HW 5 Lichen Liang

Prob 1

1 a) L2 norm is more sensitive to outliers so it's more stable and has unique solution.

b) They are uncorrelated

$e = y - Hy = (I - H)y = Hy$

$cov((I-H)y, Hy) = (I-H)\, cov(y, y) H = 0$

$\searrow H' = H$

2. a) $X = \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$

$W = (X^T X)^{-1} X^T Y$

$= \left(\begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 0 \end{pmatrix} \right)^{-1} \begin{pmatrix} 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \cdot Y$

$\begin{pmatrix} \hat{w}_1 \\ \hat{w}_2 \end{pmatrix} = \frac{1}{9} \begin{pmatrix} Y_1 + Y_2 + 2Y_3 \\ 4Y_1 - 5Y_2 - Y_3 \end{pmatrix}$

b) $cov(\hat{w}_1, \hat{w}_2) = \begin{pmatrix} 1/36 & -1/18 & -1/36 \\ -1/18 & 1/9 & 1/18 \\ -1/36 & 1/18 & 1/36 \end{pmatrix}$

Prob 2

1. a) The logistic regression is not always convex so MSE cannot be applied

b) β acts as the weight that controlls the sensitivity of the function

prob 3

RMSE:

Linear: 40.401

Ridge: 40.402

Lasso: 40.837

Lasso important features: 0, 1, 3, 4, 5, 8, 9, 13, 15, 16, 18, 19, 22, 23, 24, 50, 51, 53, 54, 60, 61