254515 0.08198921 0.08334527 0.37500003 0.37499997]
 [0.08254515 0.08198921 0.08334527 0.37499997 0.37500003]]
----------------------------------------------------------------------------------------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.0013230059513200314
--------------------------------------------------------------------------------
--------------------

################################################################################
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```
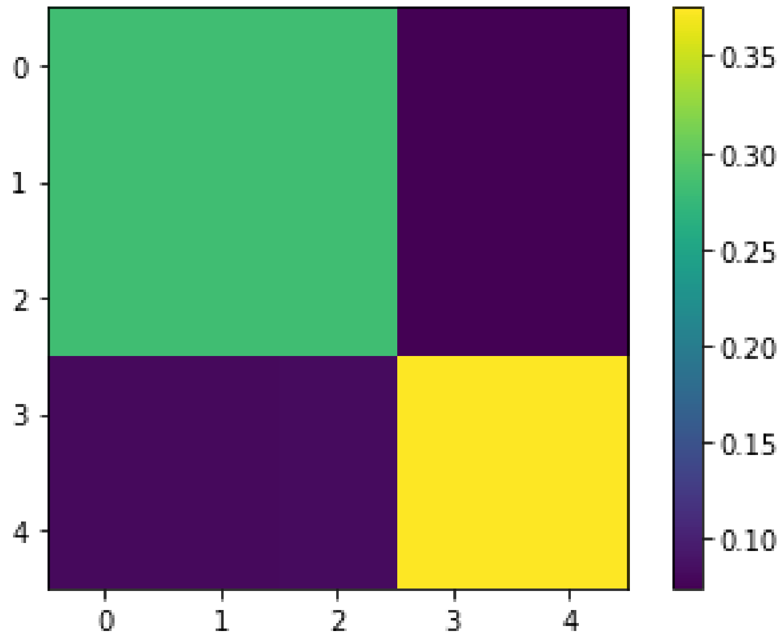
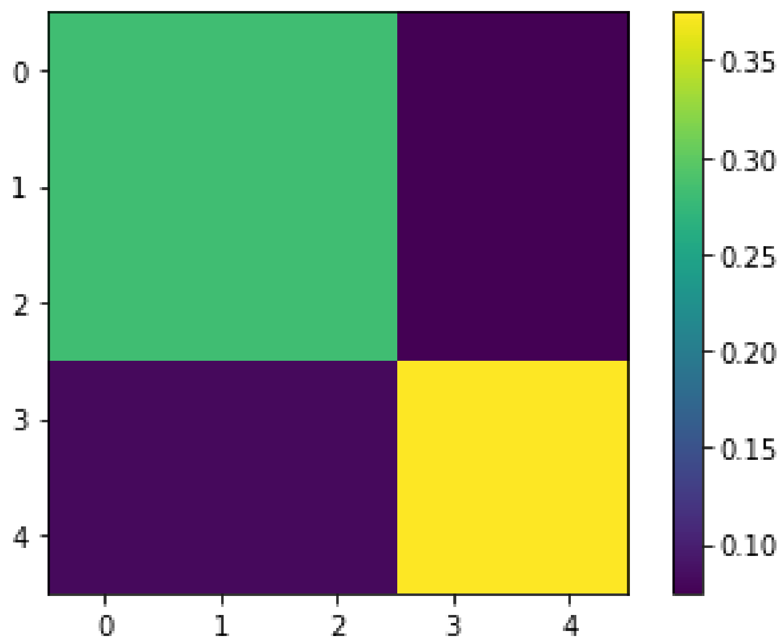

