Battery Modeling + Resistance/Impedance Matching

(for the Electric Vehicle Battery Engineering Enthusiast...)

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Summer 2023, MIT Electric Vehicle Team

1 An Informal Overview of Batteries

As suggested in the title, this notebook is an informal overview of batteries, modeling of batteries, and matching the internal resistances of batteries to improve pack-level performance for systems such as electric vehicles. But also for you, our reader, it's important to note that batteries are inherently complex systems and we cannot possibly cover all the details in this brief document. Therefore, for the reader that's less experience in electric vehicle battery design, my goal will be to give you all the terms you need to google, and some links to resources. For the more experienced designer, I hope this guide will provide understanding as to what level of detail is required when modeling a battery for an EV, what internal resistance is, how to measure it, and how it affects the performance of a battery pack.

- Battery University—great set of articles on batteries for EVs
- MIT Solar Car Battery Characterization Doc
- MIT FSAE Battery Pack Design Thesis
- FSAE Battery Pack Design
- Charle's Guan's Personal Blog
- Austin Brown's Blog
- Battery Modeling + SoC

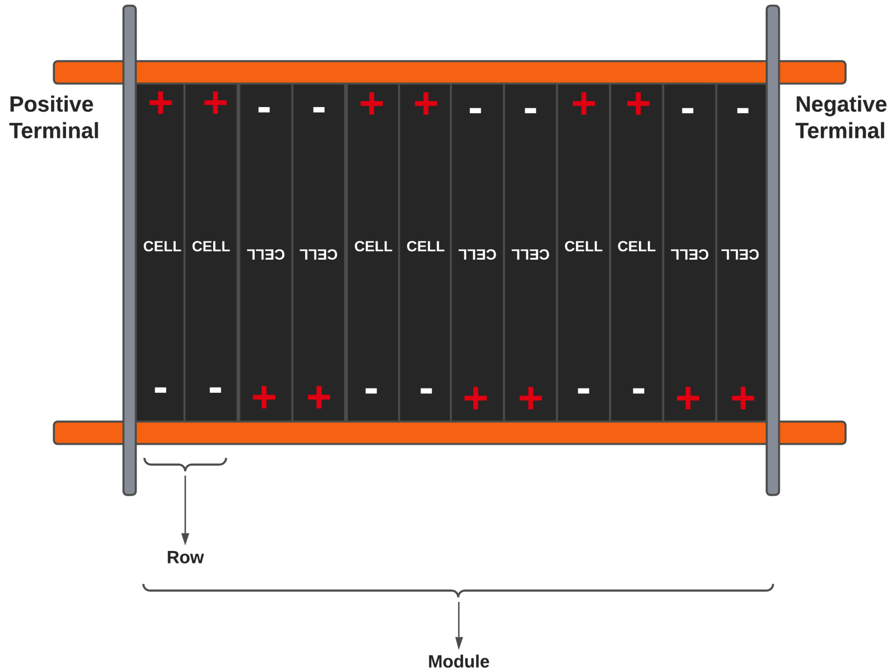

Figure 1: Visualization of a battery module with cells and rows.

1.1 Getting Initial 'Terminology' Out of the Way

A battery 'cell' is a chemical-based energy storage device that stores electrical charge. It has a voltage, and a capacity. The capacity is the amount of charge q a battery cell can store usually specified in Ah (Amp-hours). Voltage is a little more complicated, we'll discuss that later. Battery cells are assembled in parallel configuration into "rows," these "rows" are assembled in series configuration into "modules," and these modules are assembled in series or parallel into "packs." A "battery" is the combination of modules, any associated wiring, battery management system, communication, monitoring equipment, and structural components installed into an electric vehicle or similar system.

1.2 State-of-Charge and Voltage

A good explanation of all of this is contained in the MIT Solar Car Battery Characterization Doc, but we'll summarize a lot of the key points here as well.

