

# ICPC Templates

Leachim

January 28, 2021

## Contents

<b>1</b>	<b>图论</b>	<b>4</b>
1.1	最小生成树 Kruskal . . . . .	4
1.2	最小生成树 Prim . . . . .	5
1.3	最短路 Dijkstra . . . . .	6
1.4	最短路 SPFA . . . . .	8
1.5	最近公共祖先 LCA_Doubling . . . . .	9
1.6	最近公共祖先 LCA_Tarjan . . . . .	11
1.7	判断负环 SPFA_Negative_Cycle . . . . .	12
1.8	拓扑排序 Topological_Sort_Khan . . . . .	14
<b>2</b>	<b>多项式</b>	<b>16</b>
2.1	FFT 字符串匹配 String_Match_FFT . . . . .	16
2.2	FFT 递归 Fast_Fourier_Transform_Cooley-Tukey_Recursion . . . . .	17
2.3	FFT 递推 Fast_Fourier_Transform_Cooley-Tukey_Iteration . . . . .	18
2.4	NTT 递推 Number_Theoretic_Transforms . . . . .	20
2.5	多项式求逆 Polynomial_Inverse . . . . .	21
<b>3</b>	<b>字符串</b>	<b>24</b>
3.1	字符串哈希 String_Hash . . . . .	24
3.2	马拉车 Manacher . . . . .	24
3.3	字符串匹配 KMP . . . . .	25
3.4	AC 自动机 AC-Automaton . . . . .	26
3.5	后缀数组 Suffix_Array . . . . .	27
3.6	后缀自动机 Suffix-Automaton . . . . .	28
3.7	广义后缀自动机 General_Suffix-Automaton . . . . .	30
<b>4</b>	<b>数据结构</b>	<b>33</b>
4.1	并查集 Disjoin_Set_Union . . . . .	33
4.2	可撤销并查集 Disjoin_Set_Union_Withdrawable . . . . .	33
4.3	分块 Block_1 . . . . .	34
4.4	分块 Block_2 . . . . .	35
4.5	分块 Block_4 . . . . .	36
4.6	树状数组 Binary_Indexed_Tree . . . . .	38
4.7	二维树状数组 2D_Binary_Indexed_Tree . . . . .	39
4.8	线段树 Segment_Tree . . . . .	40
4.9	线段树 Segment_Tree_Multiply . . . . .	41
4.10	扫描线 Scanline . . . . .	43

4.11	zkw 线段树 ZKW_Segment_Tree	45
4.12	李超线段树 Li-Chao_Segment_Tree	46
4.13	可并堆左偏树 Leftist_Tree	48
4.14	Splay 树 Splay_Tree	50
4.15	Splay 树 Splay_Tree_Flip	52
4.16	Splay 树 Splay_Tree_Dye&Flip	54
<b>5</b>	<b>数论</b>	<b>59</b>
5.1	乘法逆元 Multiplicative_Inverse_Modulo	59
5.2	卢卡斯 Lucas	60
5.3	拓展欧几里得 Exgcd	60
5.4	拓展欧拉定理 Ex_Euler_Theorem-Automaton	61
5.5	欧拉筛 Euler_Sieve	62
5.6	杜教筛 Dujiao_Sieve	63
5.7	求原根 Get_Primitive_Root	65
5.8	贝祖引理 Bezout_Lemma	65
5.9	除法分块 Block_Division	66
<b>6</b>	<b>组合数学</b>	<b>67</b>
6.1	康托展开 Cantor	67
6.2	波利亚 Pólya	68
6.3	卡特兰数 Catalan	69
6.4	斯特林数 Stirling	70
6.4.1	第一类斯特林数	70
6.4.2	第二类斯特林数	70
6.4.3	上升幂与普通幂的相互转化	70
6.4.4	下降幂与普通幂的相互转化	71
<b>7</b>	<b>网络流</b>	<b>72</b>
7.1	最大费用流 Minimum-Cost_Flow_Edmonds-Karp	72
7.2	最大流 Maximum_Flow_Edmonds-Karp	73
7.3	最大流 Maximum_Flow_Dinic	75
7.4	二分图最大匹配 Bipartite_Graph_Maximum_Matching_Dinic	77
7.5	二分图最大匹配 Bipartite_Graph_Maximum_Matching_Hungarian	79
7.6	二分图最大匹配 Bipartite_Graph_Maximum_Matching_Hopcroft-Karp	80
<b>8</b>	<b>计算几何</b>	<b>82</b>
8.1	计算几何 Computational_Geometry	82
<b>9</b>	<b>优化算法</b>	<b>85</b>
9.1	wqs 二分 + 决策单调性 dp	85
9.2	斜率优化 Slope_Optimization	86
<b>10</b>	<b>离线算法</b>	<b>88</b>
10.1	莫队算法 Mo	88
10.2	线段树分治 Segment_Tree_Partition	89

<b>11 杂项</b>	<b>92</b>
11.1 快读快写 Fast_Read&Write . . . . .	92
11.2 快速幂 Fast_Power . . . . .	92
11.3 矩阵快速幂 Matrix_Fast_Power . . . . .	93
11.4 矩阵加速 Matrix_Acceleration . . . . .	94
11.5 最长公共子序列 Longest_Increasing_Subsequence . . . . .	95
11.6 模拟退火 Simulated_Annealing . . . . .	96
11.7 快速沃尔什变换 Fast_Walsh_Transform . . . . .	97
11.8 快速莫比乌斯变换 Fast_Mobius_Transform . . . . .	98
11.9 快速子集变换 Fast_Subset_Transform . . . . .	100
11.10 24 点 24_Point . . . . .	101

# 1 图论

## 1.1 最小生成树 Kruskal

```
1 #include <stdio>
2 #include <algorithm>
3
4 #define MAXN 5005
5 #define MAXM 200005
6 #define _for(i,a,b) for(int i=(a);i<=(b);i++)
7
8 using namespace std;
9
10 int read(){
11     int ng=0,x=0;
12     char ch=getchar();
13     for(;ch<'0' || ch>'9';ch=getchar()) ng|=ch=='-';
14     for(;ch>='0' && ch<='9';ch=getchar()) x=(x<<3)+(x<<1)+ch-'0';
15     return ng?-x:x;
16 }
17
18 struct road
19 {
20     int f,g,w;
21 }r[MAXM];
22
23 struct cmpf
24 {
25     bool operator() (const road& a, const road& b) const {
26         return a.w<b.w;
27     }
28 };
29
30 int n,m,ans=0,fa[MAXN];
31
32 int find(int x){
33     return fa[x]==x?x:fa[x]=find(fa[x]);
34 }
35
36 void kruskal(){
37     int rx,ry;
38
39     sort(r+1, r+m+1, cmpf());
40     _for(i,1,m){
41         rx=find(r[i].f);
42         ry=find(r[i].g);
43         if(rx==ry)
44             continue;
45         fa[rx]=ry;
46         ans+=r[i].w;
47     }
48 }
49
```

```

50 int main(){
51     n=read();m=read();
52     _for(i,1,m){
53         r[i].f=read();
54         r[i].g=read();
55         r[i].w=read();
56     }
57     _for(i,1,n)
58         fa[i]=i;
59     kruskal();
60     _for(i,2,n){
61         if(find(i)!=find(i-1)){
62             puts("orz");
63             return 0;
64         }
65     }
66     printf("%d", ans);
67 }

```

## 1.2 最小生成树 Prim

```

1  #include <cstdio>
2  #include <cstdlib>
3  #include <queue>
4
5  #define MAXN 5005
6  #define MAXM 200005
7
8  using namespace std;
9
10 struct edge
11 {
12     int v,to,next;
13 }e[MAXM<<1];
14
15 int n,m,f,g,w,tot=0,head[MAXN],ans=0,flag[MAXN];
16
17 void add(int x, int y, int z){
18     e[++tot].v=z;
19     e[tot].to=y;
20     e[tot].next=head[x];
21     head[x]=tot;
22 }
23
24 struct HeapNode
25 {
26     int v,u;
27     bool operator <(const HeapNode& a) const{
28         return v > a.v;
29     }
30 };
31

```

```

32 priority_queue<HeapNode> Q;
33
34 void prim(){
35     for(int i=1;i<=n;i++)
36         flag[i]=0;
37     flag[1]=1;
38     for(int p=head[1];p;p=e[p].next)
39         Q.push((HeapNode){e[p].v,e[p].to});
40     for(int i=1,u;i<n;i++){
41         while(flag[Q.top().u]){
42             Q.pop();
43             if(Q.empty()){
44                 puts("orz");
45                 exit(0);
46             }
47         }
48         ans+=Q.top().v;
49         u=Q.top().u;
50         Q.pop();
51         flag[u]=1;
52         for(int p=head[u];p;p=e[p].next)
53             if(!flag[e[p].to])
54                 Q.push((HeapNode){e[p].v,e[p].to});
55     }
56 }
57
58 int main(){
59     scanf("%d %d", &n, &m);
60     for(int i=1;i<=n;i++)
61         head[i]=0;
62     for(int i=1;i<=m;i++){
63         scanf("%d %d %d",&f,&g,&w);
64         add(f,g,w);
65         add(g,f,w);
66     }
67     prim();
68     printf("%d",ans);
69     return 0;
70 }

```

### 1.3 最短路 Dijkstra

```

1 #include <bits/stdc++.h>
2 #define LL long long
3 using namespace std;
4 const int inf=0x3f3f3f3f;
5 const LL INF=0x3f3f3f3f3f3f3f3f;
6 const int MOD=998244353;
7 const int MAXN=2000005;
8
9 struct edge {
10     int to,next,w;

```

```
11 }e[MAXN<<1];
12
13 int tot,head[MAXN];
14
15 void add(int x,int y,int z) {
16     tot++;
17     e[tot].to=y;
18     e[tot].w=z;
19     e[tot].next=head[x];
20     head[x]=tot;
21 }
22
23 struct node {
24     int u;
25     LL dis;
26     bool operator<(const node& y) const{
27         return dis>y.dis;
28     }
29 };
30
31 LL dist[MAXN];
32 bool flag[MAXN];
33 priority_queue<node> Q;
34
35 void dij(int n,int s) {
36     memset(flag+1,0,n*sizeof(flag[0]));
37     memset(dist+1,0x3f,n*sizeof(dist[0]));
38     dist[s]=0;
39     Q.push({s,0LL});
40     while(!Q.empty()) {
41         while(!Q.empty() && flag[Q.top().u]) Q.pop();
42         if(Q.empty()) break;
43         int u=Q.top().u;
44         Q.pop();
45         flag[u]=1;
46         for(int p=head[u];p;p=e[p].next) {
47             int v=e[p].to,w=e[p].w;
48             if(flag[v]) continue;
49             if(dist[u]+w<dist[v]) {
50                 dist[v]=dist[u]+w;
51                 Q.push({v,dist[v]});
52             }
53         }
54     }
55 }
56
57 void solve() {
58     int n,m,s;
59     scanf("%d %d %d", &n, &m, &s);
60     tot=0;
61     memset(head+1,0,n*sizeof(head[0]));
62     for(int i=1;i<=m;i++) {
63         int f,g,w;
```

```
64     scanf("%d %d %d", &f, &g, &w);
65     add(f,g,w);
66 }
67 dij(n,s);
68 for(int i=1;i<=n;i++)
69     printf("%lld%c", dist[i], " \n"[i==n]);
70 }
71
72 int main() {
73     int T=1,cas=1;(void)(cas);
74     // scanf("%d", &T);
75     while(T--) {
76         // printf("Case #%d: ", cas++);
77         solve();
78     }
79     return 0;
80 }
```

## 1.4 最短路 SPFA

```
1  #include <cstdio>
2  #include <cstring>
3  #include <queue>
4
5  #define MAXN 100005
6  #define MAXM 500005
7  const int inf=0x3f3f3f3f;
8
9  using namespace std;
10
11 struct edge{
12     int v,to,next;
13 }e[MAXN];
14
15 int n,m,p,tot,head[MAXN],dist[MAXN];
16 bool flag[MAXN];
17
18 void add(int x,int y,int z){
19     tot++;
20     e[tot].v=z;
21     e[tot].to=y;
22     e[tot].next=head[x];
23     head[x]=tot;
24 }
25
26 void spfa(int x){
27     queue <int> Q;
28     memset(dist+1,inf,n*sizeof(dist[0]));
29     memset(flag+1,0,n*sizeof(flag[0]));
30
31     Q.push(x);
32     flag[x]=true;
```



```

33     dist[x]=0;
34     while(!Q.empty()){
35         int u=Q.front();
36         Q.pop();
37         flag[u]=false;
38         for(int q=head[u];q;q=e[q].next){
39             int v=e[q].to;
40             if(dist[u]+e[q].v<dist[v]){
41                 dist[v]=dist[u]+e[q].v;
42                 if(!flag[v]){
43                     Q.push(v);
44                     flag[v]=true;
45                 }
46             }
47         }
48     }
49 }
50
51 int main(){
52     scanf("%d %d %d",&n,&m,&p);
53     tot=0;
54     memset(head+1,0,n*sizeof(head[0]));
55     for(int i=1;i<=m;i++){
56         int f,g,w;
57         scanf("%d %d %d", &f, &g, &w);
58         add(f,g,w);
59     }
60     spfa(p);
61     for(int i=1;i<=n;i++)
62         printf("%d ", dist[i]==inf?2147483647:dist[i]);
63     return 0;
64 }

```

## 1.5 最近公共祖先 LCA\_Doubling

```

1  #include <stdio>
2  #include <cstring>
3  #include <algorithm>
4  #define MAXN 500005
5  #define MAXM 500005
6  #define MAXLN 25
7  using namespace std;
8
9  struct edge{
10     int to,next;
11 }e[MAXM<<1];
12
13 int tot,head[MAXN];
14
15 void add(int x,int y){
16     tot++;
17     e[tot].to=y;

```

```
18     e[tot].next=head[x];
19     head[x]=tot;
20 }
21
22 int dep[MAXN],st[MAXN][MAXLN];
23
24 void dfs(int cur,int fa){
25     dep[cur]=dep[fa]+1;
26     st[cur][0]=fa;
27     for(int l=1;(1<<l)<=dep[cur];l++)
28         st[cur][l]=st[st[cur][l-1]][l-1];
29
30     for(int p=head[cur];p;p=e[p].next){
31         if(e[p].to==fa) continue;
32         dfs(e[p].to,cur);
33     }
34 }
35
36 int lca(int x,int y){
37     if(dep[x]<dep[y]) swap(x,y);
38     for(int i=0;dep[x]-dep[y];i++)
39         if((dep[x]-dep[y])&(1<<i)) x=st[x][i];
40     if(x==y) return x;
41
42     int l;
43     for(l=1;(1<<l)<=dep[x];l++);
44     for(int i=l;i>=0;i--)
45         if(st[x][i]!=st[y][i])
46             x=st[x][i], y=st[y][i];
47     return st[x][0];
48 }
49
50 int n,m,s;
51
52 int main(){
53     scanf("%d %d %d",&n,&m,&s);
54     tot=0;
55     memset(head+1,0,n*sizeof(head[0]));
56     for(int i=1;i<n;i++){
57         int f,g;
58         scanf("%d %d",&f,&g);
59         add(f,g);
60         add(g,f);
61     }
62     dep[0]=0;
63     dfs(s,0);
64     for(int i=1;i<=m;i++){
65         int f,g;
66         scanf("%d %d",&f,&g);
67         printf("%d\n",lca(f,g));
68     }
69     return 0;
70 }
```

## 1.6 最近公共祖先 LCA\_Tarjan

```

1  #include <stdio>
2  #define MAXN 500005
3  #define MAXM 500005
4
5  struct edge{
6      int to,next;
7  }e[MAXN<<1],eq[MAXN<<1];
8
9  int n,m,s,tot,totq,head[MAXN],headq[MAXN],fa[MAXN],ans[MAXM];
10
11 void add(int x, int y){
12     tot++;
13     e[tot].to=y;
14     e[tot].next=head[x];
15     head[x]=tot;
16 }
17
18 void addq(int x, int y){
19     totq++;
20     eq[totq].to=y;
21     eq[totq].next=headq[x];
22     headq[x]=totq;
23 }
24
25 int find(int x){
26     if(fa[x]==x)
27         return x;
28     return fa[x]=find(fa[x]);
29 }
30
31 void tarjan(int x){
32     fa[x]=x;
33     for(int p=head[x];p;p=e[p].next){
34         int u=e[p].to;
35         if(!fa[u]){
36             tarjan(u);
37             fa[u]=x;
38         }
39     }
40     for(int p=headq[x];p;p=eq[p].next){
41         int u=eq[p].to;
42         if(fa[u]){
43             ans[(p+1)>>1]=find(u);
44         }
45     }
46 }
47
48 int main(){
49     scanf("%d %d %d", &n, &m, &s);
50     for(int i=1;i<=n;i++){
51         head[i]=0;

```

```

52     headq[i]=0;
53 }
54 tot=0;
55 for(int i=1;i<n;i++){
56     int f,g;
57     scanf("%d %d",&f,&g);
58     add(f,g);
59     add(g,f);
60 }
61 tot=0;
62 for(int i=1;i<=m;i++){
63     int f,g;
64     scanf("%d %d",&f,&g);
65     addq(f,g);
66     addq(g,f);
67 }
68 for(int i=1;i<=n;i++){
69     fa[i]=0;
70 }
71 tarjan(s);
72 for(int i=1;i<=m;i++)
73     printf("%d\n", ans[i]);
74 return 0;
75 }

```

## 1.7 判断负环 SPFA\_Negative\_Cycle

```

1  #include <stdio>
2  #include <cstring>
3  #include <queue>
4
5  #define MAXN 100005
6  #define MAXM 500005
7  const int inf=0x3f3f3f3f;
8
9  using namespace std;
10
11 struct edge{
12     int v,to,next;
13 }e[MAXM];
14
15 int n,m,tot,head[MAXN];
16 int dist[MAXN],cnt[MAXN];
17 bool flag[MAXN];
18
19 void add(int x,int y,int z){
20     tot++;
21     e[tot].v=z;
22     e[tot].to=y;
23     e[tot].next=head[x];
24     head[x]=tot;
25 }

```

```
26
27 bool spfa(int x){//负环return false
28     queue <int> Q;
29     memset(dist+1,inf,n*sizeof(dist[0]));
30     memset(flag+1,0,n*sizeof(flag[0]));
31     memset(cnt+1,0,n*sizeof(cnt[0]));
32
33     Q.push(x);
34     flag[x]=true;
35     dist[x]=0;
36     while(!Q.empty()){
37         int u=Q.front();
38         Q.pop();
39         flag[u]=false;
40         for(int q=head[u];q;q=e[q].next){
41             int v=e[q].to;
42             if(dist[u]+e[q].v<dist[v]){
43                 dist[v]=dist[u]+e[q].v;
44                 if(!flag[v]) {
45                     Q.push(v);
46                     flag[v]=true;
47                     if(++cnt[v]>=n){
48                         return false;
49                     }
50                 }
51             }
52         }
53     }
54     return true;
55 }
56
57 void solve() {
58     scanf("%d %d",&n,&m);
59     tot=0;
60     memset(head+1,0,n*sizeof(head[0]));
61     for(int i=1;i<=m;i++){
62         int f,g,w;
63         scanf("%d %d %d", &f, &g, &w);
64         add(f,g,w);
65         if(w>=0) add(g,f,w);
66     }
67     if(spfa(1)) printf("NO\n");
68     else printf("YES\n");
69 }
70
71 int main(){
72     int T=1;
73     scanf("%d", &T);
74     while(T--){
75         solve();
76     }
77     return 0;
78 }
```

