# ICPC Templates

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## 1 图论

#### 1.1 最小生成树 Kruskal

```
1
    #include <cstdio>
    #include <algorithm>
 3
    #define MAXN 5005
 4
    #define MAXM 200005
 6
    #define _for(i,a,b) for(int i=(a);i<=(b);i++)
 7
 8
    using namespace std;
 9
10
    int read(){
11
       int ng=0,x=0;
12
        char ch=getchar();
13
       for(;ch<'0' || ch>'9';ch=getchar()) ng|=ch=='-';
14
       for(;ch>='0' && ch<='9';ch=getchar()) x=(x<<3)+(x<<1)+ch-'0';
       return ng?-x:x;
15
16
    }
17
18
    struct road
19
20
       int f,g,w;
21
    }r[MAXM];
22
23
    struct cmpf
24
25
       bool operator() (const road& a, const road& b) const {
26
           return a.w<b.w;</pre>
27
       }
28
    };
29
30
    int n,m,ans=0,fa[MAXN];
31
32
    int find(int x){
33
        return fa[x] == x?x:fa[x] = find(fa[x]);
    }
34
35
    void kruskal(){
36
37
       int rx,ry;
38
39
        sort(r+1, r+m+1, cmpf());
40
        _for(i,1,m){
41
           rx=find(r[i].f);
42
           ry=find(r[i].g);
43
           if(rx==ry)
44
               continue;
45
           fa[rx]=ry;
46
           ans+=r[i].w;
       }
47
    }
48
49
```

```
50
    int main(){
       n=read();m=read();
51
52
       _for(i,1,m){
53
           r[i].f=read();
54
           r[i].g=read();
55
           r[i].w=read();
56
       }
57
       _for(i,1,n)
58
           fa[i]=i;
59
       kruskal();
60
       _for(i,2,n){
           if(find(i)!=find(i-1)){
61
62
              puts("orz");
63
               return 0;
           }
64
65
       }
66
       printf("%d", ans);
    }
67
```

#### 1.2 最小生成树 Prim

```
#include <cstdio>
 2
    #include <cstdlib>
    #include <queue>
 4
    #define MAXN 5005
 5
 6
    #define MAXM 200005
 7
 8
    using namespace std;
10
    struct edge
11
    {
12
       int v,to,next;
    }e[MAXM<<1];</pre>
13
14
15
    int n,m,f,g,w,tot=0,head[MAXN],ans=0,flag[MAXN];
16
    void add(int x, int y, int z){
17
18
       e[++tot].v=z;
19
        e[tot].to=y;
20
        e[tot].next=head[x];
21
       head[x]=tot;
22
23
24
   struct HeapNode
25
26
        int v,u;
        bool operator <(const HeapNode& a) const{</pre>
27
28
           return v > a.v;
29
        }
30
   };
31
```

```
32
    priority_queue<HeapNode> Q;
33
34
    void prim(){
35
        for(int i=1;i<=n;i++)</pre>
36
           flag[i]=0;
        flag[1]=1;
37
38
        for(int p=head[1];p;p=e[p].next)
39
            Q.push((HeapNode){e[p].v,e[p].to});
        for(int i=1,u;i<n;i++){</pre>
40
41
            while(flag[Q.top().u]){
42
               Q.pop();
43
               if(Q.empty()){
44
                   puts("orz");
                   exit(0);
45
               }
46
47
           }
48
           ans+=Q.top().v;
49
           u=Q.top().u;
50
            Q.pop();
51
           flag[u]=1;
52
            for(int p=head[u];p;p=e[p].next)
53
               if(!flag[e[p].to])
                   Q.push((HeapNode){e[p].v,e[p].to});
54
55
        }
56
    }
57
58
    int main(){
59
        scanf("%d %d", &n, &m);
60
        for(int i=1;i<=n;i++)</pre>
61
           head[i]=0;
        for(int i=1;i<=m;i++){</pre>
62
63
            scanf("%d %d %d",&f,&g,&w);
64
            add(f,g,w);
65
            add(g,f,w);
66
        }
67
        prim();
68
        printf("%d",ans);
69
        return 0;
70
```

#### 1.3 最短路 Djikstra

```
#include <bits/stdc++.h>
2
   #define LL long long
3
   using namespace std;
    const int inf=0x3f3f3f3f;
4
   const LL INF=0x3f3f3f3f3f3f3f3f3f;
   const int MOD=998244353;
7
    const int MAXN=2000005;
8
9
   struct edge {
10
       int to,next,w;
```

```
}e[MAXN<<1];</pre>
11
12
13
    int tot,head[MAXN];
14
15
    void add(int x,int y,int z) {
16
        tot++;
17
        e[tot].to=y;
18
        e[tot].w=z;
19
        e[tot].next=head[x];
20
        head[x]=tot;
21
    }
22
23
    struct node {
24
       int u;
25
        LL dis;
26
        bool operator<(const node& y) const{</pre>
27
           return dis>y.dis;
28
        }
    };
29
30
    LL dist[MAXN];
31
32
    bool flag[MAXN];
33
    priority_queue<node> Q;
34
35
    void dij(int n,int s) {
36
        memset(flag+1,0,n*sizeof(flag[0]));
37
        memset(dist+1,0x3f,n*sizeof(dist[0]));
38
        dist[s]=0;
39
        Q.push({s,OLL});
40
        while(!Q.empty()) {
           while(!Q.empty() && flag[Q.top().u]) Q.pop();
41
42
           if(Q.empty()) break;
43
           int u=Q.top().u;
44
           Q.pop();
45
           flag[u]=1;
46
           for(int p=head[u];p;p=e[p].next) {
47
               int v=e[p].to,w=e[p].w;
48
               if(flag[v]) continue;
               if(dist[u]+w<dist[v]) {</pre>
49
50
                   dist[v]=dist[u]+w;
51
                   Q.push({v,dist[v]});
52
               }
53
           }
54
        }
55
    }
56
57
    void solve() {
58
        int n,m,s;
59
        scanf("%d %d %d", &n, &m, &s);
60
        tot=0;
61
        memset(head+1,0,n*sizeof(head[0]));
62
       for(int i=1;i<=m;i++) {</pre>
63
           int f,g,w;
```

```
64
            scanf("%d %d %d", &f, &g, &w);
65
            add(f,g,w);
66
        }
67
        dij(n,s);
68
        for(int i=1;i<=n;i++)</pre>
           printf("\%lld\%c", \ dist[i], \ " \ \"[i==n]);
69
70
    }
71
72
    int main() {
73
        int T=1,cas=1;(void)(cas);
74
        // scanf("%d", &T);
75
        while(T--) {
76
            // printf("Case #%d: ", cas++);
77
           solve();
78
        }
79
        return 0;
80
    }
```

#### 1.4 最短路 SPFA

```
#include <cstdio>
    #include <cstring>
 3
    #include <queue>
 5
    #define MAXN 100005
    #define MAXM 500005
 6
    const int inf=0x3f3f3f3f;
 8
9
    using namespace std;
10
11
    struct edge{
12
       int v,to,next;
   }e[MAXM];
13
14
    int n,m,p,tot,head[MAXN],dist[MAXN];
15
16
    bool flag[MAXN];
17
    void add(int x,int y,int z){
18
19
       tot++;
20
       e[tot].v=z;
21
       e[tot].to=y;
22
       e[tot].next=head[x];
23
       head[x]=tot;
24
   }
25
26
    void spfa(int x){
27
       queue <int> Q;
28
       memset(dist+1,inf,n*sizeof(dist[0]));
29
       memset(flag+1,0,n*sizeof(flag[0]));
30
31
       Q.push(x);
32
       flag[x]=true;
```

```
33
        dist[x]=0;
34
        while(!Q.empty()){
35
            int u=Q.front();
36
           Q.pop();
37
           flag[u]=false;
           for(int q=head[u];q;q=e[q].next){
38
39
               int v=e[q].to;
40
               if(dist[u]+e[q].v<dist[v]){</pre>
                   dist[v]=dist[u]+e[q].v;
41
42
                   if(!flag[v]){
43
                       Q.push(v);
44
                       flag[v]=true;
                   }
45
46
               }
47
            }
48
        }
49
50
51
    int main(){
52
        scanf("%d %d %d",&n,&m,&p);
53
        tot=0;
        memset(head+1,0,n*sizeof(head[0]));
54
        for(int i=1;i<=m;i++){</pre>
55
56
            int f,g,w;
57
            scanf("%d %d %d", &f, &g, &w);
58
            add(f,g,w);
59
60
        spfa(p);
61
        for(int i=1;i<=n;i++)</pre>
62
           printf("%d ", dist[i]==inf?2147483647:dist[i]);
63
        return 0;
64
    }
```

#### 1.5 最近公共祖先 LCA\_Doubling

```
#include <cstdio>
    #include <cstring>
3
    #include <algorithm>
    #define MAXN 500005
 4
    #define MAXM 500005
 6
    #define MAXLN 25
 7
    using namespace std;
8
9
    struct edge{
10
       int to,next;
11
    }e[MAXM<<1];</pre>
12
13
    int tot,head[MAXN];
14
    void add(int x,int y){
15
16
       tot++;
       e[tot].to=y;
17
```

```
18
        e[tot].next=head[x];
19
        head[x]=tot;
20
    }
21
22
    int dep[MAXN],st[MAXN][MAXLN];
23
24
    void dfs(int cur,int fa){
25
        dep[cur]=dep[fa]+1;
26
        st[cur][0]=fa;
27
        for(int l=1;(1<<1)<=dep[cur];l++)</pre>
28
            st[cur][l]=st[st[cur][l-1]][l-1];
29
30
        for(int p=head[cur];p;p=e[p].next){
31
            if(e[p].to==fa) continue;
            dfs(e[p].to,cur);
32
33
        }
34
35
36
    int lca(int x,int y){
37
        if(dep[x] < dep[y]) swap(x,y);</pre>
38
        for(int i=0;dep[x]-dep[y];i++)
39
            if((dep[x]-dep[y])&(1<<i)) x=st[x][i];</pre>
40
        if(x==y) return x;
41
        int 1;
42
43
        for(l=1;(1<<1)<=dep[x];1++);</pre>
44
        for(int i=1;i>=0;i--)
45
            if(st[x][i]!=st[y][i])
46
               x=st[x][i], y=st[y][i];
47
        return st[x][0];
    }
48
49
50
    int n,m,s;
51
52
    int main(){
53
        scanf("%d %d %d", &n, &m, &s);
54
        memset(head+1,0,n*sizeof(head[0]));
55
56
        for(int i=1;i<n;i++){</pre>
57
           int f,g;
            scanf("%d %d", &f, &g);
58
59
            add(f,g);
60
            add(g,f);
61
        }
62
        dep[0]=0;
63
        dfs(s,0);
64
        for(int i=1;i<=m;i++){</pre>
65
            int f,g;
66
            scanf("%d %d", &f, &g);
67
            printf("%d\n", lca(f,g));
68
        }
69
        return 0;
70
    }
```

#### 1.6 最近公共祖先 LCA\_Tarjan

```
#include <cstdio>
 2
    #define MAXN 500005
    #define MAXM 500005
 3
 4
 5
    struct edge{
 6
       int to,next;
    }e[MAXM<<1],eq[MAXM<<1];</pre>
 8
 9
    int n,m,s,tot,totq,head[MAXN],headq[MAXN],fa[MAXN],ans[MAXM];
10
11
    void add(int x, int y){
12
        tot++;
13
        e[tot].to=y;
14
        e[tot].next=head[x];
15
        head[x]=tot;
    }
16
17
18
    void addq(int x, int y){
19
        tot++;
20
        eq[tot].to=y;
21
        eq[tot].next=headq[x];
22
        headq[x]=tot;
23
    }
24
    int find(int x){
25
26
       if(fa[x]==x)
27
           return x;
28
        return fa[x]=find(fa[x]);
29
    }
30
31
    void tarjan(int x){
32
       fa[x]=x;
33
        for(int p=head[x];p;p=e[p].next){
34
           int u=e[p].to;
           if(!fa[u]){
35
               tarjan(u);
36
37
               fa[u]=x;
           }
38
39
       }
40
       for(int p=headq[x];p;p=eq[p].next){
           int u=eq[p].to;
41
42
           if(fa[u]){
43
               ans[(p+1)>>1]=find(u);
44
           }
       }
45
    }
46
47
    int main(){
48
49
        scanf("%d %d %d", &n, &m, &s);
50
        for(int i=1;i<=n;i++){</pre>
51
           head[i]=0;
```

```
52
            headq[i]=0;
53
        }
54
        tot=0;
55
        for(int i=1;i<n;i++){</pre>
56
            int f,g;
            scanf("%d %d",&f,&g);
57
58
            add(f,g);
59
            add(g,f);
60
        }
61
        tot=0;
62
        for(int i=1;i<=m;i++){</pre>
63
            int f,g;
64
            scanf("%d %d",&f,&g);
65
            addq(f,g);
66
            addq(g,f);
67
        }
68
        for(int i=1;i<=n;i++){</pre>
69
            fa[i]=0;
70
        }
71
        tarjan(s);
72
        for(int i=1;i<=m;i++)</pre>
73
            printf("%d\n", ans[i]);
74
        return 0;
    }
75
```

#### 1.7 判断负环 SPFA\_Negtive\_Cycle

```
#include <cstdio>
 2
    #include <cstring>
 3
    #include <queue>
 4
 5
    #define MAXN 100005
6
    #define MAXM 500005
    const int inf=0x3f3f3f3f;
8
9
    using namespace std;
10
11
    struct edge{
12
       int v,to,next;
13
    }e[MAXM];
14
    int n,m,tot,head[MAXN];
15
16
    int dist[MAXN],cnt[MAXN];
17
    bool flag[MAXN];
18
19
    void add(int x,int y,int z){
20
       tot++;
21
       e[tot].v=z;
22
       e[tot].to=y;
       e[tot].next=head[x];
23
       head[x]=tot;
24
25 }
```

```
26
27
    bool spfa(int x){//负环return false
28
        queue <int> Q;
29
        memset(dist+1,inf,n*sizeof(dist[0]));
30
        memset(flag+1,0,n*sizeof(flag[0]));
31
        memset(cnt+1,0,n*sizeof(cnt[0]));
32
33
        Q.push(x);
34
        flag[x]=true;
35
        dist[x]=0;
36
        while(!Q.empty()){
37
           int u=Q.front();
38
           Q.pop();
39
           flag[u]=false;
40
           for(int q=head[u];q;q=e[q].next){
               int v=e[q].to;
41
42
               if(dist[u]+e[q].v<dist[v]){</pre>
                   dist[v]=dist[u]+e[q].v;
43
                   if(!flag[v]) {
44
45
                      Q.push(v);
46
                      flag[v]=true;
47
                      if (++cnt[v]>=n){
48
                          return false;
49
50
                   }
51
               }
52
           }
53
        }
54
        return true;
55
56
57
    void solve() {
58
        scanf("%d %d",&n,&m);
59
        tot=0;
60
        memset(head+1,0,n*sizeof(head[0]));
        for(int i=1;i<=m;i++){</pre>
61
62
           int f,g,w;
63
           scanf("%d %d %d", &f, &g, &w);
64
           add(f,g,w);
65
           if(w>=0) add(g,f,w);
66
67
        if(spfa(1)) printf("NO\n");
68
        else printf("YES\n");
69
    }
70
71
    int main(){
72
        int T=1;
73
        scanf("%d", &T);
74
        while(T--) {
75
           solve();
76
       }
77
        return 0;
    }
78
```

#### 1.8 拓扑排序 Topological\_Sort\_Khan

```
#include <cstdio>
 2
    #include <cstring>
    #include <stack>
 3
    #define MAXN 100005
 5
    #define MAXM 200005
    using namespace std;
 8
 9
    struct edge{
10
        int to,next;
    }e[MAXM];
11
12
13
    int n,m,tot,head[MAXN],indgr[MAXN],list[MAXN];
14
    bool flag[MAXN];
15
16
    void add(int x,int y){
17
        tot++;
18
        e[tot].to=y;
19
        e[tot].next=head[x];
20
        head[x]=tot;
21
    }
22
23
    void khan(){
24
        stack<int> s;
25
        int cnt=0;
26
        memset(flag+1,0,n*sizeof(flag[0]));
27
        for(int i=1;i<=n;i++){</pre>
28
            if(!flag[i] && indgr[i]==0){
29
               s.push(i);
30
               flag[i]=true;
31
               while(!s.empty()){
32
                   int u=s.top();s.pop();
33
                  list[++cnt]=u;
34
                   flag[u]=true;
                   for(int p=head[u];p;p=e[p].next){
35
                      int v=e[p].to;
36
37
                      indgr[v]--;
38
                      if(indgr[v]==0)
39
                          s.push(v);
40
                   }
               }
41
           }
42
43
        }
44
    }
45
46
    int main(){
47
        scanf("%d %d",&n,&m);
        memset(head+1,0,n*sizeof(head[0]));
48
49
        memset(indgr+1,0,n*sizeof(indgr[0]));
50
        tot=0;
51
        for(int i=1;i<=m;i++){</pre>
```

```
52
           int f,g;
           scanf("%d %d", &f, &g);
53
54
           add(f,g);
55
           indgr[g]++;
       }
56
57
       khan();
58
       for(int i=1;i<=n;i++)</pre>
           printf("%d ", list[i]);
59
60
       return 0;
61
   }
```

### 2 多项式

#### 2.1 FFT 字符串匹配 String\_Match\_FFT

```
1
    #include <cstdio>
    #include <cstring>
    #include <cmath>
 4
    #include <algorithm>
 5
    #include <complex>
    #define MAXN 300005
 6
 7
    #define MAXL 550005
 8
 9
    using namespace std;
10
11
    const double PI=acos(-1);
12
    complex<double> omg[MAXL],iomg[MAXL];
13
14
    void init(int n){
15
        for(int i=0;i<n;i++){</pre>
16
            omg[i]=polar(1.0,2.0*PI*i/n);
17
            iomg[i]=conj(omg[i]);
18
        }
    }
19
20
21
    void FFT(int n,complex<double>* P,complex<double>* w){
22
        for(int i=0,j=0;i<n;i++){</pre>
23
            if(i<j) swap(P[i],P[j]);</pre>
24
            for(int l=n>>1;(j^=1)<1;l>>=1);
25
        }
26
27
        for(int i=2,1;i<=n;i<<=1){</pre>
28
            l=i>>1;
29
            complex<double> t;
30
            for(int j=0;j<n;j+=i){</pre>
31
               for(int k=0;k<1;k++){</pre>
32
                   t=P[j+l+k]*w[n/i*k];
33
                   P[j+l+k]=P[j+k]-t;
34
                   P[j+k]=P[j+k]+t;
35
                }
36
            }
37
        }
38
    }
39
40
    int n,m,lim;
41
    char s1[MAXN],s2[MAXN];
42
    complex<double> A[MAXL],B[MAXL],tA[MAXL],tB[MAXL],ans[MAXL];
43
44
    int main(){
        scanf("%d %d", &m, &n);
45
        scanf("%s %s",s1,s2);
46
47
        for(int i=0;i<m;i++) A[m-i-1]=(s1[i]=='*'?0:s1[i]-'a'+1);</pre>
        for(int i=0;i<n;i++) B[i]=(s2[i]=='*'?0:s2[i]-'a'+1);</pre>
48
        for(lim=1;lim<n-1;lim<<=1);</pre>
49
```

```
50
        init(lim);
51
        fill(ans,ans+lim,complex<double>(0.0,0.0));
52
53
        for(int i=0;i<m;i++) tA[i]=A[i]*A[i]*A[i];</pre>
54
        for(int i=0;i<n;i++) tB[i]=B[i];</pre>
55
        fill(tA+m,tA+lim,complex<double>(0.0,0.0));
56
        fill(tB+n,tB+lim,complex<double>(0.0,0.0));
57
        FFT(lim,tA,omg);
58
        FFT(lim,tB,omg);
        for(int i=0;i<lim;i++) ans[i]+=tA[i]*tB[i];</pre>
59
60
61
        for(int i=0;i<m;i++) tA[i]=A[i];</pre>
62
        for(int i=0;i<n;i++) tB[i]=B[i]*B[i]*B[i];</pre>
63
        fill(tA+m,tA+lim,complex<double>(0.0,0.0));
64
        fill(tB+n,tB+lim,complex<double>(0.0,0.0));
65
        FFT(lim,tA,omg);
66
        FFT(lim,tB,omg);
67
        for(int i=0;i<lim;i++) ans[i]+=tA[i]*tB[i];</pre>
68
69
        for(int i=0;i<m;i++) tA[i]=A[i]*A[i];</pre>
70
        for(int i=0;i<n;i++) tB[i]=B[i]*B[i];</pre>
71
        fill(tA+m,tA+lim,complex<double>(0.0,0.0));
72
        fill(tB+n,tB+lim,complex<double>(0.0,0.0));
73
        FFT(lim,tA,omg);
74
        FFT(lim,tB,omg);
75
        for(int i=0;i<lim;i++) ans[i]-=complex<double>(2.0,0)*tA[i]*tB[i];
76
77
        FFT(lim,ans,iomg);
78
79
        int cnt=0;
80
        for(int i=m-1;i<n;i++)</pre>
81
           if((int)(ans[i].real()/lim+0.5)==0) cnt++;
        printf("%d\n", cnt);
82
83
        for(int i=m-1;i<n;i++)</pre>
84
            if((int)(ans[i].real()/lim+0.5)==0) printf("%d ", i-m+2);
85
        return 0;
   }
```