A battery has a maximum voltage (quoted by the manufacturer) it will charge to. Trying to charge it more than this by "adding more charge" to the battery is like asking the battery to store more energy than it physically can, which can cause the battery to catch fire to release that energy. As the battery

typical battery discharge curve:

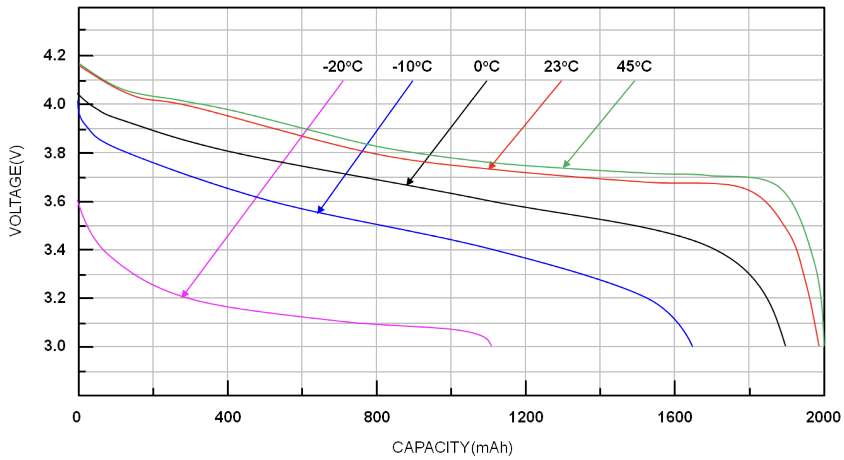

Figure 2: Typical discharge curve taken from here.

is "discharged," charge leaves the positive terminal of the battery and travels to the negative terminal (in the conventional-current assumption). As this happens the voltage decreases according to a curve called the State-of-Charge curve or discharge curve. A discharge curve for a Lithium Ion Battery is shown above. If the battery is discharged past the minimum recommended voltage (quoted by the manufacturer) permanent damage can be caused to the materials in the cell.

The State-of-Charge curve plots the $i = 0$ voltage of the cell (this will become important later) on the vertical axis and the total charge q that has left the battery at that point in the discharge cycle. The SoC curve is a parametric function of $v_{cell}(t)$ and $q_{cell}(t)$, see here for more.

Therefore, capacity rating of the battery is the amount of charge q in Ah that leaves the battery as it discharged from the maximum-manufacturer-recommended-voltage (max-MRV) to the minimum-manufacturer-recommended-voltage (min-MRV). The total energy a battery cell can store in Wh (Watt-hours) is the integral under the state-of-charge curve.

$$E_{cell} = \int_0^{q_{max}} v_{cell}(q)dq$$

A reasonable estimate of energy content, however, can be given by the following formula. Note that C is the manufacturer-quoted capacity in Ah of the cell.

$$E_{cell} \approx V_{mid}C_{battery}$$

where...

$$V_{mid} = \frac{V_{max} + V_{min}}{2}$$

The last basics we will mention in this doc are voltage of battery cells in series adds, capacity of battery cells in parallel adds, and total energy content adds whether the cells are in series and parallel. Therefore the total energy content of a battery pack is given by multiplying the total number of cells in the pack by the individual cell energy content.

$$E_{pack} = N_{cells} * E_{cell}$$

2 The Simple Battery Circuit Model

You can model a battery as a simple voltage source, but you really shouldn't. Modeling it as a charge-dependent voltage source (as a function of SoC like the discharge curve above) is better. But to develop a model that's useful for simulation of system run-time, thermal performance, and battery efficiency while also informing us how best to assemble the cells together to create a high-performing battery module, we'll consider the model in Figure 3.

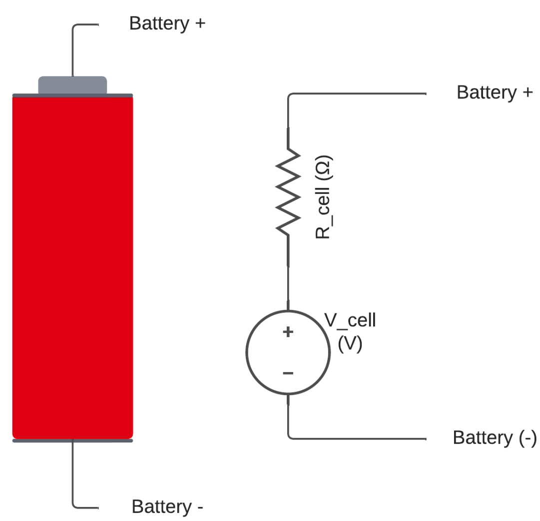

Figure 3: Simple internal-resistance model of a battery.