## 1.8 拓扑排序 Topological\_Sort\_Khan

```

1  #include <stdio>
2  #include <cstring>
3  #include <stack>
4  #define MAXN 100005
5  #define MAXM 200005
6
7  using namespace std;
8
9  struct edge{
10     int to,next;
11 }e[MAXM];
12
13 int n,m,tot,head[MAXN],indgr[MAXN],list[MAXN];
14 bool flag[MAXN];
15
16 void add(int x,int y){
17     tot++;
18     e[tot].to=y;
19     e[tot].next=head[x];
20     head[x]=tot;
21 }
22
23 void khan(){
24     stack<int> s;
25     int cnt=0;
26     memset(flag+1,0,n*sizeof(flag[0]));
27     for(int i=1;i<=n;i++){
28         if(!flag[i] && indgr[i]==0){
29             s.push(i);
30             flag[i]=true;
31             while(!s.empty()){
32                 int u=s.top();s.pop();
33                 list[++cnt]=u;
34                 flag[u]=true;
35                 for(int p=head[u];p;p=e[p].next){
36                     int v=e[p].to;
37                     indgr[v]--;
38                     if(indgr[v]==0)
39                         s.push(v);
40                 }
41             }
42         }
43     }
44 }
45
46 int main(){
47     scanf("%d %d",&n,&m);
48     memset(head+1,0,n*sizeof(head[0]));
49     memset(indgr+1,0,n*sizeof(indgr[0]));
50     tot=0;
51     for(int i=1;i<=m;i++){

```

```
52     int f,g;
53     scanf("%d %d", &f, &g);
54     add(f,g);
55     indgr[g]++;
56 }
57 khan();
58 for(int i=1;i<=n;i++)
59     printf("%d ", list[i]);
60 return 0;
61 }
```

## 2 多项式

### 2.1 FFT 字符串匹配 String\_Match\_FFT

```

1  #include <stdio>
2  #include <cstring>
3  #include <cmath>
4  #include <algorithm>
5  #include <complex>
6  #define MAXN 300005
7  #define MAXL 550005
8
9  using namespace std;
10
11 const double PI=acos(-1);
12 complex<double> omg[MAXL],iomg[MAXL];
13
14 void init(int n){
15     for(int i=0;i<n;i++){
16         omg[i]=polar(1.0,2.0*PI*i/n);
17         iomg[i]=conj(omg[i]);
18     }
19 }
20
21 void FFT(int n,complex<double>* P,complex<double>* w){
22     for(int i=0,j=0;i<n;i++){
23         if(i<j) swap(P[i],P[j]);
24         for(int l=n>>1;(j^=1)<1;l>>=1);
25     }
26
27     for(int i=2,l;i<=n;i<=<=1){
28         l=i>>1;
29         complex<double> t;
30         for(int j=0;j<n;j+=i){
31             for(int k=0;k<l;k++){
32                 t=P[j+l+k]*w[n/i*k];
33                 P[j+l+k]=P[j+k]-t;
34                 P[j+k]=P[j+k]+t;
35             }
36         }
37     }
38 }
39
40 int n,m,lim;
41 char s1[MAXN],s2[MAXN];
42 complex<double> A[MAXL],B[MAXL],tA[MAXL],tB[MAXL],ans[MAXL];
43
44 int main(){
45     scanf("%d %d", &m, &n);
46     scanf("%s %s",s1,s2);
47     for(int i=0;i<m;i++) A[m-i-1]=(s1[i]=='*'?'0':s1[i]-'a'+1);
48     for(int i=0;i<n;i++) B[i]=(s2[i]=='*'?'0':s2[i]-'a'+1);
49     for(lim=1;lim<n-1;lim<=<=1);

```



```

50     init(lim);
51     fill(ans,ans+lim,complex<double>(0.0,0.0));
52
53     for(int i=0;i<m;i++) tA[i]=A[i]*A[i]*A[i];
54     for(int i=0;i<n;i++) tB[i]=B[i];
55     fill(tA+m,tA+lim,complex<double>(0.0,0.0));
56     fill(tB+n,tB+lim,complex<double>(0.0,0.0));
57     FFT(lim,tA,omg);
58     FFT(lim,tB,omg);
59     for(int i=0;i<lim;i++) ans[i]+=tA[i]*tB[i];
60
61     for(int i=0;i<m;i++) tA[i]=A[i];
62     for(int i=0;i<n;i++) tB[i]=B[i]*B[i]*B[i];
63     fill(tA+m,tA+lim,complex<double>(0.0,0.0));
64     fill(tB+n,tB+lim,complex<double>(0.0,0.0));
65     FFT(lim,tA,omg);
66     FFT(lim,tB,omg);
67     for(int i=0;i<lim;i++) ans[i]+=tA[i]*tB[i];
68
69     for(int i=0;i<m;i++) tA[i]=A[i]*A[i];
70     for(int i=0;i<n;i++) tB[i]=B[i]*B[i];
71     fill(tA+m,tA+lim,complex<double>(0.0,0.0));
72     fill(tB+n,tB+lim,complex<double>(0.0,0.0));
73     FFT(lim,tA,omg);
74     FFT(lim,tB,omg);
75     for(int i=0;i<lim;i++) ans[i]-=complex<double>(2.0,0)*tA[i]*tB[i];
76
77     FFT(lim,ans,iomg);
78
79     int cnt=0;
80     for(int i=m-1;i<n;i++)
81         if((int)(ans[i].real()/lim+0.5)==0) cnt++;
82     printf("%d\n", cnt);
83     for(int i=m-1;i<n;i++)
84         if((int)(ans[i].real()/lim+0.5)==0) printf("%d ", i-m+2);
85     return 0;
86 }

```

## 2.2 FFT 递归 Fast\_Fourier\_Transform\_Cooley-Tukey\_Recursion

```

1  #include <cstdio>
2  #include <algorithm>
3  #include <complex>
4  #define MAXN 4000005
5
6  using namespace std;
7
8  complex<double> omg[MAXN],iomg[MAXN],temp[MAXN];
9
10 void init(int n){
11     double PI=acos(-1);
12     for(int i=0;i<n;i++){

```

```

13     omg[i]=polar(1.0,2.0*PI*i/n);
14     iomg[i]=conj(omg[i]);
15 }
16 }
17
18 void FFT(int n,complex<double>* buffer,int offset,int step,complex<double>* omg){
19     if(n==1) return;
20     int m=n>>1;
21     FFT(m,buffer,offset,step<<1,omg);
22     FFT(m,buffer,offset+step,step<<1,omg);
23     for(int i=0;i<m;i++){
24         int pos=2*i*step;
25         temp[i]=buffer[offset+pos]+omg[i*step]*buffer[offset+step+pos];
26         temp[i+m]=buffer[offset+pos]-omg[i*step]*buffer[offset+step+pos];
27     }
28     for(int i=0;i<n;i++)
29         buffer[offset+i*step]=temp[i];
30 }
31
32 int n,m,lim;
33 complex<double> A[MAXN],B[MAXN];
34
35 int main(){
36     scanf("%d %d", &n, &m);
37     for(lim=1;lim<=n+m;lim<=<=1);
38     fill(A,A+lim,complex<double>(0.0,0.0));
39     fill(B,B+lim,complex<double>(0.0,0.0));
40     for(int i=0,t;i<n+1;i++){
41         scanf("%d", &t);
42         A[i]+=t;
43     }
44     for(int i=0,t;i<m+1;i++){
45         scanf("%d", &t);
46         B[i]+=t;
47     }
48     init(lim);
49     FFT(lim,A,0,1,omg);
50     FFT(lim,B,0,1,omg);
51     for(int i=0;i<lim;i++)
52         A[i]=A[i]*B[i];
53     FFT(lim,A,0,1,iomg);
54     for(int i=0;i<n+m+1;i++)
55         printf("%d ", (int)(A[i].real()/lim+0.5));
56     return 0;
57 }

```

### 2.3 FFT 递推 Fast\_Fourier\_Transform\_Cooley-Tukey\_Iteration

```

1 #include <bits/stdc++.h>
2 #define MAXN 2100005
3
4 using namespace std;

```

```

5
6 const double PI=acos(-1);
7
8 struct Complex{
9     double real,image;
10     Complex operator+(Complex y)const{
11         return {real+y.real,image+y.image};
12     }
13     Complex operator-(Complex y)const{
14         return {real-y.real,image-y.image};
15     }
16     Complex operator*(Complex y)const{
17         return {real*y.real-image*y.image,real*y.image+image*y.real};
18     }
19 };
20
21 void FFT(int n,Complex* P,int f){
22     for(int i=0,j=0;i<n;i++){
23         if(i<j) swap(P[i],P[j]);
24         for(int l=n>>1;(j^=l)<l;l>>=1);
25     }
26
27     for(int i=2,l;i<=n;i<=1){
28         l=i>>1;
29         Complex wn={cos(2*PI/i),f*sin(2*PI/i)};
30         for(int j=0;j<n;j+=i){
31             Complex w={1,0};
32             for(int k=0;k<l;k++,w=w*wn){
33                 Complex t=P[j+l+k]*w;
34                 P[j+l+k]=P[j+k]-t;
35                 P[j+k]=P[j+k]+t;
36             }
37         }
38     }
39 }
40
41 int n,m,lim;
42 Complex A[MAXN],B[MAXN];
43
44 int main(){
45     scanf("%d %d", &n, &m);
46     for(lim=1;lim<=n+m;lim<=1);
47     memset(A,0,lim*sizeof(A[0]));
48     memset(B,0,lim*sizeof(B[0]));
49     for(int i=0,t;i<n+1;i++){
50         scanf("%d", &t);
51         A[i]={1.0*t,0};
52     }
53     for(int i=0,t;i<m+1;i++){
54         scanf("%d", &t);
55         B[i]={1.0*t,0};
56     }
57     FFT(lim,A,1);

```

```

58     FFT(lim,B,1);
59     for(int i=0;i<lim;i++)
60         A[i]=A[i]*B[i];
61     FFT(lim,A,-1);
62     for(int i=0;i<n+m+1;i++)
63         printf("%d ", (int)(A[i].real/lim+0.5));
64     return 0;
65 }

```

## 2.4 NTT 递推 Number\_Theoretic\_Transforms

```

1  #include <cstdio>
2  #include <algorithm>
3  using namespace std;
4  #define LL long long
5  const int MAXN=2100005
6  const int MOD=998244353
7  const int RT=3;
8
9  LL binpow(LL x,LL y,LL mod){
10     LL r=1%mod;
11     while(y){
12         if(y&1) r=(r*x)%mod;
13         x=(x*x)%mod;
14         y>>=1;
15     }
16     return r;
17 }
18
19 void NTT(int n,int* P,int f){
20     for(int i=0,j=0;i<n;i++){
21         if(i<j) swap(P[i],P[j]);
22         for(int l=n>>1;(j^=l)<l;l>>=1);
23     }
24
25     for(int i=2,l;i<=n;i<=<=1){
26         l=i>>1;
27         int wn=binpow(RT,(MOD-1)/i,MOD);
28         if(f== -1) wn=binpow(wn,MOD-2,MOD);
29         for(int j=0;j<n;j+=i){
30             int w=1;
31             for(int k=0;k<l;k++,w=1LL*w*wn%MOD){
32                 int t=1LL*P[j+l+k]*w%MOD;
33                 P[j+l+k]=1LL*(P[j+k]-t+MOD)%MOD;
34                 P[j+k]=1LL*(P[j+k]+t)%MOD;
35             }
36         }
37     }
38 }
39
40 int A[MAXN],B[MAXN];
41

```

```

42 int main(){
43     int n,m;
44     scanf("%d %d", &n, &m);
45     int lim;
46     for(lim=1;lim<=n+m;lim<=1);
47     for(int i=0;i<n+1;i++){
48         scanf("%d", &A[i]);
49     }
50     for(int i=0;i<m+1;i++){
51         scanf("%d", &B[i]);
52     }
53     NTT(lim,A,1);
54     NTT(lim,B,1);
55     for(int i=0;i<lim;i++)
56         A[i]=1LL*A[i]*B[i]%MOD;
57     NTT(lim,A,-1);
58     int invn=binpow(lim,MOD-2,MOD);
59     for(int i=0;i<n+m+1;i++)
60         printf("%lld ", 1LL*A[i]*invn%MOD);
61     return 0;
62 }

```

## 2.5 多项式求逆 Polynomial\_Inverse

```

1  #include <cstdio>
2  #include <algorithm>
3  #define LL long long
4  #define MAXN 270005
5  #define MOD 998244353
6  #define RT 3
7
8  using namespace std;
9
10 LL omg[MAXN],iomg[MAXN];
11
12 LL binpow(LL x,LL y,LL mod){
13     LL r=1%mod;
14     while(y){
15         if(y&1) r=(r*x)%mod;
16         x=(x*x)%mod;
17         y>>=1;
18     }
19     return r;
20 }
21
22 void init(int n){
23     omg[0]=iomg[0]=1;
24     omg[1]=binpow(RT,(MOD-1)/n,MOD);
25     iomg[1]=binpow(omg[1],MOD-2,MOD);
26     for(int i=2;i<n;i++){
27         omg[i]=omg[i-1]*omg[1]%MOD;
28         iomg[i]=iomg[i-1]*iomg[1]%MOD;

```

```

29     }
30 }
31
32 void NTT(int n, LL* P, LL* w){
33     for(int i=0, j=0; i<n; i++){
34         if(i<j) swap(P[i], P[j]);
35         for(int l=n>>1; (j^=1)<l; l>>=1);
36     }
37
38     for(int i=2, l; i<=n; i<=1){
39         l=i>>1;
40         for(int j=0; j<n; j+=i){
41             for(int k=0; k<l; k++){
42                 LL t=P[j+l+k]*w[n/i*k]%MOD;
43                 P[j+l+k]=(P[j+k]-t+MOD)%MOD;
44                 P[j+k]=(P[j+k]+t)%MOD;
45             }
46         }
47     }
48 }
49
50 void poly_inv(int dgr, LL* X, LL* Y){
51     if(dgr==1){
52         Y[0]=binpow(X[0], MOD-2, MOD);
53     }
54     else{
55         poly_inv((dgr+1)>>1, X, Y);
56
57         static LL Z[MAXN];
58         int lim;
59         for(lim=1; lim<(dgr<<1); lim<=1);
60         copy(X, X+dgr, Z);
61         fill(Z+dgr, Z+lim, 0);
62         init(lim);
63
64         NTT(lim, Z, omg);
65         NTT(lim, Y, omg);
66         for(int i=0; i<lim; i++)
67             Y[i]=(2-Z[i]*Y[i]%MOD+MOD)*Y[i]%MOD;
68         NTT(lim, Y, iomg);
69         LL invlim=binpow(lim, MOD-2, MOD);
70         for(int i=0; i<dgr; i++)
71             Y[i]=Y[i]*invlim%MOD;
72         fill(Y+dgr, Y+lim, 0);
73     }
74 }
75
76 int n;
77 LL A[MAXN], B[MAXN];
78
79 int main(){
80     scanf("%d", &n);
81     for(int i=0; i<n; i++)

```

```
82     scanf("%lld", &A[i]);
83     poly_inv(n,A,B);
84     for(int i=0;i<n;i++)
85         printf("%lld ", B[i]);
86     return 0;
87 }
```

## 3 字符串

### 3.1 字符串哈希 String\_Hash

```
1 #include <stdio>
2 #include <string>
3
4 #define BASE 307
5 #define MOD 5555567
6
7 int hsh(string x){
8     int h=0,len=x.length();
9     for(int i=0;i<len;i++){
10         h=(h*BASE+x[i])%MOD;
11     }
12     return h;
13 }
14
15 int main(){
16     return 0;
17 }
```

### 3.2 马拉车 Manacher

```
1 #include <stdio>
2 #include <cstring>
3 #include <algorithm>
4 #define MAXN 11000005
5 using namespace std;
6
7 int r[MAXN<<1],mx;
8 char st[MAXN<<1];
9
10 void manacher(char *s){
11     int len=strlen(s);
12     st[0]='$';
13     for(int i=0;i<len;i++){
14         st[i<<1|1]='#';
15         st[(i+1)<<1]=s[i];
16     }
17     len=len<<1|1;
18     st[len]='#';
19     st[len+1]='*';
20     r[1]=1;
21     mx=0;
22     for(int i=2,mid=1;i<=len;i++){
23         r[i]=min(mid+r[mid]-i,r[2*mid-i]);
24         for(;st[i-r[i]]==st[i+r[i]];r[i]++);
25         if(i+r[i]>mid+r[mid]) mid=i;
26         mx=max(mx,r[i]-1);
27     }
28 }
```



```
29
30 char s[MAXN];
31
32 int main(){
33     scanf("%s",s);
34     manacher(s);
35     printf("%d\n", mx);
36     return 0;
37 }
```

### 3.3 字符串匹配 KMP

```
1  #include <bits/stdc++.h>
2  using namespace std;
3  #define MAXN 1000005
4
5  int kmp(char *s1, char *s2, int *nxt){
6      int n=strlen(s1+1),m=strlen(s2+1);
7
8      nxt[0]=-1;
9      for(int i=1,k=-1;i<=m;i++){
10         //k初始值为-1. nxt数组可以查询boarder的boarder.
11         while(~k && s2[k+1]!=s2[i])
12             k=nxt[k];
13         nxt[i]=++k;
14     }
15
16     int cnt=0;
17     for(int i=1,k=0;i<=n;i++){
18         //匹配串前缀与模式串后缀比,上一位前缀的boarder的boarder也能匹配。
19         while(~k && s2[k+1]!=s1[i])
20             k=nxt[k];
21         if(m==++k){
22             cnt++;
23             printf("%d\n", i-m+1);
24         }
25     }
26     return cnt;
27 }
28
29 char s1[MAXN],s2[MAXN];
30 int nxt[MAXN];
31
32 int main(){
33     scanf("%s %s", s1+1, s2+1);
34     kmp(s1,s2,nxt);
35     int len=strlen(s2+1);
36     for(int i=1;i<=len;i++){
37         printf("%d ", nxt[i]);
38     }
39     return 0;
40 }
```

### 3.4 AC 自动机 AC-Automaton

```

1  #include <cstdio>
2  #include <cstring>
3  #include <map>
4  #include <queue>
5  #define MAXN 1000006
6  using namespace std;
7
8  struct trie{
9      int fail,mark,ch[26];
10 }tt[MAXN];
11
12 int tot;
13
14 void insert(char *s){
15     int len=strlen(s+1), cur=0;
16     for(int i=1;i<=len;i++){
17         int& next=tt[cur].ch[s[i]-'a'];
18         if(!next){
19             next=++tot;
20             tt[tot].mark=0;
21         }
22         cur=next;
23     }
24     tt[cur].mark++;
25 }
26
27 void getfail(){
28     queue<int> Q;
29     tt[0].fail=0;
30     tt[0].mark=0;
31     for(int i=0;i<26;i++)
32         if(tt[0].ch[i]) tt[tt[0].ch[i]].fail=0, Q.push(tt[0].ch[i]);
33     while(!Q.empty()){
34         int u=Q.front(); Q.pop();
35         for(int i=0;i<26;i++){
36             if(!tt[u].ch[i]) continue;
37             int k=tt[u].fail;
38             while(k && !tt[k].ch[i])
39                 k=tt[k].fail;
40             tt[tt[u].ch[i]].fail=tt[k].ch[i];
41             // tt[tt[u].ch[i]].mark+=tt[tt[k].ch[i]].mark;//如果需要重复统计, fail累加标记
42             Q.push(tt[u].ch[i]);
43         }
44     }
45 }
46
47 int query(char *s){
48     int len=strlen(s+1),ans=0;
49     for(int i=1,k=0;i<=len;i++){
50         while(k && !tt[k].ch[s[i]-'a'])
51             k=tt[k].fail;

```

```

52     k=tt[k].ch[s[i]-'a'];
53     ans+=tt[k].mark;
54
55     tt[k].mark=0;//清除该字符串的标记（只求
56 }
57     return ans;
58 }
59
60 int n;
61 char s[MAXN];
62
63 int main(){
64     scanf("%d", &n);
65     tot=0;
66     memset(tt,0,sizeof(tt));
67     for(int i=1;i<=n;i++){
68         scanf("%s", s+1);
69         insert(s);
70     }
71     scanf("%s", s+1);
72     getfail();
73     printf("%d\n", query(s));
74     return 0;
75 }