#### 2.2 FFT 递归 Fast\_Fourier\_Transform\_Cooley-Tukey\_Recursion

```
#include <cstdio>
    #include <algorithm>
    #include <complex>
 4
    #define MAXN 4000005
5
 6
    using namespace std;
8
    complex<double> omg[MAXN],iomg[MAXN],temp[MAXN];
 9
10
    void init(int n){
11
       double PI=acos(-1);
12
       for(int i=0;i<n;i++){</pre>
```

```
13
            omg[i]=polar(1.0,2.0*PI*i/n);
14
            iomg[i]=conj(omg[i]);
15
        }
16
    }
17
18
    void FFT(int n,complex<double>* buffer,int offset,int step,complex<double>* omg){
19
        if(n==1) return;
20
        int m=n>>1;
21
        FFT(m,buffer,offset,step<<1,omg);</pre>
22
        FFT(m,buffer,offset+step,step<<1,omg);</pre>
23
        for(int i=0;i<m;i++){</pre>
24
            int pos=2*i*step;
            temp[i]=buffer[offset+pos]+omg[i*step]*buffer[offset+step+pos];
25
            temp[i+m]=buffer[offset+pos]-omg[i*step]*buffer[offset+step+pos];
26
27
28
        for(int i=0;i<n;i++)</pre>
29
            buffer[offset+i*step]=temp[i];
30
    }
31
32
    int n,m,lim;
33
    complex<double> A[MAXN],B[MAXN];
34
35
    int main(){
36
        scanf("%d %d", &n, &m);
        for(lim=1;lim<=n+m;lim<<=1);</pre>
37
38
        fill(A,A+lim,complex<double>(0.0,0.0));
39
        fill(B,B+lim,complex<double>(0.0,0.0));
40
        for(int i=0,t;i<n+1;i++){</pre>
            scanf("%d", &t);
41
42
            A[i]+=t;
43
        }
44
        for(int i=0,t;i<m+1;i++){</pre>
            scanf("%d", &t);
45
            B[i]+=t;
46
        }
47
48
        init(lim);
49
        FFT(lim,A,0,1,omg);
50
        FFT(lim,B,0,1,omg);
51
        for(int i=0;i<lim;i++)</pre>
52
            A[i]=A[i]*B[i];
53
        FFT(lim,A,0,1,iomg);
54
        for(int i=0;i<n+m+1;i++)</pre>
55
            printf("%d ", (int)(A[i].real()/lim+0.5));
56
        return 0;
57
```

#### 2.3 FFT 递推 Fast\_Fourier\_Transform\_Cooley-Tukey\_Iteration

```
#include <bits/stdc++.h>
#define MAXN 2100005

using namespace std;
```

```
5
 6
    const double PI=acos(-1);
 7
 8
    struct Complex{
 9
        double real,image;
10
        Complex operator+(Complex y)const{
11
            return {real+y.real,image+y.image};
12
13
        Complex operator-(Complex y)const{
14
            return {real-y.real,image-y.image};
15
        }
16
        Complex operator*(Complex y)const{
17
           return {real*y.real-image*y.image,real*y.image+image*y.real};
18
        }
19
    };
20
21
    void FFT(int n,Complex* P,int f){
22
        for(int i=0,j=0;i<n;i++){</pre>
23
            if(i<j) swap(P[i],P[j]);</pre>
            for(int l=n>>1;(j^=1)<1;l>>=1);
24
        }
25
26
        for(int i=2,1;i<=n;i<<=1){</pre>
27
28
            l=i>>1;
29
           Complex wn={cos(2*PI/i),f*sin(2*PI/i)};
30
           for(int j=0;j<n;j+=i){</pre>
31
               Complex w={1,0};
32
               for(int k=0;k<1;k++,w=w*wn){</pre>
33
                   Complex t=P[j+l+k]*w;
34
                   P[j+l+k]=P[j+k]-t;
35
                   P[j+k]=P[j+k]+t;
36
               }
37
            }
38
        }
    }
39
40
41
    int n,m,lim;
42
    Complex A[MAXN],B[MAXN];
43
44
    int main(){
        scanf("%d %d", &n, &m);
45
46
        for(lim=1;lim<=n+m;lim<<=1);</pre>
47
        memset(A,0,lim*sizeof(A[0]));
48
        memset(B,0,lim*sizeof(B[0]));
49
        for(int i=0,t;i<n+1;i++){</pre>
50
            scanf("%d", &t);
51
            A[i] = \{1.0*t,0\};
52
        }
53
        for(int i=0,t;i<m+1;i++){</pre>
            scanf("%d", &t);
54
55
           B[i]={1.0*t,0};
56
        FFT(lim,A,1);
```

```
58     FFT(lim,B,1);
59     for(int i=0;i<lim;i++)
60          A[i]=A[i]*B[i];
61     FFT(lim,A,-1);
62     for(int i=0;i<n+m+1;i++)
63          printf("%d ", (int)(A[i].real/lim+0.5));
64     return 0;
65 }</pre>
```

#### 2.4 NTT 递推 Number\_Theoretic\_Transforms

```
#include <cstdio>
    #include <algorithm>
 3
    using namespace std;
 4
    #define LL long long
    const int MAXN=2100005
    const int MOD=998244353
    const int RT=3;
 9
    LL binpow(LL x,LL y,LL mod){
10
        LL r=1%mod;
11
        while(y){
12
           if(y&1) r=(r*x)%mod;
13
           x=(x*x) \mod;
14
           y>>=1;
15
        }
16
        return r;
17
    }
18
19
    void NTT(int n,int* P,int f){
        for(int i=0,j=0;i<n;i++){</pre>
20
21
            if(i<j) swap(P[i],P[j]);</pre>
22
            for(int l=n>>1;(j^=1)<1;l>>=1);
23
24
25
        for(int i=2,1;i<=n;i<<=1){</pre>
26
           l=i>>1;
27
           int wn=binpow(RT,(MOD-1)/i,MOD);
28
           if(f==-1) wn=binpow(wn,MOD-2,MOD);
29
           for(int j=0;j<n;j+=i){</pre>
30
               int w=1;
31
               for(int k=0;k<1;k++,w=1LL*w*wn%MOD){</pre>
32
                   int t=1LL*P[j+l+k]*w%MOD;
33
                   P[j+l+k]=1LL*(P[j+k]-t+MOD)%MOD;
34
                   P[j+k]=1LL*(P[j+k]+t)%MOD;
35
            }
36
37
        }
38
39
40
    int A[MAXN],B[MAXN];
41
```

```
42
    int main(){
43
        int n,m;
        scanf("%d %d", &n, &m);
44
45
        int lim:
46
        for(lim=1;lim<=n+m;lim<<=1);</pre>
        for(int i=0;i<n+1;i++){</pre>
47
48
            scanf("%d", &A[i]);
49
50
        for(int i=0;i<m+1;i++){</pre>
51
            scanf("%d", &B[i]);
52
        }
        NTT(lim,A,1);
53
54
        NTT(lim,B,1);
55
        for(int i=0;i<lim;i++)</pre>
            A[i]=1LL*A[i]*B[i]%MOD;
56
57
        NTT(lim,A,-1);
58
        int invn=binpow(lim,MOD-2,MOD);
        for(int i=0;i<n+m+1;i++)</pre>
59
60
            printf("%lld ", 1LL*A[i]*invn%MOD);
61
        return 0;
```

#### 2.5 多项式求逆 Polynomial\_Inverse

```
#include <cstdio>
 2
    #include <algorithm>
    #define LL long long
    #define MAXN 270005
 5
    #define MOD 998244353
    #define RT 3
 8
    using namespace std;
 9
    LL omg[MAXN],iomg[MAXN];
10
11
    LL binpow(LL x,LL y,LL mod){
12
13
        LL r=1%mod;
        while(y){
14
15
           if(y&1) r=(r*x)%mod;
16
           x=(x*x)\mbox{mod};
17
           y>>=1;
        }
18
19
        return r;
20
    }
21
22
    void init(int n){
        omg[0]=iomg[0]=1;
23
24
        omg[1]=binpow(RT,(MOD-1)/n,MOD);
25
        iomg[1]=binpow(omg[1],MOD-2,MOD);
       for(int i=2;i<n;i++){</pre>
26
27
           omg[i]=omg[i-1]*omg[1]%MOD;
           iomg[i]=iomg[i-1]*iomg[1]%MOD;
```

```
29
        }
30
    }
31
    void NTT(int n,LL* P,LL* w){
32
33
        for(int i=0, j=0; i<n; i++){</pre>
            if(i<j) swap(P[i],P[j]);</pre>
34
35
            for(int l=n>>1;(j^=1)<1;1>>=1);
36
        }
37
38
        for(int i=2,1;i<=n;i<<=1){</pre>
39
            l=i>>1;
40
            for(int j=0;j<n;j+=i){</pre>
41
               for(int k=0;k<1;k++){</pre>
42
                   LL t=P[j+l+k]*w[n/i*k]%MOD;
43
                   P[j+l+k]=(P[j+k]-t+MOD)%MOD;
44
                   P[j+k] = (P[j+k]+t)\%MOD;
45
               }
            }
46
        }
47
    }
48
49
50
    void poly_inv(int dgr,LL* X,LL* Y){
        if(dgr==1){
51
52
            Y[0]=binpow(X[0],MOD-2,MOD);
53
        }
54
        else{
55
            poly_inv((dgr+1)>>1,X,Y);
56
57
            static LL Z[MAXN];
58
            int lim;
            for(lim=1;lim<(dgr<<1);lim<<=1);</pre>
59
60
            copy(X,X+dgr,Z);
61
            fill(Z+dgr,Z+lim,0);
62
            init(lim);
63
64
            NTT(lim,Z,omg);
65
            NTT(lim,Y,omg);
66
            for(int i=0;i<lim;i++)</pre>
67
                Y[i] = (2-Z[i] *Y[i] %MOD+MOD) *Y[i] %MOD;
68
            NTT(lim,Y,iomg);
69
            LL invlim=binpow(lim,MOD-2,MOD);
70
            for(int i=0;i<dgr;i++)</pre>
71
                Y[i]=Y[i]*invlim%MOD;
72
            fill(Y+dgr,Y+lim,0);
73
        }
74
    }
75
76
   int n;
   LL A[MAXN],B[MAXN];
77
79
    int main(){
80
        scanf("%d", &n);
        for(int i=0;i<n;i++)</pre>
```

## 3 字符串

#### 3.1 字符串哈希 String\_Hash

```
#include <cstdio>
    #include <string>
3
    #define BASE 307
4
    #define MOD 5555567
 6
7
    int hsh(string x){
8
       int h=0,len=x.length();
9
       for(int i=0;i<len;i++){</pre>
10
           h=(h*BASE+x[i])%MOD;
11
12
       return h;
    }
13
14
15
    int main(){
16
       return 0;
17
    }
```

#### 3.2 马拉车 Manacher

```
#include <cstdio>
 2
    #include <cstring>
    #include <algorithm>
    #define MAXN 11000005
    using namespace std;
 6
 7
    int r[MAXN<<1],mx;</pre>
 8
    char st[MAXN<<1];</pre>
 9
10
    void manacher(char *s){
11
        int len=strlen(s);
12
        st[0]='$';
        for(int i=0;i<len;i++){</pre>
13
14
            st[i<<1|1]='#';
15
            st[(i+1)<<1]=s[i];
        }
16
17
        len=len<<1|1;
18
        st[len]='#';
19
        st[len+1]='*';
20
        r[1]=1;
21
        mx=0;
22
        for(int i=2,mid=1;i<=len;i++){</pre>
23
           r[i]=min(mid+r[mid]-i,r[2*mid-i]);
            for(;st[i-r[i]]==st[i+r[i]];r[i]++);
24
25
            if(i+r[i]>mid+r[mid]) mid=i;
26
           mx=max(mx,r[i]-1);
27
        }
28 }
```

```
29
    char s[MAXN];
30
31
32
    int main(){
33
       scanf("%s",s);
34
       manacher(s);
35
       printf("%d\n", mx);
36
       return 0;
37
    }
```

#### 3.3 字符串匹配 KMP

```
#include <bits/stdc++.h>
 2
    using namespace std;
 3
    #define MAXN 1000005
 4
    int kmp(char *s1, char *s2, int *nxt){
 5
       int n=strlen(s1+1),m=strlen(s2+1);
 6
 7
 8
       nxt[0]=-1;
       for(int i=1,k=-1;i<=m;i++){</pre>
9
           //k初始值为-1. nxt数组可以查询boarder的boarder.
10
11
           while(~k && s2[k+1]!=s2[i])
              k=nxt[k];
12
13
           nxt[i]=++k;
14
       }
15
16
       int cnt=0;
17
       for(int i=1,k=0;i<=n;i++){</pre>
           //匹配串前缀与模式串后缀比,上一位前缀的boarder的boarder也能匹配。
18
           while(~k && s2[k+1]!=s1[i])
19
20
              k=nxt[k];
21
           if(m==++k){
22
              cnt++;
23
              printf("%d\n", i-m+1);
24
           }
25
       }
26
       return cnt;
27
28
29
    char s1[MAXN],s2[MAXN];
30
    int nxt[MAXN];
31
32
    int main(){
33
       scanf("%s %s", s1+1, s2+1);
34
       kmp(s1,s2,nxt);
35
       int len=strlen(s2+1);
36
       for(int i=1;i<=len;i++){</pre>
           printf("%d ", nxt[i]);
37
38
       }
39
       return 0;
    }
40
```

#### 3.4 AC 自动机 AC-Automaton

```
#include <cstdio>
    #include <cstring>
 2
 3
    #include <map>
    #include <queue>
 4
 5
    #define MAXN 1000006
    using namespace std;
 7
 8
    struct trie{
 9
        int fail,mark,ch[26];
10
    }tt[MAXN];
11
12
    int tot;
13
14
    void insert(char *s){
15
        int len=strlen(s+1), cur=0;
16
        for(int i=1;i<=len;i++){</pre>
17
           int& next=tt[cur].ch[s[i]-'a'];
18
           if(!next){
19
               next=++tot;
20
               tt[tot].mark=0;
21
           }
22
           cur=next;
23
        }
24
        tt[cur].mark++;
25
    }
26
27
    void getfail(){
28
        queue<int> Q;
        tt[0].fail=0;
29
30
        tt[0].mark=0;
31
        for(int i=0;i<26;i++)</pre>
32
           if(tt[0].ch[i]) tt[tt[0].ch[i]].fail=0, Q.push(tt[0].ch[i]);
33
        while(!Q.empty()){
34
           int u=Q.front(); Q.pop();
35
           for(int i=0;i<26;i++){</pre>
               if(!tt[u].ch[i]) continue;
36
               int k=tt[u].fail;
37
38
               while(k && !tt[k].ch[i])
39
                  k=tt[k].fail;
               tt[tt[u].ch[i]].fail=tt[k].ch[i];
40
41
               // tt[tt[u].ch[i]].mark+=tt[tt[k].ch[i]].mark;//如果需要重复统计, fail累加标记
42
               Q.push(tt[u].ch[i]);
43
           }
44
        }
45
    }
46
47
    int query(char *s){
        int len=strlen(s+1),ans=0;
48
49
        for(int i=1,k=0;i<=len;i++){</pre>
50
           while(k && !tt[k].ch[s[i]-'a'])
               k=tt[k].fail;
51
```

```
52
           k=tt[k].ch[s[i]-'a'];
53
           ans+=tt[k].mark;
54
           tt[k].mark=0;//清除该字符串的标记(只求
55
56
       }
57
       return ans;
58
    }
59
60
    int n;
61
    char s[MAXN];
62
    int main(){
63
64
       scanf("%d", &n);
65
       tot=0;
66
       memset(tt,0,sizeof(tt));
67
       for(int i=1;i<=n;i++){</pre>
           scanf("%s", s+1);
68
69
           insert(s);
       }
70
71
       scanf("%s", s+1);
72
        getfail();
73
       printf("%d\n", query(s));
74
       return 0;
75
    }
```

#### 3.5 后缀数组 Suffix\_Array

```
#include <cstdio>
 2
    #include <cstring>
    #include <algorithm>
 4
    #define MAXN 1000005
 5
    using namespace std;
 6
   //sa:排名对应的前缀, rk:前缀的排名, tp:第二关键字排名对应的前缀, tax:排名对应的个数
 8
   //height:排名i与i-1后缀的LCP(最长公共前缀)
   int sa[MAXN],r1[MAXN],r2[MAXN],tax[MAXN],height[MAXN];
9
10
    int *rk=r1,*tp=r2;
11
    char s[MAXN];
12
13
   void rsort(int n,int m){
       memset(tax,0,(m+1)*sizeof(tax[0]));
14
15
       for(int i=1;i<=n;i++) tax[rk[i]]++;//当前排名装桶
16
       for(int i=1;i<=m;i++) tax[i]+=tax[i-1];//计算桶的名次
17
       for(int i=n;i>=1;i--) sa[tax[rk[tp[i]]]--]=tp[i];//按照第二关键字降序,分配排名。
18
   }
19
20
   void get_sa(char* s){
21
       //O(nlogn)
22
       int n=strlen(s+1), m=0;
23
       for(int i=1;i<=n;i++)</pre>
24
          m=max(m,rk[i]=s[i]),tp[i]=i;
25
       rsort(n,m);
```

```
26
        for(int k=1,p=0;p<n;k<<=1,m=p){</pre>
27
           p=0;
           //重制第二关键字
28
29
           for(int i=n-k+1;i<=n;i++) tp[++p]=i; //后续为空, 排前面
30
           for(int i=1;i<=n;i++) if(sa[i]>k) tp[++p]=sa[i]-k; //按照第一关键字排第二关键字
31
32
           rsort(n,m);
33
34
           swap(tp,rk);
35
           rk[sa[1]]=p=1;
36
           for(int i=2;i<=n;i++){</pre>
37
               \label{eq:rksa[i]} $$ rk[sa[i]] = tp[sa[i-1]]  \&\&  tp[sa[i]+k] = tp[sa[i-1]+k])?p:++p; 
           }
38
39
       }
40
       //利用height[rk[i+1]]>=height[rk[i]]-1
41
42
       for(int i=1,k=0;i<=n;i++){</pre>
43
44
           if(k) k--;
           while(rk[i]>1 && s[i+k]==s[sa[rk[i]-1]+k]) k++;
45
46
           height[rk[i]]=k;
        }
47
48
    }
49
    int main(){
50
51
       scanf("%s",s+1);
52
        get_sa(s);
53
       int len=strlen(s+1);
       for(int i=1;i<=len;i++)</pre>
54
55
           printf("%d ", sa[i]);
56
        return 0;
57
    }
```

#### 3.6 后缀自动机 Suffix-Automaton

```
#include <bits/stdc++.h>
    #define MAXN 1000005
    #define LL long long
 3
 4
    using namespace std;
 5
 6
    struct SAM {
 7
       int len,link,cnt;
8
       int ch[26];
9
    }sam[MAXN<<1];</pre>
10
    int sz,last;
11
12
13
    void sam_init() {
       sam[0].len=0;
14
15
       sam[0].link=-1;
16
       sam[0].cnt=0;
       memset(sam[0].ch,0,sizeof(sam[0].ch));
17
```

```
18
        sz=0;
19
        last=0;
20
    }
21
22
    void sam_extend(int c) {
23
        int cur=++sz;
24
        sam[cur].len=sam[last].len+1;
25
        memset(sam[cur].ch,0,sizeof(sam[cur].ch));
26
        int p=last;
27
28
        for(;~p && !sam[p].ch[c];p=sam[p].link)
29
           sam[p].ch[c]=cur;
30
31
        if(!~p) {
32
           sam[cur].link=0;
33
        } else {
34
           int q=sam[p].ch[c];
           if(sam[p].len+1==sam[q].len) {
35
36
               sam[cur].link=q;
37
           } else {
38
               int clone=++sz;
39
               sam[clone] = sam[q];
40
               sam[clone].len=sam[p].len+1;
41
               sam[clone].cnt=0;
42
               sam[q].link=sam[cur].link=clone;
43
               for(;~p && sam[p].ch[c]==q;p=sam[p].link)
44
                   sam[p].ch[c]=clone;
45
           }
46
       }
47
        last=cur;
48
49
        sam[cur].cnt=1;
50
    }
51
52
    struct edge{
53
        int to,next;
54
    }e[MAXN<<1];</pre>
55
56
    int tot,head[MAXN<<1];</pre>
57
58
    void add(int x,int y) {
59
        tot++;
60
        e[tot].to=y;
61
        e[tot].next=head[x];
62
        head[x]=tot;
63
    }
64
65
    char s[MAXN];
66
    LL ans;
67
68
    void dfs(int x) {
69
       for(int p=head[x];p;p=e[p].next) {
           int u=e[p].to;
70
```

```
71
            dfs(u);
72
            sam[x].cnt+=sam[u].cnt;
73
        }
74
        if(sam[x].cnt!=1) ans=max(ans,1LL*sam[x].len*sam[x].cnt);
75
76
77
     void solve() {
78
        scanf("%s", s);
79
        int len=strlen(s);
80
        sam_init();
81
        for(int i=0;i<len;i++)</pre>
82
            sam_extend(s[i]-'a');
83
        tot=0;
84
        memset(head,0,(sz+1)*sizeof(head[0]));
85
        for(int i=1;i<=sz;i++)</pre>
86
            add(sam[i].link,i);
87
        ans=0;
88
        dfs(0);
        printf("%lld\n", ans);
89
90
    }
91
92
     int main() {
93
        int T=1,cas=1;
94
        // scanf("%d", &T);
        while(T--) {
95
96
            // printf("Case #%d: ", cas++);
97
            solve();
98
        }
99
        return 0;
100
```