Iteration 8

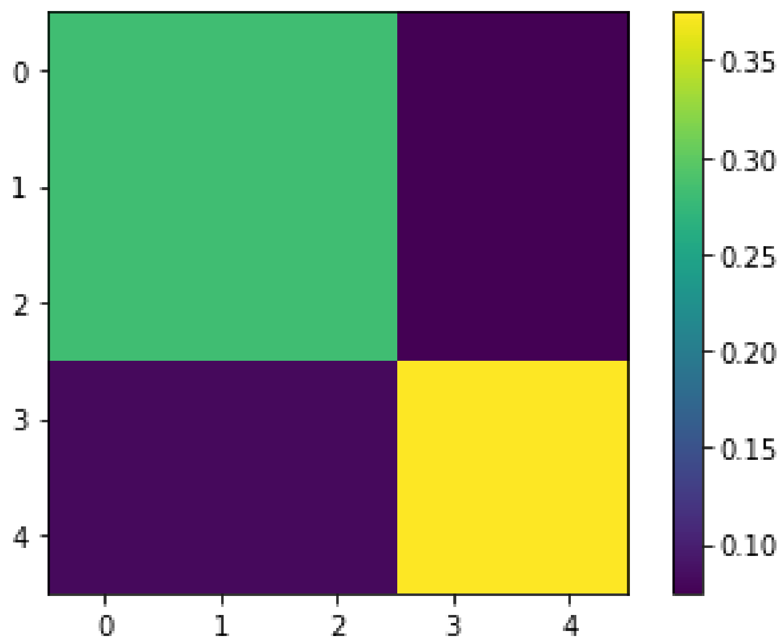
```
--------------------------------------------------------------------------------
--------------------
P_8^(c):
[[0.28301642 0.28297276 0.28312174 0.07483352 0.07483352]
 [0.2832483  0.28321767 0.28334811 0.07467201 0.07467201]
 [0.28288758 0.28283846 0.28300017 0.07505366 0.07505366]
 [0.08261834 0.08227683 0.08299755 0.375      0.375     ]
 [0.08261834 0.08227683 0.08299755 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```



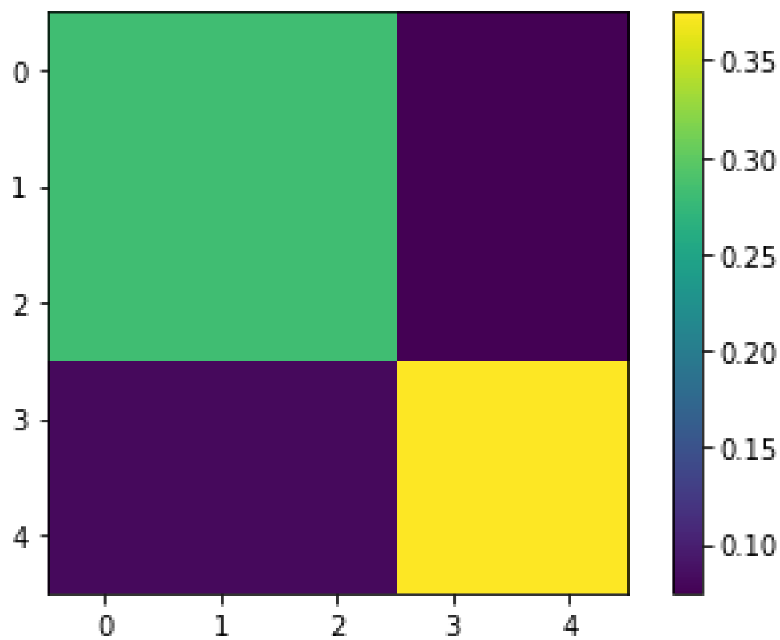
```
----------------------------------------------------------------------------------------------------
Error: E = 0.0007381780571392802
----------------------------------------------------------------------------------------------------

####################################################################################################

 Iteration 9

----------------------------------------------------------------------------------------------------
P_9^(c):
[[0.28304642 0.28304758 0.28311738 0.07485178 0.07485178]
 [0.28314702 0.28315232 0.28321634 0.07474844 0.07474844]
 [0.28295979 0.28295932 0.28303292 0.07496164 0.07496164]
 [0.08260651 0.08242507 0.08281328 0.375      0.375     ]
 [0.08260651 0.08242507 0.08281328 0.375      0.375     ]]
----------------------------------------------------------------------------------------------------
```