```

### 3.5 后缀数组 Suffix\_Array

```

1  #include <cstdio>
2  #include <cstring>
3  #include <algorithm>
4  #define MAXN 1000005
5  using namespace std;
6
7  //sa:排名对应的前缀, rk:前缀的排名, tp:第二关键字排名对应的前缀, tax:排名对应的个数
8  //height:排名i与i-1后缀的LCP(最长公共前缀)
9  int sa[MAXN], r1[MAXN], r2[MAXN], tax[MAXN], height[MAXN];
10 int *rk=r1, *tp=r2;
11 char s[MAXN];
12
13 void rsort(int n, int m){
14     memset(tax, 0, (m+1)*sizeof(tax[0]));
15     for(int i=1; i<=n; i++) tax[rk[i]]++; //当前排名装桶
16     for(int i=1; i<=m; i++) tax[i]+=tax[i-1]; //计算桶的名次
17     for(int i=n; i>=1; i--) sa[tax[rk[tp[i]]]--]=tp[i]; //按照第二关键字降序, 分配排名。
18 }
19
20 void get_sa(char* s){
21     //O(nlogn)
22     int n=strlen(s+1), m=0;
23     for(int i=1; i<=n; i++)
24         m=max(m, rk[i]=s[i]), tp[i]=i;
25     rsort(n, m);

```

```

26     for(int k=1,p=0;p<n;k<=1,m=p){
27         p=0;
28         //重制第二关键字
29         for(int i=n-k+1;i<=n;i++) tp[++p]=i; //后续为空,排前面
30         for(int i=1;i<=n;i++) if(sa[i]>k) tp[++p]=sa[i]-k; //按照第一关键字排第二关键字
31
32         rsort(n,m);
33
34         swap(tp,rk);
35         rk[sa[1]]=p=1;
36         for(int i=2;i<=n;i++){
37             rk[sa[i]]=(tp[sa[i]]==tp[sa[i-1]] && tp[sa[i]+k]==tp[sa[i-1]+k])?p:++p;
38         }
39     }
40
41     //利用height[rk[i+1]]>=height[rk[i]]-1
42     //O(n)
43     for(int i=1,k=0;i<=n;i++){
44         if(k) k--;
45         while(rk[i]>1 && s[i+k]==s[sa[rk[i]-1]+k]) k++;
46         height[rk[i]]=k;
47     }
48 }
49
50 int main(){
51     scanf("%s",s+1);
52     get_sa(s);
53     int len=strlen(s+1);
54     for(int i=1;i<=len;i++)
55         printf("%d ", sa[i]);
56     return 0;
57 }

```

### 3.6 后缀自动机 Suffix-Automaton

```

1  #include <bits/stdc++.h>
2  #define MAXN 1000005
3  #define LL long long
4  using namespace std;
5
6  struct SAM {
7      int len,link,cnt;
8      int ch[26];
9  }sam[MAXN<<1];
10
11 int sz,last;
12
13 void sam_init() {
14     sam[0].len=0;
15     sam[0].link=-1;
16     sam[0].cnt=0;
17     memset(sam[0].ch,0,sizeof(sam[0].ch));

```

```

18     sz=0;
19     last=0;
20 }
21
22 void sam_extend(int c) {
23     int cur=++sz;
24     sam[cur].len=sam[last].len+1;
25     memset(sam[cur].ch,0,sizeof(sam[cur].ch));
26
27     int p=last;
28     for(;~p && !sam[p].ch[c];p=sam[p].link)
29         sam[p].ch[c]=cur;
30
31     if(!~p) {
32         sam[cur].link=0;
33     } else {
34         int q=sam[p].ch[c];
35         if(sam[p].len+1==sam[q].len) {
36             sam[cur].link=q;
37         } else {
38             int clone=++sz;
39             sam[clone]=sam[q];
40             sam[clone].len=sam[p].len+1;
41             sam[clone].cnt=0;
42             sam[q].link=sam[cur].link=clone;
43             for(;~p && sam[p].ch[c]==q;p=sam[p].link)
44                 sam[p].ch[c]=clone;
45         }
46     }
47
48     last=cur;
49     sam[cur].cnt=1;
50 }
51
52 struct edge{
53     int to,next;
54 }e[MAXN<<1];
55
56 int tot,head[MAXN<<1];
57
58 void add(int x,int y) {
59     tot++;
60     e[tot].to=y;
61     e[tot].next=head[x];
62     head[x]=tot;
63 }
64
65 char s[MAXN];
66 LL ans;
67
68 void dfs(int x) {
69     for(int p=head[x];p;p=e[p].next) {
70         int u=e[p].to;

```

```

71     dfs(u);
72     sam[x].cnt+=sam[u].cnt;
73 }
74 if(sam[x].cnt!=1) ans=max(ans,1LL*sam[x].len*sam[x].cnt);
75 }
76
77 void solve() {
78     scanf("%s", s);
79     int len=strlen(s);
80     sam_init();
81     for(int i=0;i<len;i++)
82         sam_extend(s[i]-'a');
83     tot=0;
84     memset(head,0,(sz+1)*sizeof(head[0]));
85     for(int i=1;i<=sz;i++)
86         add(sam[i].link,i);
87     ans=0;
88     dfs(0);
89     printf("%lld\n", ans);
90 }
91
92 int main() {
93     int T=1,cas=1;
94     // scanf("%d", &T);
95     while(T--) {
96         // printf("Case #%d: ", cas++);
97         solve();
98     }
99     return 0;
100 }

```

### 3.7 广义后缀自动机 General\_Suffix-Automaton

```

1  #include <bits/stdc++.h>
2  #define MAXN 1000005
3  #define LL long long
4  using namespace std;
5  // const int inf=0x3f3f3f3f;
6
7  struct SAM {
8      int len,link,cnt;
9      int ch[26];
10 }sam[MAXN<<1];
11
12 int sz,last;
13
14 void sam_init() {
15     sam[0].len=0;
16     sam[0].link=-1;
17     sam[0].cnt=0;
18     memset(sam[0].ch,0,sizeof(sam[0].ch));
19     sz=0;

```

```
20     last=0;
21 }
22
23 void sam_extend(int c) {
24     if(sam[last].ch[c] && sam[last].len+1==sam[sam[last].ch[c]].len) {
25         last=sam[last].ch[c];
26         sam[last].cnt++;
27         return;
28     }
29
30     int cur=++sz;
31     sam[cur].len=sam[last].len+1;
32     memset(sam[cur].ch,0,sizeof(sam[cur].ch));
33
34     int p=last;
35     for(;~p && !sam[p].ch[c];p=sam[p].link)
36         sam[p].ch[c]=cur;
37
38     if(!~p) {
39         sam[cur].link=0;
40     } else {
41         int q=sam[p].ch[c];
42         if(sam[p].len+1==sam[q].len) {
43             sam[cur].link=q;
44         } else {
45             int clone;
46             if(p==last) {
47                 clone=cur;
48             } else {
49                 clone=++sz;
50                 sam[cur].link=clone;
51             }
52
53             sam[clone]=sam[q];
54             sam[clone].len=sam[p].len+1;
55             sam[q].link=clone;
56             sam[clone].cnt=0;
57             for(;~p && sam[p].ch[c]==q;p=sam[p].link)
58                 sam[p].ch[c]=clone;
59         }
60     }
61
62     last=cur;
63     sam[cur].cnt=1;
64 }
65
66 char s[MAXN];
67
68 void solve() {
69     int n;
70     scanf("%d", &n);
71     sam_init();
72     for(int i=1;i<=n;i++) {
```

```
73     scanf("%s", s);
74     int len=strlen(s);
75     last=0;
76     for(int j=0;j<len;j++) {
77         sam_extend(s[j]-'a');
78     }
79 }
80
81 LL ans=0;
82 for(int i=1;i<=sz;i++) {
83     ans+=sam[i].len-sam[sam[i].link].len;
84 }
85
86 printf("%lld\n", ans);
87 }
88
89 int main() {
90     int T=1,cas=1;
91     // scanf("%d", &T);
92     while(T--) {
93         // printf("Case #%d: ", cas++);
94         solve();
95     }
96     return 0;
97 }
```



## 4 数据结构

### 4.1 并查集 Disjoin\_Set\_Union

```
1  #include <stdio>
2  #define maxn 10005
3
4  int n,m,a,b,c,fa[maxn];
5
6  void ini(){
7      for(int i=1;i<=n;i++){
8          fa[i]=i;
9      }
10 }
11
12 int find(int x){
13     if(fa[x] == x)
14         return x;
15     return fa[x]=find(fa[x]);
16 }
17
18 void join(int x, int y){
19     fa[find(x)]=find(y);
20 }
21
22 int main(){
23     scanf("%d %d", &n, &m);
24     ini();
25     for(int i=1; i<=m; i++){
26         scanf("%d %d %d", &a, &b, &c);
27         if(a==1)
28             join(b,c);
29         else if(find(b)==find(c))
30             puts("Y");
31         else puts("N");
32     }
33     return 0;
34 }
```

### 4.2 可撤销并查集 Disjoin\_Set\_Union\_Withdrawable

```
1  struct DSU{
2      int fa[MAXN],rk[MAXN];
3      vector<pair<int*,int*>> stk;
4      void init(int n) {
5          stk.clear();
6          for(int i=1;i<=n;i++) fa[i]=i,rk[i]=1;
7      }
8      int find(int x) {
9          if(x==fa[x]) return x;
10         return find(fa[x]);
11     }
```

```

12     bool join(int x,int y) {
13         int rx=find(x),ry=find(y);
14         if(rx==ry) return false;
15
16         if(rk[rx]>rk[ry]) swap(rx,ry);
17         stk.emplace_back(fa+rx,rx);
18         fa[rx]=ry;
19         stk.emplace_back(rk+ry,rk[ry]);
20         rk[ry]+=rk[rx];
21         return true;
22     }
23     void withdraw() {
24         *stk.back().first=stk.back().second;
25         stk.pop_back();
26         *stk.back().first=stk.back().second;
27         stk.pop_back();
28     }
29 }dsu;

```

### 4.3 分块 Block\_1

```

1  #include <cstdio>
2  #include <cstring>
3  #include <cmath>
4  #include <algorithm>
5  #define MAXN 50005
6
7  using namespace std;
8
9  int n,blo,v[MAXN],bl[MAXN],atag[MAXN];
10
11 void add(int l,int r,int x){
12     if(bl[l]==bl[r]){
13         for(int i=l;i<=r;i++)
14             v[i]+=x;
15     }
16     else{
17         for(int i=l;i<=bl[l]*blo;i++)
18             v[i]+=x;
19         for(int i=(bl[r]-1)*blo+1;i<=r;i++)
20             v[i]+=x;
21     }
22     for(int i=bl[l]+1;i<=bl[r]-1;i++)
23         atag[i]+=x;
24 }
25
26 int main(){
27     scanf("%d",&n);blo=sqrt(n);
28     for(int i=1;i<=n;i++)
29         scanf("%d",&v[i]);
30     for(int i=1;i<=n;i++)
31         bl[i]=(i-1)/blo+1;

```

```

32     memset(atag+1,0,b1[n]*sizeof(atag[0]));
33     for(int i=1;i<=n;i++){
34         int opt,l,r,c;
35         scanf("%d %d %d %d", &opt, &l, &r, &c);
36         if(opt==0){
37             add(l,r,c);
38         }
39         else printf("%d\n", v[r]+atag[b1[r]]);
40     }
41     return 0;
42 }

```

#### 4.4 分块 Block\_2

```

1  #include <cstdio>
2  #include <cstring>
3  #include <cmath>
4  #include <algorithm>
5  #include <vector>
6  #define MAXN 50005
7  #define MAXB 505
8
9  using namespace std;
10
11 int n,blo,v[MAXN],b1[MAXN],atag[MAXB];
12 vector<int> ve[MAXB];
13
14 void reset(int x){
15     ve[x].clear();
16     for(int i=(x-1)*blo+1;i<=min(x*blo,n);i++){
17         ve[x].push_back(v[i]);
18     }
19     sort(ve[x].begin(),ve[x].end());
20 }
21
22 void add(int l,int r,int x){
23     if(b1[l]==b1[r]){
24         for(int i=l;i<=r;i++){
25             v[i]+=x;
26             reset(b1[l]);
27         }
28     }
29     else{
30         for(int i=l;i<=b1[l]*blo;i++)
31             v[i]+=x;
32         reset(b1[l]);
33         for(int i=(b1[r]-1)*blo+1;i<=r;i++){
34             v[i]+=x;
35             reset(b1[r]);
36             atag[i]+=x;
37         }
38     }

```

```

39
40 int query(int l,int r,int x){
41     int cnt=0;
42     if(bl[l]==bl[r]){
43         for(int i=l;i<=r;i++)
44             if(v[i]+atag[bl[i]]<x) cnt++;
45     }
46     else{
47         for(int i=l;i<=bl[l]*blo;i++)
48             if(v[i]+atag[bl[i]]<x) cnt++;
49         for(int i=(bl[r]-1)*blo+1;i<=r;i++)
50             if(v[i]+atag[bl[i]]<x) cnt++;
51         for(int i=bl[l]+1;i<=bl[r]-1;i++)
52             cnt+=lower_bound(ve[i].begin(),ve[i].end(),x-atag[i])-ve[i].begin();
53     }
54     return cnt;
55 }
56
57 int main(){
58     scanf("%d", &n);blo=sqrt(n);
59     for(int i=1;i<=n;i++)
60         scanf("%d", &v[i]);
61     for(int i=1;i<=n;i++){
62         bl[i]=(i-1)/blo+1;
63         ve[bl[i]].push_back(v[i]);
64     }
65     memset(atag+1,0,bl[n]*sizeof(atag[0]));
66     for(int i=1;i<=bl[n];i++)
67         sort(ve[i].begin(),ve[i].end());
68
69     for(int i=1;i<=n;i++){
70         int opt,l,r,c;
71         scanf("%d %d %d %d", &opt, &l, &r, &c);
72         if(opt==0){
73             add(l,r,c);
74         }
75         else{
76             printf("%d\n", query(l,r,c*c));
77         }
78     }
79     return 0;
80 }

```

## 4.5 分块 Block\_4

```

1 #include <stdio>
2 #include <cstring>
3 #include <cmath>
4 #include <algorithm>
5 #define LL long long
6 #define MAXN 50005
7 #define MAXB 505

```

```

8
9 using namespace std;
10
11 int n,blo;
12 LL v[MAXN],bl[MAXN],atag[MAXB],sum[MAXB];
13
14 void add(int l,int r,int x){
15     if(bl[l]==bl[r]){
16         for(int i=l;i<=r;i++)
17             v[i]+=x,sum[bl[i]]+=x;
18     }
19     else{
20         for(int i=l;i<=bl[l]*blo;i++)
21             v[i]+=x,sum[bl[i]]+=x;
22         for(int i=(bl[r]-1)*blo+1;i<=r;i++)
23             v[i]+=x,sum[bl[i]]+=x;
24         for(int i=bl[l]+1;i<=bl[r]-1;i++)
25             atag[i]+=x;
26     }
27 }
28
29 LL query(int l,int r){
30     LL ans=0;
31     if(bl[l]==bl[r]){
32         for(int i=l;i<=r;i++)
33             ans+=v[i]+atag[bl[i]];
34     }
35     else{
36         for(int i=l;i<=bl[l]*blo;i++)
37             ans+=v[i]+atag[bl[i]];
38         for(int i=(bl[r]-1)*blo+1;i<=r;i++)
39             ans+=v[i]+atag[bl[i]];
40         for(int i=bl[l]+1;i<=bl[r]-1;i++)
41             ans+=sum[i]+atag[i]*blo;
42     }
43     return ans;
44 }
45
46 int main(){
47     scanf("%d",&n);blo=sqrt(n);
48     for(int i=1;i<=n;i++)
49         bl[i]=(i-1)/blo+1;
50     memset(atag+1,0,bl[n]*sizeof(atag[0]));
51     memset(sum+1,0,bl[n]*sizeof(sum[0]));
52     for(int i=1;i<=n;i++){
53         scanf("%lld",&v[i]);
54         sum[bl[i]]+=v[i];
55     }
56     for(int i=1;i<=n;i++){
57         int opt,l,r,c;
58         scanf("%d %d %d %d",&opt,&l,&r,&c);
59         if(opt==0){
60             add(l,r,c);

```

```

61     }
62     else printf("%lld\n", query(1,r)%(c+1));
63 }
64 return 0;
65 }

```

## 4.6 树状数组 Binary\_Indexed\_Tree

```

1  #include <bits/stdc++.h>
2  #define LL long long
3  using namespace std;
4  const int MAXN=2000005;
5
6  struct BIT {
7      int n,b[MAXN];
8
9      int lowbit(int x){
10         return x&(-x);
11     }
12
13     void change(int x,int y){
14         for(;x<=n;x+=lowbit(x))
15             b[x]+=y;
16     }
17
18     int sum(int x){
19         int s=0;
20         for(;x>0;x-=lowbit(x))
21             s+=b[x];
22         return s;
23     }
24
25     void build(int len,int *a) {
26         n=len;
27         for(int i=1;i<=n;i++)
28             b[i]=a[i];
29         for(int x=1;x<<1<=n;x<<=1)
30             for(int i=x;i+x<=n;i+=x<<1)
31                 b[i+x]+=b[i];
32     }
33 }bit;
34
35 int a[MAXN];
36
37 void solve() {
38     int n,m;
39     scanf("%d %d", &n, &m);
40     for(int i=1;i<=n;i++)
41         scanf("%d", &a[i]);
42     bit.build(n,a);
43     for(int i=1;i<=m;i++) {
44         int opt,x,y;

```

```

45     scanf("%d %d %d", &opt, &x, &y);
46     if(opt==1) {
47         bit.change(x,y);
48     } else {
49         printf("%d\n", bit.sum(y)-bit.sum(x-1));
50     }
51 }
52 }
53
54 int main() {
55     int T=1,cas=1;(void)(cas);
56     // scanf("%d", &T);
57     while(T--) {
58         // printf("Case #%d: ", cas++);
59         solve();
60     }
61     return 0;
62 }

```

#### 4.7 二维树状数组 2D\_Binary\_Indexed\_Tree

```

1  #include <bits/stdc++.h>
2  const int MAXN=2005;
3
4  struct BIT {
5      int n,m;
6      int a[MAXN][MAXN];
7
8      int lowbit(int x){
9          return x&(-x);
10     }
11
12     void change(int x,int y,int k){
13         for(int i=x;i<=n;i+=lowbit(i)){
14             for(int j=y;j<=m;j+=lowbit(j)){
15                 a[i][j]+=k;
16             }
17         }
18     }
19
20     int sum(int x,int y){
21         int s=0;
22         for(int i=x;i>0;i-=lowbit(i)){
23             for(int j=y;j>0;j-=lowbit(j)){
24                 s+=a[i][j];
25             }
26         }
27         return s;
28     }
29 }bit;
30
31 int main(){

```

```

32     int n,m,H;
33     scanf("%d %d %d",&n, &m, &H);
34     bit.n=n;
35     bit.m=m;
36     for(int i=1;i<=n;i++)
37         for(int j=1;j<=m;j++)
38             bit.a[i][j]=0;
39     for(int i=1;i<=H;i++) {
40         int x,y;
41         scanf("%d %d", &x, &y);
42         bit.change(1,1,1);
43         bit.change(x+1,1,-1);
44         bit.change(1,y+1,-1);
45         bit.change(x+1,y+1,1);//1-2+1
46     }
47     for(int i=1;i<=n;i++)
48         for(int j=1;j<=m;j++)
49             printf("%d\n", bit.sum(i,j));
50 }