#### 3.7 广义后缀自动机 General\_Suffix-Automaton

```
#include <bits/stdc++.h>
    #define MAXN 1000005
    #define LL long long
3
    using namespace std;
5
    // const int inf=0x3f3f3f3f;
6
    struct SAM {
       int len,link,cnt;
8
9
       int ch[26];
10
    }sam[MAXN<<1];</pre>
11
12
    int sz,last;
13
    void sam_init() {
14
15
       sam[0].len=0;
16
       sam[0].link=-1;
17
       sam[0].cnt=0;
18
       memset(sam[0].ch,0,sizeof(sam[0].ch));
19
       sz=0;
```

```
20
        last=0;
21
    }
22
23
    void sam_extend(int c) {
24
        if(sam[last].ch[c] && sam[last].len+1==sam[sam[last].ch[c]].len) {
25
           last=sam[last].ch[c];
26
           sam[last].cnt++;
27
           return;
        }
28
29
30
        int cur=++sz;
        sam[cur].len=sam[last].len+1;
31
        memset(sam[cur].ch,0,sizeof(sam[cur].ch));
32
33
34
        int p=last;
35
        for(;~p && !sam[p].ch[c];p=sam[p].link)
36
           sam[p].ch[c]=cur;
37
38
        if(!~p) {
39
           sam[cur].link=0;
40
        } else {
41
           int q=sam[p].ch[c];
           if(sam[p].len+1==sam[q].len) {
42
43
               sam[cur].link=q;
           } else {
44
45
               int clone;
46
               if(p==last) {
47
                   clone=cur;
               } else {
48
49
                   clone=++sz;
50
                   sam[cur].link=clone;
51
               }
52
53
               sam[clone] = sam[q];
               sam[clone].len=sam[p].len+1;
54
55
               sam[q].link=clone;
56
               sam[clone].cnt=0;
57
               for(;~p && sam[p].ch[c]==q;p=sam[p].link)
58
                   sam[p].ch[c]=clone;
59
           }
       }
60
61
62
        last=cur;
63
        sam[cur].cnt=1;
64
    }
65
    char s[MAXN];
66
67
    void solve() {
68
69
        int n;
70
        scanf("%d", &n);
71
       sam_init();
        for(int i=1;i<=n;i++) {</pre>
```

```
73
           scanf("%s", s);
74
           int len=strlen(s);
75
           last=0;
76
           for(int j=0;j<len;j++) {
77
              sam_extend(s[j]-'a');
78
           }
79
       }
80
81
       LL ans=0;
82
       for(int i=1;i<=sz;i++) {</pre>
           ans+=sam[i].len-sam[sam[i].link].len;
83
84
85
86
       printf("%lld\n", ans);
    }
87
88
89
   int main() {
90
       int T=1,cas=1;
91
       // scanf("%d", &T);
92
       while(T--) {
           // printf("Case #%d: ", cas++);
93
94
           solve();
95
       }
96
       return 0;
97
   }
```

## 4 数据结构

#### 4.1 并查集 Disjoin\_Set\_Union

```
#include <cstdio>
    #define maxn 10005
 3
 4
    int n,m,a,b,c,fa[maxn];
 5
 6
    void ini(){
 7
       for(int i=1;i<=n;i++){</pre>
 8
           fa[i]=i;
 9
       }
10
    }
11
12
    int find(int x){
       if(fa[x] == x)
13
14
           return x;
15
       return fa[x]=find(fa[x]);
    }
16
17
18
    void join(int x, int y){
       fa[find(x)]=find(y);
19
20
    }
21
22
    int main(){
23
       scanf("%d %d", &n, &m);
24
        ini();
        for(int i=1; i<=m; i++){</pre>
25
           scanf("%d %d %d", &a, &b, &c);
26
27
           if(a==1)
28
               join(b,c);
           else if(find(b)==find(c))
29
30
               puts("Y");
31
           else puts("N");
32
       }
33
       return 0;
34
```

#### 4.2 可撤销并查集 Disjoin\_Set\_Union\_Withdrawable

```
struct DSU{
 2
       int fa[MAXN],rk[MAXN];
3
       vector<pair<int*,int>> stk;
       void init(int n) {
 4
 5
           stk.clear();
 6
           for(int i=1;i<=n;i++) fa[i]=i,rk[i]=1;</pre>
8
       int find(int x) {
9
           if(x==fa[x]) return x;
10
           return find(fa[x]);
11
       }
```

```
12
       bool join(int x,int y) {
13
           int rx=find(x),ry=find(y);
           if(rx==ry) return false;
14
15
16
           if(rk[rx]>rk[ry]) swap(rx,ry);
17
           stk.emplace_back(fa+rx,rx);
18
           fa[rx]=ry;
19
           stk.emplace_back(rk+ry,rk[ry]);
           rk[ry]+=rk[rx];
20
21
           return true;
22
       }
23
       void withdraw() {
           *stk.back().first=stk.back().second;
24
25
           stk.pop_back();
26
           *stk.back().first=stk.back().second;
27
           stk.pop_back();
       }
28
29
    }dsu;
```

#### 4.3 分块 Block\_1

```
#include <cstdio>
 2
    #include <cstring>
    #include <cmath>
    #include <algorithm>
 4
    #define MAXN 50005
 5
 6
 7
    using namespace std;
 8
 9
    int n,blo,v[MAXN],bl[MAXN],atag[MAXN];
10
11
    void add(int l,int r,int x){
12
        if(bl[l]==bl[r]){
13
            for(int i=1;i<=r;i++)</pre>
14
               v[i]+=x;
15
        }
16
        else{
            for(int i=1;i<=bl[1]*blo;i++)</pre>
17
18
               v[i]+=x;
19
            for(int i=(bl[r]-1)*blo+1;i<=r;i++)</pre>
20
               v[i]+=x;
        }
21
22
        for(int i=bl[l]+1;i<=bl[r]-1;i++)</pre>
23
            atag[i]+=x;
24
    }
25
    int main(){
26
27
        scanf("%d", &n);blo=sqrt(n);
28
        for(int i=1;i<=n;i++)</pre>
29
            scanf("%d", &v[i]);
        for(int i=1;i<=n;i++)</pre>
30
            bl[i]=(i-1)/blo+1;
31
```

```
32
        memset(atag+1,0,bl[n]*sizeof(atag[0]));
33
        for(int i=1;i<=n;i++){</pre>
34
           int opt,1,r,c;
35
           scanf("%d %d %d %d", &opt, &l, &r, &c);
36
           if(opt==0){
               add(1,r,c);
37
38
           }
39
           else printf("%d\n", v[r]+atag[bl[r]]);
40
41
        return 0;
42
```

#### 4.4 分块 Block\_2

```
1
    #include <cstdio>
    #include <cstring>
 3
    #include <cmath>
    #include <algorithm>
    #include <vector>
    #define MAXN 50005
 6
    #define MAXB 505
 8
 9
    using namespace std;
10
11
    int n,blo,v[MAXN],bl[MAXN],atag[MAXB];
12
    vector<int> ve[MAXB];
13
14
    void reset(int x){
15
        ve[x].clear();
16
        for(int i=(x-1)*blo+1;i<=min(x*blo,n);i++){</pre>
17
            ve[x].push_back(v[i]);
        }
18
        sort(ve[x].begin(),ve[x].end());
19
    }
20
21
22
    void add(int 1,int r,int x){
23
        if(bl[1]==bl[r]){
            for(int i=1;i<=r;i++)</pre>
24
25
               v[i]+=x;
26
           reset(bl[1]);
27
        }
28
        else{
29
           for(int i=1;i<=bl[1]*blo;i++)</pre>
30
               v[i] +=x;
31
           reset(bl[1]);
32
           for(int i=(bl[r]-1)*blo+1;i<=r;i++)</pre>
33
               v[i] +=x;
34
           reset(bl[r]);
35
            for(int i=bl[l]+1;i<=bl[r]-1;i++)</pre>
36
            atag[i]+=x;
        }
37
38 }
```

```
39
40
     int query(int 1,int r,int x){
41
        int cnt=0;
42
        if(bl[l]==bl[r]){
43
            for(int i=1;i<=r;i++)</pre>
44
                if(v[i]+atag[bl[i]]<x) cnt++;</pre>
45
        }
46
        else{
            for(int i=1;i<=bl[l]*blo;i++)</pre>
47
                if(v[i]+atag[bl[i]]<x) cnt++;</pre>
48
49
            for(int i=(bl[r]-1)*blo+1;i<=r;i++)</pre>
50
                if(v[i]+atag[bl[i]]<x) cnt++;</pre>
51
            for(int i=bl[1]+1;i<=bl[r]-1;i++)</pre>
52
                cnt+=lower_bound(ve[i].begin(),ve[i].end(),x-atag[i])-ve[i].begin();
53
54
        return cnt;
55
    }
56
57
    int main(){
58
        scanf("%d", &n);blo=sqrt(n);
59
        for(int i=1;i<=n;i++)</pre>
60
            scanf("%d", &v[i]);
61
        for(int i=1;i<=n;i++){</pre>
62
            bl[i]=(i-1)/blo+1;
63
            ve[bl[i]].push_back(v[i]);
64
65
        memset(atag+1,0,bl[n]*sizeof(atag[0]));
66
        for(int i=1;i<=bl[n];i++)</pre>
67
            sort(ve[i].begin(),ve[i].end());
68
69
        for(int i=1;i<=n;i++){</pre>
70
            int opt,1,r,c;
71
            scanf("%d %d %d %d", &opt, &l, &r, &c);
72
            if(opt==0){
73
                add(1,r,c);
74
            }
75
            else{
76
                printf("%d\n", query(1,r,c*c));
77
            }
78
        }
79
        return 0;
80
    }
```

#### 4.5 分块 Block\_4

```
#include <cstdio>
#include <cstring>
#include <cmath>
#include <algorithm>
#define LL long long
#define MAXN 50005
#define MAXB 505
```

```
8
 9
    using namespace std;
10
11
    int n,blo;
12
    LL v[MAXN],bl[MAXN],atag[MAXB],sum[MAXB];
13
14
    void add(int 1,int r,int x){
15
        if(bl[1]==bl[r]){
16
            for(int i=1;i<=r;i++)</pre>
17
                v[i]+=x,sum[bl[i]]+=x;
18
        }
19
        else{
20
            for(int i=1;i<=b1[1]*blo;i++)</pre>
21
                v[i]+=x,sum[bl[i]]+=x;
            for(int i=(bl[r]-1)*blo+1;i<=r;i++)</pre>
22
23
                v[i]+=x,sum[bl[i]]+=x;
24
            for(int i=bl[l]+1;i<=bl[r]-1;i++)</pre>
25
                atag[i]+=x;
26
        }
27
    }
28
    LL query(int 1,int r){
29
30
        LL ans=0;
31
        if(bl[l]==bl[r]){
32
            for(int i=1;i<=r;i++)</pre>
33
                ans+=v[i]+atag[bl[i]];
34
        }
35
        else{
36
            for(int i=1;i<=b1[1]*blo;i++)</pre>
37
                ans+=v[i]+atag[bl[i]];
            for(int i=(bl[r]-1)*blo+1;i<=r;i++)</pre>
38
39
                ans+=v[i]+atag[bl[i]];
40
            for(int i=bl[l]+1;i<=bl[r]-1;i++)</pre>
41
                ans+=sum[i]+atag[i]*blo;
        }
42
43
        return ans;
44
    }
45
46
    int main(){
47
        scanf("%d", &n);blo=sqrt(n);
48
        for(int i=1;i<=n;i++)</pre>
49
            bl[i]=(i-1)/blo+1;
50
        memset(atag+1,0,bl[n]*sizeof(atag[0]));
51
        memset(sum+1,0,bl[n]*sizeof(sum[0]));
52
        for(int i=1;i<=n;i++){</pre>
53
            scanf("%lld", &v[i]);
54
            sum[bl[i]]+=v[i];
55
        }
        for(int i=1;i<=n;i++){</pre>
56
57
            int opt,1,r,c;
            \verb|scanf("%d %d %d %d", &opt, &l, &r, &c);|\\
58
59
            if(opt==0){
60
                add(1,r,c);
```

## 4.6 树状数组 Binary\_Indexed\_Tree

```
#include <bits/stdc++.h>
 2
    #define LL long long
 3
    using namespace std;
    const int MAXN=2000005;
 5
 6
    struct BIT {
        int n,b[MAXN];
 8
 9
        int lowbit(int x){
10
           return x&(-x);
11
12
13
        void change(int x,int y){
           for(;x<=n;x+=lowbit(x))</pre>
14
15
               b[x] +=y;
16
        }
17
18
        int sum(int x){
19
           int s=0;
20
           for(;x>0;x-=lowbit(x))
21
               s+=b[x];
22
           return s;
23
        }
24
25
        void build(int len,int *a) {
26
           n=len;
27
           for(int i=1;i<=n;i++)</pre>
28
               b[i]=a[i];
29
           for(int x=1;x<<1<=n;x<<=1)</pre>
30
               for(int i=x;i+x<=n;i+=x<<1)</pre>
31
                   b[i+x]+=b[i];
32
        }
33
    }bit;
34
35
    int a[MAXN];
36
37
    void solve() {
38
        int n,m;
        scanf("%d %d", &n, &m);
39
40
        for(int i=1;i<=n;i++)</pre>
41
           scanf("%d", &a[i]);
42
        bit.build(n,a);
        for(int i=1;i<=m;i++) {</pre>
43
44
           int opt,x,y;
```

```
45
           scanf("%d %d %d", &opt, &x, &y);
46
           if(opt==1) {
               bit.change(x,y);
47
48
           } else {
49
              printf("%d\n", bit.sum(y)-bit.sum(x-1));
           }
50
51
       }
52
    }
53
54
    int main() {
55
       int T=1,cas=1;(void)(cas);
       // scanf("%d", &T);
56
       while(T--) {
57
           // printf("Case #%d: ", cas++);
58
59
           solve();
       }
60
61
       return 0;
    }
62
```

# 4.7 二维树状数组 2D\_Binary\_Indexed\_Tree

```
#include <bits/stdc++.h>
 2
    const int MAXN=2005;
    struct BIT {
 4
 5
        int n,m;
 6
        int a[MAXN][MAXN];
 8
        int lowbit(int x){
 9
           return x&(-x);
10
11
12
        void change(int x,int y,int k){
13
           for(int i=x;i<=n;i+=lowbit(i)){</pre>
               for(int j=y;j<=m;j+=lowbit(j)){</pre>
14
15
                   a[i][j]+=k;
16
17
           }
        }
18
19
20
        int sum(int x,int y){
21
           int s=0;
22
           for(int i=x;i>0;i-=lowbit(i)){
23
               for(int j=y;j>0;j-=lowbit(j)){
24
                   s+=a[i][j];
25
               }
           }
26
27
           return s;
28
29
    }bit;
30
    int main(){
```

```
32
        int n,m,H;
33
        scanf("%d %d %d",&n, &m, &H);
34
        bit.n=n;
35
        bit.m=m;
36
        for(int i=1;i<=n;i++)</pre>
37
            for(int j=1;j<=m;j++)</pre>
38
                bit.a[i][j]=0;
39
        for(int i=1;i<=H;i++) {</pre>
            int x,y;
40
41
            scanf("%d %d", &x, &y);
42
            bit.change(1,1,1);
43
            bit.change(x+1,1,-1);
44
            bit.change(1,y+1,-1);
45
            bit.change(x+1,y+1,1);//1-2+1
46
47
        for(int i=1;i<=n;i++)</pre>
48
            for(int j=1;j<=m;j++)</pre>
49
                printf("%d\n", bit.sum(i,j));
50
    }
```

## 4.8 线段树 Segment\_Tree

```
#include <cstdio>
    #include <algorithm>
 3
    #define LL long long
 4
    #define MAXN 100005
 5
    using namespace std;
 6
 7
    struct SGT{
 8
       LL sum[MAXN<<2],tag[MAXN<<2];</pre>
 9
10
       void pushup(int x) {sum[x]=sum[x<<1]+sum[x<<1|1];}</pre>
11
       void pushdown(int x,int l,int r) {
12
           int m=(l+r)>>1;
13
           sum[x<<1]+=tag[x]*(m-l+1);
14
           tag[x<<1]+=tag[x];
15
           sum[x<<1|1]+=tag[x]*(r-m);
16
           tag[x<<1|1]+=tag[x];
17
           tag[x]=0;
18
       }
19
       void build(int x,int 1,int r,LL *a) {
20
21
           tag[x]=0;
22
           if(l==r) sum[x]=a[1];
23
           else {
24
               int m=(1+r)>>1;
               build(x<<1,1,m,a);
25
26
               build(x<<1|1,m+1,r,a);
27
               pushup(x);
28
           }
29
       }
30
```

```
31
        void modify(int x,int l,int r,int ql,int qr,LL delta) {
32
            if(ql<=1 && r<=qr) {</pre>
33
               sum[x]+=delta*(r-l+1);
34
               tag[x]+=delta;
35
               return;
           }
36
37
            if(tag[x]) pushdown(x,1,r);
38
            int m=(1+r)>>1;
39
            if(ql<=m) modify(x<<1,1,m,ql,qr,delta);</pre>
40
            if(m<qr) modify(x<<1|1,m+1,r,ql,qr,delta);</pre>
41
            // sum[x]=tag[x]*(r-l+1)+sum[x<<1]+sum[x<<1|1];
42
           pushup(x);
43
        }
44
45
        LL query(int x,int l,int r,int ql,int qr){
46
            if(q1<=1 && r<=qr) return sum[x];</pre>
47
48
            if(tag[x]) pushdown(x,1,r);
            int m=(1+r)>>1;
49
           LL res=0;
50
51
            if(ql<=m) res+=query(x<<1,1,m,ql,qr);</pre>
52
            if(m<qr) res+=query(x<<1|1,m+1,r,ql,qr);</pre>
53
            // res+=tag[x]*(min(qr,r)-max(ql,l)+1);
54
           return res;
        }
55
56
    }sgt;
57
58
    LL a[MAXN];
59
60
    int main(){
61
        int n,m;
62
        scanf("%d %d",&n,&m);
63
        for(int i=1;i<=n;i++) scanf("%lld", &a[i]);</pre>
64
        sgt.build(1,1,n,a);
65
        for(int i=1;i<=m;i++){</pre>
66
            int op,1,r;
67
            scanf("%d", &op);
68
            if(op==1){
               LL k;
69
70
               scanf("%d %d %lld",&l,&r,&k);
71
               sgt.modify(1,1,n,l,r,k);
           }
72
73
            else{
74
               scanf("%d %d",&l,&r);
75
               printf("\%lld\n", sgt.query(1,1,n,l,r));
76
            }
77
        }
78
        return 0;
79
    }
```

#### 4.9 线段树 Segment\_Tree\_Multiply

```
#include <bits/stdc++.h>
   #define LL long long
 3
   using namespace std;
 4
    const int MAXN=100005;
    const int MOD=571373;
 6
 7
    struct SGT{
 8
        LL sum [MAXN<<2], ts [MAXN<<2], tm [MAXN<<2];
 9
10
        void pushup(int x) {sum[x]=(sum[x<<1]+sum[x<<1|1])%MOD;}</pre>
11
        void pushdown(int x,int l,int r) {
12
           int m=(1+r)>>1;
13
           sum[x<<1]=(sum[x<<1]*tm[x]%MOD + ts[x]*(m-1+1)%MOD)%MOD;
14
           sum[x<<1|1]=(sum[x<<1|1]*tm[x]%MOD + ts[x]*(r-m)%MOD)%MOD;
15
           tm[x<<1]=tm[x<<1]*tm[x]%MOD;
16
           tm[x<<1|1]=tm[x<<1|1]*tm[x]%MOD;
17
           ts[x<<1]=(ts[x<<1]*tm[x]%MOD+ts[x])%MOD;
18
           ts[x<<1|1]=(ts[x<<1|1]*tm[x]%MOD+ts[x])%MOD;
19
           tm[x]=1;
20
           ts[x]=0;
        }
21
22
        void build(int x,int 1,int r,LL* a) {
23
24
           ts[x]=0,tm[x]=1;
25
           if(l==r) sum[x]=a[1];
26
           else {
27
               int m=(1+r)>>1;
28
               build(x<<1,1,m,a);
29
               build(x<<1|1,m+1,r,a);
30
               pushup(x);
31
           }
32
        }
33
34
        void plus(int x,int l,int r,int ql,int qr,LL d) {
35
           if(ql<=l && r<=qr) {</pre>
36
               sum[x] = (sum[x]+d*(r-l+1)%MOD)%MOD;
37
               ts[x]=(ts[x]+d)%MOD;
38
               return;
           }
39
40
41
           pushdown(x,1,r);
42
           int m=(1+r)>>1;
43
           if(q1<=m) plus(x<<1,1,m,q1,qr,d);</pre>
44
           if(m<qr) plus(x<<1|1,m+1,r,ql,qr,d);</pre>
45
           pushup(x);
46
        }
47
48
        void multi(int x,int l,int r,int ql,int qr,LL d) {
49
           if(ql<=1 && r<=qr) {</pre>
50
               sum[x]=(sum[x]*d)%MOD;
51
               ts[x]=(ts[x]*d)%MOD;
52
               tm[x]=(tm[x]*d)%MOD;
53
               return;
```

```
54
            }
55
56
            pushdown(x,1,r);
57
            int m=(1+r)>>1;
58
            if(ql<=m) multi(x<<1,1,m,ql,qr,d);</pre>
59
            if(m<qr) multi(x<<1|1,m+1,r,ql,qr,d);</pre>
60
            pushup(x);
61
        }
62
63
        LL query(int x,int l,int r,int ql,int qr) {
64
            if(ql<=1 && r<=qr) {</pre>
65
                return sum[x];
66
            }
67
68
            pushdown(x,1,r);
69
            int m=(1+r)>>1;
70
            LL res=0;
            if(q1<=m) res=(res+query(x<<1,1,m,q1,qr))%MOD;</pre>
71
72
            if(m<qr) res=(res+query(x<<1|1,m+1,r,q1,qr))%MOD;</pre>
73
            return res;
74
75
     }sgt;
76
77
     LL a[MAXN];
78
79
     int main(){
80
         int n,m,p;
81
        scanf("%d %d %d", &n, &m, &p);
82
        for(int i=1;i<=n;i++) scanf("%lld", &a[i]);</pre>
83
        sgt.build(1,1,n,a);
        for(int i=1;i<=m;i++){</pre>
84
85
            int op,1,r;
86
            scanf("%d", &op);
87
            if(op==1){
88
                LL k;
89
                scanf("%d %d %lld", &l, &r, &k);
90
                sgt.multi(1,1,n,1,r,k);
91
            } else if(op==2) {
92
                LL k;
93
                scanf("%d %d %lld", &l, &r, &k);
94
                sgt.plus(1,1,n,1,r,k);
95
            } else {
96
                scanf("%d %d",&l,&r);
97
                printf("%lld\n", sgt.query(1,1,n,1,r));
98
            }
99
100
        return 0;
101
     }
```