```
----------------------------------------------------------------------------------
--------------------
Error: E = 0.0003844320425488355
----------------------------------------------------------------------------------
--------------------

##################################################################################
####################

 Iteration 10

----------------------------------------------------------------------------------
--------------------
P_10^(c):
[[0.28306519 0.2830473  0.28309027 0.07484787 0.07484787]
 [0.28313411 0.28311728 0.28315868 0.07479519 0.07479519]
 [0.28302978 0.28301138 0.28305536 0.0749052  0.0749052 ]
 [0.08261889 0.08251012 0.08271782 0.375      0.375     ]
 [0.08261889 0.08251012 0.08271782 0.375      0.375     ]]
----------------------------------------------------------------------------------
--------------------
```



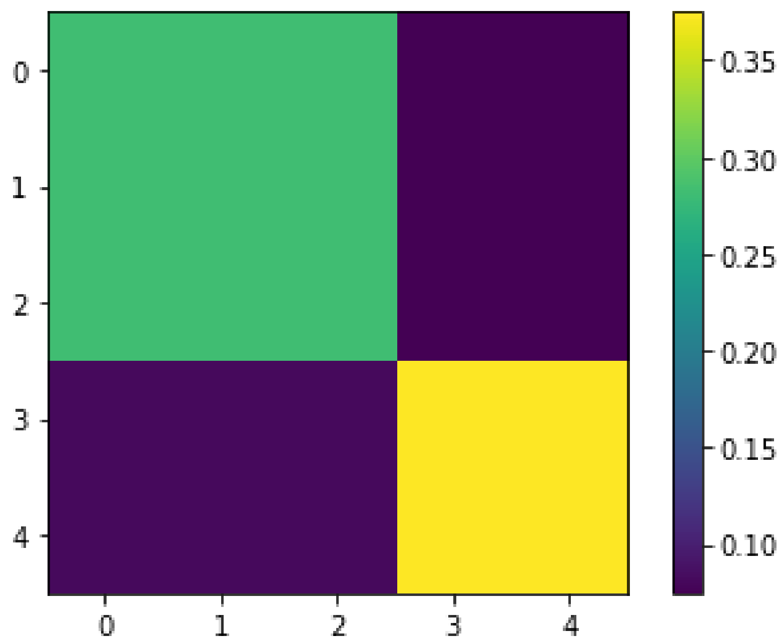

```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.00020608992047994139
--------------------------------------------------------------------------------
--------------------

################################################################################
####################

 Iteration 11

--------------------------------------------------------------------------------
--------------------
P_11^(c):
[[0.28307243 0.28307037 0.28309021 0.07485072 0.07485072]
 [0.28310207 0.28310033 0.2831197  0.07481835 0.07481835]
 [0.28304855 0.28304634 0.28306648 0.07487939 0.07487939]
 [0.08261148 0.08255481 0.08266589 0.375      0.375     ]
 [0.08261148 0.08255481 0.08266589 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 0.00011199314926600407
--------------------------------------------------------------------------------
--------------------

################################################################################
####################
```

Iteration 12

