TD 4: Datalog Programs with or without Negation

Exercise 1. (Datalog Evaluation)

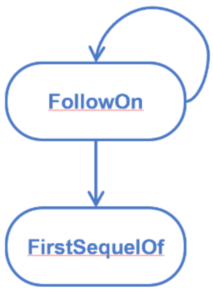
Given the Datalog program Π with the following rules:

```
FirstSequelOf(x,y):- SequelOf(x,y)
FollowOn(x,y):- FirstSequelOf(x,y)
FollowOn(x,y):- FirstSequelOf(x,z),FollowOn(z,y)
```

and EDB (`SequelOf`) with the facts:

```
SequelOf := {(t1,t2),(t2,t3),(t3,t4),(t5,t6),(t6,t7),(t7,t8)}
```

1. Produce the dependency graph for Π.
   Solution:



2. Apply the naïve evaluation algorithm to Π.
   Solution:
   We will proceed in rounds to infer FirstSequel facts and then FollowOn facts.
   Initial `FirstSequelof:={}, FollowOn := {}`
   Round1 `FirstSequelOf := {(t1,t2),(t2,t3),(t3,t4),(t5,t6),(t6,t7),(t7,t8)},`
   `FollowOn ={}`
   Round2 `FollowOn := {(t1,t2),(t2,t3),(t3,t4),(t5,t6),(t6,t7),(t7,t8)}`
   Round3 `FollowOn := {(t1,t2),(t2,t3),(t3,t4),(t5,t6),(t6,t7),(t7,t8),`
   `(t1,t3),(t2,t4),(t5,t7),(t6,t8)}`
   Round4 `FollowOn := {(t1,t2),(t2,t3),(t3,t4),(t5,t6),(t6,t7),(t7,t8),`
   `(t1,t3),(t2,t4),(t5,t7),(t6,t8),(t1,t4),(t5,t8)}`
   Round5 no change in FollowOn. STOP

Exercise 2. (Datalog with Negation)

Given the Datalog program Π with the following rules:

```
P(x,y)  :- A(x,y)
P(x,y)  :- A(x,z), P(z,y)
Q(x,y)  :- P(x,y), ¬Q(y,x)
```

1. Produce the dependency graph for Π.
   Solution:

2. Is Π stratifiable? Justify the response.
   Solution: No, it doesn't. This is due to the negative cycle on 'Q'.

Exercise 3. (Datalog with Negation)

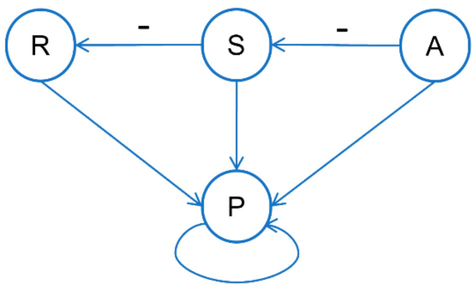
Given the Datalog program Π with the following rules:

```
S(x) :- P(x,x), ¬R(x,x)
R(x,y) :- P(x,y), C(x,'R')
R(x,y) :- P(x,y), C(y,'R')
P(x,y) :- F(x,y)
P(x,z) :- P(x,y), F(y,z)
A(X) :- P(x,x), ¬S(x)
```

and EDB with the facts:

```
F(1,2)
F(2,3)
F(3,1)
C(1,2)
C(1,'R')
C(2,'B')
C(3,'J')
```

1. Produce the dependency graph for Π.
   Solution:



2. Is Π stratifiable? Justify the response. **Solution**: Yes, it is. In particular, there are no negative cycles.

3. Provide the stratification for Π. **Solution**: `[(P,1); (R,1); (S,2); (A,3)]`

4. Provide the partitioning of Π.
   Solution:

```
P1 = {P(x,y) :- F(x,y)
      P(x,z) :- P(x,y), F(y,z)
      R(x,y) :- P(x,y), C(x,'R')
      R(x,y) :- P(x,y), C(y,'R')}
P2 = {S(x) :- P(x,x), ¬R(x,x)}
P3 = {A(X) :- P(x,x), ¬S(x)}
```

5. Compute the minimal model.

Solution: `P(1,2),P(2,3),P(3,1),R(1,2),R(3,1),P(1,3),P(2,1),P(3,2),R(1,3),`
`R(2,1),P(1,1),P(2,2),P(3,3),R(1,1),S(2),S(3),A(1)`

Stratification Algorithm

The algorithm for the stratification is provided below[1].

```
for each predicate p do
    stratum[p] := 1;
repeat
    for each rule r with head predicate p do begin
        for each negated subgoal of r with predicate q do
            stratum[p] := max(stratum[p], 1+stratum[q]);
        for each nonnegated subgoal of r with predicate q do
            stratum[p] := max(stratum[p], stratum[q])
    end
until there are no changes to any stratum
    or some stratum exceeds the number of predicates
```

[1] Jeffrey Ullman, Principles of database and knowledge-base systems volume 1, 1980, page 134