The equivalent circuit diagram shows a resistance in series with an ideal voltage source. This is called the internal resistance of the battery. It's a first-order approximation for battery cells and generally works for DC and other systems with large dynamic time-constants. This internal resistance of the cell, or R_{cell}, comes from the resistance of the electrolyte within the cell itself, and it can be easily measured (but we'll get to that later). This internal resistance is usually on the order of milli-ohms but still must be considered for thermal and electrical reasons. Note that the resistance R_{cell} is not a "real" or "physical" resistance like the voltage V_{cell} is not and "real" or "physical" ideal voltage source either. Neither of these can be directly measured because they are internal to a battery and cannot be physically separated for measurement purposes. The only voltage that can be measured is the terminal voltage $V_{terminal}$, which is the voltage across the positive and negative terminals of the battery, like you'd measure a cell with a multi-meter.

The terminal voltage of a cell drawing a current i is defined as

$$V_{terminal} = V_{cell} - iR_{cell}$$

And the thermal power generated by the cell under load is

$$P_{thermal} = i^2 R_{cell}$$

This is especially important for electric vehicles and other systems that see high current draws. There are many other models for a battery including the Randles model (that we won't consider here).

3 Battery Management Systems, in Brief

Battery management systems essentially prevent a battery pack from catching fire. For the sake of brevity, we won't go through them in huge amounts of detail here. However, a passing knowledge of what a BMS does might be very useful to understanding the following sections. Check out this resource from the Art + Science of PCB Design @ MIT. We recommend you at least read up to the portion of "what a BMS does" but the schematics and other details are not necessary.

Simply, it's important to understand that a BMS has a bunch of sensors that read certain characteristics of the battery cell to ensure safe operation of the pack. **One important note is that when reading voltages, the BMS can only read the terminal voltage of the battery for the same reasons as above.**

4 Measuring the Internal Resistance of a Battery Cell

The procedure to measure the internal resistance of a battery cell is fairly simple and can be broken down into the following steps.

- Measure the open-circuit voltage of the battery cell.

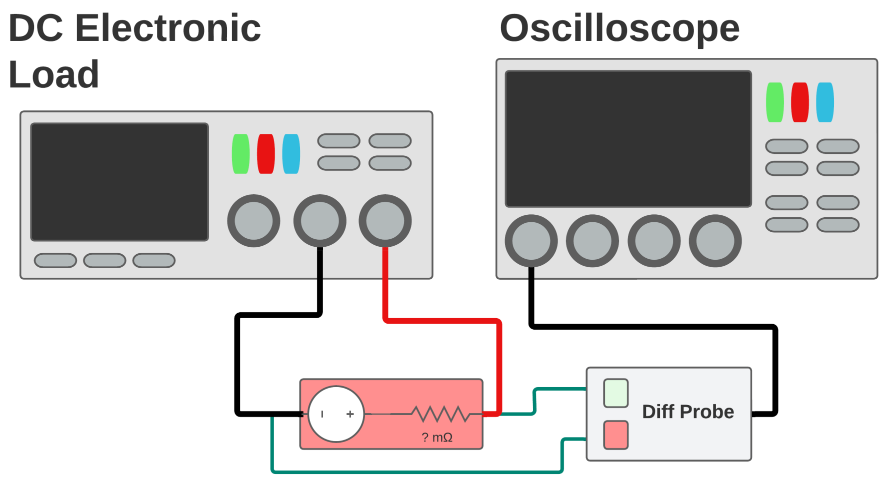

Figure 4: Test setup diagram for measuring battery cells internal resistance.

- Connect the cell to a DC Electronic Load and command a current (depending on the cell data-sheet).
- Read the voltage drop seen when that current is commanded using an oscilloscope (preferably with a differential probe).
- Use the formula $V_{terminal} = V_{OCV} - i_{test}R_{cell}$ to calculate the internal resistance of the cell.

4.1 Reading the Voltage Drop

It's important to get an accurate reading on the voltage of the battery cell during this test, while also eliminating as much noise as possible from the reading. Otherwise the final value will be difficult to discern.