```

## 4.8 线段树 Segment\_Tree

```

1  #include <cstdio>
2  #include <algorithm>
3  #define LL long long
4  #define MAXN 100005
5  using namespace std;
6
7  struct SGT{
8      LL sum[MAXN<<2],tag[MAXN<<2];
9
10     void pushup(int x) {sum[x]=sum[x<<1]+sum[x<<1|1];}
11     void pushdown(int x,int l,int r) {
12         int m=(l+r)>>1;
13         sum[x<<1]+=tag[x]*(m-l+1);
14         tag[x<<1]+=tag[x];
15         sum[x<<1|1]+=tag[x]*(r-m);
16         tag[x<<1|1]+=tag[x];
17         tag[x]=0;
18     }
19
20     void build(int x,int l,int r,LL *a) {
21         tag[x]=0;
22         if(l==r) sum[x]=a[l];
23         else {
24             int m=(l+r)>>1;
25             build(x<<1,l,m,a);
26             build(x<<1|1,m+1,r,a);
27             pushup(x);
28         }
29     }
30 }

```



```

31 void modify(int x,int l,int r,int ql,int qr,LL delta) {
32     if(ql<=l && r<=qr) {
33         sum[x]+=delta*(r-l+1);
34         tag[x]+=delta;
35         return;
36     }
37     if(tag[x]) pushdown(x,l,r);
38     int m=(l+r)>>1;
39     if(ql<=m) modify(x<<1,l,m,ql,qr,delta);
40     if(m<qr) modify(x<<1|1,m+1,r,ql,qr,delta);
41     // sum[x]=tag[x]*(r-l+1)+sum[x<<1]+sum[x<<1|1];
42     pushup(x);
43 }
44
45 LL query(int x,int l,int r,int ql,int qr){
46     if(ql<=l && r<=qr) return sum[x];
47
48     if(tag[x]) pushdown(x,l,r);
49     int m=(l+r)>>1;
50     LL res=0;
51     if(ql<=m) res+=query(x<<1,l,m,ql,qr);
52     if(m<qr) res+=query(x<<1|1,m+1,r,ql,qr);
53     // res+=tag[x]*(min(qr,r)-max(ql,l)+1);
54     return res;
55 }
56 }sgt;
57
58 LL a[MAXN];
59
60 int main(){
61     int n,m;
62     scanf("%d %d",&n,&m);
63     for(int i=1;i<=n;i++) scanf("%lld", &a[i]);
64     sgt.build(1,1,n,a);
65     for(int i=1;i<=m;i++){
66         int op,l,r;
67         scanf("%d", &op);
68         if(op==1){
69             LL k;
70             scanf("%d %d %lld",&l,&r,&k);
71             sgt.modify(1,1,n,l,r,k);
72         }
73         else{
74             scanf("%d %d",&l,&r);
75             printf("%lld\n", sgt.query(1,1,n,l,r));
76         }
77     }
78     return 0;
79 }

```

#### 4.9 线段树 Segment\_Tree\_Multiply

```

1  #include <bits/stdc++.h>
2  #define LL long long
3  using namespace std;
4  const int MAXN=100005;
5  const int MOD=571373;
6
7  struct SGT{
8      LL sum[MAXN<<2],ts[MAXN<<2],tm[MAXN<<2];
9
10     void pushup(int x) {sum[x]=(sum[x<<1]+sum[x<<1|1])%MOD;}
11     void pushdown(int x,int l,int r) {
12         int m=(l+r)>>1;
13         sum[x<<1]=(sum[x<<1]*tm[x]%MOD + ts[x]*(m-l+1)%MOD)%MOD;
14         sum[x<<1|1]=(sum[x<<1|1]*tm[x]%MOD + ts[x]*(r-m)%MOD)%MOD;
15         tm[x<<1]=tm[x<<1]*tm[x]%MOD;
16         tm[x<<1|1]=tm[x<<1|1]*tm[x]%MOD;
17         ts[x<<1]=(ts[x<<1]*tm[x]%MOD+ts[x])%MOD;
18         ts[x<<1|1]=(ts[x<<1|1]*tm[x]%MOD+ts[x])%MOD;
19         tm[x]=1;
20         ts[x]=0;
21     }
22
23     void build(int x,int l,int r,LL* a) {
24         ts[x]=0,tm[x]=1;
25         if(l==r) sum[x]=a[l];
26         else {
27             int m=(l+r)>>1;
28             build(x<<1,l,m,a);
29             build(x<<1|1,m+1,r,a);
30             pushup(x);
31         }
32     }
33
34     void plus(int x,int l,int r,int ql,int qr,LL d) {
35         if(ql<=l && r<=qr) {
36             sum[x]=(sum[x]+d*(r-l+1)%MOD)%MOD;
37             ts[x]=(ts[x]+d)%MOD;
38             return;
39         }
40
41         pushdown(x,l,r);
42         int m=(l+r)>>1;
43         if(ql<=m) plus(x<<1,l,m,ql,qr,d);
44         if(m<qr) plus(x<<1|1,m+1,r,ql,qr,d);
45         pushup(x);
46     }
47
48     void multi(int x,int l,int r,int ql,int qr,LL d) {
49         if(ql<=l && r<=qr) {
50             sum[x]=(sum[x]*d)%MOD;
51             ts[x]=(ts[x]*d)%MOD;
52             tm[x]=(tm[x]*d)%MOD;
53             return;

```

```

54     }
55
56     pushdown(x,l,r);
57     int m=(l+r)>>1;
58     if(ql<=m) multi(x<<1,l,m,ql,qr,d);
59     if(m<qr) multi(x<<1|1,m+1,r,ql,qr,d);
60     pushup(x);
61 }
62
63 LL query(int x,int l,int r,int ql,int qr) {
64     if(ql<=l && r<=qr) {
65         return sum[x];
66     }
67
68     pushdown(x,l,r);
69     int m=(l+r)>>1;
70     LL res=0;
71     if(ql<=m) res=(res+query(x<<1,l,m,ql,qr))%MOD;
72     if(m<qr) res=(res+query(x<<1|1,m+1,r,ql,qr))%MOD;
73     return res;
74 }
75 }sgt;
76
77 LL a[MAXN];
78
79 int main(){
80     int n,m,p;
81     scanf("%d %d %d", &n, &m, &p);
82     for(int i=1;i<=n;i++) scanf("%lld", &a[i]);
83     sgt.build(1,1,n,a);
84     for(int i=1;i<=m;i++){
85         int op,l,r;
86         scanf("%d", &op);
87         if(op==1){
88             LL k;
89             scanf("%d %d %lld", &l, &r, &k);
90             sgt.multi(1,1,n,l,r,k);
91         } else if(op==2) {
92             LL k;
93             scanf("%d %d %lld", &l, &r, &k);
94             sgt.plus(1,1,n,l,r,k);
95         } else {
96             scanf("%d %d",&l,&r);
97             printf("%lld\n", sgt.query(1,1,n,l,r));
98         }
99     }
100     return 0;
101 }

```

## 4.10 扫描线 Scanline

```
1 #include <stdio>
```

```

2  #include <algorithm>
3  #define LL long long
4  #define MAXN 100005
5  using namespace std;
6
7  struct line{
8      int x,y1,y2,sign;
9      bool operator<(line b)const{
10         if(x!=b.x) return x<b.x;
11         else return sign>b.sign;
12     }
13 }li[MAXN<<1];
14
15 struct node{
16     int le,ri;
17     int cnt,len;
18 }sgt[MAXN<<3];
19
20 int dy[MAXN<<1];
21
22 void pushup(int cur){
23     if(sgt[cur].cnt)
24         sgt[cur].len=dy[sgt[cur].ri]-dy[sgt[cur].le];
25     else if(sgt[cur].le<sgt[cur].ri-1)
26         sgt[cur].len=sgt[cur<<1].len+sgt[cur<<1|1].len;
27     else sgt[cur].len=0;
28 }
29
30 void build(int cur,int l,int r){
31     sgt[cur].le=l, sgt[cur].ri=r;
32     sgt[cur].cnt=sgt[cur].len=0;
33     if(l<r-1){
34         build(cur<<1,l,(l+r)>>1);
35         build(cur<<1|1,(l+r)>>1,r);
36     }
37 }
38
39 void modify(int cur,int l,int r,int sign){
40     if(l<=sgt[cur].le && sgt[cur].ri<=r){
41         sgt[cur].cnt+=sign;
42     }
43     else{
44         int mid=(sgt[cur].le+sgt[cur].ri)>>1;
45         if(l<mid) modify(cur<<1,l,r,sign);
46         if(r>mid) modify(cur<<1|1,l,r,sign);
47     }
48     pushup(cur);
49 }
50
51
52 int main(){
53     int n,cnt;
54     scanf("%d", &n);

```

```

55     cnt=0;
56     for(int i=1;i<=n;i++){
57         int x1,y1,x2,y2;
58         scanf("%d %d %d %d", &x1, &y1, &x2, &y2);
59         li[(i<<1)-1].x=x1, li[i<<1].x=x2;
60         li[(i<<1)-1].y1=li[i<<1].y1=y1;
61         li[(i<<1)-1].y2=li[i<<1].y2=y2;
62         li[(i<<1)-1].sign=1, li[i<<1].sign=-1;
63         dy[++cnt]=y1, dy[++cnt]=y2;
64     }
65     sort(dy+1,dy+cnt+1);
66     cnt=unique(dy+1,dy+cnt+1)-dy-1;
67     for(int i=1;i<=(n<<1);i++){
68         li[i].y1=lower_bound(dy+1,dy+cnt+1,li[i].y1)-dy;
69         li[i].y2=lower_bound(dy+1,dy+cnt+1,li[i].y2)-dy;
70     }
71     sort(li+1,li+(n<<1)+1);
72     build(1,1,cnt);
73     LL sum=0;
74     for(int i=1;i<(n<<1);i++){
75         modify(1,li[i].y1,li[i].y2,li[i].sign);
76         sum+=1LL*sgt[1].len*(li[i+1].x-li[i].x);
77     }
78     printf("%lld\n", sum);
79     return 0;
80 }

```

#### 4.11 zkw 线段树 ZKW\_Segment\_Tree

```

1  #include <stdio>
2  #include <cstring>
3  #define LL long long
4  #define MAXN 100005
5
6  struct node{
7      LL sum,tag;
8  }sgt[MAXN<<2];
9
10 int M;
11
12 int a[MAXN];
13
14 void built(int n){
15     for(M=1;M<n+2;M<=<1);
16     memset(sgt+M,0,M*sizeof(sgt[0]));
17     for(int i=1;i<=n;i++)
18         sgt[M+i].sum=a[i];
19     for(int i=M-1;i;i--)
20         sgt[i].sum=sgt[i<<1].sum+sgt[i<<1|1].sum;
21 }
22
23 void modify(int l,int r,LL del){

```

```

24 LL len=1,lc=0,rc=0;
25 for(l=l+M-1,r=r+M+1;l^r^1;l>>=1,r>>=1,len<=1){
26     if(~l&1) sgt[l+1].tag+=del, lc+=len;
27     if(r&1) sgt[r-1].tag+=del, rc+=len;
28     sgt[l>>1].sum+=del*lc;
29     sgt[r>>1].sum+=del*rc;
30 }
31 for(lc+=rc,l>>=1;l>>=1)
32     sgt[l].sum+=del*lc;
33 }
34
35 LL query(int l,int r){
36     LL res=0,len=1,lc=0,rc=0;
37     for(l=l+M-1,r=r+M+1;l^r^1;l>>=1,r>>=1,len<=1){
38         if(~l&1) res+=sgt[l+1].sum+sgt[l+1].tag*len, lc+=len;
39         if(r&1) res+=sgt[r-1].sum+sgt[r-1].tag*len, rc+=len;
40         res+=sgt[l>>1].tag*lc;
41         res+=sgt[r>>1].tag*rc;
42     }
43     for(lc+=rc,l>>=1;l>>=1)
44         res+=sgt[l].tag*lc;
45     return res;
46 }
47
48 int main(){
49     int n,m;
50     scanf("%d %d", &n, &m);
51     for(int i=1;i<=n;i++)
52         scanf("%d", &a[i]);
53     built(n);
54     for(int i=1;i<=m;i++){
55         int opt,x,y;
56         scanf("%d %d %d", &opt, &x, &y);
57         if(opt==1){
58             int k;
59             scanf("%d", &k);
60             modify(x,y,k);
61         }
62         else{
63             printf("%lld\n", query(x,y));
64         }
65     }
66     return 0;
67 }

```

#### 4.12 李超线段树 Li-Chao\_Segment\_Tree

```

1 #include <stdio>
2 #include <algorithm>
3 #define N 39989
4 #define MAXN 40005
5 #define MAXT 160005

```

```

6  const double eps=1e-12;
7  const double inf=1e9;
8
9  using namespace std;
10
11 struct line{
12     int l,r;
13     double k,b;
14     int id;
15 }sgt[MAXT];
16
17 double calc(line l,int x){return l.k*x+l.b;}
18
19 void modify(int cur,int l,int r,line li){
20     if(li.l<=l && r<=li.r){
21         if(calc(li,l)-calc(sgt[cur],l)>eps && calc(li,r)-calc(sgt[cur],r)>eps)
22             sgt[cur]=li;
23         else if(calc(li,l)-calc(sgt[cur],l)>eps || calc(li,r)-calc(sgt[cur],r)>eps){
24             int mid=(l+r)>>1;
25             if(calc(li,mid)-calc(sgt[cur],mid)>eps)
26                 swap(li,sgt[cur]);
27             if(calc(li,l)-calc(sgt[cur],l)>eps)
28                 modify(cur<<1,l,mid,li);
29             else modify(cur<<1|1,mid+1,r,li);
30         }
31     }
32     else{
33         int mid=(l+r)>>1;
34         if(li.l<=mid) modify(cur<<1,l,mid,li);
35         if(li.r>mid) modify(cur<<1|1,mid+1,r,li);
36     }
37 }
38
39 line query(int cur,int l,int r,int x){
40     if(l==r) return sgt[cur];
41     else{
42         int mid=(l+r)>>1;
43         line t;
44         if(x<=mid) t=query(cur<<1,l,mid,x);
45         else t=query(cur<<1|1,mid+1,r,x);
46         if(!t.id || calc(sgt[cur],x)-calc(t,x)>eps) return sgt[cur];
47         else return t;
48     }
49 }
50
51 void built(int cur,int l,int r){
52     sgt[cur].k=sgt[cur].b=0;
53     sgt[cur].l=1; sgt[cur].r=N;
54     sgt[cur].id=0;
55     if(l<r){
56         int mid=(l+r)>>1;
57         built(cur<<1,l,mid);
58         built(cur<<1|1,mid+1,r);

```

```

59     }
60 }
61
62 int n;
63
64 int main(){
65     scanf("%d", &n);
66     built(1,1,N);
67     int last=0,id=0;
68     for(int i=1;i<=n;i++){
69         int opt;
70         scanf("%d", &opt);
71         if(opt==0){
72             int x;
73             scanf("%d", &x);
74             x=(x+last-1)%N+1;
75             printf("%d\n", last=query(1,1,N,x).id);
76         }
77         else{
78             int x0,x1,y0,y1;
79             scanf("%d %d %d %d", &x0,&y0,&x1,&y1);
80             x0=(x0+last-1)%N+1;
81             x1=(x1+last-1)%N+1;
82             y0=(y0+last-1)%1000000000+1;
83             y1=(y1+last-1)%1000000000+1;
84             line t;
85             t.id=++id;
86             t.l=min(x0,x1); t.r=max(x0,x1);
87             t.k=x1==x0?(double)(y1-y0)/(x1-x0);
88             t.b=x1==x0?max(y0,y1):y0-t.k*x0;
89             modify(1,1,N,t);
90         }
91     }
92     return 0;
93 }

```

### 4.13 可并堆左偏树 Leftist\_Tree

```

1  #include <cstdio>
2  #include <algorithm>
3  #define MAXN 100005
4
5  using namespace std;
6
7  int n,m;
8
9  struct node{
10     int rt,lc,rc,dis,v;
11 }lt[MAXN];
12
13 int find(int x){
14     if(lt[x].rt==x)

```



```

15     return x;
16     return lt[x].rt=find(lt[x].rt);
17 }
18
19 int merge(int x,int y){
20     if(!x || !y) return x+y;
21     if(lt[x].v>lt[y].v || (lt[x].v==lt[y].v && x>y)) swap(x,y); //后一个条件蜜汁优化?
22     lt[x].rc=merge(lt[x].rc,y);
23     lt[lt[x].rc].rt=x;
24     if(lt[lt[x].lc].dis<lt[lt[x].rc].dis) swap(lt[x].lc,lt[x].rc);
25     lt[x].dis=lt[lt[x].rc].dis+1;
26     return x;
27 }
28
29 void pop(int x){
30     lt[x].v=-1;
31     lt[lt[x].lc].rt=lt[x].lc;
32     lt[lt[x].rc].rt=lt[x].rc;
33     lt[x].rt=merge(lt[x].lc,lt[x].rc);
34 }
35
36 int main(){
37     scanf("%d %d", &n, &m);
38     for(int i=1;i<=n;i++){
39         scanf("%d", &lt[i].v);
40         lt[i].rt=i;
41         lt[i].lc=lt[i].rc=0;
42         lt[i].dis=0;
43     }
44     lt[0].dis=0;
45     for(int i=1;i<=m;i++){
46         int opt;
47         scanf("%d",&opt);
48         if(opt==1){
49             int x,y;
50             scanf("%d %d", &x, &y);
51             int rx=find(x),ry=find(y);
52             if(lt[x].v== -1 || lt[y].v== -1 || rx==ry)
53                 continue;
54             merge(rx,ry);
55         }
56         else{
57             int x;
58             scanf("%d", &x);
59             if(lt[x].v== -1)
60                 printf("-1\n");
61             else{
62                 int rx=find(x);
63                 printf("%d\n", lt[rx].v);
64                 pop(rx);
65             }
66         }
67     }

```

```

68     return 0;
69 }

```

#### 4.14 Splay 树 Splay\_Tree

```

1  #include <cstdio>
2  #define MAXN 100005
3  const int inf=0x3f3f3f3f;
4
5  int root,len;
6
7  struct node{
8      int v,fa,ch[2],size,cnt;
9  }sp[MAXN];
10
11 int getch(int x) {return sp[x].fa.ch[1]==x;}
12 void pushup(int x) {sp[x].size=sp[x].cnt+sp[sp[x].ch[0]].size+sp[sp[x].ch[1]].size;}
13
14 void rotate(int x){
15     int f=sp[x].fa, ff=sp[f].fa;
16     int k=getch(x);
17     sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
18     sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
19     sp[x].ch[k^1]=f; sp[f].fa=x;
20     pushup(f); pushup(x);
21 }
22
23 void splay(int x,int goal=0){
24     for(int f;(f=sp[x].fa)!=goal;rotate(x)){
25         if(sp[f].fa!=goal)
26             rotate(getch(x)==getch(f)?f:x);
27     }
28     if(!goal) root=x;
29 }
30
31 void insert(int x){
32     int cur=root,f=0;
33     while(cur&&sp[cur].v!=x){
34         f=cur;
35         cur=sp[cur].ch[x>sp[cur].v];
36     }
37     if(cur)
38         sp[cur].cnt++;
39     else{
40         cur=++len;
41         sp[f].ch[x>sp[f].v]=cur;
42         sp[cur].ch[0]=sp[cur].ch[1]=0;
43         sp[cur].fa=f;
44         sp[cur].v=x;
45         sp[cur].cnt=sp[cur].size=1;
46     }
47     splay(cur);