```
--------------------------------------------------------------------------------
--------------------
P_12^(c):
[[0.28307718 0.28307081 0.28308324 0.07484853 0.07484853]
 [0.28309734 0.28309106 0.28310336 0.07483209 0.07483209]
 [0.28306733 0.28306092 0.28307343 0.07486373 0.07486373]
 [0.0826132  0.08257973 0.08263945 0.375      0.375     ]
 [0.0826132  0.08257973 0.08263945 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```




```
--------------------------------------------------------------------------------
--------------------
Error: E = 5.808816317746934e-05
--------------------------------------------------------------------------------
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```
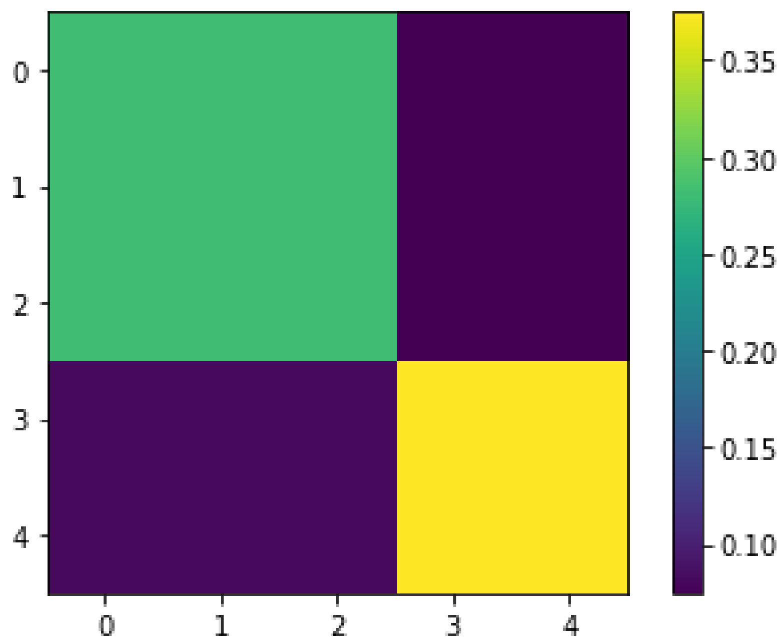

Iteration 13

```
--------------------------------------------------------------------------------
--------------------
P_13^(c):
[[0.28307893 0.28307779 0.28308347 0.07484885 0.07484885]
 [0.28308755 0.28308645 0.28309208 0.07483899 0.07483899]
 [0.28307228 0.28307114 0.28307683 0.07485645 0.07485645]
 [0.08261022 0.082593   0.08262477 0.375      0.375     ]
 [0.08261022 0.082593   0.08262477 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```




```
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--------------------
Error: E = 3.255008690844517e-05
--------------------------------------------------------------------------------
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```

Iteration 14

```
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--------------------
P_14^(c):
[[0.28308014 0.28307804 0.28308164 0.07484799 0.07484799]
 [0.28308599 0.28308391 0.28308749 0.07484299 0.07484299]
 [0.28307738 0.28307529 0.28307888 0.07485208 0.07485208]
 [0.08261032 0.08260025 0.08261739 0.375      0.375     ]
 [0.08261032 0.08260025 0.08261739 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```

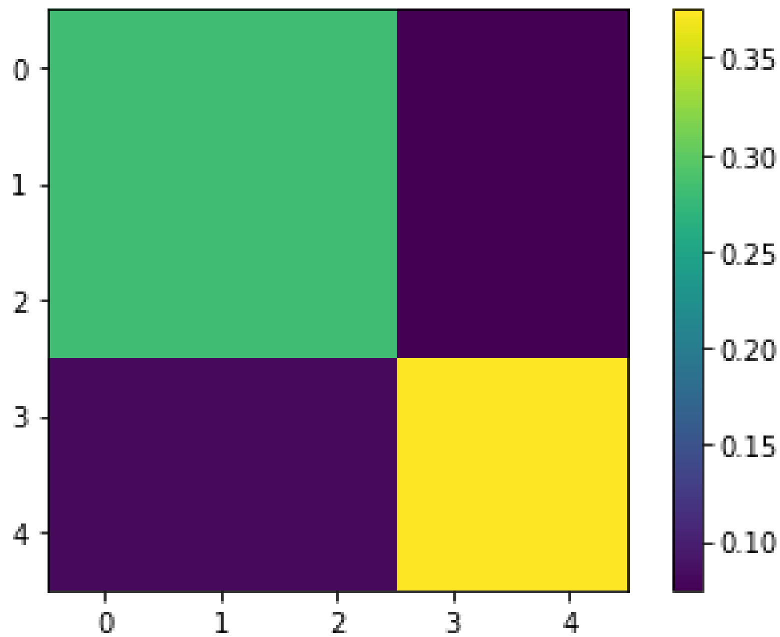


```
----------------------------------------------------------------------------------
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Error: E = 1.646191710555071e-05
----------------------------------------------------------------------------------
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 Iteration 15

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--------------------
P_15^(c):
[[0.28308057 0.28308013 0.28308175 0.07484797 0.07484797]
 [0.28308307 0.28308262 0.28308424 0.07484503 0.07484503]
 [0.28307871 0.28307826 0.28307989 0.07485002 0.07485002]
 [0.08260929 0.08260415 0.08261323 0.375      0.375     ]
 [0.08260929 0.08260415 0.08261323 0.375      0.375     ]]
----------------------------------------------------------------------------------
--------------------
```