Looking at figure 5, we see the oscilloscope voltage curve has three distinct regions. A flat region that represents the open-circuit-voltage, the voltage when the cell has no current flowing through it. A region with a sharp voltage drop due to the internal resistance. And a region with a slow decay in voltage due to the discharge of the cell. It's important to only take the initial voltage drop when calculating the cell's internal resistance, as voltage drop in the decay region is due to charge leaving the cell's positive terminal. **The main idea here is to take the voltage reading of the initial drop quickly.**

Below is one of the procedures we used to get an accurate reading on the internal resistance of the SPIM08HP cells used in our battery pack.

Also note that for all tests, we don't really recommend the use of a hand-held multi-meter. Generally, they do not have a quick enough or stable enough voltage measurement to accurately characterize a cell.

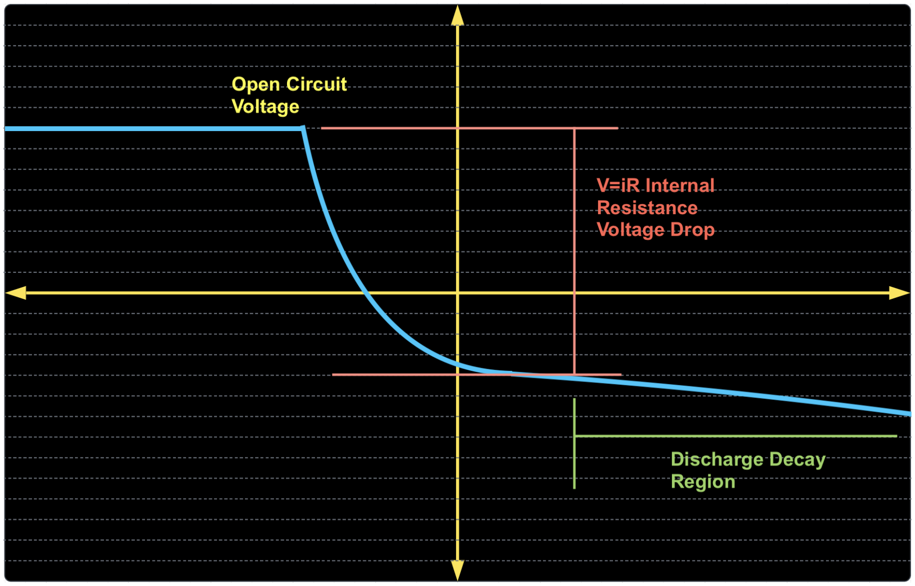

Figure 5: Typical scope output from cell V=iR test.

4.2 Specific Example w/ SPIM08HP Cells

This is the general procedure, we'll now show you a specific example of the test with the SPIM08HP Lithium-Ion cells.

4.2.1 Materials and Setup

For this test we used the following pieces of equipment.

- A Rigol DL3031A High Precision Electronic Load
- Rigol UltraLoad Software
- The battery cell you want to measure
- Various test leads

In this case we will *not be using an oscilloscope*, because our electronic load comes with a software that lets you output a waveform for voltage, and log that data to a CSV. The main idea, however, remains the same. You can use any measurement system that outputs this waveform with the electronic load.[1]