```

```

48 }
49
50 void find(int x){
51     int cur=root;
52     while(x!=sp[cur].v && sp[cur].ch[x>sp[cur].v])
53         cur=sp[cur].ch[x>sp[cur].v];
54     splay(cur);
55 }
56
57 int kth(int x){
58     if(sp[root].size<x) return 0;
59     int cur=root;
60     while(1){
61         if(x<=sp[sp[cur].ch[0]].size)
62             cur=sp[cur].ch[0];
63         else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
64             x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
65             cur=sp[cur].ch[1];
66         }
67         else return sp[cur].v;
68     }
69 }
70
71 int pre(int x){
72     find(x);
73     if(x>sp[root].v) return root;
74     int cur=sp[root].ch[0];
75     while(sp[cur].ch[1])
76         cur=sp[cur].ch[1];
77     return cur;
78 }
79
80 int succ(int x){
81     find(x);
82     if(x<sp[root].v) return root;
83     int cur=sp[root].ch[1];
84     while(sp[cur].ch[0])
85         cur=sp[cur].ch[0];
86     return cur;
87 }
88
89 void erase(int x){
90     int last=pre(x),next=succ(x),del;
91     splay(last);splay(next,last);
92     del=sp[next].ch[0];
93     if(sp[del].cnt>1){
94         sp[del].cnt--;
95         splay(del);
96     }
97     else{
98         sp[next].ch[0]=0;
99         sp[del].fa=0;
100        sp[del]=sp[len];

```

```

101     int f=sp[del].fa;
102     sp[f].ch[(sp[f].ch[1]==len)]=del;
103     sp[sp[del].ch[0]].fa=del;
104     sp[sp[del].ch[1]].fa=del;
105     if(root==len) root=del;
106     len--;
107
108 }
109 }
110
111
112 int n;
113 int main(){
114     scanf("%d", &n);
115     root=0;len=0;
116     insert(-inf);insert(inf);
117     sp[0].size=0;
118     for(int i=1;i<=n;i++){
119         int opt,x;
120         scanf("%d %d", &opt, &x);
121         if(opt==1){
122             insert(x);
123         }
124         else if(opt==2){
125             erase(x);
126         }
127         else if(opt==3){
128             find(x);
129             printf("%d\n", sp[sp[root].ch[0]].size);
130         }
131         else if(opt==4){
132             printf("%d\n", kth(x+1));
133         }
134         else if(opt==5){
135             printf("%d\n", sp[pre(x)].v);
136         }
137         else{
138             printf("%d\n", sp[succ(x)].v);
139         }
140     }
141     return 0;
142 }

```

#### 4.15 Splay 树 Splay\_Tree\_Flip

```

1 #include <cstdio>
2 #include <algorithm>
3 #define MAXN 100005
4 const int inf=0x3f3f3f3f;
5
6 using namespace std;
7

```

```

8  int root,len,a[MAXN];
9
10 struct node{
11     int v,fa,ch[2],size,cnt,tag;
12 }sp[MAXN];
13
14 int getch(int x) {return sp[sp[x].fa].ch[1]==x;}
15 void pushup(int x) {sp[x].size=sp[x].cnt+sp[sp[x].ch[0]].size+sp[sp[x].ch[1]].size;}
16
17 void pushdown(int x){
18     if(sp[x].tag){
19         sp[sp[x].ch[0]].tag^=1;
20         sp[sp[x].ch[1]].tag^=1;
21         swap(sp[x].ch[0],sp[x].ch[1]);
22         sp[x].tag=0;
23     }
24 }
25
26 void rotate(int x){
27     int f=sp[x].fa, ff=sp[f].fa;
28     int k=getch(x);
29     sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
30     sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
31     sp[x].ch[k^1]=f; sp[f].fa=x;
32     pushup(f); pushup(x);
33 }
34
35 void splay(int x,int goal=0){
36     for(int f;(f=sp[x].fa)!=goal;rotate(x)){
37         if(sp[f].fa!=goal)
38             rotate(getch(x)==getch(f)?f:x);
39     }
40     if(!goal) root=x;
41 }
42
43 int find(int x){
44     int cur=root;
45     while(1){
46         pushdown(cur);
47         if(x<=sp[sp[cur].ch[0]].size)
48             cur=sp[cur].ch[0];
49         else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
50             x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
51             cur=sp[cur].ch[1];
52         }
53         else return cur;
54     }
55 }
56
57 int built(int f,int l,int r){
58     if(l>r) return 0;
59     int mid=(l+r)>>1, cur=++len;
60     sp[cur].fa=f;

```

```

61     sp[cur].cnt=1;
62     sp[cur].v=a[mid];
63     sp[cur].tag=0;
64     sp[cur].ch[0]=built(cur,l,mid-1);
65     sp[cur].ch[1]=built(cur,mid+1,r);
66     pushup(cur);
67     return cur;
68 }
69
70 void flip(int l,int r){
71     int last=find(l-1),next=find(r+1);
72     splay(last);splay(next,last);
73     sp[sp[sp[root].ch[1]].ch[0]].tag^=1;
74 }
75
76 void dfs(int cur){
77     pushdown(cur);
78     if(sp[cur].ch[0]) dfs(sp[cur].ch[0]);
79     if(sp[cur].v!=-inf && sp[cur].v!=inf) printf("%d ", sp[cur].v);
80     if(sp[cur].ch[1]) dfs(sp[cur].ch[1]);
81 }
82
83 int n,m;
84
85 int main(){
86     scanf("%d %d", &n, &m);
87     for(int i=1;i<=n;i++) a[i+1]=i;
88     a[1]=-inf;a[n+2]=inf;
89     len=0;
90     root=built(0,1,n+2);
91     sp[0].size=0;
92     for(int i=1;i<=m;i++){
93         int l,r;
94         scanf("%d %d", &l, &r);
95         flip(l+1,r+1);
96     }
97     dfs(root);
98     return 0;
99 }

```

#### 4.16 Splay 树 Splay\_Tree\_Dye&Flip

```

1  #include <cstdio>
2  #include <cstring>
3  #include <algorithm>
4  #define MAXN 500005
5  const int inf=0x3f3f3f3f;
6  using namespace std;
7
8  struct node{
9      int v,fa,ch[2],cnt;//basic
10     int size,sum,lm,rm,mm;//pushup

```

```

11     int flip,color;//pushdown
12 }sp[MAXN];
13
14 int a[MAXN],len,root,recy[MAXN],rlen;
15
16 int getch(int x){return sp[sp[x].fa].ch[1]==x;}
17
18 void pushup(int x){
19     int lc=sp[x].ch[0],rc=sp[x].ch[1];
20     sp[x].size=sp[lc].size+sp[rc].size+sp[x].cnt;
21     sp[x].sum=sp[lc].sum+sp[rc].sum+sp[x].v;
22     sp[x].lm=max(sp[lc].lm, sp[lc].sum+sp[x].v+sp[rc].lm);
23     sp[x].rm=max(sp[rc].rm, sp[rc].sum+sp[x].v+sp[lc].rm);
24     sp[x].mm=max(max(sp[lc].mm,sp[rc].mm),sp[lc].rm+sp[x].v+sp[rc].lm);
25 }
26
27 void pushdown(int x){
28     int lc=sp[x].ch[0],rc=sp[x].ch[1];
29     if(sp[x].color!=inf){
30         if(lc){
31             sp[lc].v=sp[lc].color=sp[x].color;
32             sp[lc].sum=sp[lc].size*sp[x].color;
33         }
34         if(rc){
35             sp[rc].v=sp[rc].color=sp[x].color;
36             sp[rc].sum=sp[rc].size*sp[x].color;
37         }
38         if(sp[x].color>0){
39             if(lc) sp[lc].lm=sp[lc].rm=sp[lc].mm=sp[lc].sum;
40             if(rc) sp[rc].lm=sp[rc].rm=sp[rc].mm=sp[rc].sum;
41         }
42         else{
43             if(lc) {sp[lc].lm=sp[lc].rm=0; sp[lc].mm=sp[lc].v;}
44             if(rc) {sp[rc].lm=sp[rc].rm=0; sp[rc].mm=sp[rc].v;}
45         }
46         sp[x].color=inf;
47         sp[x].flip=0;
48     }
49     else if(sp[x].flip){
50         if(lc){
51             sp[lc].flip^=1;
52             swap(sp[lc].ch[0],sp[lc].ch[1]);
53             swap(sp[lc].lm,sp[lc].rm);
54         }
55         if(rc){
56             sp[rc].flip^=1;
57             swap(sp[rc].ch[0],sp[rc].ch[1]);
58             swap(sp[rc].lm,sp[rc].rm);
59         }
60         sp[x].flip=0;
61     }
62 }
63

```

```

64 void rotate(int x){
65     int f=sp[x].fa, ff=sp[f].fa;
66     int k=getch(x);
67     sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
68     sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
69     sp[x].ch[k^1]=f; sp[f].fa=x;
70     pushup(f); pushup(x);
71 }
72
73 void splay(int x,int goal=0){
74     for(int f;(f=sp[x].fa)!=goal;rotate(x)){
75         if(sp[f].fa!=goal)
76             rotate(getch(x)==getch(f)?f:x);
77     }
78     if(!goal) root=x;
79 }
80
81 int find(int x){
82     int cur=root;
83     while(1){
84         pushdown(cur);
85         if(x<=sp[sp[cur].ch[0]].size)
86             cur=sp[cur].ch[0];
87         else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
88             x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
89             cur=sp[cur].ch[1];
90         }
91         else return cur;
92     }
93 }
94
95 int built(int f,int l,int r){
96     if(l>r) return 0;
97     int mid=(l+r)>>1, cur=rlen?recy[rlen--]:++len;
98     sp[cur].v=a[mid];
99     sp[cur].fa=f;
100    sp[cur].cnt=1;
101    sp[cur].flip=0;
102    sp[cur].color=inf;
103    sp[cur].ch[0]=built(cur,l,mid-1);
104    sp[cur].ch[1]=built(cur,mid+1,r);
105    pushup(cur);
106    return cur;
107 }
108
109 void insert(int pos,int tot){
110     int l=find(pos),r=find(pos+1);
111     splay(l);splay(r,l);
112     sp[r].ch[0]=built(r,1,tot);
113     pushup(r); pushup(l);
114 }
115
116 void recycle(int x){

```



```
117     if(!x) return;
118     recycle(sp[x].ch[0]);
119     recycle(sp[x].ch[1]);
120     sp[sp[x].fa].ch[getch(x)]=0;
121     recy[++rlen]=x;
122 }
123
124 void erase(int pos,int tot){
125     int l=find(pos-1),r=find(pos+tot);
126     splay(l);splay(r,l);
127     recycle(sp[r].ch[0]);
128     pushup(r); pushup(l);
129 }
130
131 void dye(int pos,int tot,int c){
132     int l=find(pos-1),r=find(pos+tot);
133     splay(l);splay(r,l);
134     int x=sp[r].ch[0];
135     sp[x].color=c;
136     sp[x].v=c;
137     sp[x].sum=sp[x].size*c;
138     if(c>0)
139         sp[x].lm=sp[x].rm=sp[x].mm=sp[x].sum;
140     else{
141         sp[x].lm=sp[x].rm=0;
142         sp[x].mm=sp[x].v;
143     }
144     pushup(r); pushup(l);
145 }
146
147 void reverse(int pos,int tot){
148     int l=find(pos-1),r=find(pos+tot);
149     splay(l);splay(r,l);
150     int x=sp[r].ch[0];
151     sp[x].flip^=1;
152     swap(sp[x].ch[0],sp[x].ch[1]);
153     swap(sp[x].lm,sp[x].rm);
154     pushup(r); pushup(l);
155 }
156
157 int getsum(int pos,int tot){
158     int l=find(pos-1),r=find(pos+tot);
159     splay(l);splay(r,l);
160     return sp[sp[r].ch[0]].sum;
161 }
162
163 int n,m;
164
165 int main(){
166     scanf("%d %d", &n, &m);
167     for(int i=1;i<=n;i++){
168         scanf("%d", &a[i+1]);
169     }
```

```

170     memset(sp,0,sizeof(sp[0]));
171     sp[0].mm=a[1]=a[n+2]=-inf;
172     rlen=0;
173     len=0;
174     root=built(0,1,n+2);
175     for(int i=1;i<=m;i++){
176         char opt[10];
177         scanf("%s", opt);
178         if(opt[0]=='I'){//Insert
179             int pos,tot;
180             scanf("%d %d", &pos, &tot);
181             for(int i=1;i<=tot;i++)
182                 scanf("%d", &a[i]);
183             insert(pos+1,tot);
184         }
185         else if(opt[0]=='D'){//Delete
186             int pos,tot;
187             scanf("%d %d", &pos, &tot);
188             erase(pos+1,tot);
189         }
190         else if(opt[2]=='K'){//Make-Same
191             int pos,tot,c;
192             scanf("%d %d %d", &pos, &tot, &c);
193             dye(pos+1,tot,c);
194         }
195         else if(opt[0]=='R'){//Reverse
196             int pos,tot;
197             scanf("%d %d", &pos, &tot);
198             reverse(pos+1,tot);
199         }
200         else if(opt[0]=='G'){//Get-Sum
201             int pos,tot;
202             scanf("%d %d", &pos, &tot);
203             printf("%d\n", getsum(pos+1,tot));
204         }
205         else if(opt[0]=='M'){//Max-Sum
206             printf("%d\n", sp[root].mm);
207         }
208     }
209     return 0;
210 }

```

## 5 数论

### 5.1 乘法逆元 Multiplicative\_Inverse\_Modulo

```

1  /*
2     Coded with Leachim's ACM Template.
3     No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"
9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const LL INF=0x3f3f3f3f3f3f3f3f;
13 const double eps=1e-7;
14 const int dx[4]={1,-1,0,0};
15 const int dy[4]={0,0,1,-1};
16 const int RT=3;
17 const int MOD=998244353;
18 const int MAXN=20000005;
19
20 int inv[MAXN],f[MAXN],fi[MAXN];
21
22 void pre_inv(int n,int p) {
23     inv[1]=1;
24     f[0]=fi[0]=f[1]=fi[1]=1;
25     for(int i=2;i<=n;i++) {
26         inv[i]=1LL*(p-p/i)*inv[p/i]%p;
27         f[i]=1LL*f[i-1]*i%p;
28         fi[i]=1LL*fi[i-1]*inv[i]%p;
29     }
30 }
31
32 void solve() {
33     int n,p;
34     scanf("%d %d", &n, &p);
35     pre_inv(n,p);
36     for(int i=1;i<=n;i++)
37         printf("%d\n", inv[i]);
38 }
39
40 int main() {
41     int T=1,cas=1;(void)(cas);
42     // scanf("%d", &T);
43     while(T--) {
44         // printf("Case #%d: ", cas++);
45         solve();
46     }
47     return 0;
48 }

```

## 5.2 卢卡斯 Lucas

```

1  #include <stdio>
2  #define LL long long
3  #define MAXP 100005
4
5  int f[MAXP],fi[MAXP];
6
7  LL bincpow(LL x,LL y,LL m){
8      LL r=1%m;
9      for(;y;y>>=1,x=x*x%m)
10         if(y&1) r=r*x%m;
11     return r;
12 }
13
14 void pre(LL p){
15     f[0]=1;
16     for(int i=1;i<=p-1;i++)
17         f[i]=1LL*f[i-1]*i%p;
18     fi[p-1]=bincpow(f[p-1],p-2,p);
19     for(int i=p-1;i;i--)
20         fi[i-1]=1LL*fi[i]*i%p;
21 }
22
23 LL C(LL x,LL y,LL p) {
24     if(x<y) return 0;
25     return 1LL*f[x]*fi[y]%p*fi[x-y]%p;
26 }
27
28 LL lucas(LL x,LL y,LL p){
29     if(!y) return 1;
30     return C(x%p,y%p,p)*lucas(x/p,y/p,p)%p;
31 }
32
33 int main(){
34     int T;
35     scanf("%d", &T);
36     while(T--){
37         LL n,m,p;
38         scanf("%lld %lld %lld", &n, &m, &p);
39         pre(p);
40         printf("%lld\n", lucas(n+m,m,p));
41     }
42     return 0;
43 }

```

## 5.3 拓展欧几里得 Exgcd

```

1  #include <stdio>
2  #include <cmath>
3  #define LL long long
4

```

```

5 LL exgcd(LL a,LL b,LL &x,LL &y){
6     if(!b || !a){
7         x=(a!=0);y=(b!=0);
8         return a+b;
9     }
10    LL g=exgcd(b,a%b,y,x);
11    y=y-(a/b)*x;
12    return g;
13 }
14
15 int main(){
16     int T;
17     scanf("%d", &T);
18     while(T--){
19         LL a,b,c,x,y,g;
20         scanf("%lld %lld %lld", &a, &b, &c);
21         g=exgcd(a,b,x,y);
22         if(c%g){
23             printf("-1\n");
24             continue;
25         }
26         a/=g;b/=g;c/=g;x*=c;y*=c;
27         LL kl=ceil((double)(-x+1)/b),kr=floor((double)(y-1)/a);
28         if(kr<kl){
29             printf("%lld %lld\n", (x+kl*b), (y-kr*a));
30         }
31         else{
32             printf("%lld %lld %lld %lld %lld\n",kr-kl+1, (x+kl*b), (y-kr*a), (x+kr*b), (y-kl*
33                 a));
34         }
35     }
36     return 0;
37 }

```

#### 5.4 拓展欧拉定理 Ex\_Euler\_Theorem-Automaton

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 #define LL long long
4 #define MAXN 100000005
5
6 LL binpow(LL x,LL y,LL m){
7     int r=1%m;
8     x%=m;
9     while(y){
10         if(y&1) r=1LL*r*x%m;
11         x=1LL*x*x%m;
12         y>>=1;
13     }
14     return r;
15 }
16

```

```

17 void solve() {
18     int a,m;
19     scanf("%d %d", &a, &m);
20     int mm=m,phi=m;
21     for(int i=2;i*i<=m;i++) {
22         if(mm%i==0){
23             while(mm%i==0) mm/=i;
24             phi=phi/i*(i-1);
25         }
26     }
27     if(mm>1) phi=phi/mm*(mm-1);
28     char c;
29     while(!isdigit(c=getchar()));
30     int b=c-'0';
31     bool flag=0;
32     while(isdigit(c=getchar())){
33         b=10*b+c-'0';
34         if(b>=phi) b%=phi, flag=1;
35     }
36     if(flag) b+=phi;
37     printf("%lld\n", binpow(a,b,m));
38 }
39 }
40
41 int main() {
42     int T=1;
43     // scanf("%d", &T);
44     while(T--) {
45         solve();
46     }
47     return 0;
48 }

```

## 5.5 欧拉筛 Euler\_Sieve

```

1  #include <stdio>
2  #include <cstring>
3  #define MAXN 2000005
4
5  int cnt,p[MAXN];
6  bool inp[MAXN];
7  int phi[MAXN],mu[MAXN];
8
9  void euler_sieve(int n){
10     cnt=0;
11     memset(inp,0,(n+1)*sizeof(inp[0]));
12     inp[0]=inp[1]=1;
13     phi[1]=1;
14     mu[1]=1;
15     for(int i=2;i<=n;i++){
16         if(!inp[i]) p[++cnt]=i, phi[i]=i-1, mu[i]=-1;
17         for(int j=1;j<=cnt && i*p[j]<=n;j++){

```

```

18     inp[i*p[j]]=1;
19     if(i%p[j]){
20         phi[i*p[j]]=phi[i]*(p[j]-1);
21         mu[i*p[j]]=-mu[i];
22     }
23     else{
24         phi[i*p[j]]=phi[i]*p[j];
25         mu[i*p[j]]=0;
26         break;
27     }
28 }
29 }
30 }
31
32 int main(){
33     int n,m;
34     scanf("%d %d", &n, &m);
35     euler_sieve(n);
36     for(int i=1;i<=m;i++){
37         int k;
38         scanf("%d",&k);
39         printf("%d\n", p[k]);
40     }
41 }

```

## 5.6 杜教筛 Dujiao\_Sieve