#### 4.10 扫描线 Scanline

```
1 #include <cstdio>
```

```
#include <algorithm>
    #define LL long long
    #define MAXN 100005
 4
 5
    using namespace std;
 6
 7
    struct line{
 8
        int x,y1,y2,sign;
 9
        bool operator<(line b)const{</pre>
10
            if(x!=b.x) return x<b.x;</pre>
11
            else return sign>b.sign;
12
        }
    }li[MAXN<<1];</pre>
13
14
    struct node{
15
16
        int le,ri;
17
        int cnt,len;
    }sgt[MAXN<<3];</pre>
18
19
20
    int dy[MAXN<<1];</pre>
21
22
    void pushup(int cur){
23
        if(sgt[cur].cnt)
            sgt[cur].len=dy[sgt[cur].ri]-dy[sgt[cur].le];
24
25
        else if(sgt[cur].le<sgt[cur].ri-1)</pre>
26
            sgt[cur].len=sgt[cur<<1].len+sgt[cur<<1|1].len;</pre>
27
        else sgt[cur].len=0;
28
29
30
    void build(int cur,int l,int r){
31
        sgt[cur].le=1, sgt[cur].ri=r;
        sgt[cur].cnt=sgt[cur].len=0;
32
33
        if(1<r-1){
34
            build(cur<<1,1,(1+r)>>1);
35
            build(cur<<1|1,(1+r)>>1,r);
36
        }
37
    }
38
39
    void modify(int cur,int l,int r,int sign){
40
        if(l<=sgt[cur].le && sgt[cur].ri<=r){</pre>
41
            sgt[cur].cnt+=sign;
        }
42
43
        else{
44
            int mid=(sgt[cur].le+sgt[cur].ri)>>1;
45
            if(l<mid) modify(cur<<1,l,r,sign);</pre>
46
            if(r>mid) modify(cur<<1|1,1,r,sign);</pre>
47
48
        pushup(cur);
49
    }
50
51
52
    int main(){
53
        int n,cnt;
        scanf("%d", &n);
```

```
55
       cnt=0;
56
        for(int i=1;i<=n;i++){</pre>
57
           int x1,y1,x2,y2;
58
           scanf("%d %d %d %d", &x1, &y1, &x2, &y2);
59
           li[(i<<1)-1].x=x1, li[i<<1].x=x2;
60
           li[(i<<1)-1].y1=li[i<<1].y1=y1;
61
           li[(i<<1)-1].y2=li[i<<1].y2=y2;
62
           li[(i<<1)-1].sign=1, li[i<<1].sign=-1;
63
           dy[++cnt]=y1, dy[++cnt]=y2;
64
       }
65
       sort(dy+1,dy+cnt+1);
66
        cnt=unique(dy+1,dy+cnt+1)-dy-1;
       for(int i=1;i<=(n<<1);i++){</pre>
67
68
           li[i].y1=lower_bound(dy+1,dy+cnt+1,li[i].y1)-dy;
69
           li[i].y2=lower_bound(dy+1,dy+cnt+1,li[i].y2)-dy;
70
       }
71
       sort(li+1,li+(n<<1)+1);
72
       build(1,1,cnt);
73
       LL sum=0;
74
       for(int i=1;i<(n<<1);i++){</pre>
75
           modify(1,li[i].y1,li[i].y2,li[i].sign);
76
           sum+=1LL*sgt[1].len*(li[i+1].x-li[i].x);
77
       }
78
       printf("%lld\n", sum);
79
       return 0;
80
    }
```

### 4.11 zkw 线段树 ZKW\_Segment\_Tree

```
#include <cstdio>
    #include <cstring>
    #define LL long long
 3
    #define MAXN 100005
 4
 6
    struct node{
 7
        LL sum, tag;
 8
    }sgt[MAXN<<2];</pre>
 9
10
    int M;
11
12
    int a[MAXN];
13
14
    void built(int n){
15
        for (M=1; M<n+2; M<<=1);</pre>
16
        memset(sgt+M,0,M*sizeof(sgt[0]));
        for(int i=1;i<=n;i++)</pre>
17
18
            sgt[M+i].sum=a[i];
19
        for(int i=M-1;i;i--)
            sgt[i].sum=sgt[i<<1].sum+sgt[i<<1|1].sum;
20
21
    }
22
   void modify(int l,int r,LL del){
```

```
24
        LL len=1,1c=0,rc=0;
25
        for(l=l+M-1,r=r+M+1;l^r^1;l>>=1,r>>=1,len<<=1){</pre>
26
            if(~l&1) sgt[l+1].tag+=del, lc+=len;
27
           if(r&1) sgt[r-1].tag+=del, rc+=len;
28
            sgt[1>>1].sum+=del*lc;
            sgt[r>>1].sum+=del*rc;
29
30
        }
31
        for(lc+=rc,l>>=1;l;l>>=1)
32
            sgt[1].sum+=del*lc;
33
    }
34
    LL query(int l,int r){
35
36
        LL res=0,len=1,lc=0,rc=0;
        for(l=1+M-1,r=r+M+1;l^r^1;l>>=1,r>>=1,len<<=1){</pre>
37
38
            if(~l&1) res+=sgt[l+1].sum+sgt[l+1].tag*len, lc+=len;
39
           if(r&1) res+=sgt[r-1].sum+sgt[r-1].tag*len, rc+=len;
40
           res+=sgt[l>>1].tag*lc;
41
           res+=sgt[r>>1].tag*rc;
42
        }
43
        for(lc+=rc,l>>=1;l;l>>=1)
44
           res+=sgt[1].tag*lc;
45
        return res;
46
    }
47
48
    int main(){
49
        int n,m;
50
        scanf("%d %d", &n, &m);
51
        for(int i=1;i<=n;i++)</pre>
52
           scanf("%d", &a[i]);
53
        built(n);
        for(int i=1;i<=m;i++){</pre>
54
55
           int opt,x,y;
           scanf("%d %d %d", &opt, &x, &y);
56
57
           if(opt==1){
58
               int k;
59
               scanf("%d", &k);
60
               modify(x,y,k);
           }
61
           else{
62
63
               printf("\%lld\n", query(x,y));
           }
64
65
        }
66
        return 0;
67
    }
```

### 4.12 李超线段树 Li-Chao\_Segment\_Tree

```
#include <cstdio>
#include <algorithm>
#define N 39989
#define MAXN 40005
#define MAXT 160005
```

```
const double eps=1e-12;
    const double inf=1e9;
 9
    using namespace std;
10
    struct line{
11
12
        int 1,r;
13
        double k,b;
        int id;
14
15
    }sgt[MAXT];
16
17
    double calc(line l,int x){return l.k*x+l.b;}
18
    void modify(int cur,int 1,int r,line li){
19
20
        if(li.1<=1 && r<=li.r){</pre>
21
            if(calc(li,1)-calc(sgt[cur],1)>eps && calc(li,r)-calc(sgt[cur],r)>eps)
22
               sgt[cur]=li;
23
           else if(calc(li,1)-calc(sgt[cur],1)>eps || calc(li,r)-calc(sgt[cur],r)>eps){
24
               int mid=(1+r)>>1;
25
               if(calc(li,mid)-calc(sgt[cur],mid)>eps)
26
                   swap(li,sgt[cur]);
27
               if(calc(li,1)-calc(sgt[cur],1)>eps)
28
                   modify(cur<<1,1,mid,li);</pre>
29
               else modify(cur<<1|1,mid+1,r,li);</pre>
30
           }
31
        }
32
        else{
33
           int mid=(l+r)>>1;
34
           if(li.l<=mid) modify(cur<<1,1,mid,li);</pre>
35
            if(li.r>mid) modify(cur<<1|1,mid+1,r,li);</pre>
36
        }
37
    }
38
39
    line query(int cur,int l,int r,int x){
40
        if(l==r) return sgt[cur];
41
        else{
42
           int mid=(1+r)>>1;
43
           line t;
44
           if(x<=mid) t=query(cur<<1,1,mid,x);</pre>
45
           else t=query(cur<<1|1,mid+1,r,x);</pre>
46
           if(!t.id || calc(sgt[cur],x)-calc(t,x)>eps) return sgt[cur];
47
            else return t;
48
        }
49
    }
50
51
    void built(int cur,int l,int r){
52
        sgt[cur].k=sgt[cur].b=0;
53
        sgt[cur].l=1; sgt[cur].r=N;
54
        sgt[cur].id=0;
        if(l<r){
55
           int mid=(1+r)>>1;
56
57
           built(cur<<1,1,mid);</pre>
           built(cur<<1|1,mid+1,r);
```

```
59
        }
60
    }
61
62
    int n;
63
    int main(){
64
65
        scanf("%d", &n);
66
        built(1,1,N);
67
        int last=0,id=0;
68
       for(int i=1;i<=n;i++){</pre>
69
           int opt;
           scanf("%d", &opt);
70
71
           if(opt==0){
72
               int x;
73
               scanf("%d", &x);
74
               x=(x+last-1)%N+1;
75
               printf("%d\n", last=query(1,1,N,x).id);
           }
76
77
           else{
78
               int x0,x1,y0,y1;
               scanf("%d %d %d %d", &x0,&y0,&x1,&y1);
79
               x0=(x0+last-1)%N+1;
80
               x1=(x1+last-1)%N+1;
81
82
               y0=(y0+last-1)%100000000+1;
               y1=(y1+last-1)%1000000000+1;
83
84
               line t;
85
               t.id=++id;
86
               t.l=min(x0,x1); t.r=max(x0,x1);
87
               t.k=x1==x0?0:(double)(y1-y0)/(x1-x0);
88
               t.b=x1==x0?max(y0,y1):y0-t.k*x0;
               modify(1,1,N,t);
89
90
           }
91
92
        return 0;
93
    }
```

## 4.13 可并堆左偏树 Leftist\_Tree

```
#include <cstdio>
   #include <algorithm>
   #define MAXN 100005
4
5
    using namespace std;
6
7
   int n,m;
8
9
   struct node{
10
       int rt,lc,rc,dis,v;
11
   }lt[MAXN];
12
   int find(int x){
13
       if(lt[x].rt==x)
14
```

```
15
           return x;
16
       return lt[x].rt=find(lt[x].rt);
17
    }
18
19
    int merge(int x,int y){
20
       if(!x || !y) return x+y;
21
       if(lt[x].v>lt[y].v || (lt[x].v==lt[y].v && x>y)) swap(x,y);//后一个条件蜜汁优化?
22
       lt[x].rc=merge(lt[x].rc,y);
23
       lt[lt[x].rc].rt=x;
24
       if(lt[lt[x].lc].dis<lt[lt[x].rc].dis) swap(lt[x].lc,lt[x].rc);</pre>
25
       lt[x].dis=lt[lt[x].rc].dis+1;
26
       return x;
27
   }
28
    void pop(int x){
29
       lt[x].v=-1;
30
       lt[lt[x].lc].rt=lt[x].lc;
31
32
       lt[lt[x].rc].rt=lt[x].rc;
33
       lt[x].rt=merge(lt[x].lc,lt[x].rc);
34
   |}
35
36
    int main(){
       scanf("%d %d", &n, &m);
37
38
       for(int i=1;i<=n;i++){</pre>
39
           scanf("%d", &lt[i].v);
40
           lt[i].rt=i;
41
           lt[i].lc=lt[i].rc=0;
           lt[i].dis=0;
42
43
       }
44
       lt[0].dis=0;
       for(int i=1;i<=m;i++){</pre>
45
46
           int opt;
47
           scanf("%d",&opt);
48
           if(opt==1){
49
               int x,y;
50
               scanf("%d %d", &x, &y);
51
               int rx=find(x),ry=find(y);
52
               if(lt[x].v==-1||lt[y].v==-1||rx==ry)
53
                  continue;
54
              merge(rx,ry);
           }
55
56
           else{
57
               int x;
58
               scanf("%d", &x);
59
               if(lt[x].v==-1)
60
                  printf("-1\n");
61
               else{
62
                  int rx=find(x);
                  printf("%d\n", lt[rx].v);
63
64
                  pop(rx);
65
              }
66
           }
       }
```

```
68 return 0;
69 }
```

## 4.14 Splay 树 Splay\_Tree

```
#include <cstdio>
 2
    #define MAXN 100005
 3
    const int inf=0x3f3f3f3f;
 4
5
   int root,len;
 6
    struct node{
 8
       int v,fa,ch[2],size,cnt;
 9
    }sp[MAXN];
10
    int getch(int x) {return sp[sp[x].fa].ch[1]==x;}
11
12
    void pushup(int x) {sp[x].size=sp[x].cnt+sp[sp[x].ch[0]].size+sp[sp[x].ch[1]].size;}
13
14
    void rotate(int x){
15
       int f=sp[x].fa, ff=sp[f].fa;
16
       int k=getch(x);
       sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
17
18
       sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
19
       sp[x].ch[k^1]=f; sp[f].fa=x;
20
       pushup(f); pushup(x);
21
    }
22
23
    void splay(int x,int goal=0){
24
       for(int f;(f=sp[x].fa)!=goal;rotate(x)){
25
           if(sp[f].fa!=goal)
26
               rotate(getch(x)==getch(f)?f:x);
27
       }
28
       if(!goal) root=x;
29
    }
30
    void insert(int x){
31
32
        int cur=root,f=0;
33
       while(cur&&sp[cur].v!=x){
34
           f=cur;
35
           cur=sp[cur].ch[x>sp[cur].v];
36
       }
37
       if(cur)
38
           sp[cur].cnt++;
39
       else{
40
           cur=++len;
           sp[f].ch[x>sp[f].v]=cur;
41
           sp[cur].ch[0]=sp[cur].ch[1]=0;
42
43
           sp[cur].fa=f;
44
           sp[cur].v=x;
45
           sp[cur].cnt=sp[cur].size=1;
46
47
       splay(cur);
```

```
48
    }
49
    void find(int x){
50
51
        int cur=root;
52
        while(x!=sp[cur].v && sp[cur].ch[x>sp[cur].v])
            cur=sp[cur].ch[x>sp[cur].v];
53
54
        splay(cur);
55
    }
56
57
     int kth(int x){
58
        if(sp[root].size<x) return 0;</pre>
        int cur=root;
59
        while(1){
60
            if(x<=sp[sp[cur].ch[0]].size)</pre>
61
62
                cur=sp[cur].ch[0];
63
            else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
               x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
64
65
                cur=sp[cur].ch[1];
            }
66
67
            else return sp[cur].v;
68
        }
69
    }
70
71
     int pre(int x){
72
        find(x);
73
        if(x>sp[root].v) return root;
74
        int cur=sp[root].ch[0];
75
        while(sp[cur].ch[1])
76
            cur=sp[cur].ch[1];
77
        return cur;
    }
78
79
80
     int succ(int x){
81
        find(x);
82
        if(x<sp[root].v) return root;</pre>
83
        int cur=sp[root].ch[1];
84
        while(sp[cur].ch[0])
85
            cur=sp[cur].ch[0];
86
        return cur;
87
    }
88
89
     void erase(int x){
90
        int last=pre(x),next=succ(x),del;
91
        splay(last);splay(next,last);
92
        del=sp[next].ch[0];
93
        if(sp[del].cnt>1){
94
            sp[del].cnt--;
95
            splay(del);
96
        }
97
        else{
98
            sp[next].ch[0]=0;
99
            sp[del].fa=0;
100
            sp[del]=sp[len];
```

```
101
            int f=sp[del].fa;
102
            sp[f].ch[(sp[f].ch[1]==len)]=del;
103
            sp[sp[del].ch[0]].fa=del;
104
            sp[sp[del].ch[1]].fa=del;
105
            if(root==len) root=del;
106
            len--;
107
108
        }
109
     }
110
111
112
     int n;
113
     int main(){
114
        scanf("%d", &n);
115
        root=0;len=0;
116
        insert(-inf);insert(inf);
117
        sp[0].size=0;
        for(int i=1;i<=n;i++){</pre>
118
            int opt,x;
119
120
            scanf("%d %d", &opt, &x);
121
            if(opt==1){
122
                insert(x);
123
124
            else if(opt==2){
125
                erase(x);
126
            }
127
            else if(opt==3){
128
                find(x);
129
                printf("%d\n", sp[sp[root].ch[0]].size);
130
            else if(opt==4){
131
132
                printf("%d\n", kth(x+1));
133
            }
134
            else if(opt==5){
135
                printf("%d\n", sp[pre(x)].v);
136
            }
137
            else{
138
                printf("%d\n", sp[succ(x)].v);
139
            }
140
        }
141
        return 0;
142
```

## 4.15 Splay 树 Splay\_Tree\_Flip

```
#include <cstdio>
#include <algorithm>
#define MAXN 100005
const int inf=0x3f3f3f3f;

using namespace std;
```

```
8
    int root,len,a[MAXN];
9
10
    struct node{
11
        int v,fa,ch[2],size,cnt,tag;
12
    }sp[MAXN];
13
14
    int getch(int x) {return sp[sp[x].fa].ch[1]==x;}
15
    void pushup(int x) {sp[x].size=sp[x].cnt+sp[sp[x].ch[0]].size+sp[sp[x].ch[1]].size;}
16
17
    void pushdown(int x){
18
        if(sp[x].tag){
           sp[sp[x].ch[0]].tag^=1;
19
           sp[sp[x].ch[1]].tag^=1;
20
           swap(sp[x].ch[0],sp[x].ch[1]);
21
22
           sp[x].tag=0;
23
       }
24
25
26
    void rotate(int x){
27
       int f=sp[x].fa, ff=sp[f].fa;
28
       int k=getch(x);
29
       sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
30
       sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
31
       sp[x].ch[k^1]=f; sp[f].fa=x;
       pushup(f); pushup(x);
32
33
    }
34
35
    void splay(int x,int goal=0){
36
       for(int f;(f=sp[x].fa)!=goal;rotate(x)){
37
           if(sp[f].fa!=goal)
               rotate(getch(x)==getch(f)?f:x);
38
39
       }
40
        if(!goal) root=x;
41
    }
42
43
    int find(int x){
44
       int cur=root;
45
       while(1){
46
           pushdown(cur);
47
           if(x<=sp[sp[cur].ch[0]].size)</pre>
48
               cur=sp[cur].ch[0];
49
           else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
50
               x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
51
               cur=sp[cur].ch[1];
           }
52
53
           else return cur;
       }
54
55
    }
56
57
    int built(int f,int l,int r){
58
       if(1>r) return 0;
59
       int mid=(l+r)>>1, cur=++len;
60
        sp[cur].fa=f;
```

```
61
        sp[cur].cnt=1;
62
        sp[cur].v=a[mid];
63
        sp[cur].tag=0;
64
        sp[cur].ch[0]=built(cur,1,mid-1);
65
        sp[cur].ch[1]=built(cur,mid+1,r);
66
        pushup(cur);
67
        return cur;
68
    }
69
70
    void flip(int l,int r){
        int last=find(l-1),next=find(r+1);
71
        splay(last); splay(next, last);
72
        sp[sp[sp[root].ch[1]].ch[0]].tag^=1;
73
74
    }
75
    void dfs(int cur){
76
77
       pushdown(cur);
        if(sp[cur].ch[0]) dfs(sp[cur].ch[0]);
78
79
        if(sp[cur].v!=-inf && sp[cur].v!=inf) printf("%d ", sp[cur].v);
        if(sp[cur].ch[1]) dfs(sp[cur].ch[1]);
80
81
82
83
    int n,m;
84
    int main(){
85
86
        scanf("%d %d", &n, &m);
87
        for(int i=1;i<=n;i++) a[i+1]=i;</pre>
88
        a[1]=-inf;a[n+2]=inf;
89
        len=0;
90
        root=built(0,1,n+2);
        sp[0].size=0;
91
92
        for(int i=1;i<=m;i++){</pre>
93
           int 1,r;
94
           scanf("%d %d", &l, &r);
95
           flip(l+1,r+1);
96
        }
97
        dfs(root);
98
        return 0;
99
```