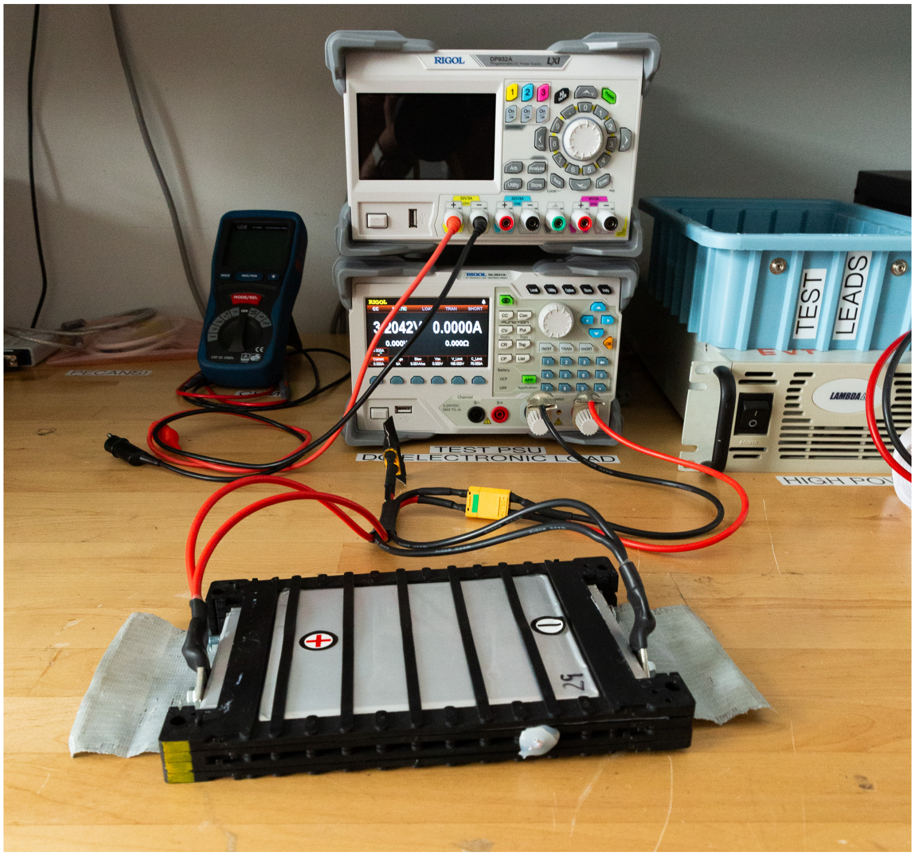

Figure 6: Bench setup with battery cell hooked up to DC electronic load, and load hooked up to PC running UltraLoad software via USB.

4.2.2 Results

Looking at the plots in 7 and 8, we can see a very clear initial voltage drop section, and discharge-decay section of the graph. We can easily read off these voltages and perform the calculation described above to determine the internal resistance.

$$V_{OCV} = 3.2042V$$

$$V_{terminal} = 2.9500V$$

$$i_{test} = 20.050A$$

[1] If you need super precise measurement of battery parameters, entire pack measurement, or measurement of a Randles model of a battery, we would recommend using a battery analyzer instead.

Figure 7: Load graph at 20A.

$$V_{terminal} = V_{OCV} - i_{test} R_{cell}$$

$$2.9500V = 3.2042V - (20.050A) R_{cell}$$

$$R_{cell} = 0.0128\Omega$$

This results in a cell resistance of 12.8 mΩ, which is in the expected range for these cells.

4.3 Selecting a Test Current + Measurement Precision

Note that the test current we selected to measure our cells, 20A, is a high test current. Some cells (such as the Panasonic NCR18650 cells) have recommended max discharge currents per cell of around 10A total. **In this case, a 20A test current could damage the cell and start a fire, so check the data-sheet from your cell manufacturer.**

Generally, you need a current high enough to see a significant drop in voltage for the following equation.

$$V_{terminal} = V_{cell} - i R_{cell}$$

This really depends on your measurement capabilities. If your voltage measurement method is only good to 0.1V, you might need much higher currents to get the voltage drop to be high enough where you can be confident that measurement noise is not affecting your reading. **You likely want a current where**