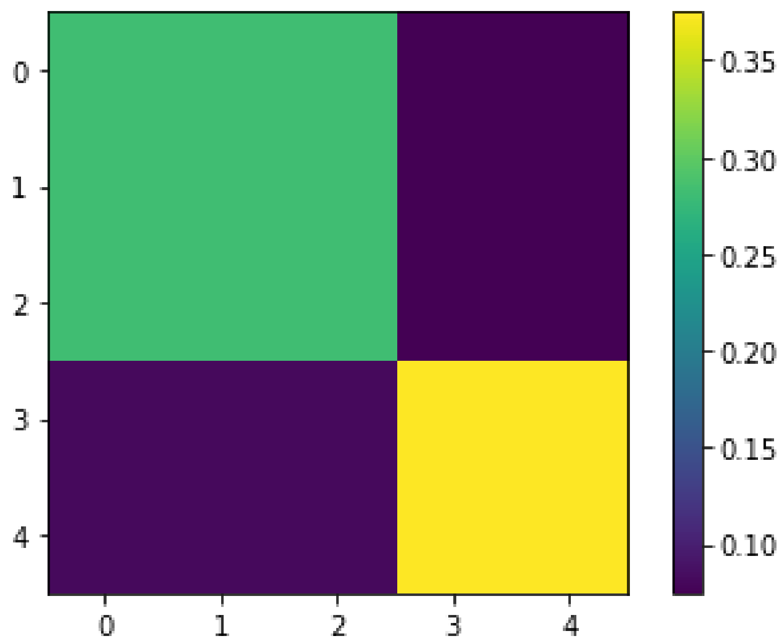


```
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Error: E = 9.431521149778312e-06
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```

Iteration 16

```
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--------------------
P_16^(c):
[[0.28308088 0.28308023 0.28308126 0.07484768 0.07484768]
 [0.28308258 0.28308192 0.28308295 0.07484619 0.07484619]
 [0.28308011 0.28307945 0.28308049 0.07484879 0.07484879]
 [0.08260923 0.08260625 0.08261116 0.375      0.375     ]
 [0.08260923 0.08260625 0.08261116 0.375      0.375     ]]
-----------------------------------------------------------------------------------
--------------------
```

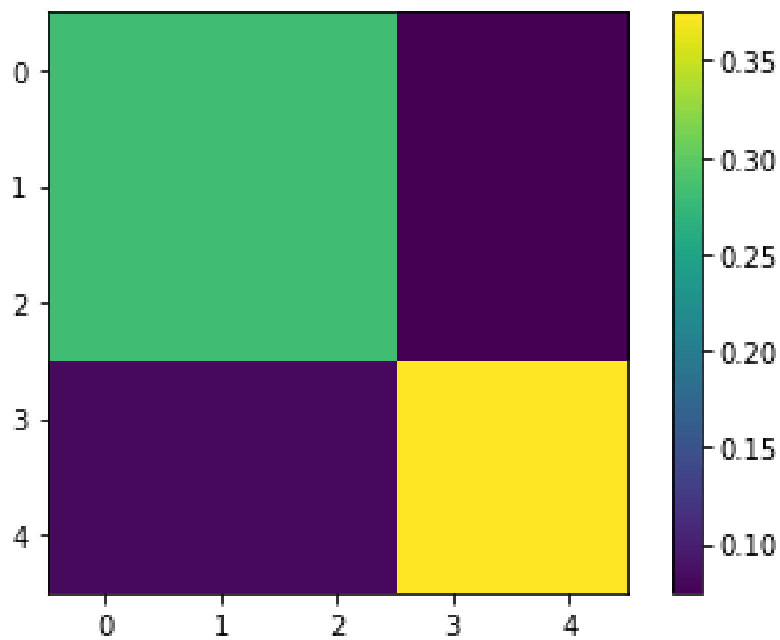