```

1  #include <cstdio>
2  #include <cstring>
3  #define LL long long
4  #define MAXR R+5
5  #define R 2000000//r=n^(2/3)
6
7  int n,r,p[MAXR],cnt;
8  bool inp[MAXR];
9
10 LL phi[MAXR],sphi[MAXR],sphir[MAXR],mu[MAXR],smu[MAXR],smur[MAXR];
11
12 void pre(){
13     memset(inp+1,0,r*sizeof(inp[0]));
14     inp[0]=inp[1]=1;
15     cnt=0;
16     sphi[1]=phi[1]=1;
17     smu[1]=mu[1]=1;
18     for(int i=2;i<=r;i++){
19         if(!inp[i]) p[++cnt]=i, phi[i]=i-1, mu[i]=-1;
20         for(int j=1;j<=cnt&&i*p[j]<=r;j++){
21             inp[i*p[j]]=1;
22             if(i%p[j]){
23                 phi[i*p[j]]=phi[i]*(p[j]-1);
24                 mu[i*p[j]]=-mu[i];
25             }

```

```

26         else{
27             phi[i*p[j]]=phi[i]*p[j];
28             mu[i*p[j]]=0;
29             break;
30         }
31     }
32     sph[i]=sph[i-1]+phi[i];
33     smu[i]=smu[i-1]+mu[i];
34 }
35 }
36
37 LL sumphi(LL x){
38     if(x<=r) return sph[x];
39     else if(sphir[n/x]) return sphir[n/x];
40     LL &sx=sphir[n/x];
41     sx=x*(x+1)/2;
42     for(LL i=2;i<=x;i++){
43         LL t=x/i,j=x/t;
44         sx-=(j-i+1)*sumphi(t);
45         i=j;
46     }
47     return sx;
48 }
49
50 LL summu(LL x){
51     if(x<=r) return smu[x];
52     else if(smur[n/x]) return smur[n/x];
53     LL &sx=smur[n/x];
54     sx=1;
55     for(LL i=2;i<=x;i++){
56         LL t=x/i,j=x/t;
57         sx-=(j-i+1)*summu(t);
58         i=j;
59     }
60     return sx;
61 }
62
63 int main(){
64     int T;
65     scanf("%d", &T);
66     r=R;
67     pre();
68     while(T--){
69         scanf("%d", &n);
70         memset(sphir+1,0,(n/r)*sizeof(sphir[0]));
71         memset(smur+1,0,(n/r)*sizeof(smur[0]));
72         printf("%lld %lld\n", sumphi(n), summu(n));
73     }
74     return 0;
75 }

```



## 5.7 求原根 Get\_Primitive\_Root

```
1  #include <cstdio>
2  #include <vector>
3
4  using namespace std;
5
6  int p;
7  vector<int> v;
8
9  int binpow(int x,int y,int mod){
10     int r=1%mod;
11     while(y){
12         if(y&1) r=(1LL*r*x)%mod;
13         x=(1LL*x*x)%mod;
14         y>>=1;
15     }
16     return r;
17 }
18
19 int main(){
20     scanf("%d", &p);
21     int pp=p-1;
22     for(int i=2;i*i<=pp;i++){
23         if(pp%i==0){
24             v.push_back(i);
25             while(pp%i==0) pp/=i;
26         }
27     }
28     if(pp>1) v.push_back(pp);
29     for(int g=2;;g++){
30         bool isg=true;
31         for(int d:v){
32             if(binpow(g,(p-1)/d,p)==1){
33                 isg=false;
34                 break;
35             }
36         }
37         if(isg){
38             printf("%d\n", g);
39             break;
40         }
41     }
42     return 0;
43 }
```

## 5.8 贝祖引理 Bezout\_Lemma

```
1  #include <cstdio>
2
3  int gcd(int x,int y){
4     if(!x || !y) return x+y;
```

```
5     return gcd(y,x%y);
6 }
7
8 int n;
9
10 int main(){
11     scanf("%d", &n);
12     int g;
13     scanf("%d", &g);
14     if(g<0)g=-g;
15     for(int i=2;i<=n;i++){
16         int t;
17         scanf("%d", &t);
18         if(t<0) t=-t;
19         g=gcd(g,t);
20     }
21     printf("%d\n", g);
22     return 0;
23 }
```

## 5.9 除法分块 Block\_Division

```
1 #include <cstdio>
2 #define LL long long
3
4 LL n;
5
6 int main(){
7     scanf("%lld", &n);
8     LL ans=0;
9     for(LL i=1;i<=n;i++){
10         LL t=n/i,j=n/t;
11         ans+=(j-i+1)*t;
12         i=j;
13     }
14     printf("%lld\n", ans);
15     return 0;
16 }
```

## 6 组合数学

### 6.1 康托展开 Cantor

```

1  /*
2     康托 (Cantor) 展开：求全排列是第几个。
3  */
4  #include <bits/stdc++.h>
5  #define LL long long
6  using namespace std;
7  const int MAXN=2000005;
8  const int MOD=998244353;
9
10 struct BIT {
11     int n,b[MAXN];
12
13     int lowbit(int x){
14         return x&(-x);
15     }
16
17     void change(int x,int y){
18         for(;x<=n;x+=lowbit(x))
19             b[x]+=y;
20     }
21
22     int sum(int x){
23         int s=0;
24         for(;x>0;x-=lowbit(x))
25             s+=b[x];
26         return s;
27     }
28 }bit;
29
30 int a[MAXN],f[MAXN];
31
32 void solve() {
33     int n;
34     scanf("%d", &n);
35     for(int i=1;i<=n;i++)
36         scanf("%d", &a[i]);
37
38     f[0]=1;
39     for(int i=1;i<=n;i++)
40         f[i]=1LL*f[i-1]*i%MOD;
41
42     bit.n=n;
43
44     int ans=1;
45     for(int i=1;i<n;i++) {
46         bit.change(a[i],1);
47         ans=(ans+1LL*f[n-i]*(a[i]-bit.sum(a[i]))%MOD)%MOD;
48     }
49     printf("%d\n", ans);

```

```

50 }
51
52 int main() {
53     int T=1,cas=1;(void)(cas);
54     // scanf("%d", &T);
55     while(T--) {
56         // printf("Case #d: ", cas++);
57         solve();
58     }
59     return 0;
60 }

```

## 6.2 波利亚 Pólya

```

1  /*
2   Poly定理
3   本质不同个数=sigma(颜色个数~等价置换的划分个数)/(等价置换总个数)
4
5   例题: n个点围成一圈, m种颜色。求染色旋转本质不同个数。
6   ans = sigma(m^gcd(n,i))/n
7         = sigma(m^d * phi(n/d))/n ----- 莫比乌斯反演
8  */
9  #include <bits/stdc++.h>
10 #define LL long long
11 using namespace std;
12 const int MOD=1e9+7;
13 const int MAXN=2000005;
14
15 int binpow(int x,int y,int m) {
16     int r=1%m;
17     while(y) {
18         if(y&1) r=1LL*r*x%m;
19         x=1LL*x*x%m;
20         y>>=1;
21     }
22     return r;
23 }
24
25 int phi(int x) {
26     int r=x;
27     for(int i=2;i*i<=x;i++) {
28         if(x%i==0) {
29             r=r/i*(i-1);
30             while(x%i==0) x/=i;
31         }
32     }
33     if(x>1) r=r/x*(x-1);
34     return r;
35 }
36
37 void solve() {
38     int n,m; //n个点, m种颜色。

```

```

39     scanf("%d", &n);
40     m=n;
41     int ans=0;
42     for(int i=1;i*i<=n;i++) {
43         if(n%i==0) {
44             ans=(ans+1LL*binpow(m,i,MOD)*phi(n/i)%MOD)%MOD;
45             if(i*i!=n) {
46                 ans=(ans+1LL*binpow(m,n/i,MOD)*phi(i)%MOD)%MOD;
47             }
48         }
49     }
50     ans=1LL*ans*binpow(n,MOD-2,MOD)%MOD;
51     printf("%d\n", ans);
52 }
53
54 int main() {
55     int T=1,cas=1;(void)(cas);
56     scanf("%d", &T);
57     while(T--) {
58         // printf("Case #d: ", cas++);
59         solve();
60     }
61     return 0;
62 }

```

### 6.3 卡特兰数 Catalan

1. 有  $2n$  个人排成一行进入剧场。入场费 5 元。其中只有  $n$  个人有一张 5 元钞票，另外  $n$  人只有 10 元钞票，剧院无其它钞票，问有多少中方法使得只要有 10 元的人买票，售票处就有 5 元的钞票找零？
2. 一位大城市的律师在她住所以北  $n$  个街区和以东  $n$  个街区处工作。每天她走  $2n$  个街区去上班。如果他从不穿越（但可以碰到）从家到办公室的对角线，那么有多少条可能的道路？
3. 在圆上选择  $2n$  个点，将这些点成对连接起来使得所得到的  $n$  条线段不相交的方法数？
4. 对角线不相交的情况下，将一个凸多边形区域分成三角形区域的方法数？
5. 一个栈（无穷大）的进栈序列为  $1, 2, 3, \dots, n$  有多少个不同的出栈序列？
6.  $n$  个结点可构造多少个不同的二叉树？
7.  $n$  个不同的数依次进栈，求不同的出栈结果的种数？
8.  $n$  个  $+1$  和  $n$  个  $-1$  构成  $2n$  项  $a_1, a_2, \dots, a_{2n}$ ，其部分和满足  $a_1 + a_2 + \dots + a_k \geq 0 (k = 1, 2, 3, \dots, 2n)$  对与  $n$  该数列为？

H0	H1	H2	H3	H4	H5	H6
1	1	2	5	14	42	132

关于 Catalan 数的常见公式：

$$H_n = \begin{cases} \sum_{i=1}^n H_{i-1}H_{n-i} & n \geq 2, n \in \mathbf{N}_+ \\ 1 & n = 0, 1 \end{cases}$$

$$H_n = \frac{H_{n-1}(4n-2)}{n+1}$$

$$H_n = \binom{2n}{n} - \binom{2n}{n-1}$$

$$H_n = \frac{\binom{2n}{n}}{n+1} (n \geq 2, n \in \mathbf{N}_+)$$

## 6.4 斯特林数 Stirling

### 6.4.1 第一类斯特林数

(斯特林轮换数)

$\left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right]$  表示将  $n$  个两两不同的元素, 划分为  $k$  个非空圆排列的方案数。  
递推式

$$\left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] = \left[ \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right] + (n-1) \left[ \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right]$$

边界是  $\left[ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right] = [n=0]$ 。

### 6.4.2 第二类斯特林数

(斯特林子集数)

$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\}$  表示将  $n$  个两两不同的元素, 划分为  $k$  个非空子集的方案数。  
递推式

$$\left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} = \left\{ \begin{smallmatrix} n-1 \\ k-1 \end{smallmatrix} \right\} + k \left\{ \begin{smallmatrix} n-1 \\ k \end{smallmatrix} \right\}$$

边界是  $\left\{ \begin{smallmatrix} n \\ 0 \end{smallmatrix} \right\} = [n=0]$ 。

### 6.4.3 上升幂与普通幂的相互转化

我们记上升阶乘幂  $x^{\overline{n}} = \prod_{k=0}^{n-1} (x+k)$ 。

则可以利用下面的恒等式将上升幂转化为普通幂:

$$x^{\overline{n}} = \sum_k \left[ \begin{smallmatrix} n \\ k \end{smallmatrix} \right] x^k$$

如果将普通幂转化为上升幂, 则有下列的恒等式:

$$x^n = \sum_k \left\{ \begin{smallmatrix} n \\ k \end{smallmatrix} \right\} (-1)^{n-k} x^{\overline{k}}$$

#### 6.4.4 下降幂与普通幂的相互转化

我们记下降阶乘幂  $x^{\underline{n}} = \frac{x!}{(x-n)!} = \prod_{k=0}^{n-1} (x-k)$ 。

则可以利用下面的恒等式将普通幂转化为下降幂：

$$x^n = \sum_k \left\{ \begin{matrix} n \\ k \end{matrix} \right\} x^{\underline{k}}$$

如果将下降幂转化为普通幂，则有下列恒等式：

$$x^{\underline{n}} = \sum_k \left[ \begin{matrix} n \\ k \end{matrix} \right] (-1)^{n-k} x^k$$

## 7 网络流

### 7.1 最大费用流 Minimum-Cost\_Flow\_Edmonds-Karp

```

1  #include <cstdio>
2  #include <cstring>
3  #include <queue>
4  #include <algorithm>
5  #define MAXN 5005
6  #define MAXM 50005
7
8  const int inf = 0x3f3f3f3f;
9
10 using namespace std;
11
12 int n,m,s,t,tot,head[MAXN],dis[MAXN],maxflow,mincost;
13 bool inque[MAXN];
14
15 struct edge{//残量网络 residual network
16     int to,cf,next,dis;//Cf:residual capacity
17 }e[(MAXM<<1)+1];
18
19 struct node{
20     int fr,edge;
21 }pre[MAXN];
22
23 void add(int x,int y,int f,int d){
24     tot++;
25     e[tot].cf=f;
26     e[tot].dis=d;
27     e[tot].to=y;
28     e[tot].next=head[x];
29     head[x]=tot;
30 }
31
32 bool spfa(){//SPFA
33     memset(dis+1,inf,n*sizeof(dis[0]));
34     memset(inque+1,0,n*sizeof(inque[0]));
35     pre[t].fr=0;
36     queue<int> q;
37     dis[s]=0;
38     q.push(s);
39     inque[s]=1;
40     while(!q.empty()){
41         int u=q.front();q.pop();
42         inque[u]=0;
43         for(int p=head[u];p;p=e[p].next){
44             int v=e[p].to;
45             if(e[p].cf && dis[v]>dis[u]+e[p].dis){
46                 dis[v]=dis[u]+e[p].dis;
47                 pre[v].fr=u;
48                 pre[v].edge=p;
49                 if(!inque[v]){

```



```

50         q.push(v);
51         inque[v]=1;
52     }
53 }
54 }
55 }
56 return pre[t].fr!=0;
57 }
58
59 int min_flow(){
60     int mn=inf;
61     for(int u=t;u!=s;u=pre[u].fr){
62         mn=min(mn,e[pre[u].edge].cf);
63     }
64     for(int u=t;u!=s;u=pre[u].fr){
65         e[pre[u].edge].cf-=mn;
66         e[pre[u].edge^1].cf+=mn;
67     }
68     return mn;
69 }
70
71 void edmonds_karp(){
72     maxflow=0,mincost=0;
73     while(spfa()){
74         int flow=min_flow();
75         maxflow+=flow;
76         mincost+=flow*dis[t];
77     }
78 }
79
80 int main(){
81     scanf("%d %d %d %d", &n, &m, &s, &t);
82     tot=1;
83     memset(head+1,0,n*sizeof(head[0]));
84     for(int i=1;i<=m;i++){
85         int f,g,w,d;
86         scanf("%d %d %d %d",&f,&g,&w,&d);
87         add(f,g,w,d);
88         add(g,f,0,-d);
89     }
90     edmonds_karp();
91     printf("%d %d\n",maxflow,mincost);
92     return 0;
93 }

```

## 7.2 最大流 Maximum\_Flow\_Edmonds-Karp

```

1 #include <stdio>
2 #include <cstring>
3 #include <queue>
4 #include <algorithm>
5 #define MAXN 10005

```

```
6 #define MAXM 100005
7
8 using namespace std;
9
10 const int inf = 0x3f3f3f3f;
11
12 int n,m,s,t,tot,head[MAXN],vis[MAXN];
13
14 struct edge{
15     int to,cf,next;
16 }e[MAXM<<1];
17
18 struct node{
19     int fr,edge;
20 }pre[MAXN];
21
22 void add(int x,int y,int z){
23     tot++;
24     e[tot].cf=z;
25     e[tot].to=y;
26     e[tot].next=head[x];
27     head[x]=tot;
28 }
29
30 bool find_augment(){
31     memset(pre+1,0,n*sizeof(pre[0]));
32     memset(vis+1,0,n*sizeof(vis[0]));
33     queue<int> q;
34     vis[s]=1;
35     q.push(s);
36     while(!q.empty()){
37         int u=q.front();q.pop();
38         for(int p=head[u];p;p=e[p].next){
39             int v=e[p].to;
40             if(!vis[v] && e[p].cf){
41                 pre[v].fr=u;
42                 pre[v].edge=p;
43                 vis[v]=1;
44                 q.push(v);
45                 if(v==t) return true;
46             }
47         }
48     }
49     return false;
50 }
51
52 int min_flow(){
53     int mn=inf;
54     for(int u=t;u!=s;u=pre[u].fr){
55         mn=min(mn,e[pre[u].edge].cf);
56     }
57     for(int u=t;u!=s;u=pre[u].fr){
58         e[pre[u].edge].cf-=mn;
```

```

59     e[pre[u].edge^1].cf+=mn;
60 }
61 return mn;
62 }
63
64 int edmonds_karp(){
65     int flow=0;
66     while(find_augment()){
67         flow+=min_flow();
68     }
69     return flow;
70 }
71
72 int main(){
73     scanf("%d %d %d %d", &n, &m, &s, &t);
74     tot=1;
75     memset(head+1,0,n*sizeof(head[0]));
76     for(int i=1;i<=m;i++){
77         int f,g,w;
78         scanf("%d %d %d",&f,&g,&w);
79         add(f,g,w);
80         add(g,f,0);
81     }
82     printf("%d\n", edmonds_karp());
83     return 0;
84 }

```

### 7.3 最大流 Maximum\_Flow\_Dinic

```

1  #include <cstdio>
2  #include <cstring>
3  #include <queue>
4  #include <algorithm>
5  #define MAXN 10005
6  #define MAXM 100005
7
8  const int inf = 0x3f3f3f3f;
9
10 using namespace std;
11
12 int n,m,s,t,tot,head[MAXN],lb[MAXN],cur[MAXN];
13
14 struct edge{//残量网络 residual network
15     int to,cf,next;//Cf:residual capacity
16 }e[(MAXM<<1)+1];
17
18 void add(int x,int y,int z){
19     tot++;
20     e[tot].cf=z;
21     e[tot].to=y;
22     e[tot].next=head[x];
23     head[x]=tot;

```

```

24 }
25
26 bool label_vertex(){//BFS
27     memset(lb+1,0,n*sizeof(lb[0]));
28     queue<int> q;
29     lb[s]=1;
30     q.push(s);
31     while(!q.empty()){
32         int u=q.front();q.pop();
33         for(int p=head[u];p;p=e[p].next){
34             int v=e[p].to;
35             if(e[p].cf && !lb[v]){
36                 lb[v]=lb[u]+1;
37                 q.push(v);
38                 if(v==t) return true;
39             }
40         }
41     }
42     return false;
43 }
44
45 int multi_augment(int u,int lim){//DFS 多路增广
46     if(u == t) return lim;
47
48     int used=0;
49     for(int& p=cur[u];p;p=e[p].next){
50         int v=e[p].to;
51         if(e[p].cf && lb[v]==lb[u]+1){
52             int rest=multi_augment(v,min(lim-used,e[p].cf));
53             used+=rest;
54             e[p].cf-=rest;
55             e[p^1].cf+=rest;
56             if(used==lim) break;
57         }
58     }
59     return used;
60 }
61
62 int dinic(){
63     int flow=0;
64     while(label_vertex()){//BFS 标记
65         for(int i=1;i<=n;i++) cur[i]=head[i];//当前弧优化
66         flow+=multi_augment(s,inf);//DFS 顺着标记找增广路
67     }
68     return flow;
69 }
70
71 int main(){
72     scanf("%d %d %d %d", &n, &m, &s, &t);
73     tot=1;
74     memset(head+1,0,n*sizeof(head[0]));
75     for(int i=1;i<=m;i++){
76         int f,g,w;

```

```

77     scanf("%d %d %d",&f,&g,&w);
78     add(f,g,w);
79     add(g,f,0);
80 }
81 printf("%d\n", dinic());
82 return 0;
83 }

```

## 7.4 二分题最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Dinic