### 4.16 Splay 树 Splay\_Tree\_Dye&Flip

```
#include <cstdio>
#include <cstring>
#include <algorithm>
#define MAXN 500005
const int inf=0x3f3f3f3f;
using namespace std;

struct node{
int v,fa,ch[2],cnt;//basic
int size,sum,lm,rm,mm;//pushup
```

```
11
        int flip,color;//pushdown
12
    }sp[MAXN];
13
14
    int a[MAXN],len,root,recy[MAXN],rlen;
15
16
    int getch(int x){return sp[sp[x].fa].ch[1]==x;}
17
18
    void pushup(int x){
19
        int lc=sp[x].ch[0],rc=sp[x].ch[1];
20
        sp[x].size=sp[lc].size+sp[rc].size+sp[x].cnt;
21
        sp[x].sum=sp[lc].sum+sp[rc].sum+sp[x].v;
22
        sp[x].lm=max(sp[lc].lm, sp[lc].sum+sp[x].v+sp[rc].lm);
23
        sp[x].rm=max(sp[rc].rm, sp[rc].sum+sp[x].v+sp[lc].rm);
24
        sp[x].mm=max(max(sp[lc].mm,sp[rc].mm),sp[lc].rm+sp[x].v+sp[rc].lm);
25
26
27
    void pushdown(int x){
28
        int lc=sp[x].ch[0],rc=sp[x].ch[1];
29
        if(sp[x].color!=inf){
30
           if(lc){
31
               sp[lc].v=sp[lc].color=sp[x].color;
32
               sp[lc].sum=sp[lc].size*sp[x].color;
           }
33
34
           if(rc){
               sp[rc].v=sp[rc].color=sp[x].color;
35
36
               sp[rc].sum=sp[rc].size*sp[x].color;
37
           }
38
           if(sp[x].color>0){
39
               if(lc) sp[lc].lm=sp[lc].rm=sp[lc].mm=sp[lc].sum;
40
               if(rc) sp[rc].lm=sp[rc].rm=sp[rc].mm=sp[rc].sum;
           }
41
42
           else{
               if(lc) {sp[lc].lm=sp[lc].rm=0; sp[lc].mm=sp[lc].v;}
43
44
               if(rc) {sp[rc].lm=sp[rc].rm=0; sp[rc].mm=sp[rc].v;}
           }
45
46
           sp[x].color=inf;
47
           sp[x].flip=0;
48
49
       else if(sp[x].flip){
50
           if(lc){
51
               sp[lc].flip^=1;
52
               swap(sp[lc].ch[0],sp[lc].ch[1]);
53
               swap(sp[lc].lm,sp[lc].rm);
54
           }
55
           if(rc){
56
               sp[rc].flip^=1;
57
               swap(sp[rc].ch[0],sp[rc].ch[1]);
58
               swap(sp[rc].lm,sp[rc].rm);
59
60
           sp[x].flip=0;
61
       }
62
    }
63
```

```
void rotate(int x){
 65
        int f=sp[x].fa, ff=sp[f].fa;
 66
         int k=getch(x);
 67
         sp[ff].ch[getch(f)]=x; sp[x].fa=ff;
 68
         sp[sp[x].ch[k^1]].fa=f; sp[f].ch[k]=sp[x].ch[k^1];
 69
        sp[x].ch[k^1]=f; sp[f].fa=x;
 70
        pushup(f); pushup(x);
 71
     }
 72
 73
     void splay(int x,int goal=0){
 74
        for(int f;(f=sp[x].fa)!=goal;rotate(x)){
 75
            if(sp[f].fa!=goal)
                rotate(getch(x)==getch(f)?f:x);
 76
        }
 77
 78
         if(!goal) root=x;
 79
     }
 80
     int find(int x){
 81
        int cur=root;
 82
 83
        while(1){
 84
            pushdown(cur);
            if(x<=sp[sp[cur].ch[0]].size)</pre>
 85
 86
                cur=sp[cur].ch[0];
 87
            else if(x>sp[sp[cur].ch[0]].size+sp[cur].cnt){
                x-=sp[sp[cur].ch[0]].size+sp[cur].cnt;
 88
 89
                cur=sp[cur].ch[1];
 90
            }
 91
            else return cur;
92
        }
 93
     }
94
95
     int built(int f,int l,int r){
 96
         if(l>r) return 0;
97
         int mid=(l+r)>>1, cur=rlen?recy[rlen--]:++len;
98
        sp[cur].v=a[mid];
99
        sp[cur].fa=f;
100
        sp[cur].cnt=1;
101
        sp[cur].flip=0;
102
        sp[cur].color=inf;
103
         sp[cur].ch[0]=built(cur,1,mid-1);
104
        sp[cur].ch[1]=built(cur,mid+1,r);
105
        pushup(cur);
106
        return cur;
107
     }
108
109
     void insert(int pos,int tot){
110
         int l=find(pos),r=find(pos+1);
111
        splay(1);splay(r,1);
112
        sp[r].ch[0]=built(r,1,tot);
113
        pushup(r); pushup(1);
114
     }
115
    void recycle(int x){
```

```
117
        if(!x) return;
118
        recycle(sp[x].ch[0]);
119
        recycle(sp[x].ch[1]);
120
        sp[sp[x].fa].ch[getch(x)]=0;
121
        recy[++rlen]=x;
122
     }
123
124
     void erase(int pos,int tot){
125
        int l=find(pos-1),r=find(pos+tot);
126
        splay(1);splay(r,1);
127
        recycle(sp[r].ch[0]);
128
        pushup(r); pushup(1);
129
     }
130
131
     void dye(int pos,int tot,int c){
132
        int l=find(pos-1),r=find(pos+tot);
133
        splay(1);splay(r,1);
134
        int x=sp[r].ch[0];
135
        sp[x].color=c;
136
        sp[x].v=c;
        sp[x].sum=sp[x].size*c;
137
138
        if(c>0)
139
            sp[x].lm=sp[x].rm=sp[x].mm=sp[x].sum;
140
        else{
141
            sp[x].lm=sp[x].rm=0;
142
            sp[x].mm=sp[x].v;
143
144
        pushup(r); pushup(1);
145
     }
146
147
     void reverse(int pos,int tot){
148
        int l=find(pos-1),r=find(pos+tot);
149
        splay(1);splay(r,1);
150
        int x=sp[r].ch[0];
151
        sp[x].flip^=1;
152
        swap(sp[x].ch[0],sp[x].ch[1]);
153
        swap(sp[x].lm,sp[x].rm);
154
        pushup(r); pushup(1);
155
     }
156
157
     int getsum(int pos,int tot){
158
        int l=find(pos-1),r=find(pos+tot);
159
        splay(1); splay(r,1);
160
        return sp[sp[r].ch[0]].sum;
161
     }
162
163
    int n,m;
164
165
     int main(){
        scanf("%d %d", &n, &m);
166
167
        for(int i=1;i<=n;i++){</pre>
168
            scanf("%d", &a[i+1]);
169
```

```
170
        memset(sp,0,sizeof(sp[0]));
171
        sp[0].mm=a[1]=a[n+2]=-inf;
172
        rlen=0;
173
        len=0;
174
        root=built(0,1,n+2);
175
        for(int i=1;i<=m;i++){</pre>
176
            char opt[10];
177
            scanf("%s", opt);
178
            if(opt[0]=='I'){//Insert
179
                int pos,tot;
180
                scanf("%d %d", &pos, &tot);
181
                for(int i=1;i<=tot;i++)</pre>
182
                    scanf("%d", &a[i]);
183
                insert(pos+1,tot);
            }
184
185
            else if(opt[0]=='D'){//Delete
186
                int pos,tot;
                scanf("%d %d", &pos, &tot);
187
188
                erase(pos+1,tot);
189
            else if(opt[2]=='K'){//Make-Same
190
191
                int pos,tot,c;
192
                scanf("%d %d %d", &pos, &tot, &c);
193
                dye(pos+1,tot,c);
            }
194
195
            else if(opt[0] == 'R'){//Reverse
196
                int pos,tot;
197
                scanf("%d %d", &pos, &tot);
198
                reverse(pos+1,tot);
199
200
            else if(opt[0] == 'G'){//Get-Sum
201
                int pos,tot;
202
                scanf("%d %d", &pos, &tot);
203
                printf("%d\n", getsum(pos+1,tot));
204
            }
205
            else if(opt[0] == 'M'){//Max-Sum
206
                printf("%d\n", sp[root].mm);
207
            }
208
209
        return 0;
210
     }
```

## 5 数论

### 5.1 乘法逆元 Multiplicative\_Inverse\_Modulo

```
1
 2
        Coded with Leachim's ACM Template.
 3
        No errors. No warnings. ~~
 4
    */
 5
    #include <bits/stdc++.h>
 6
    #pragma GCC diagnostic ignored "-Wunused-const-variable"
 7
    #pragma GCC diagnostic ignored "-Wsign-conversion"
    #pragma GCC diagnostic ignored "-Wsign-compare"
    #define LL long long
 9
10
    using namespace std;
11
    const int inf=0x3f3f3f3f;
12
    const LL INF=0x3f3f3f3f3f3f3f3f3f3f;
13
    const double eps=1e-7;
14
    const int dx[4]=\{1,-1,0,0\};
15
    const int dy[4]=\{0,0,1,-1\};
16
    const int RT=3;
17
    const int MOD=998244353;
18
    const int MAXN=20000005;
19
20
   int inv[MAXN],f[MAXN],fi[MAXN];
21
22
    void pre_inv(int n,int p) {
23
        inv[1]=1;
24
        f[0]=fi[0]=f[1]=fi[1]=1;
25
        for(int i=2;i<=n;i++) {</pre>
26
           inv[i]=1LL*(p-p/i)*inv[p%i]%p;
27
           f[i]=1LL*f[i-1]*i%p;
28
           fi[i]=1LL*fi[i-1]*inv[i]%p;
        }
29
    }
30
31
32
    void solve() {
33
       int n,p;
34
        scanf("%d %d", &n, &p);
35
        pre_inv(n,p);
36
        for(int i=1;i<=n;i++)</pre>
37
           printf("%d\n", inv[i]);
38
    }
39
40
    int main() {
41
        int T=1, cas=1; (void) (cas);
        // scanf("%d", &T);
42
43
        while(T--) {
           // printf("Case #%d: ", cas++);
44
45
           solve();
46
47
        return 0;
48
    }
```

## 5.2 数论分块 Block\_Division

```
#include <cstdio>
 2
    #define LL long long
3
4
    LL n;
5
    int main(){
       scanf("%lld", &n);
8
       LL ans=0;
9
       for(LL i=1;i<=n;i++){</pre>
10
           LL t=n/i,j=n/t;
11
           ans+=(j-i+1)*t;
12
           i=j;
13
14
       printf("%lld\n", ans);
15
       return 0;
16
    }
```

# 5.3 贝祖引理 Bezout\_Lemma

```
1
    #include <cstdio>
 2
    int gcd(int x,int y){
 4
       if(!x || !y) return x+y;
 5
       return gcd(y,x%y);
 6
    }
 7
8
    int n;
10
    int main(){
11
        scanf("%d", &n);
12
       int g;
13
       scanf("%d", &g);
14
       if(g<0)g=-g;
15
       for(int i=2;i<=n;i++){</pre>
16
           int t;
           scanf("%d", &t);
17
18
           if(t<0) t=-t;</pre>
19
           g=gcd(g,t);
20
21
       printf("%d\n", g);
22
        return 0;
23
```

## 5.4 卢卡斯 Lucas

```
#include <cstdio>
#define LL long long
#define MAXP 100005
```

```
int f[MAXP],fi[MAXP];
    LL binpow(LL x,LL y,LL m){
 8
       LL r=1\%m;
 9
        for(;y;y>>=1,x=x*x%m)
10
           if(y&1) r=r*x%m;
11
        return r;
12
    }
13
14
    void pre(LL p){
15
       f[0]=1;
       for(int i=1;i<=p-1;i++)</pre>
16
           f[i]=1LL*f[i-1]*i%p;
17
       fi[p-1]=binpow(f[p-1],p-2,p);
18
19
        for(int i=p-1;i;i--)
20
           fi[i-1]=1LL*fi[i]*i%p;
21
    }
22
    LL C(LL x,LL y,LL p) {
23
       if(x<y) return 0;</pre>
24
25
        return 1LL*f[x]*fi[y]%p*fi[x-y]%p;
26
    }
27
28
    LL lucas(LL x,LL y,LL p){
29
       if(!y) return 1;
30
        return C(x\%p,y\%p,p)*lucas(x/p,y/p,p)\%p;
31
32
33
   int main(){
34
       int T;
        scanf("%d", &T);
35
36
        while(T--){
37
           LL n,m,p;
38
           scanf("%11d %11d %11d", &n, &m, &p);
39
           pre(p);
40
           printf("%lld\n", lucas(n+m,m,p));
       }
41
42
        return 0;
43
```

### 5.5 拓展欧几里得 Exgcd

```
#include <cstdio>
    #include <cmath>
 2
3
    #define LL long long
 4
5
    LL exgcd(LL a, LL b, LL &x, LL &y){
 6
       if(!b || !a){
 7
           x=(a!=0);y=(b!=0);
8
           return a+b;
9
       }
       LL g=exgcd(b,a%b,y,x);
10
```

```
11
      y=y-(a/b)*x;
12
      return g;
13
   }
14
15
   int main(){
16
      int T;
17
      scanf("%d", &T);
18
      while(T--){
19
         LL a,b,c,x,y,g;
20
         scanf("%11d %11d %11d", &a, &b, &c);
21
         g=exgcd(a,b,x,y);
22
         if(c%g){
23
            printf("-1\n");
24
            continue;
         }
25
26
         a/=g;b/=g;c/=g;x*=c;y*=c;
27
         LL kl=ceil((double)(-x+1)/b),kr=floor((double)(y-1)/a);
28
         if(kr<kl){</pre>
29
            printf("%lld %lld\n",(x+kl*b),(y-kr*a));
30
         }
31
         else{
32
            }
33
34
      }
35
      return 0;
36
   }
```

## 5.6 拓展欧拉定理 Ex\_Euler\_Theorem-Automaton

```
#include <bits/stdc++.h>
 2
    using namespace std;
 3
    #define LL long long
    #define MAXN 10000005
 5
    LL binpow(LL x,LL y,LL m){
 6
       int r=1%m;
 8
       x\%=m;
9
       while(y){
10
           if(y&1) r=1LL*r*x%m;
           x=1LL*x*x%m;
11
12
           y>>=1;
13
14
       return r;
15
    }
16
    void solve() {
17
18
       int a,m;
19
       scanf("%d %d", &a, &m);
20
       int mm=m,phi=m;
       for(int i=2;i*i<=m;i++) {</pre>
21
           if(mm\%i==0){
22
```

```
23
               while(mm%i==0) mm/=i;
24
               phi=phi/i*(i-1);
25
           }
26
       }
27
       if(mm>1) phi=phi/mm*(mm-1);
28
       char c;
29
       while(!isdigit(c=getchar()));
30
       int b=c-'0';
31
       bool flag=0;
32
       while(isdigit(c=getchar())){
33
           b=10*b+c-'0';
           if(b>=phi) b%=phi, flag=1;
34
35
       }
36
       if(flag) b+=phi;
       printf("%lld\n", binpow(a,b,m));
37
38
39
    }
40
41
    int main() {
42
       int T=1;
       // scanf("%d", &T);
43
       while(T--) {
44
45
           solve();
46
       }
47
       return 0;
48
    }
```

## 5.7 中国剩余定理 Chinese\_Remainder\_Theorem

```
/*
 1
 2
       中国剩余定理
 3
       x%ai=bi;
       sigma(ai*PI/ai*inv(PI/ai,ai))%PI;
 4
 5
 6
    #include <bits/stdc++.h>
 7
    #define LL long long
    using namespace std;
9
    const int MAXN=2000005;
10
    LL exgcd(LL a,LL b,LL &x,LL &y){
11
12
       if(!b || !a){
           x=(a!=0);y=(b!=0);
13
14
           return a+b;
15
       }
16
       LL g=exgcd(b,a%b,y,x);
       y=y-(a/b)*x;
17
18
       return g;
    }
19
20
21
    LL inv(LL a,LL m) {
22
       LL x,y;
23
       exgcd(a,m,x,y);
```

```
24
        return (x%m+m)%m;
25
    }
26
27
    int a[MAXN],b[MAXN];
28
29
    void solve() {
30
        int n;
31
        scanf("%d", &n);
32
        LL sum=1;
33
        for(int i=1;i<=n;i++) {//x%a==b</pre>
34
            scanf("%d %d", &a[i], &b[i]);
            sum=sum*a[i];
35
36
        }
37
        LL ans=0;
38
        for(int i=1;i<=n;i++) {</pre>
39
            ans = (ans + (\_int128)b[i] * (sum/a[i]) % sum * inv(sum/a[i],a[i]) % sum) % sum;
40
41
        printf("%lld\n", ans);
42
    }
```

## 5.8 拓展中国剩余定理 Ex\_Chinese\_Remainder\_Theorem

```
/*
 1
 2
       拓展中国剩余定理
 3
       x%ai=bi;
    */
 4
 5
    #include <bits/stdc++.h>
 6
    #define LL long long
 7
    using namespace std;
    const int MAXN=2000005;
 9
10
    LL exgcd(LL a, LL b, LL &x, LL &y){
11
       if(!b || !a){
           x=(a!=0);y=(b!=0);
12
13
           return a+b;
14
15
       LL g=exgcd(b,a%b,y,x);
16
       y=y-(a/b)*x;
17
       return g;
18
    }
19
   LL inv(LL a,LL m) {
20
21
       LL x,y;
22
       exgcd(a,m,x,y);
23
       return (x%m+m)%m;
24
    }
25
26
   LL a[MAXN],b[MAXN];
27
28
    void solve() {
29
       int n;
       scanf("%d", &n);
30
```

```
31
        for(int i=1;i<=n;i++) {//x%a==b</pre>
32
           scanf("%1ld %1ld", &a[i], &b[i]);
33
       }
34
35
       LL M=a[1],R=b[1];//通解 R+C*M, 特解 R
        for(int i=2;i<=n;i++) {</pre>
36
37
           LL x,y;
38
           LL g=exgcd(M,a[i],x,y);//R' = R+M*x=y*a[i]+b
39
           if(abs(b[i]-R)%g) {
40
               printf("-1\n");
41
               return;
           }
42
43
           x=((\_int128)(b[i]-R)/g*x\%(a[i]/g)+a[i]/g)\%(a[i]/g);
44
           R=R+M*x;
           M=M/g*a[i];
45
46
        }
47
        printf("%lld\n", R);
    }
48
```

## 5.9 欧拉筛 Eular\_Sieve

```
1
    #include <cstdio>
 2
    #include <cstring>
    #define MAXN 2000005
 4
    int cnt,p[MAXN];
 5
    bool inp[MAXN];
 7
    int phi[MAXN],mu[MAXN];
 8
 9
    void eular_sieve(int n){
10
        cnt=0;
11
        memset(inp,0,(n+1)*sizeof(inp[0]));
12
        inp[0]=inp[1]=1;
13
        phi[1]=1;
14
       mu[1]=1;
15
        for(int i=2;i<=n;i++){</pre>
            if(!inp[i]) p[++cnt]=i, phi[i]=i-1, mu[i]=-1;
16
17
           for(int j=1;j<=cnt && i*p[j]<=n;j++){</pre>
18
               inp[i*p[j]]=1;
19
               if(i%p[j]){
20
                   phi[i*p[j]]=phi[i]*(p[j]-1);
21
                  mu[i*p[j]]=-mu[i];
22
               }
23
               else{
24
                   phi[i*p[j]]=phi[i]*p[j];
25
                   mu[i*p[j]]=0;
26
                   break;
27
               }
28
           }
29
        }
30
    }
31
```

```
32
    int main(){
33
        int n,m;
34
        scanf("%d %d", &n, &m);
35
        eular_sieve(n);
36
        for(int i=1;i<=m;i++){</pre>
37
            int k;
38
            scanf("%d",&k);
39
            printf("%d\n", p[k]);
40
        }
41
```

## 5.10 杜教筛 Dujiao\_Sieve

```
#include <cstdio>
 2
    #include <cstring>
 3
    #define LL long long
 4
    #define MAXR R+5
    #define R 2000000//r=n^(2/3)
 5
 7
    int n,r,p[MAXR],cnt;
 8
    bool inp[MAXR];
 9
10
    LL phi[MAXR], sphi[MAXR], sphir[MAXR], mu[MAXR], smu[MAXR], smur[MAXR];
11
12
    void pre(){
13
        memset(inp+1,0,r*sizeof(inp[0]));
14
        inp[0]=inp[1]=1;
15
        cnt=0;
16
        sphi[1]=phi[1]=1;
17
        smu[1]=mu[1]=1;
        for(int i=2;i<=r;i++){</pre>
18
19
            if(!inp[i]) p[++cnt]=i, phi[i]=i-1, mu[i]=-1;
20
           for(int j=1;j<=cnt&&i*p[j]<=r;j++){</pre>
21
               inp[i*p[j]]=1;
22
               if(i%p[j]){
23
                   phi[i*p[j]]=phi[i]*(p[j]-1);
24
                   mu[i*p[j]]=-mu[i];
               }
25
26
               else{
27
                   phi[i*p[j]]=phi[i]*p[j];
28
                   mu[i*p[j]]=0;
29
                   break;
30
31
           }
32
           sphi[i]=sphi[i-1]+phi[i];
33
            smu[i]=smu[i-1]+mu[i];
34
        }
    }
35
36
37
    LL sumphi(LL x){
38
        if(x<=r) return sphi[x];</pre>
        else if(sphir[n/x]) return sphir[n/x];
39
```

```
40
        LL &sx=sphir[n/x];
41
        sx=x*(x+1)/2;
42
        for(LL i=2;i<=x;i++){</pre>
43
           LL t=x/i, j=x/t;
44
           sx=(j-i+1)*sumphi(t);
45
46
        }
47
        return sx;
48
    }
49
50
    LL summu(LL x){
        if(x<=r) return smu[x];</pre>
51
52
        else if(smur[n/x]) return smur[n/x];
53
        LL &sx=smur[n/x];
54
        sx=1;
        for(LL i=2;i<=x;i++){</pre>
55
56
           LL t=x/i, j=x/t;
           sx=(j-i+1)*summu(t);
57
58
           i=j;
59
        }
60
        return sx;
61
    }
62
63
    int main(){
64
        int T;
65
        scanf("%d", &T);
66
        r=R;
67
        pre();
        while(T--){
68
69
           scanf("%d", &n);
70
           memset(sphir+1,0,(n/r)*sizeof(sphir[0]));
71
           memset(smur+1,0,(n/r)*sizeof(smur[0]));
72
           printf("%11d %11d\n", sumphi(n), summu(n));
73
        }
74
        return 0;
75
    }
```

## 5.11 求原根 Get\_Primitive\_Root

```
#include <cstdio>
 2
    #include <vector>
3
4
    using namespace std;
5
6
    int p;
 7
    vector<int> v;
8
9
    int binpow(int x,int y,int mod){
10
       int r=1%mod;
11
       while(y){
12
           if(y&1) r=(1LL*r*x)%mod;
           x=(1LL*x*x)\mbox{mod};
13
```

```
14
           y>>=1;
        }
15
16
        return r;
17
18
    int main(){
19
20
        scanf("%d", &p);
21
        int pp=p-1;
22
        for(int i=2;i*i<=pp;i++){</pre>
23
           if (pp%i==0) {
24
               v.push_back(i);
25
               while(pp%i==0) pp/=i;
26
           }
27
28
        if(pp>1) v.push_back(pp);
29
        for(int g=2;;g++){
30
           bool isg=true;
           for(int d:v){
31
32
               if(binpow(g,(p-1)/d,p)==1){
33
                   isg=false;
34
                   break;
               }
35
36
           }
37
           if(isg){
38
               printf("%d\n", g);
39
               break;
40
           }
41
        }
42
        return 0;
43
```