```
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Error: E = 4.680440454801369e-06
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 Iteration 17

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--------------------
P_17^(c):
[[0.28308099 0.28308084 0.2830813  0.07484765 0.07484765]
 [0.28308171 0.28308156 0.28308202 0.07484678 0.07484678]
 [0.28308047 0.28308031 0.28308078 0.07484821 0.07484821]
 [0.0826089  0.08260738 0.08260998 0.375      0.375     ]
 [0.0826089  0.08260738 0.08260998 0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```

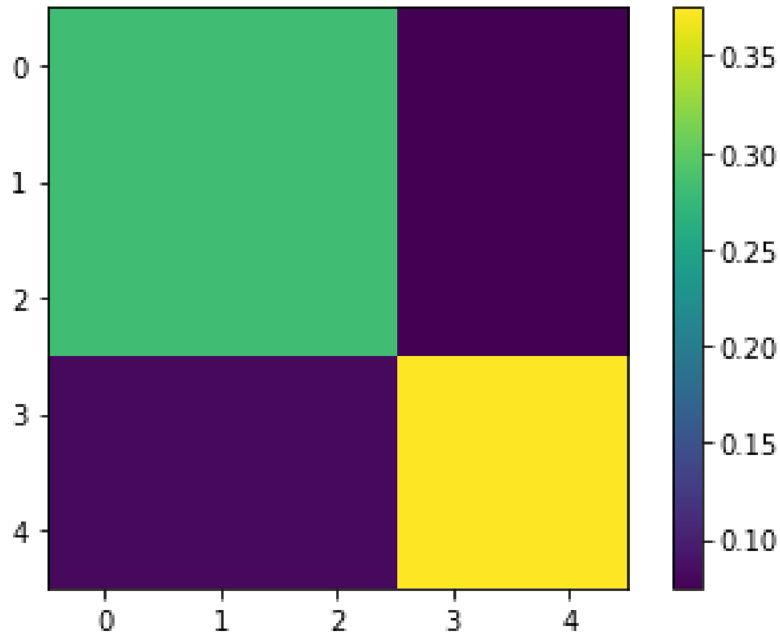


```
--------------------------------------------------------------------------------
--------------------
Error: E = 2.724907137148452e-06
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```

Iteration 18