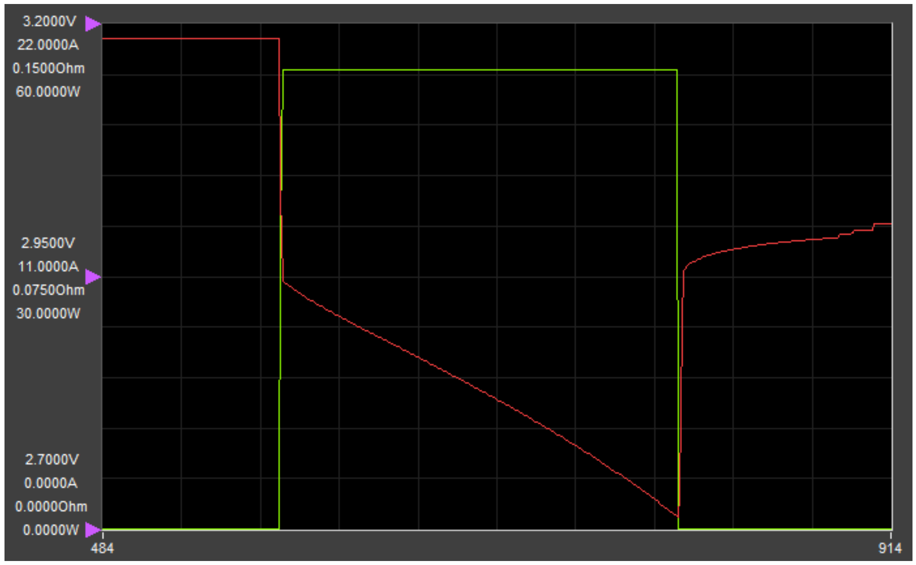

Figure 8: Load graph at 20A with current included.

the measurement precision is 100x smaller than the voltage drop you see, that is, you should be measuring to two decimal places finer than the expected voltage drop.

The Panasonic NCR18650 cell has a current limit of 10A. It also has a datasheet quoted internal impedance of 50mΩ. Let's set the test current to 8A for this cell.

$$8A * 0.05\Omega = 0.4V$$

What this means is our multi-meter/scope, or other form of measurement equipment for voltage needs a precision of 0.005V or smaller. **In this case we rounded to the nearest multiple of 5. So a system with +/-0.005V of precision would be good enough for measuring the resistance of this cell.**

5 Considering Internal Resistance During Battery Pack Assembly

Ok, so as we've seen, batteries have an internal resistance and this resistance can be measured fairly easily. We've also seen that it can vary a little bit from cell-to-cell, sometimes significantly so. So what does this mean for when we're building a battery pack? Let's say you're building a battery with multiple cells. The way you arrange these cells in the pack matters a lot, especially if their internal resistances are different. Let's look at two examples.

5.1 Batteries Arranged in Parallel

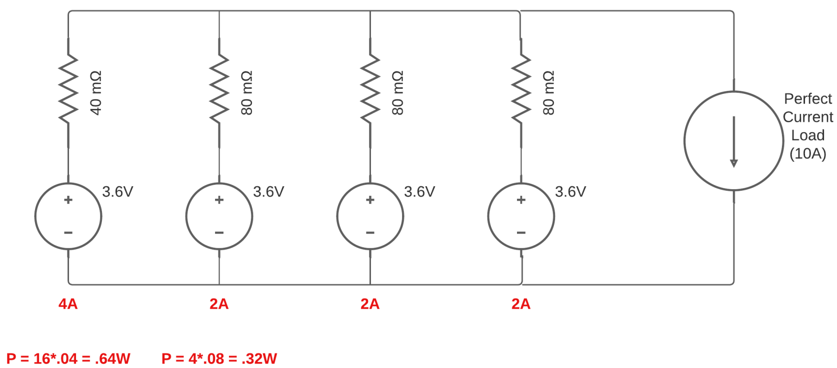

Figure 9: Battery cells with impedance difference arranged in parallel.

Let's assume we have four battery cells arranged in parallel, one of which has an internal resistance of 40mΩ and the rest with an internal resistance of 80mΩ. Now suppose these cells are loaded with a perfect current source of 10A.

In this case, we can ignore the voltage sources in the circuit diagram, and note that the current splits between the branches only due to the internal resistance of the cells. The cell with the lower resistance will take more current than its buddies in parallel. In this case, the 40mΩ cell takes twice as much current. This causes a few problems.

- Since thermal power produced in a cell is related to the i^2R of the cell's internal resistance, the 40mΩ cell dissipates twice the energy of the other cells and will "heat up" twice as much. If the cooling system of the battery assumes all cells heat equally, then a fire could start because the 40mΩ cell is simply not being cooled enough since it's heating more than expected. Additionally, most battery systems monitor temperature "regions" rather than the temperature of individual cells in order to reduce system complexity. If one cell is heating more than its neighbors, the protection systems may not kick in in time to save the pack from thermal runaway.

- This doesn't matter so much in a circuit diagram, but in a real, physical system, if the designer assumes all cells will share current equally when sizing the copper bus-bars or connection tabs in a battery pack, this battery pack could fail because the copper tabs connecting the 40mΩ cell to the battery are taking higher current loads than expected.