```

1  #include <cstdio>
2  #include <cstring>
3  #include <queue>
4  #define MAXN 2005
5  #define MAXM 1000005
6
7  using namespace std;
8
9  const int inf=0x3f3f3f3f;
10
11 struct edge{
12     int to,cf,next;
13 }e[MAXM<<1];
14
15 int n,n1,n2,m,s,t;
16 int tot,head[MAXN],cur[MAXN],lbl[MAXN];
17
18 void add(int x,int y,int z){
19     tot++;
20     e[tot].to=y;
21     e[tot].cf=z;
22     e[tot].next=head[x];
23     head[x]=tot;
24 }
25
26 bool bfs(){
27     memset(lbl+1,0,n*sizeof(lbl[0]));
28     lbl[t]=1;
29     queue<int> q;
30     q.push(t);
31     while(!q.empty()){
32         int u=q.front();q.pop();
33         for(int p=head[u];p;p=e[p].next){
34             int v=e[p].to;
35             if(e[p].cf && !lbl[v]){
36                 lbl[v]=lbl[u]+1;
37                 q.push(v);
38                 if(v==s) return true;
39             }
40         }
41     }
42     return lbl[s]!=0;

```

```

43 }
44
45 int dfs(int u,int lim){
46     if(u==t)return lim;
47
48     int used=0;
49     for(int& p=cur[u];p;p=e[p].next){
50         int v=e[p].to;
51         if(e[p].cf && lbl[v]==lbl[u]-1){
52             int rest=dfs(v,min(lim-used,e[p].cf));
53             used+=rest;
54             e[p].cf-=rest;
55             e[p^1].cf+=rest;
56             if(used==lim) break;
57         }
58     }
59     return used;
60 }
61
62 int dinic(){
63     int flow=0;
64     while(bfs()){
65         for(int i=1;i<=n;i++){
66             cur[i]=head[i];
67             flow+=dfs(s,inf);
68         }
69     }
70     return flow;
71 }
72
73 int main(){
74     scanf("%d %d %d", &n1, &n2, &m);
75     n=n1+n2+2;//n个点
76     s=n-1;t=n;
77     tot=1;
78     memset(head+1,0,n*sizeof(head[0]));
79     for(int i=1;i<=m;i++){
80         int f,g;
81         scanf("%d %d",&f,&g);
82         if(f>n1 || g>n2) continue;
83         add(f,n1+g,1);
84         add(n1+g,f,0);
85     }
86     for(int i=1;i<=n1;i++){
87         add(s,i,1);
88         add(i,s,0);
89     }
90     for(int i=n1+1;i<=n1+n2;i++){
91         add(i,t,1);
92         add(t,i,0);
93     }
94     printf("%d\n", dinic());
95     return 0;
96 }

```

## 7.5 二分图最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Hungarian

```

1  #include <stdio>
2  #include <cstring>
3  #define MAXN 1005
4  #define MAXM 1000005
5
6  struct node{
7      int to,next;
8  }e[MAXM];
9
10 int n1,n2,m,head[MAXN],tot,dfn[MAXN],mat[MAXN];
11
12 void add(int x,int y){
13     tot++;
14     e[tot].to=y;
15     e[tot].next=head[x];
16     head[x]=tot;
17 }
18
19 bool augment(int x,int stamp){
20     for(int p=head[x];p;p=e[p].next){
21         int u=e[p].to;
22         if(dfn[u] == stamp) continue;
23         dfn[u]=stamp;
24         if(!mat[u] || augment(mat[u],stamp)){
25             mat[u]=x;
26             return true;
27         }
28     }
29     return false;
30 }
31
32 int match(){
33     memset(mat+1,0,n2*sizeof(mat[0]));
34     memset(dfn+1,0,n2*sizeof(dfn[0]));
35     int cnt=0;
36     for(int i=1;i<=n1;i++){
37         if(augment(i,i))
38             cnt++;
39     }
40     return cnt;
41 }
42
43 int main(){
44     scanf("%d %d %d", &n1,&n2,&m);
45     tot=0;
46     memset(head+1,0,n1*sizeof(head[0]));
47     for(int i=1;i<=m;i++){
48         int f,g;
49         scanf("%d %d", &f, &g);
50         if(f>n1 || g>n2)
51             continue;

```

```

52     add(f,g);
53 }
54 printf("%d\n", match());
55 return 0;
56 }

```

## 7.6 二分图最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Hopcroft-Karp

```

1  #include <cstdio>
2  #include <cstring>
3  #include <queue>
4  #define MAXN 2005
5  #define MAXM 1000005
6
7  const int inf=0x3f3f3f3f;
8
9  using namespace std;
10
11 int n1,n2,n,m,tot,head[MAXN];
12 int mat[MAXN],lb[MAXN],dfn[MAXN];
13
14 struct edge{
15     int to,next;
16 }e[MAXM];
17
18 void add(int x,int y){
19     tot++;
20     e[tot].to=y;
21     e[tot].next=head[x];
22     head[x]=tot;
23 }
24
25 bool bfs(){
26     memset(lb+1,0,n*sizeof(lb[0]));
27     queue<int> q;
28     for(int i=1;i<=n1;i++){
29         if(!mat[i]){
30             q.push(i);
31             lb[i]=1;
32         }
33     }
34     int dis=inf;
35     while(!q.empty()){
36         int u=q.front();q.pop();
37         for(int p=head[u];p;p=e[p].next){
38             int v=e[p].to;
39             if(!lb[v]){
40                 lb[v]=lb[u]+1;
41                 if(!mat[v]) dis=lb[v];
42                 else if(lb[v]<dis){
43                     lb[mat[v]]=lb[v]+1;

```



```

44         q.push(mat[v]);
45     }
46 }
47 }
48 }
49 return dis!=inf;
50 }
51
52 bool dfs(int u,int stamp){
53     for(int p=head[u];p;p=e[p].next){
54         int v=e[p].to;
55         if(dfn[v]!=stamp && lb[v]==lb[u]+1){
56             dfn[v]=stamp;
57             if(!mat[v] || (lb[mat[v]]==lb[v]+1 && dfs(mat[v],stamp))){
58                 mat[v]=u;
59                 mat[u]=v;
60                 return true;
61             }
62         }
63     }
64     return false;
65 }
66
67 int hopcroft_karp(){
68     int cnt=0,stamp=0;
69     memset(dfn+1,0,n*sizeof(dfn[0]));
70     memset(mat+1,0,n*sizeof(mat[0]));
71     while(bfs()){
72         stamp++;
73         for(int i=1;i<=n1;i++){
74             if(!mat[i] && dfs(i,stamp)){
75                 cnt++;
76             }
77         }
78     }
79     return cnt;
80 }
81
82 int main(){
83     scanf("%d %d %d", &n1, &n2, &m);
84     n=n1+n2;
85     memset(head+1,0,n*sizeof(head[0]));
86     tot=0;
87     for(int i=1;i<=m;i++){
88         int f,g;
89         scanf("%d %d",&f,&g);
90         if(f>n1 || g>n2) continue;
91         add(f,n1+g);
92     }
93     printf("%d\n", hopcroft_karp());
94     return 0;
95 }

```

## 8 计算几何

### 8.1 计算几何 Computational\_Geometry

```

1  #include <bits/stdc++.h>
2  #define MAXN 2000005
3  #define LL long long
4  using namespace std;
5  const double PI=acos(-1.0);
6  const double inf=1e100;
7  const double eps=1e-7;
8
9  int sgn(double d) {
10     if(abs(d)<eps) return 0;
11     if(d>0) return 1;
12     return -1;
13 }
14
15 int dcmp(double x,double y) {
16     if(abs(x-y)<eps) return 0;
17     if(x>y) return 1;
18     return -1;
19 }
20
21 struct Point{
22     double x,y;
23     Point(double x=0, double y=0):x(x),y(y){}
24
25     Point operator + (const Point& B) const{
26         return Point(x+B.x,y+B.y);
27     }
28     Point operator - (const Point& B) const{
29         return Point(x-B.x,y-B.y);
30     }
31     Point operator * (const double k) const{
32         return Point(x*k,y*k);
33     }
34     Point operator / (const double k) const{
35         return Point(x/k,y/k);
36     }
37     bool operator < (const Point B) {
38         if(dcmp(x,B.x)==0)
39             return dcmp(y,B.y)<0;
40         else return dcmp(x,B.x)<0;
41     }
42
43     double operator * (const Point& B) const{//点积
44         return x*B.x+y*B.y;
45     }
46     double operator ^ (const Point& B) const{//叉积
47         return x*B.y-y*B.x;
48     }
49 };

```

```

50
51 typedef Point Vector;
52
53 double Length(Vector A) {
54     return sqrt(A*A);
55 }
56
57 double Angle(Vector A, Vector B) { //弧度
58     return acos(A*B/Length(A)/Length(B));
59 }
60
61 double Area2(Vector A, Vector B){ //求平行四边形面积
62     return A^B;
63 }
64
65 Vector Rotate(Vector A, double rad) { //逆时针
66     return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
67 }
68
69 Vector Normal(Vector A) { //逆时针转90度, 单位法向量
70     double L=Length(A);
71     return Vector(-A.y/L, A.x/L);
72 }
73
74 bool ToLeftTest(Vector A, Vector B) { //B是不是在A左边
75     return sgn(A^B)>0;
76 }
77
78 struct Line { //点向式+两点。既可以line也可以seg
79     Point p1,p2; //p1->p2
80     Vector v;
81     Line(Point p1, Point p2):p1(p1),p2(p2),v((p2-p1)/Length(p2-p1)){}
82     Point point(double t) { //给t求点
83         return p1+v*t;
84     }
85 };
86 typedef Line Segment;
87
88 bool OnLine(Point P, Line l) { //判断点P是否在直线L上
89     return sgn((P-l.p1)^l.v);
90 }
91
92 Point GetIntersection(Line l1, Line l2) { //求直线交点
93     double t = (l2.v^(l1.p1-l2.p1))/(l1.v^l2.v);
94     return l1.point(t);
95 }
96
97 double DistanceToLine(Point P, Line l) { //点到直线距离
98     return abs(l.v^(P-l.p1));
99 }
100
101 Point GetProjection(Point P, Line l) { //求投影点
102     return l.point((P-l.p1)*l.v);

```

```
103 }
104
105 bool OnSegment(Point P, Segment s) {
106     return (OnLine(P,s) && sgn((s.p1-P)*(s.p2-P))<0);
107 }
108
109 bool InSegmentIntersection(Segment s1, Segment s2) { //不允许端点相交
110     double c1=(s1.p2-s1.p1)^(s2.p1-s1.p1), c2=(s1.p2-s1.p1)^(s2.p2-s1.p1);
111     double c3=(s2.p2-s2.p1)^(s1.p1-s2.p1), c4=(s2.p2-s2.p1)^(s1.p2-s2.p1);
112     return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);
113 }
114
115 void solve() {
116
117 }
118
119 int main() {
120     int T=1,cas=1;
121     // scanf("%d", &T);
122     while(T--) {
123         // printf("Case #%d: ", cas++);
124         solve();
125     }
126     return 0;
127 }
```

## 9 优化算法

### 9.1 wqs 二分 + 决策单调性 dp

```

1  /*
2   决策单调性 + WQS二分
3
4   给定数列{a}。设立k个关键点。
5   让sigma min|a_i-b_k| 最小（每个a_i到最近的关键点距离之和）
6  */
7  #include <bits/stdc++.h>
8  #define LL long long
9  using namespace std;
10 const int MAXN=2000005;
11
12 LL a[MAXN],f[MAXN];
13 int cnt[MAXN],lb[MAXN],rb[MAXN],p[MAXN];
14
15 LL w(int x,int y){
16     return a[y]+a[x]-a[(x+y+1)>>1]-a[(x+y)>>1];
17 }
18
19 int k,n;
20
21 int calc(LL d) {
22     int h,t;
23     h=t=1;
24     lb[t]=1,rb[t]=n,p[t]=0;
25     f[0]=cnt[0]=0;
26     for(int i=1;i<=n;i++) {
27         f[i]=f[p[h]]+w(p[h],i)+d;
28         cnt[i]=cnt[p[h]]+1;
29
30         while(rb[h]<=i) h++;
31         if(h<=t && lb[h]<=i) lb[h]=i+1;
32
33         if(h<=t && f[i]+w(i,n)>f[p[t]]+w(p[t],n)) continue;
34         while(h<=t && f[i]+w(i,lb[t])<=f[p[t]]+w(p[t],lb[t])) t--;
35         if(h<=t) {
36             int l=lb[t],r=rb[t]+1;
37             while(l<r) {
38                 int mid=(l+r)>>1;
39                 if(f[i]+w(i,mid)<=f[p[t]]+w(p[t],mid)) r=mid;
40                 else l=mid+1;
41             }
42             rb[t]=l-1;
43             ++t;
44             lb[t]=1,rb[t]=n,p[t]=i;
45         } else {
46             ++t;
47             lb[t]=i,rb[t]=n,p[t]=i;
48         }
49     }
50 }

```

```

50     return cnt[n];
51 }
52
53 void solve() {
54     scanf("%d %d", &n, &k);
55     a[0]=0;
56     for(int i=1;i<=n;i++) {
57         scanf("%lld", &a[i]);
58         a[i]+=a[i-1];
59     }
60
61     LL l=0,r=a[n];
62     while(l<r) {
63         LL mid=(l+r+1)>>1;
64         if(calc(mid)>=k) l=mid;
65         else r=mid-1;
66     }
67     calc(l);
68
69     printf("%lld\n", f[n]-1LL*k*1);
70 }
71
72 int main() {
73     int T=1,cas=1;(void)(cas);
74     // scanf("%d", &T);
75     while(T--) {
76         // printf("Case #%d: ", cas++);
77         solve();
78     }
79     return 0;
80 }

```

## 9.2 斜率优化 Slope\_Optimization

```

1  #include <bits/stdc++.h>
2  #define LL long long
3  using namespace std;
4  const int MAXN=2000005;
5
6  //似乎会爆LL
7  LL c[MAXN],dp[MAXN];
8  LL X[MAXN],Y[MAXN];
9
10 void solve() {
11     int n,L;
12     scanf("%d %d", &n, &L);
13     L++;
14     c[0]=0;
15     for(int i=1;i<=n;i++) {
16         scanf("%lld", &c[i]);
17         c[i]+=c[i-1]+1;//c[i] = sum(c[j])+i;
18     }

```

```
19     int h,t;
20     h=t=1;
21     dp[0]=0;
22     X[t]=c[0];
23     Y[t]=dp[0]+c[0]*c[0];
24     for(int i=1;i<=n;i++) {
25         while(h<t && Y[h+1]-Y[h]<=2*(X[h+1]-X[h])*(c[i]-L)) h++;
26         dp[i]=Y[h]-2*(c[i]-L)*X[h]+(c[i]-L)*(c[i]-L);
27         LL x=c[i],y=dp[i]+c[i]*c[i];
28         while(h<t && (y-Y[t])*(X[t]-X[t-1])<=(Y[t]-Y[t-1])*(x-X[t])) t--;
29         t++;
30         X[t]=x,Y[t]=y;
31     }
32     printf("%lld\n", dp[n]);
33 }
34
35 int main() {
36     int T=1,cas=1;(void)(cas);
37     // scanf("%d", &T);
38     while(T--) {
39         // printf("Case #%d: ", cas++);
40         solve();
41     }
42     return 0;
43 }
```

## 10 离线算法

### 10.1 莫队算法 Mo

```

1  /*
2     Coded with Leachim's ACM Template.
3     No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"
9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const LL INF=0x3f3f3f3f3f3f3f3f;
13 const double eps=1e-7;
14 const int dx[4]={1,-1,0,0};
15 const int dy[4]={0,0,1,-1};
16 const int MOD=998244353;
17 const int MAXN=2000005;
18
19 LL gcd(LL x,LL y) {
20     if(!x || !y) return x+y;
21     return gcd(y,x%y);
22 }
23
24 /*
25     莫队:
26     sqrt(m) 个块
27     块大小 n/sqrt(m);
28     卡常大小 n/sqrt(m*2/3);
29 */
30
31 struct query {
32     int l,r,id;
33 }q[MAXN];
34
35 int a[MAXN],cnt[MAXN];
36 int block,bl[MAXN];
37 LL ans1[MAXN],ans2[MAXN],sum;
38
39 bool cmp(const query &x, const query &y) {
40     if(bl[x.l]!=bl[y.l])
41         return bl[x.l]<bl[y.l];
42     else if(bl[x.l]&1)
43         return x.r<y.r;
44     else return x.r>y.r;
45 }
46
47 void modify(int x,int f) {
48     sum-=cnt[a[x]]*cnt[a[x]];
49     cnt[a[x]]+=f;

```



```

50     sum+=cnt[a[x]]*cnt[a[x]];
51 }
52
53 void solve() {
54     int n,m;
55     scanf("%d %d", &n, &m);
56     block=n/sqrt(m*2/3);
57     for(int i=1;i<=n;i++) {
58         scanf("%d", &a[i]);
59         bl[i]=(i-1)/block+1;
60     }
61     for(int i=1;i<=m;i++) {
62         scanf("%d %d", &q[i].l, &q[i].r);
63         q[i].id=i;
64     }
65     sort(q+1,q+1+m,cmp);
66     memset(cnt+1,0,n*sizeof(cnt[0]));
67     sum=0;
68     for(int i=1,l=1,r=0;i<=m;i++) {
69         for(;r<q[i].r;r++) modify(r+1,1);
70         for(;r>q[i].r;r--) modify(r,-1);
71         for(;l<q[i].l;l++) modify(l,-1);
72         for(;l>q[i].l;l--) modify(l-1,1);
73         ans1[q[i].id]=sum-(r-l+1);
74         ans2[q[i].id]=1LL*(q[i].r-q[i].l+1)*(q[i].r-q[i].l);
75     }
76     for(int i=1;i<=m;i++) {
77         LL g=gcd(ans1[i],ans2[i]);
78         if(!g) printf("0/1\n");
79         else printf("%lld/%lld\n", ans1[i]/g, ans2[i]/g);
80     }
81 }
82
83 int main() {
84     int T=1,cas=1;(void)(cas);
85     // scanf("%d", &T);
86     while(T--) {
87         // printf("Case #%d: ", cas++);
88         solve();
89     }
90     return 0;
91 }

```

## 10.2 线段树分治 Segment\_Tree\_Partition

```

1  /*
2     可撤销并查集+线段树分治
3     离线操作：增边，删边，询问图是否联通
4  */
5  #include <bits/stdc++.h>
6  #define LL long long
7  using namespace std;

```

```

8  const int MAXN=5005;
9  const int MAXM=500005;
10
11 struct DSU{
12     int fa[MAXN],rk[MAXN];
13     vector<pair<int*,int>> stk;
14     void init(int n) {
15         stk.clear();
16         for(int i=1;i<=n;i++) fa[i]=i,rk[i]=1;
17     }
18     int find(int x) {
19         if(x==fa[x]) return x;
20         return find(fa[x]);
21     }
22     bool join(int x,int y) {
23         int rx=find(x),ry=find(y);
24         if(rx==ry) return false;
25
26         if(rk[rx]>rk[ry]) swap(rx,ry);
27         stk.emplace_back(fa+rx,rx);
28         fa[rx]=ry;
29         stk.emplace_back(rk+ry,rk[ry]);
30         rk[ry]+=rk[rx];
31         return true;
32     }
33     void withdraw() {
34         *stk.back().first=stk.back().second;
35         stk.pop_back();
36         *stk.back().first=stk.back().second;
37         stk.pop_back();
38     }
39 }dsu;
40
41 int qf[MAXM],qg[MAXM];
42
43 struct SGT{
44     vector<pair<int,int>> eg[MAXM<<2];
45
46     void modify(int x,int l,int r,int ql,int qr,int f,int g) {
47         if(ql<=l && r<=qr) {
48             eg[x].emplace_back(f,g);
49             return;
50         }
51         int m=(l+r)>>1;
52         if(ql<=m) modify(x<<1,l,m,ql,qr,f,g);
53         if(m<qr) modify(x<<1|1,m+1,r,ql,qr,f,g);
54     }
55
56     void dfs(int x,int l,int r) {
57         int cnt=0;
58         for(auto pr:eg[x])
59             if(dsu.join(pr.first,pr.second))
60                 cnt++;

```