```
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--------------------
P_18^(c):
[[0.28308107 0.28308087 0.28308117 0.07484756 0.07484756]
 [0.28308156 0.28308136 0.28308166 0.07484712 0.07484712]
 [0.28308085 0.28308065 0.28308095 0.07484786 0.07484786]
 [0.08260886 0.08260799 0.0826094  0.375      0.375     ]
 [0.08260886 0.08260799 0.0826094  0.375      0.375     ]]
--------------------------------------------------------------------------------
--------------------
```

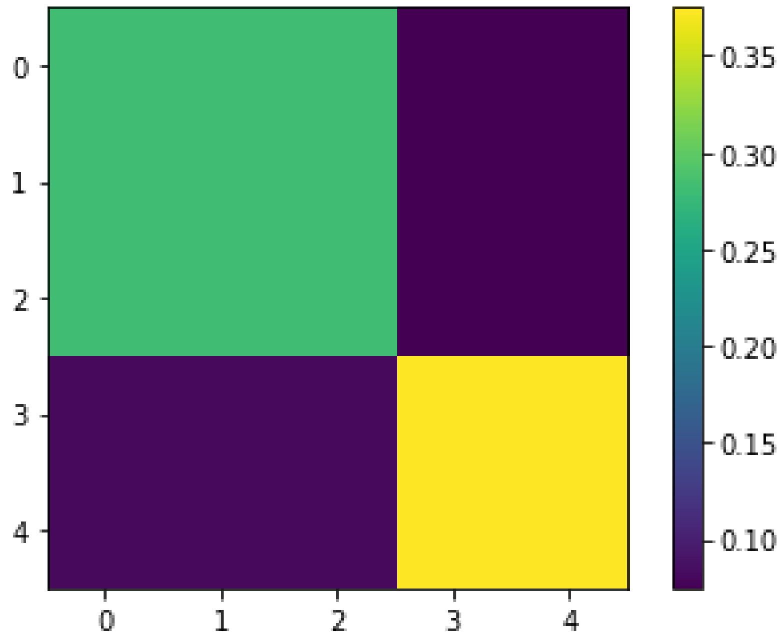


```
------------------------------------------------------------------------------------------------------
Error: E = 1.3333189024757566e-06
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####################################################################################################

 Iteration 19

------------------------------------------------------------------------------------------------------
P_19^(c):
[[0.2830811  0.28308105 0.28308118 0.07484754 0.07484754]
 [0.28308131 0.28308126 0.28308139 0.07484729 0.07484729]
 [0.28308095 0.2830809  0.28308104 0.0748477  0.0748477 ]
 [0.08260876 0.08260832 0.08260906 0.375      0.375     ]
 [0.08260876 0.08260832 0.08260906 0.375      0.375     ]]
------------------------------------------------------------------------------------------------------
```

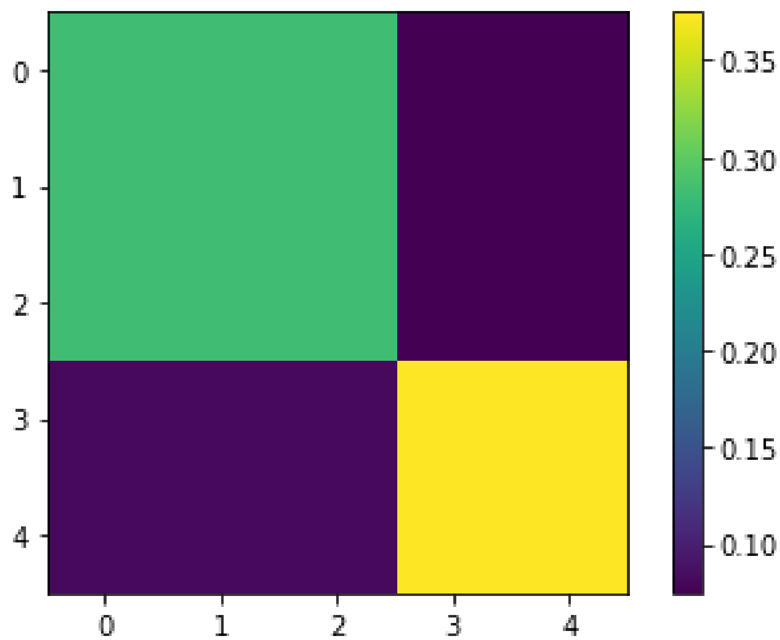


```
--------------------------------------------------------------------------------
--------------------
Error: E = 7.853202634581425e-07
--------------------------------------------------------------------------------
--------------------

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```

One can clearly see, that the P matrix still changes after iteration $t > 2$, but at $t = 2$ it is already rather obvious, how the final result is going to look. It takes 19 iterations to reach the convergence threshold, but visually it does not really change anymore after iteration 5.