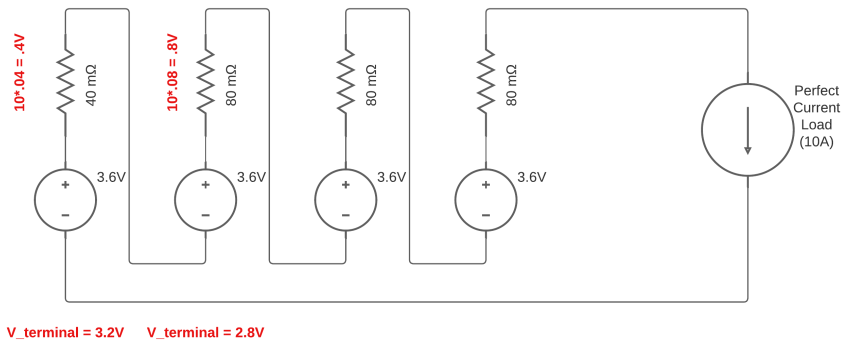

Figure 10: Battery cells with impedance difference arranged in series.

5.2 Batteries Arranged in Series

Now let's look at batteries arranged in series, again with one cell having half the internal resistance of the rest. In this case all cells take the same amount of current, but the voltage drop across them is different. The cell with the lower internal resistance will read a higher terminal resistance than the other cells. This has a few implications.

- Now of course, the same idea that some cells will heat up more than others applies, but in this case the higher resistance cells will heat up more since the current i is the same through all cells. This is something to consider when designing a cooling system. Certain regions of the battery will heat more than others. We'll explain why this is less of a concern in series than in parallel later.

- The voltage drop across some cells will be much higher than others. In the case above, a BMS reading the voltage of all the cells with a protection set-point of 3.0V would trigger. But the internal chemical voltage when $i = 0$ of the cell that triggered the BMS is 3.6V, which means it could technically discharge more. This means during this load case, the battery would shut off prematurely, leading to decreased range if this was an electric vehicle.

5.3 Discussion, Matching in Series vs. Parallel?

It's probably clear to you by now that different internal resistances in a battery can cause all sorts of issues, some worse than others. But it's physically impossible for all batteries in a pack to have the exact same internal resistance. This is also complicated by the fact that cells are often assembled in parallel and then in series, so the resistances in 11 are actually combinations of all the resistances in that parallel row of cells. This means "matching" resistances in series is extremely computationally intensive, and time consuming, especially for larger battery packs.

So what do we do? To answer this, we need to go back to how battery packs are assembled. Cells are assembled in parallel into rows, these rows are assembled in series into modules, and these modules are assembled in series into packs. This means the pack is more "modular" in series than it is in parallel. BMS systems also tend to monitor "rows" of parallel cells rather than individual cells because of the assumption that the voltage of cells in parallel is the same, and the current going through them is distributed evenly.

What this means is that it's much more critical to match cell resistances in parallel. The issues presented by different resistances in series can be overcome with clever distributed cooling systems, or by placing higher-resistance modules closer to the main air inlet so they get more direct cooling. But in parallel, it's very difficult to cool one cell differently than its neighbors or design one tab to take more current than the rest. This has to do with the fact that cells in parallel MUST be proximally close to each other in battery packs, but cells in series don't have to be. **Therefore, it's critical to match cells in parallel and not critical to match them in series, though you should strive to minimize overall battery pack internal resistance.**

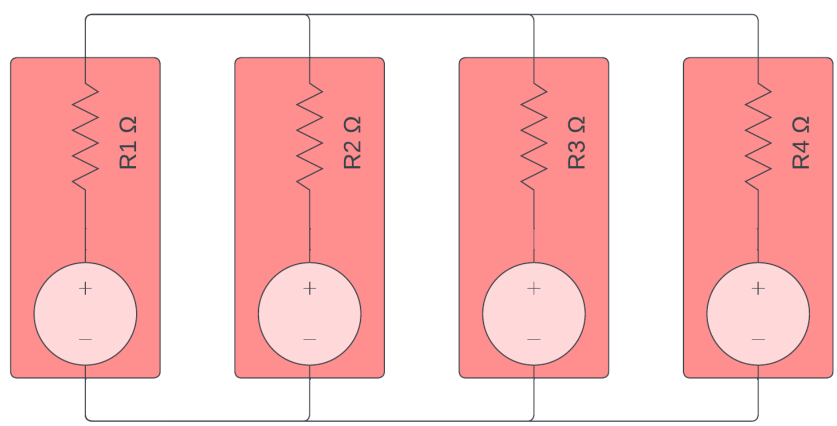

Figure 11: We want the values of R1, R2, R3, and R4 to be as close as possible in parallel.