### 5.12 素数测试 Miller\_Rabin

```
#include <bits/stdc++.h>
    #define LL long long
3
   using namespace std;
   const int MAXN=2000005;
4
5
6
   LL binpow(LL x,LL y,LL m) {
7
       LL r=1\%m;
8
       while(y) {
9
           if(y&1) r=(__int128)r*x\m;
10
           x=(__int128)x*x%m;
11
           y>>=1;
12
       }
13
       return r;
14
   }
15
16
    bool Miller_Rabin(LL x) {
       if(x==1) return false;
17
18
19
       const int p[9]={2,3,5,7,11,13,17,19,23};
```

```
20
        for(int i=0;i<9;i++) {</pre>
21
           if(x==p[i]) return true;
22
23
           LL t=x-1;
24
           while(!(t&1)) t>>=1;
25
           LL a=binpow(p[i],t,x),lst;
26
           while(t!=x-1) {
27
               lst=a;
               a=(__int128)a*a%x;
28
29
               t<<=1;
30
               if(a==1 && lst!=x-1 && lst!=1) return false;
           }
31
32
           if(a!=1) return false;
33
34
        return true;
35
    }
```

## 5.13 大数分解 Pollard\_Rho

```
/*
 1
 2
       Pollard_Rho分解因数,要保证放入的为合数。
 3
       期望复杂度O(n^(1/4)),要随机化。
 4
 5
    #include <bits/stdc++.h>
 6
    #define LL long long
 7
    using namespace std;
8
9
    LL gcd(LL x,LL y) {
10
       if(!x || !y) return x+y;
11
       return gcd(y,x%y);
12
13
    LL binpow(LL x,LL y,LL m) {
14
15
       LL r=1\%m;
       while(y) {
16
           if(y&1) r=(__int128)r*x\m;
17
18
           x=(__int128)x*x%m;
19
           y>>=1;
       }
20
21
       return r;
22
   }
23
24
    bool Miller_Rabin(LL x) {
25
       if(x==1) return false;
26
27
       const int p[9]={2,3,5,7,11,13,17,19,23};
       for(int i=0;i<9;i++) {</pre>
28
29
           if(x==p[i]) return true;
30
31
           LL t=x-1;
           while(!(t&1)) t>>=1;
32
           LL a=binpow(p[i],t,x),lst;
33
```

```
34
           while(t!=x-1) {
35
               lst=a;
36
               a=(__int128)a*a%x;
37
               t<<=1:
38
               if(a==1 && lst!=x-1 && lst!=1) return false;
           }
39
40
           if(a!=1) return false;
41
       }
42
        return true;
43
44
    LL f(LL x,LL c,LL m) {return ((__int128)x*x+c)\m;}
45
46
    LL Pollard_Rho(LL x) {
47
48
        if(x==1) return x;
49
50
       LL c=rand()\%(x-1)+1;
51
        LL s=0,t=0,val=1;
52
        for(int goal=1;;goal<<=1,s=t,val=1) {</pre>
           for(int step=1;step<=goal;step++) {</pre>
53
54
               t=f(t,c,x);
               val=(__int128)val*abs(t-s)%x;
55
               if(step%127==0) {
56
57
                   LL g=gcd(val,x);
58
                   if(g>1) return g;
59
               }
60
61
           LL g=gcd(val,x);
62
           if(g>1) return g;
63
       }
64
    }
65
66
    LL max_factor(LL x) {
67
        if(x==1 || Miller_Rabin(x)) {
68
           return x;
69
       } else {
70
           LL g=Pollard_Rho(x);
71
           while(x%g==0) x/=g;
72
           return max(max_factor(x),max_factor(g));
73
        }
74
    }
75
76
    void solve() {
77
       LL n;
78
        scanf("%lld", &n);
79
        if(Miller_Rabin(n)) {
           printf("Prime\n");
80
81
        } else {
           printf("%lld\n", max_factor(n));
82
83
        }
84
    }
85
   int main() {
```

```
87
       srand(time(0));
88
       int T=1,cas=1;(void)(cas);
89
       scanf("%d", &T);
90
       while(T--) {
91
          // printf("Case #%d: ", cas++);
92
          solve();
93
       }
94
       return 0;
95
   }
```

Xidian University 6 组合数学

# 6 组合数学

## 6.1 康托展开 Cantor

```
1
 2
        康托 (Cantor) 展开: 求全排列是第几个。
 3
 4
    #include <bits/stdc++.h>
 5
    #define LL long long
    using namespace std;
 6
 7
    const int MAXN=2000005;
 8
    const int MOD=998244353;
 9
10
    struct BIT {
       int n,b[MAXN];
11
12
13
       int lowbit(int x){
14
           return x&(-x);
15
16
17
       void change(int x,int y){
18
           for(;x<=n;x+=lowbit(x))</pre>
19
               b[x] += y;
20
       }
21
22
       int sum(int x){
23
           int s=0;
24
           for(;x>0;x-=lowbit(x))
25
               s+=b[x];
26
           return s;
27
28
    }bit;
29
30
   int a[MAXN],f[MAXN];
31
32
    void solve() {
33
       int n;
        scanf("%d", &n);
34
35
        for(int i=1;i<=n;i++)</pre>
           scanf("%d", &a[i]);
36
37
38
       f[0]=1;
39
        for(int i=1;i<=n;i++)</pre>
40
           f[i]=1LL*f[i-1]*i%MOD;
41
42
       bit.n=n;
43
       int ans=1;
44
45
       for(int i=1;i<n;i++) {</pre>
46
           bit.change(a[i],1);
47
           ans=(ans+1LL*f[n-i]*(a[i]-bit.sum(a[i]))%MOD)%MOD;
48
       printf("%d\n", ans);
49
```

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```
50
   }
51
52
    int main() {
53
       int T=1,cas=1;(void)(cas);
54
       // scanf("%d", &T);
55
       while(T--) {
56
           // printf("Case #%d: ", cas++);
57
           solve();
58
       }
59
       return 0;
60
```

#### 6.2 波利亚 Pólya

```
/*
 1
 2
       Polya定理
 3
       本质不同个数=sigma(颜色个数~等价置换的划分个数)/(等价置换总个数)
 4
 5
       例题: n条个点围成一圈, m种颜色。求染色旋转本质不同个数。
 6
       ans = sigma(m^{cd}(n,i))/n
 7
          = sigma(m<sup>d</sup> * phi(n/d))/n ----- 莫比乌斯反演
8
   */
9
   #include <bits/stdc++.h>
10
   #define LL long long
11
   using namespace std;
12
    const int MOD=1e9+7;
13
   const int MAXN=2000005;
14
15
   int binpow(int x,int y,int m) {
16
      int r=1%m;
       while(y) {
17
18
          if(y&1) r=1LL*r*x%m;
19
          x=1LL*x*x%m;
20
          y>>=1;
21
       }
22
       return r;
23
   }
24
   int phi(int x) {
25
26
       int r=x;
       for(int i=2;i*i<=x;i++) {</pre>
27
          if(x%i==0) {
28
29
             r=r/i*(i-1);
30
             while(x\%i==0) x/=i;
31
          }
32
       if(x>1) r=r/x*(x-1);
33
34
       return r;
35
36
   void solve() {
37
       int n,m; //n个点, m种颜色。
```

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```
39
        scanf("%d", &n);
40
        m=n;
41
        int ans=0;
42
        for(int i=1;i*i<=n;i++) {</pre>
43
           if(n%i==0) {
44
               ans=(ans+1LL*binpow(m,i,MOD)*phi(n/i)%MOD)%MOD;
               if(i*i!=n) {
45
46
                   ans=(ans+1LL*binpow(m,n/i,MOD)*phi(i)%MOD)%MOD;
47
           }
48
49
50
        ans=1LL*ans*binpow(n,MOD-2,MOD)%MOD;
        printf("%d\n", ans);
51
52
    }
53
54
   int main() {
55
       int T=1,cas=1;(void)(cas);
56
        scanf("%d", &T);
        while(T--) {
57
58
           // printf("Case #%d: ", cas++);
59
           solve();
60
        }
61
        return 0;
    }
```

#### 6.3 卡特兰数 Catalan

- 1. 有 2n 个人排成一行进入剧场。人场费 5 元。其中只有 n 个人有一张 5 元钞票,另外 n 人只有 10 元钞票,剧院无其它钞票,问有多少中方法使得只要有 10 元的人买票,售票处就有 5 元的钞票找零?
- 2. 一位大城市的律师在她住所以北n个街区和以东n个街区处工作。每天她走2n个街区去上班。如果他从不穿越(但可以碰到)从家到办公室的对角线,那么有多少条可能的道路?
- 3. 在圆上选择 2n 个点,将这些点成对连接起来使得所得到的 n 条线段不相交的方法数?
- 4. 对角线不相交的情况下,将一个凸多边形区域分成三角形区域的方法数?
- 5. 一个栈(无穷大)的进栈序列为  $1,2,3,\cdots,n$  有多少个不同的出栈序列?
- 6. n 个结点可构造多少个不同的二叉树?
- 7. n 个不同的数依次进栈, 求不同的出栈结果的种数?
- 8.  $n \uparrow +1$  和  $n \uparrow -1$  构成  $2n \bar{y}$   $a_1, a_2, \cdots, a_{2n}$  , 其部分和满足  $a_1 + a_2 + \cdots + a_k \geq 0 (k = 1, 2, 3, \cdots, 2n)$  对与 n 该数列为?

Н0	H1	H2	НЗ	H4	H5	Н6
1	1	2	5	14	42	132

关于 Catalan 数的常见公式:

$$H_n = \begin{cases} \sum_{i=1}^n H_{i-1} H_{n-i} & n \ge 2, n \in \mathbb{N}_+ \\ 1 & n = 0, 1 \end{cases}$$

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$$H_n = \frac{H_{n-1}(4n-2)}{n+1}$$

$$H_n = {2n \choose n} - {2n \choose n-1}$$

$$H_n = \frac{{2n \choose n}}{n+1} (n \ge 2, n \in \mathbf{N}_+)$$

### 6.4 斯特林数 Stirling

#### 6.4.1 第一类斯特林数

(斯特林轮换数)

表示将 n 个两两不同的元素,划分为 k 个非空圆排列的方案数。 递推式

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} + (n-1) \begin{bmatrix} n-1 \\ k \end{bmatrix}$$

边界是 
$$\begin{bmatrix} n \\ 0 \end{bmatrix} = [n=0]$$
 。

#### 6.4.2 第二类斯特林数

(斯特林子集数)

 ${n \brace k}$  表示将 n 个两两不同的元素,划分为 k 个非空子集的方案数。

$${n \brace k} = {n-1 \brace k-1} + k {n-1 \brace k}$$

边界是 
$$\begin{Bmatrix} n \\ 0 \end{Bmatrix} = [n=0]$$
 。

#### 6.4.3 上升幂与普通幂的相互转化

我们记上升阶乘幂  $x^{\overline{n}} = \prod_{k=0}^{n-1} (x+k)$ 。 则可以利用下面的恒等式将上升幂转化为普通幂:

$$x^{\overline{n}} = \sum_{k} \begin{bmatrix} n \\ k \end{bmatrix} x^k$$

如果将普通幂转化为上升幂,则有下面的恒等式:

$$x^{n} = \sum_{k} \begin{Bmatrix} n \\ k \end{Bmatrix} (-1)^{n-k} x^{\overline{k}}$$

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#### 6.4.4 下降幂与普通幂的相互转化

我们记下降阶乘幂  $x^n=\frac{x!}{(x-n)!}=\prod_{k=0}^{n-1}(x-k)$  。 则可以利用下面的恒等式将普通幂转化为下降幂:

$$x^n = \sum_{k} \begin{Bmatrix} n \\ k \end{Bmatrix} x^{\underline{k}}$$

如果将下降幂转化为普通幂,则有下面的恒等式:

$$x^{\underline{n}} = \sum_{k} \begin{bmatrix} n \\ k \end{bmatrix} (-1)^{n-k} x^k$$

# 6.5 范德蒙德卷积 Vandermonde\_Convolution

$$\sum_{i=0}^{k} \binom{n}{i} \binom{m}{k-i} = \binom{n+m}{k}$$

从数量分别为 n 和 m 的石堆中总共选取 k 个石子。

## 7 网络流

#### 7.1 最大费用流 Minimum-Cost\_Flow\_Edmonds-Karp

```
#include <cstdio>
    #include <cstring>
 3
    #include <queue>
 4
    #include <algorithm>
    #define MAXN 5005
 6
    #define MAXM 50005
 7
 8
    const int inf = 0x3f3f3f3f;
9
10
    using namespace std;
11
12
    int n,m,s,t,tot,head[MAXN],dis[MAXN],maxflow,mincost;
    bool inque[MAXN];
13
14
15
    struct edge{//残量网络 residual network
        int to,cf,next,dis;//Cf:residual capacity
16
17
    }e[(MAXM<<1)+1];</pre>
18
19
    struct node{
20
       int fr,edge;
21
    }pre[MAXN];
22
23
    void add(int x,int y,int f,int d){
24
       tot++;
       e[tot].cf=f;
25
26
       e[tot].dis=d;
27
       e[tot].to=y;
28
       e[tot].next=head[x];
       head[x]=tot;
29
   }
30
31
32
    bool spfa(){//SPFA
       memset(dis+1,inf,n*sizeof(dis[0]));
33
34
       memset(inque+1,0,n*sizeof(inque[0]));
35
       pre[t].fr=0;
36
       queue<int> q;
37
       dis[s]=0;
38
       q.push(s);
39
        inque[s]=1;
40
       while(!q.empty()){
41
           int u=q.front();q.pop();
42
           inque[u]=0;
43
           for(int p=head[u];p;p=e[p].next){
44
               int v=e[p].to;
               if(e[p].cf && dis[v]>dis[u]+e[p].dis){
45
                  dis[v]=dis[u]+e[p].dis;
46
47
                  pre[v].fr=u;
48
                  pre[v].edge=p;
49
                  if(!inque[v]){
```

```
50
                      q.push(v);
51
                      inque[v]=1;
52
                   }
53
               }
54
           }
       }
55
56
        return pre[t].fr!=0;
57
58
59
    int min_flow(){
60
        int mn=inf;
61
        for(int u=t;u!=s;u=pre[u].fr){
           mn=min(mn,e[pre[u].edge].cf);
62
63
       }
        for(int u=t;u!=s;u=pre[u].fr){
64
65
            e[pre[u].edge].cf-=mn;
66
            e[pre[u].edge^1].cf+=mn;
       }
67
68
        return mn;
69
    }
70
71
    void edmonds_karp(){
72
       maxflow=0,mincost=0;
73
        while(spfa()){
74
            int flow=min_flow();
75
           maxflow+=flow;
76
           mincost+=flow*dis[t];
77
        }
78
    }
79
80
    int main(){
81
        scanf("%d %d %d %d", &n, &m, &s, &t);
82
        tot=1;
83
        memset(head+1,0,n*sizeof(head[0]));
        for(int i=1;i<=m;i++){</pre>
84
85
           int f,g,w,d;
86
           scanf("%d %d %d %d",&f,&g,&w,&d);
87
            add(f,g,w,d);
88
            add(g,f,0,-d);
89
        }
90
        edmonds_karp();
91
        printf("%d %d\n",maxflow,mincost);
92
        return 0;
93
    }
```

#### 7.2 最大流 Maximum\_Flow\_Edmonds-Karp

```
#include <cstdio>
#include <cstring>
#include <queue>
#include <algorithm>
#define MAXN 10005
```

```
#define MAXM 100005
 8
    using namespace std;
 9
10
    const int inf = 0x3f3f3f3f;
11
12
    int n,m,s,t,tot,head[MAXN],vis[MAXN];
13
14
    struct edge{
15
        int to,cf,next;
16
    }e[MAXM<<1];</pre>
17
    struct node{
18
19
       int fr,edge;
    }pre[MAXN];
20
21
22
    void add(int x,int y,int z){
23
        tot++;
24
        e[tot].cf=z;
25
        e[tot].to=y;
26
        e[tot].next=head[x];
27
        head[x]=tot;
28
    }
29
30
    bool find_augment(){
31
       memset(pre+1,0,n*sizeof(pre[0]));
32
        memset(vis+1,0,n*sizeof(vis[0]));
33
        queue<int> q;
34
        vis[s]=1;
35
        q.push(s);
        while(!q.empty()){
36
37
           int u=q.front();q.pop();
38
           for(int p=head[u];p;p=e[p].next){
39
               int v=e[p].to;
40
               if(!vis[v] && e[p].cf){
41
                  pre[v].fr=u;
42
                  pre[v].edge=p;
43
                  vis[v]=1;
44
                   q.push(v);
45
                   if(v==t) return true;
               }
46
           }
47
48
        }
49
        return false;
50
    }
51
52
    int min_flow(){
53
        int mn=inf;
        for(int u=t;u!=s;u=pre[u].fr){
54
55
           mn=min(mn,e[pre[u].edge].cf);
56
       }
57
        for(int u=t;u!=s;u=pre[u].fr){
           e[pre[u].edge].cf-=mn;
```

```
59
           e[pre[u].edge^1].cf+=mn;
60
61
        return mn;
62
    }
63
64
    int edmonds_karp(){
65
       int flow=0;
66
        while(find_augment()){
67
           flow+=min_flow();
68
       }
69
        return flow;
    }
70
71
72
    int main(){
73
        scanf("%d %d %d %d", &n, &m, &s, &t);
74
75
       memset(head+1,0,n*sizeof(head[0]));
76
       for(int i=1;i<=m;i++){</pre>
77
           int f,g,w;
           scanf("%d %d %d",&f,&g,&w);
78
79
           add(f,g,w);
80
           add(g,f,0);
81
       }
        printf("%d\n", edmonds_karp());
        return 0;
83
84
    }
```

## 7.3 最大流 Maximum\_Flow\_Dinic

```
#include <cstdio>
    #include <cstring>
    #include <queue>
3
    #include <algorithm>
4
    #define MAXN 10005
 6
    #define MAXM 100005
 7
8
    const int inf = 0x3f3f3f3f;
9
10
    using namespace std;
11
    int n,m,s,t,tot,head[MAXN],lb[MAXN],cur[MAXN];
12
13
14
    struct edge{//残量网络 residual network
15
       int to,cf,next;//Cf:residual capacity
    }e[(MAXM<<1)+1];</pre>
16
17
18
    void add(int x,int y,int z){
19
       tot++;
20
       e[tot].cf=z;
21
       e[tot].to=y;
22
       e[tot].next=head[x];
       head[x]=tot;
23
```

```
24
    }
25
26
    bool label_vertex(){//BFS
27
       memset(lb+1,0,n*sizeof(lb[0]));
28
       queue<int> q;
29
       lb[s]=1;
30
       q.push(s);
31
       while(!q.empty()){
32
           int u=q.front();q.pop();
33
           for(int p=head[u];p;p=e[p].next){
34
               int v=e[p].to;
               if(e[p].cf && !lb[v]){
35
                  lb[v]=lb[u]+1;
36
37
                  q.push(v);
38
                  if(v==t) return true;
39
              }
40
           }
       }
41
42
       return false;
43
    }
44
    int multi_augment(int u,int lim){//DFS 多路增广
45
       if(u == t) return lim;
46
47
       int used=0;
48
49
       for(int& p=cur[u];p;p=e[p].next){
50
           int v=e[p].to;
51
           if(e[p].cf && lb[v]==lb[u]+1){
52
               int rest=multi_augment(v,min(lim-used,e[p].cf));
53
              used+=rest;
54
               e[p].cf-=rest;
55
               e[p^1].cf+=rest;
56
               if(used==lim) break;
57
           }
       }
58
59
       return used;
60
    }
61
62
    int dinic(){
63
       int flow=0;
64
       while(label_vertex()){//BFS 标记
65
           for(int i=1;i<=n;i++) cur[i]=head[i];//当前弧优化
66
           flow+=multi_augment(s,inf);//DFS 顺着标记找增广路
67
       }
68
       return flow;
69
    }
70
71
    int main(){
       scanf("%d %d %d %d", &n, &m, &s, &t);
72
73
74
       memset(head+1,0,n*sizeof(head[0]));
75
       for(int i=1;i<=m;i++){</pre>
76
           int f,g,w;
```

```
77 | scanf("%d %d %d",&f,&g,&w);
78 | add(f,g,w);
79 | add(g,f,0);
80 | }
81 | printf("%d\n", dinic());
82 | return 0;
83 |
```