```

61     if(l==r) {
62         if(qf[l]) printf("%c\n", "NY"[dsu.find(qf[l])==dsu.find(qg[l])]);
63     } else {
64         int m=(l+r)>>1;
65         dfs(x<<1,l,m);
66         dfs(x<<1|1,m+1,r);
67     }
68     while(cnt--) dsu.withdraw();
69 }
70 }sgt;
71
72 int mp[MAXN][MAXN];
73
74 void solve() {
75     int n,m;
76     scanf("%d %d", &n, &m);
77     memset(qf+1,0,m*sizeof(qf[0]));
78     memset(qg+1,0,m*sizeof(qg[0]));
79     for(int i=1;i<=m;i++) {
80         int op,f,g;
81         scanf("%d %d %d", &op, &f, &g);
82         if(op==0) {
83             mp[f][g]=mp[g][f]=i;
84         } else if(op==1) {
85             sgt.modify(1,1,m+1,mp[f][g],i,f,g);
86             mp[f][g]=mp[g][f]=0;
87         } else qf[i]=f,qg[i]=g;
88     }
89
90     for(int f=1;f<n;f++) {
91         for(int g=f+1;g<=n;g++) {
92             if(mp[f][g]) {
93                 sgt.modify(1,1,m+1,mp[f][g],m+1,f,g);
94                 mp[f][g]=mp[g][f]=0;
95             }
96         }
97     }
98
99     dsu.init(n);
100    sgt.dfs(1,1,m+1);
101 }
102
103 int main() {
104     int T=1,cas=1;(void)(cas);
105     // scanf("%d", &T);
106     while(T--) {
107         // printf("Case #%d: ", cas++);
108         solve();
109     }
110     return 0;
111 }

```

## 11 杂项

### 11.1 快读快写 Fast\_Read&Write

```
1 #include <stdio>
2
3 int read(){
4     int ng=0,x=0;
5     char ch=getchar();
6     for(;ch<'0' || ch>'9';ch=getchar()) ng|=ch=='-';
7     for(;ch>='0' && ch<='9';ch=getchar()) x=(x<<3)+(x<<1)+ch-'0';
8     return ng?-x:x;
9 }
10
11 int write(){
12
13 }
14
15 int main(){
16     return 0;
17 }
```

### 11.2 快速幂 Fast\_Power

```
1 #include <stdio>
2
3 #define LL long long
4
5 LL a,b,n;
6
7 #include <stdio>
8
9 #define LL long long
10
11 LL a,b,n;
12
13 LL binpow(LL x,LL y,LL m){
14     LL r=1%m;
15     for(;y>=1,x=x*x%m)
16         if(y&1) r=r*x%m;
17     return r;
18 }
19
20 int main(){
21     scanf("%lld %lld %lld",&a,&b,&n);
22     printf("%lld^%lld mod %lld=%lld\n",a,b,n,binpow(a,b,n));
23     return 0;
24 }
25
26
27 int main(){
28     scanf("%lld %lld %lld",&a,&b,&n);
```

```

29     printf("%lld~%lld mod %lld=%lld\n",a,b,n,binpow(a,b,n));
30     return 0;
31 }

```

### 11.3 矩阵快速幂 Matrix\_Fast\_Power

```

1  #include <stdio>
2  #include <cstring>
3  #define LL long long
4  #define MAXN 105
5  #define MOD 1000000007
6
7  LL n,k;
8
9  struct Matrix{
10     LL num[MAXN][MAXN];
11
12     Matrix(int opt){
13         memset(num,0,sizeof(num));
14         if(opt==1)
15             for(int i=1;i<=n;i++)
16                 num[i][i]=1;
17     }
18
19     Matrix operator*(const Matrix &y) const{
20         Matrix x=*this,ans(0);
21         for(int i=1;i<=n;i++){
22             for(int j=1;j<=n;j++){
23                 for(int k=1;k<=n;k++){
24                     ans.num[i][j]=(ans.num[i][j]+x.num[i][k]*y.num[k][j]%MOD)%MOD;
25                 }
26             }
27         }
28         return ans;
29     }
30 };
31
32 Matrix binpow(Matrix x,LL y){
33     Matrix r(1);
34     while(y){
35         if(y&1) r=r*x;
36         x=x*x;
37         y>>=1;
38     }
39     return r;
40 }
41
42 int main(){
43     scanf("%lld %lld", &n, &k);
44     Matrix A(0);
45     for(int i=1;i<=n;i++){
46         for(int j=1;j<=n;j++){

```

```

47         scanf("%lld", &A.num[i][j]);
48     }
49 }
50 A=binpow(A,k);
51 for(int i=1;i<=n;i++){
52     for(int j=1;j<=n;j++){
53         printf("%lld ", A.num[i][j]);
54     }
55     puts("");
56 }
57 return 0;
58 }

```

## 11.4 矩阵加速 Matrix\_Acceleration

```

1  #include <stdio>
2  #include <cstring>
3  #define LL long long
4  #define MAXN 5
5  #define MOD 1000000007
6
7  struct Matrix{
8      int num[MAXN][MAXN];
9      int n,m;
10
11      Matrix(int nn,int mm){
12          n=nn; m=mm;
13          memset(num,0,sizeof(num));
14      }
15      Matrix(int nn){
16          n=m=nn;
17          memset(num,0,sizeof(num));
18          for(int i=1;i<=nn;i++) num[i][i]=1;
19      }
20
21      Matrix operator*(const Matrix &y) const{
22          Matrix x=*this,ans(x.n,y.m);
23          if(x.m!=y.n) return ans;
24          for(int i=1;i<=x.n;i++){
25              for(int j=1;j<=y.m;j++){
26                  for(int k=1;k<=x.m;k++){
27                      ans.num[i][j]=(ans.num[i][j]+1LL*x.num[i][k]*y.num[k][j]%MOD)%MOD;
28                  }
29              }
30          }
31          return ans;
32      }
33 };
34
35 Matrix binpow(Matrix x,LL y){
36     Matrix r(x.n);
37     while(y){

```

```

38     if(y&1) r=r*x;
39     x=x*x;
40     y>>=1;
41 }
42 return r;
43 }
44
45 int main(){
46     int T;
47     scanf("%d", &T);
48     while(T--){
49         int n;
50         scanf("%d", &n);
51         if(n<=3) printf("1\n");
52         else{
53             Matrix x(3,3);
54             x.num[1][1]=x.num[2][1]=x.num[1][3]=x.num[3][2]=1;
55             x=binpow(x,n-3);
56             Matrix y(3,1);
57             for(int i=1;i<=3;i++) y.num[i][1]=1;
58             printf("%d\n", (x*y).num[1][1]);
59         }
60     }
61     return 0;
62 }

```

## 11.5 最长公共子序列 Longest\_Increasing\_Subsequence

```

1  #include <cstdio>
2  #include <functional>
3  #include <algorithm>
4  #include <map>
5  #include <utility>
6  #define MAXN 100005
7
8  using namespace std;
9
10 int n,a[MAXN],len;
11 map <int,int> mp,rmp;
12
13 int main(){
14     scanf("%d", &n);
15     for(int i=1,t;i<=n;i++){
16         scanf("%d", &t);
17         mp.insert(make_pair(t,i));
18     }
19     a[len=0]=0;
20     map <int,int>::iterator ite;
21     for(int i=1,t,v;i<=n;i++){
22         scanf("%d",&t);
23         ite=mp.find(t);
24         v=ite->second;

```

```
25     if(v>a[len])
26         a[++len]=v;
27     else{
28         int *p=upper_bound(a+1,a+len+1,v,less<int>());
29         *p=v;
30     }
31 }
32 printf("%d\n",len);
33 return 0;
34 }
```

## 11.6 模拟退火 Simulated\_Annealing

```
1  #include <bits/stdc++.h>
2  #define MAXN 1005
3
4  int n;
5
6  double ansx,ansy;
7
8  struct point{
9      double x,y,w;
10 }p[MAXN];
11
12 double f(double x,double y){
13     double sum=0.0;
14     for(int i=1;i<=n;i++){
15         double dx=x-p[i].x,dy=y-p[i].y;
16         sum+=sqrt(dx*dx+dy*dy)*p[i].w;
17     }
18     return sum;
19 }
20
21 void SA(){
22     double T=3000,d=0.999,tt=1e-15;
23     while(T>tt){
24         double newx=ansx+(2*rand()-RAND_MAX)*T;
25         double newy=ansy+(2*rand()-RAND_MAX)*T;
26         double delta=f(newx,newy)-f(ansx,ansy);
27         if(delta<0 || exp(-delta/T)*RAND_MAX>rand()){
28             ansx=newx;
29             ansy=newy;
30         }
31         T*=d;
32     }
33 }
34
35 int main(){
36     srand(time(0));
37     scanf("%d", &n);
38     for(int i=1;i<=n;i++){
39         scanf("%lf %lf %lf", &p[i].x, &p[i].y, &p[i].w);
```



```

40     }
41     ansx=ansy=0.0;
42     for(int i=1;i<=n;i++){
43         ansx+=p[i].x;
44         ansy+=p[i].y;
45     }
46     ansx/=n;
47     ansy/=n;
48     SA();
49     printf("%.3lf %.3lf\n", ansx, ansy);
50     return 0;
51 }

```

## 11.7 快速沃尔什变换 Fast\_Walsh\_Transform

```

1  /*
2     Coded with Leachim's ACM Template.
3     No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"
9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const double eps=1e-7;
13 const int dx[4]={1,-1,0,0};
14 const int dy[4]={0,0,1,-1};
15 const int MAXN=2000005;
16 const int MOD=998244353;
17
18 void FWTor(int n,int* P,int f) {
19     for (int l=1;(l<<1)<=n;l<<=1)
20         for(int i=0;i<n;i+=(l<<1))
21             for(int j=0;j<l;j++)
22                 P[i+j+1]=((P[i+j+1]+P[i+j]*f)%MOD+MOD)%MOD;
23 }
24
25 void FWTand(int n,int* P,int f) {
26     for (int l=1;(l<<1)<=n;l<<=1)
27         for(int i=0;i<n;i+=(l<<1))
28             for(int j=0;j<l;j++)
29                 P[i+j]=((P[i+j]+P[i+j+1]*f)%MOD+MOD)%MOD;
30 }
31
32 void FWTxor(int n,int* P,int f) {
33     for (int l=1;(l<<1)<=n;l<<=1)
34         for(int i=0;i<n;i+=(l<<1))
35             for(int j=0;j<l;j++) {
36                 int t=P[i+j];
37                 P[i+j]=(1LL*f*(t+P[i+j+1])%MOD+MOD)%MOD;

```

```

38         P[i+j+1]=(1LL*f*(t-P[i+j+1]))%MOD+MOD)%MOD;
39     }
40 }
41
42 int A[MAXN],B[MAXN],C[MAXN];
43
44 void solve() {
45     int n;
46     scanf("%d", &n);
47     int lim=1<<n;
48     for(int i=0;i<lim;i++) scanf("%d", &A[i]);
49     for(int i=0;i<lim;i++) scanf("%d", &B[i]);
50
51     FWTor(lim,A,1);FWTor(lim,B,1);
52     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
53     FWTor(lim,A,-1);FWTor(lim,B,-1);FWTor(lim,C,-1);
54     for(int i=0;i<lim;i++) printf("%d ", C[i]);
55     puts("");
56
57     FWTand(lim,A,1);FWTand(lim,B,1);
58     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
59     FWTand(lim,A,-1);FWTand(lim,B,-1);FWTand(lim,C,-1);
60     for(int i=0;i<lim;i++) printf("%d ", C[i]);
61     puts("");
62
63     FWTxor(lim,A,1);FWTxor(lim,B,1);
64     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
65     FWTxor(lim,A,(MOD+1)/2);FWTxor(lim,B,(MOD+1)/2);FWTxor(lim,C,(MOD+1)/2);
66     for(int i=0;i<lim;i++) printf("%d ", C[i]);
67     puts("");
68 }
69
70 int main() {
71     int T=1,cas=1;(void)(cas);
72     // scanf("%d", &T);
73     while(T--) {
74         // printf("Case #%d: ", cas++);
75         solve();
76     }
77     return 0;
78 }

```

## 11.8 快速莫比乌斯变换 Fast\_Mobius\_Transform

```

1  /*
2     Coded with Leachim's ACM Template.
3     No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"

```

```

9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const LL INF=0x3f3f3f3f3f3f3f3f;
13 const double eps=1e-7;
14 const int dx[4]={1,-1,0,0};
15 const int dy[4]={0,0,1,-1};
16 const int MOD=998244353;
17 const int MAXN=2000005;
18
19 void FMTor(int n,int* P,int f) {
20     for (int i=1;i<n;i<=&1)
21         for(int j=0;j<n;j++)
22             if(i&j) P[j]=(P[j]+f*P[i^j])%MOD+MOD)%MOD;
23 }
24
25 void FMTand(int n,int* P,int f) {
26     for (int i=1;i<n;i<=&1)
27         for(int j=0;j<n;j++)
28             if(i&~j) P[j]=(P[j]+f*P[i|j])%MOD+MOD)%MOD;
29 }
30
31 void FWTxor(int n,int* P,int f) {
32     for (int l=1;(l<<1)<=n;l<=&1)
33         for(int i=0;i<n;i+=(l<<1))
34             for(int j=0;j<l;j++) {
35                 int t=P[i+j];
36                 P[i+j]=(1LL*f*(t+P[i+j+1])%MOD+MOD)%MOD;
37                 P[i+j+1]=(1LL*f*(t-P[i+j+1])%MOD+MOD)%MOD;
38             }
39 }
40
41 int A[MAXN],B[MAXN],C[MAXN];
42
43 void solve() {
44     int n;
45     scanf("%d", &n);
46     int lim=1<<n;
47     for(int i=0;i<lim;i++) scanf("%d", &A[i]);
48     for(int i=0;i<lim;i++) scanf("%d", &B[i]);
49
50     FMTor(lim,A,1);FMTor(lim,B,1);
51     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
52     FMTor(lim,A,-1);FMTor(lim,B,-1);FMTor(lim,C,-1);
53     for(int i=0;i<lim;i++) printf("%d ", C[i]);
54     puts("");
55
56     FMTand(lim,A,1);FMTand(lim,B,1);
57     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
58     FMTand(lim,A,-1);FMTand(lim,B,-1);FMTand(lim,C,-1);
59     for(int i=0;i<lim;i++) printf("%d ", C[i]);
60     puts("");
61

```

```

62     FWThor(lim,A,1);FWThor(lim,B,1);
63     for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;
64     FWThor(lim,A,(MOD+1)/2);FWThor(lim,B,(MOD+1)/2);FWThor(lim,C,(MOD+1)/2);
65     for(int i=0;i<lim;i++) printf("%d ", C[i]);
66     puts("");
67 }
68
69 int main() {
70     int T=1,cas=1;(void)(cas);
71     // scanf("%d", &T);
72     while(T--) {
73         // printf("Case #%d: ", cas++);
74         solve();
75     }
76     return 0;
77 }

```

## 11.9 快速子集变换 Fast\_Subset\_Transform

```

1  /*
2      Coded with Leachim's ACM Template.
3      No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"
9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const LL INF=0x3f3f3f3f3f3f3f3f;
13 const double eps=1e-7;
14 const int dx[4]={1,-1,0,0};
15 const int dy[4]={0,0,1,-1};
16 const int MOD=1e9+9;
17 const int MAXN=2000005;
18
19 void FWThor(int n,int* P,int f) {
20     for (int l=1;(l<<1)<=n;l<<=1)
21         for(int i=0;i<n;i+=(l<<1))
22             for(int j=0;j<l;j++)
23                 P[i+j+l]=(P[i+j+l]+P[i+j]*f)%MOD+MOD)%MOD;
24 }
25
26 int A[21][MAXN],B[21][MAXN],C[21][MAXN];
27
28 void solve() {
29     int n;
30     scanf("%d", &n);
31     int lim=1<<n;
32     for(int i=0;i<=n;i++) {
33         memset(A[i],0,lim*sizeof(A[i][0]));

```

```

34     memset(B[i],0,lim*sizeof(B[i][0]));
35     memset(C[i],0,lim*sizeof(C[i][0]));
36 }
37 for(int i=0;i<lim;i++)
38     scanf("%d", &A[__builtin_popcount(i)][i]);
39 for(int i=0;i<lim;i++)
40     scanf("%d", &B[__builtin_popcount(i)][i]);
41
42 for(int i=0;i<=n;i++) {
43     FWTor(lim,A[i],1);
44     FWTor(lim,B[i],1);
45 }
46
47 for(int i=0;i<=n;i++) {
48     for(int j=0;j<=i;j++) {
49         for(int s=0;s<lim;s++) {
50             C[i][s]=(C[i][s]+1LL*A[j][s]*B[i-j][s]%MOD)%MOD;
51         }
52     }
53     FWTor(lim,C[i],-1);
54 }
55
56 for(int i=0;i<lim;i++) printf("%d ", C[__builtin_popcount(i)][i]);
57 puts("");
58 }
59
60 int main() {
61     int T=1,cas=1;(void)(cas);
62     // scanf("%d", &T);
63     while(T--) {
64         // printf("Case #%d: ", cas++);
65         solve();
66     }
67     return 0;
68 }

```

## 11.10 24 点 24\_Point

```

1  /*
2     Coded with Leachim's ACM Template.
3     No errors. No warnings. ~~
4  */
5  #include <bits/stdc++.h>
6  #pragma GCC diagnostic ignored "-Wunused-const-variable"
7  #pragma GCC diagnostic ignored "-Wsign-conversion"
8  #pragma GCC diagnostic ignored "-Wsign-compare"
9  #define LL long long
10 using namespace std;
11 const int inf=0x3f3f3f3f;
12 const LL INF=0x3f3f3f3f3f3f3f3f;
13 const double eps=1e-7;
14 const int dx[4]={1,-1,0,0};

```

```
15 const int dy[4]={0,0,1,-1};
16 const int MOD=998244353;
17 const int MAXN=2000005;
18
19 int a[5],p[5];
20
21 set<pair<double,string>> dfs(int l, int r) {
22     set<pair<double,string>> s;
23     if(l==r) {
24         s.emplace(a[p[l]],to_string(a[p[l]]));
25     }
26     for(int i=l;i<r;i++) {
27         auto s1=dfs(l,i);
28         auto s2=dfs(i+1,r);
29         for(auto p1:s1) {
30             for(auto p2:s2) {
31                 s.emplace(p1.first+p2.first,"("+p1.second+"+"+p2.second+")");
32                 s.emplace(p1.first-p2.first,"("+p1.second+"-"+p2.second+")");
33                 s.emplace(p1.first*p2.first,"("+p1.second+"*"+p2.second+")");
34                 if(abs(p2.first)>eps) s.emplace(p1.first/p2.first,"("+p1.second+"/"+p2.
                    second+")");
35             }
36         }
37     }
38     return s;
39 }
40
41 void solve() {
42     for(int i=1;i<=4;i++) {
43         scanf("%d", &a[i]);
44         p[i]=i;
45     }
46     do {
47         auto s=dfs(1,4);
48         for(auto p:s) {
49             if(abs(p.first-24)<eps) {
50                 cout<<p.second.substr(1,p.second.size()-2)<<endl;
51                 return;
52             }
53         }
54     }while(next_permutation(p+1,p+5));
55
56     printf("-1\n");
57 }
58
59 int main() {
60     int T=1,cas=1;(void)(cas);
61     // scanf("%d", &T);
62     while(T--) {
63         // printf("Case #%d: ", cas++);
64         solve();
65     }
66     return 0;
```

67 }