Mathematically this looks something like the below equations.

$$\min_{R1,R2,R3,R4 \in R_{cells}} [(R1 - Ra)^2 + (R2 - Ra)^2 + (R3 - Ra)^2 + (R4 - Ra)^2]$$

$$Ra = \frac{R1 + R2 + R3 + R4}{2}$$

You take the average resistance of the four cells, and make the values as close as possible. This is like minimizing the standard deviation of the four resistance value (the given example uses four cells, for any pack you want to generalize this to n cells).

5.4 How to Match Cells in a Pack

For the Version 1 battery pack for the hydrogen bike project, we bought 36 of the SPIM08HP lithium-ion cells and measured their internal resistances using the procedure described in section 4.2. We then created a python script to match the resistances based on minimizing the standard deviations of groups of cells. The current method is not necessarily *optimal* but it is *close enough* for most pack purposes.

The python script can be found in our battery design repository on Github. The output is displayed in 12.

```
Multiple solutions possible... generating groups
sorting groups...
iterating through groups to find a solution...


Solution found! Printing...]

-----Final Cell Groups-----
Cells in Group: [33, 35]
Cells in Group: [9, 12]
Cells in Group: [4, 6]
Cells in Group: [23, 19]
Cells in Group: [16, 24]
Cells in Group: [5, 3]
Cells in Group: [36, 13]
Cells in Group: [21, 26]
Cells in Group: [32, 30]
Cells in Group: [28, 31]
Cells in Group: [27, 11]
Cells in Group: [14, 8]


-----Pack Specifications-----
Total Calculated Pack Resistance: 54.040541675540716

Resistances by Group: [[[33, 35], 1.5850000000000002], [[9, 12], 6.179994559303591], [[4, 6], 11.2
62483491915493], [[23, 19], 17.5224675174746], [[16, 24], 26.07744121706548], [[5, 3], 31.94733474
3470932], [[36, 13], 33.89201340671001], [[21, 26], 35.326159748173424], [[32, 30], 37.53453078889
7406], [[28, 31], 44.53037930622502], [[27, 11], 50.07782640212777], [[14, 8], 54.040541675540716]
]
```

Figure 12: Output of the impedance matching of our cells with measured cell resistance data for a 12s2p battery pack.

5.5 Performance Estimation + Modeling

Based on the above output, we can create a simple model for the entire battery pack as an ideal voltage source and series resistance. But there are a couple of things to note first.

- Battery internal resistance changes with both temperature and state-of-charge. For precise characterization, more detailed models will need to be built. For now, we recommend performing tests at 50-percent SoC and room temperature as a *good-enough* mode for the battery.

- When measuring internal resistances for impedance matching, **it is important to ensure all cells are at similar SoC and temperature for impedance matching to be accurate.**

5.5.1 Final Battery Model

The final battery model is described by calculating the nominal voltage of the battery, and the total internal resistance of the battery pack.

$$V_{nom} = 12 * 3.7V = 44.4V$$

$$R_{pack} = 54m\Omega$$

A very simple thermal model of the pack can be written as follows.

$$P_{thermal} = i^2 R_{pack}$$

Therefore the minimum cooling power required for any system cooling the battery pack can be estimated using the above thermal model.

6 References + Acknowledgements

Special thanks to Chris Evagora @ MIT for his help in understanding batteries as well as Elijah B. Stanger-Jones, Francis Wang, and Quang Kieu for teaching me everything I know about EE. Special shout-out to Fischer J. Moseley for being a great friend. Additional shout-out to Rigol Technologies for sponsoring the MIT EVT with some awesome equipment that helps us test and characterize batteries, and write these guides.