## 7.4 二分图最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Dinic

```
#include <cstdio>
 2
    #include <cstring>
 3
    #include <queue>
 4
    #define MAXN 2005
 5
    #define MAXM 1000005
 6
 7
    using namespace std;
 8
 9
    const int inf=0x3f3f3f3f;
10
11
    struct edge{
12
        int to,cf,next;
13
    }e[MAXM<<1];</pre>
14
15
    int n,n1,n2,m,s,t;
16
    int tot,head[MAXN],cur[MAXN],lbl[MAXN];
17
    void add(int x,int y,int z){
18
19
        tot++;
20
        e[tot].to=y;
21
        e[tot].cf=z;
22
        e[tot].next=head[x];
23
        head[x]=tot;
    }
24
25
    bool bfs(){
26
27
        memset(lbl+1,0,n*sizeof(lbl[0]));
        lbl[t]=1;
28
29
        queue<int> q;
30
        q.push(t);
        while(!q.empty()){
31
32
           int u=q.front();q.pop();
33
           for(int p=head[u];p;p=e[p].next){
34
               int v=e[p].to;
35
               if(e[p^1].cf && !lbl[v]){
36
                   lbl[v]=lbl[u]+1;
37
                   q.push(v);
38
                   if(v==s) return true;
39
               }
40
           }
       }
41
42
        return lbl[s]!=0;
```

```
43
    }
44
    int dfs(int u,int lim){
45
        if(u==t)return lim;
46
47
48
        int used=0;
49
        for(int& p=cur[u];p;p=e[p].next){
50
           int v=e[p].to;
51
            if(e[p].cf && lbl[v]==lbl[u]-1){
52
               int rest=dfs(v,min(lim-used,e[p].cf));
53
               used+=rest;
               e[p].cf-=rest;
54
55
               e[p^1].cf+=rest;
56
               if(used==lim) break;
57
           }
58
        }
59
        return used;
    }
60
61
62
    int dinic(){
63
        int flow=0;
64
        while(bfs()){
65
           for(int i=1;i<=n;i++)</pre>
66
               cur[i]=head[i];
67
           flow+=dfs(s,inf);
        }
68
69
        return flow;
70
    }
71
72
    int main(){
73
        scanf("%d %d %d", &n1, &n2, &m);
74
        n=n1+n2+2;//n个点
75
        s=n-1;t=n;
76
77
        memset(head+1,0,n*sizeof(head[0]));
78
        for(int i=1;i<=m;i++){</pre>
79
           int f,g;
80
           scanf("%d %d",&f,&g);
81
           if(f>n1 || g>n2) continue;
82
           add(f,n1+g,1);
83
           add(n1+g,f,0);
84
        }
85
        for(int i=1;i<=n1;i++){</pre>
86
           add(s,i,1);
87
            add(i,s,0);
88
89
        for(int i=n1+1;i<=n1+n2;i++){</pre>
90
           add(i,t,1);
91
            add(t,i,0);
92
93
        printf("%d\n", dinic());
94
        return 0;
    }
95
```

# 7.5 二分图最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Hungarian

```
#include <cstdio>
 2
    #include <cstring>
    #define MAXN 1005
 3
    #define MAXM 1000005
 4
 5
 6
    struct node{
 7
        int to,next;
 8
    }e[MAXM];
 9
10
    int n1,n2,m,head[MAXN],tot,dfn[MAXN],mat[MAXN];
11
12
    void add(int x,int y){
13
        tot++;
14
        e[tot].to=y;
15
        e[tot].next=head[x];
16
        head[x]=tot;
17
    }
18
19
    bool augment(int x,int stamp){
20
        for(int p=head[x];p;p=e[p].next){
21
           int u=e[p].to;
22
           if(dfn[u] == stamp) continue;
           dfn[u]=stamp;
23
24
           if(!mat[u] || augment(mat[u],stamp)){
25
               mat[u]=x;
26
               return true;
           }
27
28
        }
29
        return false;
30
    }
31
32
    int match(){
33
       memset(mat+1,0,n2*sizeof(mat[0]));
34
        memset(dfn+1,0,n2*sizeof(dfn[0]));
35
        int cnt=0;
        for(int i=1;i<=n1;i++){</pre>
36
37
           if(augment(i,i))
38
               cnt++;
39
       }
40
        return cnt;
41
    }
42
43
    int main(){
44
        scanf("%d %d %d", &n1,&n2,&m);
        tot=0;
45
46
        memset(head+1,0,n1*sizeof(head[0]));
47
        for(int i=1;i<=m;i++){</pre>
48
           int f,g;
49
           scanf("%d %d", &f, &g);
50
           if(f>n1 || g>n2)
51
               continue;
```

```
52     add(f,g);
53     }
54     printf("%d\n", match());
55     return 0;
56 }
```

# 7.6 二分图最大匹配 Bipartite\_Graph\_Maximum\_Matching\_Hopcroft-Karp

```
1
    #include <cstdio>
    #include <cstring>
    #include <queue>
 4
    #define MAXN 2005
 5
    #define MAXM 1000005
    const int inf=0x3f3f3f3f;
 8
 9
    using namespace std;
10
11
    int n1,n2,n,m,tot,head[MAXN];
12
    int mat[MAXN],lb[MAXN],dfn[MAXN];
13
14
    struct edge{
15
        int to,next;
16
    }e[MAXM];
17
18
    void add(int x,int y){
19
        tot++;
20
        e[tot].to=y;
21
        e[tot].next=head[x];
22
        head[x]=tot;
23
   }
24
25
    bool bfs(){
26
        memset(lb+1,0,n*sizeof(lb[0]));
27
        queue<int> q;
28
        for(int i=1;i<=n1;i++){</pre>
29
           if(!mat[i]){
30
               q.push(i);
31
               lb[i]=1;
           }
32
       }
33
        int dis=inf;
34
35
        while(!q.empty()){
36
           int u=q.front();q.pop();
           for(int p=head[u];p;p=e[p].next){
37
38
               int v=e[p].to;
39
               if(!lb[v]){
40
                  lb[v]=lb[u]+1;
41
                   if(!mat[v]) dis=lb[v];
42
                   else if(lb[v]<dis){</pre>
43
                      lb[mat[v]]=lb[v]+1;
```

```
44
                      q.push(mat[v]);
                   }
45
46
               }
47
           }
48
        }
49
        return dis!=inf;
50
    }
51
52
    bool dfs(int u,int stamp){
53
        for(int p=head[u];p;p=e[p].next){
54
           int v=e[p].to;
            if(dfn[v]!=stamp && lb[v]==lb[u]+1){
55
56
               dfn[v]=stamp;
               if(!mat[v] || (lb[mat[v]]==lb[v]+1 && dfs(mat[v],stamp))){
57
58
                   mat[v]=u;
59
                   mat[u]=v;
60
                   return true;
               }
61
           }
62
63
        }
64
        return false;
65
    }
66
67
    int hopcroft_karp(){
68
        int cnt=0,stamp=0;
69
        memset(dfn+1,0,n*sizeof(dfn[0]));
70
        memset(mat+1,0,n*sizeof(mat[0]));
71
        while(bfs()){
72
           stamp++;
73
           for(int i=1;i<=n1;i++){</pre>
74
               if(!mat[i] && dfs(i,stamp)){
75
                   cnt++;
76
               }
77
           }
78
        }
79
        return cnt;
80
    }
81
82
    int main(){
83
        scanf("%d %d %d", &n1, &n2, &m);
84
        n=n1+n2;
85
        memset(head+1,0,n*sizeof(head[0]));
86
        tot=0;
87
        for(int i=1;i<=m;i++){</pre>
88
           int f,g;
89
           scanf("%d %d",&f,&g);
90
           if(f>n1 || g>n2) continue;
           add(f,n1+g);
91
92
93
        printf("%d\n", hopcroft_karp());
94
        return 0;
95
    }
```

Xidian University 8 计算几何

# 8 计算几何

#### 8.1 计算几何 Computational\_Geometry

```
#include <bits/stdc++.h>
    #define MAXN 2000005
 3
    #define LL long long
 4
    using namespace std;
    const double PI=acos(-1.0);
 6
    const double inf=1e100;
 7
    const double eps=1e-7;
 8
 9
    int sgn(double d) {
10
        if(abs(d)<eps) return 0;</pre>
11
        if(d>0) return 1;
12
        return -1;
13
    }
14
15
    int dcmp(double x,double y) {
16
        if(abs(x-y)<eps) return 0;</pre>
17
        if(x>y) return 1;
18
        return -1;
19
    }
20
21
    struct Point{
22
        double x,y;
23
        Point(double x=0, double y=0):x(x),y(y){}
24
25
        Point operator + (const Point& B) const{
26
           return Point(x+B.x,y+B.y);
27
        }
28
        Point operator - (const Point& B) const{
29
           return Point(x-B.x,y-B.y);
30
31
        Point operator * (const double k) const{
32
           return Point(x*k,y*k);
33
        Point operator / (const double k) const{
34
35
           return Point(x/k,y/k);
36
        }
37
        bool operator < (const Point B) {</pre>
38
           if(dcmp(x,B.x)==0)
39
               return dcmp(y,B.y)<0;</pre>
40
           else return dcmp(x,B.x)<0;</pre>
41
        }
42
43
        double operator * (const Point& B) const{//点积
44
           return x*B.x+y*B.y;
        }
45
        double operator \hat{\ } (const Point& B) const \{//\mathbb{Z}
46
47
           return x*B.y-y*B.x;
48
        }
49 };
```

Xidian University 8 计算几何

```
50
51
    typedef Point Vector;
52
53
    double Length(Vector A) {
54
        return sqrt(A*A);
55
    }
56
57
    double Angle(Vector A, Vector B) {//弧度
58
        return acos(A*B/Length(A)/Length(B));
59
    }
60
    double Area2(Vector A, Vector B){//求平行四边形面积
61
62
        return A^B;
63
    }
64
65
    Vector Rotate(Vector A, double rad) {//逆时针
66
       return Vector(A.x*cos(rad)-A.y*sin(rad), A.x*sin(rad)+A.y*cos(rad));
67
    }
68
    Vector Normal(Vector A) {//逆时针转90度,单位法向量
69
70
        double L=Length(A);
71
        return Vector(-A.y/L, A.x/L);
72
    }
73
    bool ToLeftTest(Vector A, Vector B) {//B是不是在A左边
74
75
        return sgn(A^B)>0;
76
77
78
    struct Line {//点向式+两点。既可以line也可以seg
79
       Point p1,p2;//p1->p2
80
        Vector v;
81
       Line(Point p1, Point p2):p1(p1),p2(p2),v((p2-p1)/Length(p2-p1)){}
        Point point(double t) {//给t求点
82
83
           return p1+v*t;
84
        }
85
    };
86
    typedef Line Segment;
87
88
    bool OnLine(Point P, Line 1) {//判断点P是否在直线L上
89
        return sgn((P-1.p1)^l.v);
90
    |}
91
92
    Point GetIntersection(Line 11, Line 12) {//求直线交点
        double t = (12.v^(11.p1-12.p1))/(11.v^12.v);
93
94
        return 11.point(t);
95
    }
96
97
    double DistanceToLine(Point P, Line 1) {//点到直线距离
98
        return abs(1.v^(P-1.p1));
99
    }
100
101
    Point GetProjection(Point P, Line 1) {//求投影点
102
        return 1.point((P-1.p1)*1.v);
```

Xidian University 8 计算几何

```
103
    }
104
105
     bool OnSegment(Point P, Segment s) {
106
        return (OnLine(P,s) && sgn((s.p1-P)*(s.p2-P))<0);</pre>
107
    }
108
109
     bool InSegmentIntersection(Segment s1, Segment s2) {//不允许端点相交
        double c1=(s1.p2-s1.p1)^(s2.p1-s1.p1), c2=(s1.p2-s1.p1)^(s2.p2-s1.p1);
110
111
        double c3=(s2.p2-s2.p1)^(s1.p1-s2.p1), c4=(s2.p2-s2.p1)^(s1.p2-s2.p1);
112
        return (sgn(c1)*sgn(c2)<0 && sgn(c3)*sgn(c4)<0);</pre>
113
    }
114
115
    void solve() {
116
117
     }
118
119
    int main() {
120
        int T=1,cas=1;
121
        // scanf("%d", &T);
122
        while(T--) {
123
            // printf("Case #%d: ", cas++);
124
            solve();
125
        }
126
        return 0;
127
    }
```

Xidian University 9 优化算法

# 9 优化算法

## 9.1 wqs 二分 + 决策单调性 dp

```
1
 2
        决策单调性 + WQS二分
 3
 4
        给定数列{a}。设立k个关键点。
 5
        让sigma min|a_i-b_k| 最小 (每个a_i到最近的关键点距离之和)
 6
    */
 7
   #include <bits/stdc++.h>
 8
    #define LL long long
    using namespace std;
10
   const int MAXN=2000005;
11
12
    LL a[MAXN],f[MAXN];
    int cnt[MAXN],1b[MAXN],rb[MAXN],p[MAXN];
13
14
15
    LL w(int x,int y){
        return a[y]+a[x]-a[(x+y+1)>>1]-a[(x+y)>>1];
16
17
    }
18
19
    int k,n;
20
21
    int calc(LL d) {
22
       int h,t;
23
       h=t=1;
       lb[t]=1,rb[t]=n,p[t]=0;
24
25
       f[0]=cnt[0]=0;
26
       for(int i=1;i<=n;i++) {</pre>
           f[i]=f[p[h]]+w(p[h],i)+d;
27
28
           cnt[i]=cnt[p[h]]+1;
29
           while(rb[h]<=i) h++;</pre>
30
31
           if(h<=t && lb[h]<=i) lb[h]=i+1;</pre>
32
           if(h<=t && f[i]+w(i,n)>f[p[t]]+w(p[t],n)) continue;
33
34
           while (h \le t \&\& f[i] + w(i, lb[t]) \le f[p[t]] + w(p[t], lb[t])) t --;
35
           if(h<=t) {
36
               int l=lb[t],r=rb[t]+1;
37
               while(l<r) {</pre>
                  int mid=(l+r)>>1;
38
                   if(f[i]+w(i,mid)<=f[p[t]]+w(p[t],mid)) r=mid;</pre>
39
40
                   else l=mid+1;
41
               }
42
               rb[t]=l-1;
43
               ++t;
               lb[t]=1,rb[t]=n,p[t]=i;
44
45
           } else {
46
               ++t;
47
               lb[t]=i,rb[t]=n,p[t]=i;
           }
48
49
        }
```

Xidian University 9 优化算法

```
50
        return cnt[n];
51
    }
52
53
    void solve() {
54
        scanf("%d %d", &n, &k);
55
        a[0]=0;
56
       for(int i=1;i<=n;i++) {</pre>
57
           scanf("%lld", &a[i]);
58
           a[i]+=a[i-1];
59
       }
60
       LL l=0,r=a[n];
61
62
        while(l<r) {</pre>
63
           LL mid=(l+r+1)>>1;
           if(calc(mid)>=k) l=mid;
64
65
           else r=mid-1;
66
       }
67
        calc(1);
68
69
       printf("\%lld\n", f[n]-1LL*k*l);\\
    }
70
71
72
   int main() {
73
       int T=1,cas=1;(void)(cas);
74
        // scanf("%d", &T);
75
        while(T--) {
76
           // printf("Case #%d: ", cas++);
77
           solve();
78
       }
79
       return 0;
80
    }
```

## 9.2 斜率优化 Slope\_Optimization

```
#include <bits/stdc++.h>
    #define LL long long
    using namespace std;
4
    const int MAXN=2000005;
5
   //似乎会爆LL
    LL c[MAXN],dp[MAXN];
   LL X[MAXN],Y[MAXN];
8
9
10
    void solve() {
11
       int n,L;
12
       scanf("%d %d", &n, &L);
13
       L++;
14
       c[0]=0;
15
       for(int i=1;i<=n;i++) {</pre>
16
           scanf("%lld", &c[i]);
17
           c[i]+=c[i-1]+1;//c[i] = sum(c[j])+i;
18
       }
```

Xidian University 9 优化算法

```
19
        int h,t;
20
        h=t=1;
21
        dp[0]=0;
22
        X[t]=c[0];
        Y[t]=dp[0]+c[0]*c[0];
23
        for(int i=1;i<=n;i++) {</pre>
24
25
            \label{eq:while(h<t && Y[h+1]-Y[h]<=2*(X[h+1]-X[h])*(c[i]-L)) h++;} \\
            dp[i]=Y[h]-2*(c[i]-L)*X[h]+(c[i]-L)*(c[i]-L);
26
27
            LL x=c[i],y=dp[i]+c[i]*c[i];
28
            \label{eq:while} \mbox{while} (\mbox{h<t \&\& (y-Y[t])*(X[t]-X[t-1])<=(Y[t]-Y[t-1])*(x-X[t])) t--;}
29
            t++;
30
            X[t]=x,Y[t]=y;
31
        }
32
        printf("%lld\n", dp[n]);
33
    }
34
35
   int main() {
36
        int T=1,cas=1;(void)(cas);
37
        // scanf("%d", &T);
38
        while(T--) {
            // printf("Case #%d: ", cas++);
39
40
            solve();
41
        }
42
        return 0;
43
   }
```

# 10 离线算法

#### 10.1 莫队算法 Mo

```
1
 2
       Coded with Leachim's ACM Template.
 3
       No errors. No warnings. ~~
 4
    */
 5
    #include <bits/stdc++.h>
 6
    #pragma GCC diagnostic ignored "-Wunused-const-variable"
 7
    #pragma GCC diagnostic ignored "-Wsign-conversion"
    #pragma GCC diagnostic ignored "-Wsign-compare"
9
    #define LL long long
10
    using namespace std;
    const int inf=0x3f3f3f3f;
11
12
    const LL INF=0x3f3f3f3f3f3f3f3f3f;
   const double eps=1e-7;
13
14
    const int dx[4]=\{1,-1,0,0\};
15
    const int dy[4]=\{0,0,1,-1\};
    const int MOD=998244353;
16
17
    const int MAXN=2000005;
18
19
    LL gcd(LL x,LL y) {
20
       if(!x || !y) return x+y;
21
       return gcd(y,x%y);
22
    }
23
24
25
        莫队:
26
       sqrt(m)个块
27
        块大小n/sqrt(m);
28
        卡常大小n/sqrt(m*2/3);
29
    */
30
31
    struct query {
32
       int 1,r,id;
33
    }q[MAXN];
34
35
    int a[MAXN],cnt[MAXN];
36
   int block,bl[MAXN];
37
    LL ans1[MAXN],ans2[MAXN],sum;
38
39
    bool cmp(const query &x, const query &y) {
40
       if(bl[x.1]!=bl[y.1])
41
           return bl[x.1] < bl[y.1];</pre>
       else if(bl[x.1]&1)
42
43
           return x.r<y.r;</pre>
44
       else return x.r>y.r;
    }
45
46
47
    void modify(int x,int f) {
       sum-=cnt[a[x]]*cnt[a[x]];
48
49
       cnt[a[x]]+=f;
```

```
50
        sum+=cnt[a[x]]*cnt[a[x]];
51
    }
52
53
    void solve() {
54
        int n,m;
        scanf("%d %d", &n, &m);
55
56
        block=n/sqrt(m*2/3);
57
        for(int i=1;i<=n;i++) {</pre>
            scanf("%d", &a[i]);
58
59
           bl[i]=(i-1)/block+1;
60
61
        for(int i=1;i<=m;i++) {</pre>
62
            scanf("%d %d", &q[i].1, &q[i].r);
63
            q[i].id=i;
64
65
        sort(q+1,q+1+m,cmp);
66
        memset(cnt+1,0,n*sizeof(cnt[0]));
67
        sum=0;
68
        for(int i=1,l=1,r=0;i<=m;i++) {</pre>
69
           for(;r<q[i].r;r++) modify(r+1,1);</pre>
70
           for(;r>q[i].r;r--) modify(r,-1);
71
           for(;l<q[i].1;l++) modify(1,-1);</pre>
72
           for(;l>q[i].1;l--) modify(l-1,1);
73
           ans1[q[i].id]=sum-(r-l+1);
74
            ans2[q[i].id]=1LL*(q[i].r-q[i].l+1)*(q[i].r-q[i].l);
75
        }
76
        for(int i=1;i<=m;i++) {</pre>
77
           LL g=gcd(ans1[i],ans2[i]);
78
            if(!g) printf("0/1\n");
79
            else printf("%lld/%lld\n", ans1[i]/g, ans2[i]/g);
80
        }
    }
81
82
83
    int main() {
84
        int T=1,cas=1;(void)(cas);
85
        // scanf("%d", &T);
86
        while(T--) {
            // printf("Case #%d: ", cas++);
87
88
            solve();
89
        }
90
        return 0;
91
```

#### 10.2 线段树分治 Segment\_Tree\_Partition

```
      1
      /*

      2
      可撤销并查集+线段树分治

      3
      离线操作:增边,删边,询问图是否联通

      4
      */

      5
      #include <bits/stdc++.h>

      6
      #define LL long long

      7
      using namespace std;
```

```
const int MAXN=5005;
 9
    const int MAXM=500005;
10
11
    struct DSU{
12
        int fa[MAXN],rk[MAXN];
13
        vector<pair<int*,int>> stk;
14
        void init(int n) {
15
           stk.clear();
16
           for(int i=1;i<=n;i++) fa[i]=i,rk[i]=1;
17
       }
18
        int find(int x) {
19
           if(x==fa[x]) return x;
20
           return find(fa[x]);
21
       }
22
        bool join(int x,int y) {
23
           int rx=find(x),ry=find(y);
24
           if(rx==ry) return false;
25
26
           if(rk[rx]>rk[ry]) swap(rx,ry);
27
           stk.emplace_back(fa+rx,rx);
28
           fa[rx]=ry;
29
           stk.emplace_back(rk+ry,rk[ry]);
30
           rk[ry]+=rk[rx];
31
           return true;
32
       }
33
        void withdraw() {
34
           *stk.back().first=stk.back().second;
35
           stk.pop_back();
36
           *stk.back().first=stk.back().second;
37
           stk.pop_back();
        }
38
39
    }dsu;
40
41
    int qf[MAXM],qg[MAXM];
42
43
    struct SGT{
44
        vector<pair<int,int>> eg[MAXM<<2];</pre>
45
46
        void modify(int x,int l,int r,int ql,int qr,int f,int g) {
47
           if(ql<=1 && r<=qr) {</pre>
48
               eg[x].emplace_back(f,g);
49
               return;
           }
50
51
           int m=(1+r)>>1;
52
           if(ql<=m) modify(x<<1,1,m,ql,qr,f,g);</pre>
53
           if(m<qr) modify(x<<1|1,m+1,r,ql,qr,f,g);</pre>
54
55
56
        void dfs(int x,int l,int r) {
57
           int cnt=0;
58
           for(auto pr:eg[x])
59
               if(dsu.join(pr.first,pr.second))
60
                   cnt++;
```

```
61
            if(l==r) {
                if(qf[l]) \ printf("%c\n", "NY"[dsu.find(qf[l])==dsu.find(qg[l])]);\\
 62
63
            } else {
64
                int m=(1+r)>>1;
 65
                dfs(x<<1,1,m);
                dfs(x<<1|1,m+1,r);
 66
67
 68
            while(cnt--) dsu.withdraw();
 69
 70
     }sgt;
 71
 72
     int mp[MAXN][MAXN];
 73
 74
     void solve() {
 75
         int n,m;
 76
        scanf("%d %d", &n, &m);
 77
        memset(qf+1,0,m*sizeof(qf[0]));
 78
        memset(qg+1,0,m*sizeof(qg[0]));
        for(int i=1;i<=m;i++) {</pre>
 79
 80
            int op,f,g;
            scanf("%d %d %d", &op, &f, &g);
 81
 82
            if(op==0) {
 83
                mp[f][g]=mp[g][f]=i;
 84
            } else if(op==1) {
                sgt.modify(1,1,m+1,mp[f][g],i,f,g);
 85
 86
                mp[f][g]=mp[g][f]=0;
 87
            } else qf[i]=f,qg[i]=g;
 88
        }
 89
90
        for(int f=1;f<n;f++) {</pre>
91
            for(int g=f+1;g<=n;g++) {</pre>
92
                if(mp[f][g]) {
93
                    sgt.modify(1,1,m+1,mp[f][g],m+1,f,g);
94
                   mp[f][g]=mp[g][f]=0;
95
                }
96
            }
97
        }
98
99
        dsu.init(n);
100
         sgt.dfs(1,1,m+1);
101
     }
102
103
     int main() {
104
        int T=1,cas=1;(void)(cas);
105
        // scanf("%d", &T);
106
        while(T--) {
107
            // printf("Case #%d: ", cas++);
108
            solve();
109
        }
110
        return 0;
111
     }
```

# 11 杂项

## 11.1 快读快写 Fast\_Read&Write

```
1
    #include <cstdio>
 2
3
    int read(){
 4
       int ng=0,x=0;
 5
       char ch=getchar();
 6
       for(;ch<'0' || ch>'9';ch=getchar()) ng|=ch=='-';
 7
       for(;ch>='0' && ch<='9';ch=getchar()) x=(x<<3)+(x<<1)+ch-'0';
 8
       return ng?-x:x;
9
    }
10
11
    int write(){
12
13
   }
14
   int main(){
15
16
       return 0;
17
```

### 11.2 快速幂 Fast\_Power

```
#include <cstdio>
 2
3
    #define LL long long
 4
 5
    LL a,b,n;
 7
    #include <cstdio>
8
9
    #define LL long long
10
11
    LL a,b,n;
12
    LL binpow(LL x,LL y,LL m){
13
14
       LL r=1\%m;
15
       for(;y;y>>=1,x=x*x%m)
16
           if(y&1) r=r*x%m;
17
       return r;
    }
18
19
20
    int main(){
21
       scanf("%lld %lld %lld",&a,&b,&n);
22
       printf("%11d^%11d mod %11d=%11d\n",a,b,n,binpow(a,b,n));
23
       return 0;
24
    }
25
26
27
    int main(){
       scanf("%lld %lld %lld",&a,&b,&n);
```

```
29     printf("%lld^%lld mod %lld=%lld\n",a,b,n,binpow(a,b,n));
30     return 0;
31 }
```

# 11.3 矩阵快速幂 Matrix\_Fast\_Power

```
#include <cstdio>
    #include <cstring>
    #define LL long long
    #define MAXN 105
 4
    #define MOD 1000000007
 7
    LL n,k;
 8
 9
    struct Matrix{
10
        LL num[MAXN][MAXN];
11
        Matrix(int opt){
12
13
           memset(num,0,sizeof(num));
14
           if (opt==1)
15
               for(int i=1;i<=n;i++)</pre>
16
                   num[i][i]=1;
17
        }
18
19
        Matrix operator*(const Matrix &y) const{
20
           Matrix x=*this,ans(0);
21
           for(int i=1;i<=n;i++){</pre>
22
               for(int j=1;j<=n;j++){</pre>
23
                   for(int k=1;k<=n;k++){</pre>
24
                       ans.num[i][j]=(ans.num[i][j]+x.num[i][k]*y.num[k][j]%MOD)%MOD;
25
                   }
26
               }
27
           }
28
           return ans;
29
        }
30
    };
32
    Matrix binpow(Matrix x,LL y){
33
        Matrix r(1);
34
        while(y){
35
           if(y&1) r=r*x;
36
           x=x*x;
37
           y>>=1;
38
        }
39
        return r;
40
    }
41
42
    int main(){
        scanf("%11d %11d", &n, &k);
43
44
        Matrix A(0);
        for(int i=1;i<=n;i++){</pre>
45
           for(int j=1;j<=n;j++){</pre>
46
```

```
47
                scanf("%lld", &A.num[i][j]);
            }
48
49
        }
50
        A=binpow(A,k);
51
        for(int i=1;i<=n;i++){</pre>
            for(int j=1;j<=n;j++){</pre>
52
53
                printf("%lld ", A.num[i][j]);
54
            }
55
            puts("");
56
        }
        return 0;
57
    }
58
```

## 11.4 矩阵加速 Matrix\_Acceleration

```
1
    #include <cstdio>
 2
    #include <cstring>
    #define LL long long
 3
    #define MAXN 5
 4
    #define MOD 100000007
 5
 6
 7
    struct Matrix{
 8
        int num[MAXN][MAXN];
 9
        int n,m;
10
11
        Matrix(int nn,int mm){
12
           n=nn; m=mm;
13
           memset(num,0,sizeof(num));
14
        }
15
        Matrix(int nn){
16
17
           memset(num,0,sizeof(num));
18
           for(int i=1;i<=nn;i++) num[i][i]=1;</pre>
19
        }
20
21
        Matrix operator*(const Matrix &y) const{
22
           Matrix x=*this,ans(x.n,y.m);
23
           if(x.m!=y.n) return ans;
24
           for(int i=1;i<=x.n;i++){</pre>
25
               for(int j=1;j<=y.m;j++){</pre>
26
                   for(int k=1;k<=x.m;k++){</pre>
                       ans.num[i][j]=(ans.num[i][j]+1LL*x.num[i][k]*y.num[k][j]%MOD)%MOD;
27
28
                   }
29
               }
30
           }
31
           return ans;
32
        }
33
    };
34
35
    Matrix binpow(Matrix x,LL y){
36
        Matrix r(x.n);
        while(y){
37
```

```
38
            if(y&1) r=r*x;
39
           x=x*x;
40
            y>>=1;
41
        }
42
        return r;
43
    }
44
45
    int main(){
46
        int T;
47
        scanf("%d", &T);
48
        while(T--){
49
            int n;
50
           scanf("%d", &n);
           if(n<=3) printf("1\n");</pre>
51
52
53
               Matrix x(3,3);
               x.num[1][1]=x.num[2][1]=x.num[1][3]=x.num[3][2]=1;
54
55
               x=binpow(x,n-3);
56
               Matrix y(3,1);
57
               for(int i=1;i<=3;i++) y.num[i][1]=1;</pre>
58
               printf("%d\n", (x*y).num[1][1]);
           }
59
60
        }
61
        return 0;
62
    }
```

## 11.5 最长公共子序列 Longest\_Increasing\_Subsequence

```
#include <cstdio>
    #include <functional>
 3
    #include <algorithm>
 4
    #include <map>
 5
    #include <utility>
    #define MAXN 100005
 6
 7
8
    using namespace std;
 9
    int n,a[MAXN],len;
10
11
    map <int,int> mp,rmp;
12
13
    int main(){
        scanf("%d", &n);
14
15
        for(int i=1,t;i<=n;i++){</pre>
16
           scanf("%d", &t);
17
           mp.insert(make_pair(t,i));
18
        a[len=0]=0;
19
20
        map <int,int>::iterator ite;
        for(int i=1,t,v;i<=n;i++){</pre>
21
22
           scanf("%d",&t);
23
           ite=mp.find(t);
           v=ite->second;
24
```

```
25
           if(v>a[len])
26
               a[++len]=v;
27
           else{
28
               int *p=upper_bound(a+1,a+len+1,v,less<int>());
29
               *p=v;
           }
30
31
       }
32
       printf("%d\n",len);
33
       return 0;
34
```

## 11.6 模拟退火 Simulated\_Annealing

```
#include <bits/stdc++.h>
 2
    #define MAXN 1005
 3
 4
    int n;
 5
 6
    double ansx,ansy;
 7
 8
    struct point{
 9
        double x,y,w;
10
    }p[MAXN];
11
12
    double f(double x,double y){
13
        double sum=0.0;
14
        for(int i=1;i<=n;i++){</pre>
15
           double dx=x-p[i].x,dy=y-p[i].y;
16
            sum+=sqrt(dx*dx+dy*dy)*p[i].w;
17
18
        return sum;
19
20
21
    void SA(){
22
       double T=3000,d=0.999,tt=1e-15;
23
        while(T>tt){
24
           double newx=ansx+(2*rand()-RAND_MAX)*T;
           double newy=ansy+(2*rand()-RAND_MAX)*T;
25
26
           double delta=f(newx,newy)-f(ansx,ansy);
27
           if(delta<0 || exp(-delta/T)*RAND_MAX>rand()){
28
               ansx=newx;
29
               ansy=newy;
30
           }
31
           T*=d;
       }
32
33
    }
34
35
    int main(){
36
        srand(time(0));
37
        scanf("%d", &n);
        for(int i=1;i<=n;i++){</pre>
38
           scanf("%lf %lf %lf", &p[i].x, &p[i].y, &p[i].w);
39
```

```
40
41
        ansx=ansy=0.0;
        for(int i=1;i<=n;i++){</pre>
42
43
            ansx+=p[i].x;
44
            ansy+=p[i].y;
45
        }
46
        ansx/=n;
47
        ansy/=n;
        SA();
48
49
        printf("%.31f %.31f\n", ansx, ansy);
50
        return 0;
51
    }
```

#### 11.7 快速沃尔什变换 Fast\_Walsh\_Transform

```
1
 2
        Coded with Leachim's ACM Template.
 3
        No errors. No warnings. ~~
    */
 4
 5
    #include <bits/stdc++.h>
    #pragma GCC diagnostic ignored "-Wunused-const-variable"
    #pragma GCC diagnostic ignored "-Wsign-conversion"
    #pragma GCC diagnostic ignored "-Wsign-compare"
 8
    #define LL long long
    using namespace std;
10
    const int inf=0x3f3f3f3f;
11
12
    const double eps=1e-7;
    const int dx[4]=\{1,-1,0,0\};
13
    const int dy[4]={0,0,1,-1};
14
15
    const int MAXN=2000005;
16
    const int MOD=998244353;
17
18
    void FWTor(int n,int* P,int f) {
19
        for (int l=1;(1<<1)<=n;1<<=1)</pre>
20
           for(int i=0;i<n;i+=(1<<1))</pre>
21
               for(int j=0;j<1;j++)</pre>
22
                   P[i+j+1] = ((P[i+j+1]+P[i+j]*f)%MOD+MOD)%MOD;
23
    }
24
25
    void FWTand(int n,int* P,int f) {
        for (int l=1;(1<<1)<=n;1<<=1)</pre>
26
27
           for(int i=0;i<n;i+=(1<<1))</pre>
28
               for(int j=0;j<1;j++)</pre>
29
                   P[i+j]=((P[i+j]+P[i+j+1]*f)MOD+MOD)MOD;
30
    }
31
    void FWTxor(int n,int* P,int f) {
32
33
        for (int l=1;(1<<1)<=n;1<<=1)</pre>
34
           for(int i=0;i<n;i+=(1<<1))</pre>
35
               for(int j=0;j<1;j++) {</pre>
36
                   int t=P[i+j];
37
                   P[i+j]=(1LL*f*(t+P[i+j+l])%MOD+MOD)%MOD;
```

```
38
                   P[i+j+1]=(1LL*f*(t-P[i+j+1])%MOD+MOD)%MOD;
39
               }
40
    }
41
42
    int A[MAXN],B[MAXN],C[MAXN];
43
    void solve() {
44
45
        int n;
        scanf("%d", &n);
46
47
        int lim=1<<n;</pre>
        for(int i=0;i<lim;i++) scanf("%d", &A[i]);</pre>
48
49
        for(int i=0;i<lim;i++) scanf("%d", &B[i]);</pre>
50
51
        FWTor(lim,A,1);FWTor(lim,B,1);
52
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
53
        FWTor(lim,A,-1);FWTor(lim,B,-1);FWTor(lim,C,-1);
54
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
55
        puts("");
56
57
        FWTand(lim,A,1);FWTand(lim,B,1);
58
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
59
        FWTand(lim,A,-1);FWTand(lim,B,-1);FWTand(lim,C,-1);
60
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
61
        puts("");
62
63
        FWTxor(lim,A,1);FWTxor(lim,B,1);
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
64
65
        FWTxor(lim,A,(MOD+1)/2);FWTxor(lim,B,(MOD+1)/2);FWTxor(lim,C,(MOD+1)/2);
66
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
67
        puts("");
68
    }
69
70
    int main() {
71
        int T=1, cas=1; (void) (cas);
        // scanf("%d", &T);
72.
73
        while(T--) {
74
            // printf("Case #%d: ", cas++);
75
            solve();
76
        }
77
        return 0;
78
    }
```

#### 11.8 快速莫比乌斯变换 Fast\_Mobius\_Transform

```
/*
Coded with Leachim's ACM Template.
No errors. No warnings. ~~

*/

#include <bits/stdc++.h>
#pragma GCC diagnostic ignored "-Wunused-const-variable"
#pragma GCC diagnostic ignored "-Wsign-conversion"
#pragma GCC diagnostic ignored "-Wsign-compare"
```

```
9
    #define LL long long
10
    using namespace std;
    const int inf=0x3f3f3f3f;
11
12
    const LL INF=0x3f3f3f3f3f3f3f3f3f3f;
13
    const double eps=1e-7;
14
    const int dx[4]=\{1,-1,0,0\};
    const int dy[4]=\{0,0,1,-1\};
15
16
    const int MOD=998244353;
    const int MAXN=2000005;
17
18
19
    void FMTor(int n,int* P,int f) {
        for (int i=1;i<n;i<<=1)</pre>
20
21
            for(int j=0;j<n;j++)</pre>
22
                if(i&j) P[j]=((P[j]+f*P[i^j])%MOD+MOD)%MOD;
23
24
25
    void FMTand(int n,int* P,int f) {
26
        for (int i=1;i<n;i<<=1)</pre>
27
            for(int j=0;j<n;j++)</pre>
28
               if(i&~j) P[j]=((P[j]+f*P[i|j])%MOD+MOD)%MOD;
29
30
    void FWTxor(int n,int* P,int f) {
31
32
        for (int l=1;(1<<1)<=n;1<<=1)</pre>
            for(int i=0;i<n;i+=(1<<1))</pre>
33
34
               for(int j=0;j<1;j++) {</pre>
35
                   int t=P[i+j];
36
                   P[i+j]=(1LL*f*(t+P[i+j+1])MOD+MOD)MOD;
37
                   P[i+j+1]=(1LL*f*(t-P[i+j+1])%MOD+MOD)%MOD;
38
               }
39
    }
40
    int A[MAXN],B[MAXN],C[MAXN];
41
42
43
    void solve() {
44
        int n:
45
        scanf("%d", &n);
46
        int lim=1<<n;</pre>
47
        for(int i=0;i<lim;i++) scanf("%d", &A[i]);</pre>
48
        for(int i=0;i<lim;i++) scanf("%d", &B[i]);</pre>
49
50
        FMTor(lim,A,1);FMTor(lim,B,1);
51
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
52
        FMTor(lim,A,-1);FMTor(lim,B,-1);FMTor(lim,C,-1);
53
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
54
        puts("");
55
56
        FMTand(lim,A,1);FMTand(lim,B,1);
57
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
58
        FMTand(lim,A,-1);FMTand(lim,B,-1);FMTand(lim,C,-1);
59
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
60
        puts("");
61
```

```
62
        FWTxor(lim,A,1);FWTxor(lim,B,1);
63
        for(int i=0;i<lim;i++) C[i]=1LL*A[i]*B[i]%MOD;</pre>
64
        FWTxor(lim,A,(MOD+1)/2);FWTxor(lim,B,(MOD+1)/2);FWTxor(lim,C,(MOD+1)/2);
65
        for(int i=0;i<lim;i++) printf("%d ", C[i]);</pre>
66
        puts("");
67
    }
68
69
    int main() {
70
        int T=1,cas=1;(void)(cas);
71
        // scanf("%d", &T);
72
        while(T--) {
73
           // printf("Case #%d: ", cas++);
74
           solve();
       }
75
76
        return 0;
77
```

# 11.9 快速子集变换 Fast\_Subset\_Transform

```
1
 2
        Coded with Leachim's ACM Template.
 3
        No errors. No warnings. ~~
 4
 5
    #include <bits/stdc++.h>
    #pragma GCC diagnostic ignored "-Wunused-const-variable"
 6
 7
    #pragma GCC diagnostic ignored "-Wsign-conversion"
    #pragma GCC diagnostic ignored "-Wsign-compare"
   #define LL long long
10
   using namespace std;
11
    const int inf=0x3f3f3f3f;
12
    const LL INF=0x3f3f3f3f3f3f3f3f3f3f;
13
    const double eps=1e-7;
14
   const int dx[4]=\{1,-1,0,0\};
15
    const int dy[4]=\{0,0,1,-1\};
    const int MOD=1e9+9;
16
17
    const int MAXN=2000005;
18
19
    void FWTor(int n,int* P,int f) {
20
        for (int l=1;(1<<1)<=n;1<<=1)</pre>
21
           for(int i=0;i<n;i+=(1<<1))</pre>
22
               for(int j=0;j<1;j++)</pre>
23
                   P[i+j+1] = ((P[i+j+1]+P[i+j]*f)%MOD+MOD)%MOD;
24
25
   int A[21][MAXN],B[21][MAXN],C[21][MAXN];
26
27
28
   void solve() {
29
        int n;
30
        scanf("%d", &n);
31
        int lim=1<<n;</pre>
32
        for(int i=0;i<=n;i++) {</pre>
33
           memset(A[i],0,lim*sizeof(A[i][0]));
```

```
memset(B[i],0,lim*sizeof(B[i][0]));
34
35
            memset(C[i],0,lim*sizeof(C[i][0]));
36
        }
37
        for(int i=0;i<lim;i++)</pre>
38
            scanf("%d", &A[__builtin_popcount(i)][i]);
39
        for(int i=0;i<lim;i++)</pre>
40
            scanf("%d", &B[__builtin_popcount(i)][i]);
41
42
        for(int i=0;i<=n;i++) {</pre>
43
            FWTor(lim,A[i],1);
44
            FWTor(lim,B[i],1);
45
        }
46
        for(int i=0;i<=n;i++) {</pre>
47
48
            for(int j=0;j<=i;j++) {</pre>
49
                for(int s=0;s<lim;s++) {</pre>
50
                   C[i][s]=(C[i][s]+1LL*A[j][s]*B[i-j][s]%MOD)%MOD;
51
            }
52
53
            FWTor(lim,C[i],-1);
54
        }
55
56
        for(int i=0;i<lim;i++) printf("%d ", C[__builtin_popcount(i)][i]);</pre>
57
        puts("");
58
    }
59
60
    int main() {
61
        int T=1, cas=1; (void) (cas);
62
        // scanf("%d", &T);
63
        while(T--) {
64
            // printf("Case #%d: ", cas++);
65
            solve();
        }
66
67
        return 0;
68
    }
```

#### 11.10 24 点 24\_Point

```
1
2
       Coded with Leachim's ACM Template.
3
       No errors. No warnings. ~~
4
   */
    #include <bits/stdc++.h>
5
   #pragma GCC diagnostic ignored "-Wunused-const-variable"
7
   #pragma GCC diagnostic ignored "-Wsign-conversion"
   #pragma GCC diagnostic ignored "-Wsign-compare"
9
   #define LL long long
10
   using namespace std;
   const int inf=0x3f3f3f3f;
11
12 const LL INF=0x3f3f3f3f3f3f3f3f3f3f;
13 | const double eps=1e-7;
14 | const int dx[4]=\{1,-1,0,0\};
```

```
const int dy[4]=\{0,0,1,-1\};
15
16
    const int MOD=998244353;
    const int MAXN=2000005;
17
18
19
    int a[5],p[5];
20
21
    set<pair<double,string>> dfs(int 1, int r) {
22
        set<pair<double,string>> s;
23
        if(l==r) {
24
            s.emplace(a[p[1]],to_string(a[p[1]]));
25
        for(int i=1;i<r;i++) {</pre>
26
27
           auto s1=dfs(l,i);
28
           auto s2=dfs(i+1,r);
29
           for(auto p1:s1) {
               for(auto p2:s2) {
30
                   s.emplace(p1.first+p2.first,"("+p1.second+"+"+p2.second+")");
31
                   s.emplace(p1.first-p2.first,"("+p1.second+"-"+p2.second+")");
32
                   s.emplace(p1.first*p2.first,"("+p1.second+"*"+p2.second+")");
33
34
                   if(abs(p2.first)>eps) s.emplace(p1.first/p2.first,"("+p1.second+"/"+p2.
                       second+")");
35
               }
36
           }
37
        }
38
        return s;
39
    }
40
41
    void solve() {
42
        for(int i=1;i<=4;i++) {</pre>
43
           scanf("%d", &a[i]);
44
           p[i]=i;
45
       }
46
        do {
47
           auto s=dfs(1,4);
           for(auto p:s) {
48
49
               if(abs(p.first-24) < eps) {</pre>
50
                   cout<<p.second.substr(1,p.second.size()-2)<<end1;</pre>
51
                   return;
52
               }
53
           }
54
        }while(next_permutation(p+1,p+5));
55
56
        printf("-1\n");
57
    }
58
59
    int main() {
60
        int T=1,cas=1;(void)(cas);
61
        // scanf("%d", &T);
        while(T--) {
62
            // printf("Case #%d: ", cas++);
63
64
           solve();
65
        }
        return 0;
```

67 |}