CSC418: Assignment #1

Due on Tuesday, March 8, 2016

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Problem 1

a) A unit vector perpendicular to t and $c - e_m$ will be the cross product of the two divided by the norm of t and $c - e_m$. So $\vec{d} = \frac{t \times (c - e_m)}{|t \times c - (e_m)|}$

b)$e_L = e_m - \frac{s}{2} \cdot d, e_R = e_m + \frac{s}{2} \cdot d$

c)For the right eye, we use e_R as e, and our gaze vector will be $\vec{g_R} = c - e_R$. So then $\vec{w_R} = -\vec{g_R} = -(c - e_R)$ Then $\vec{u_R} = \frac{\vec{t} \times \vec{w_R}}{||\vec{t} \times \vec{w_R}||} = \frac{\vec{t} \times -(c - e_R)}{||\vec{t} \times -(c - e_R)||}$ Then $\vec{v_R} = \vec{w_R} \times \vec{u_R} = -(c - e_R) \times \frac{\vec{t} \times -(c - e_R)}{||\vec{t} \times -(c - e_R)||}$. So the coordinate frame for the right eye is $(\vec{u_R}, \vec{v_R}, \vec{w_R}) = (\frac{\vec{t} \times -(c - e_R)}{||\vec{t} \times -(c - e_R)||}, -(c - e_R) \times \frac{\vec{t} \times -(c - e_R)}{||\vec{t} \times -(c - e_R)||}, -(c - e_R))$ and the coordinate frame for the left eye is $(\vec{u_L}, \vec{v_L}, \vec{w_L}) = (\frac{\vec{t} \times -(c - e_L)}{||\vec{t} \times -(c - e_L)||}, -(c - e_L) \times \frac{\vec{t} \times -(c - e_L)}{||\vec{t} \times -(c - e_L)||}, -(c - e_L))$

d) $p_R = e_L + p_L^1 \vec{u_R} + p_L^2 \vec{v_R} + p_L^3 \vec{w_R} = [\vec{u_R} \vec{v_R} \vec{w_R}] p_L + e_L$. So we can then write $M_{LR} = \begin{bmatrix} u_R^1 & v_R^1 & w_R^1 & e_1 \\ u_R^2 & v_R^2 & w_R^2 & e_2 \\ u_R^3 & v_R^3 & w_R^3 & e_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

e) To determine whether a polygon face with normal n is to be culled, we put it through the culling criterion. From camera e_R, if $(c - e_R) \cdot \vec{n} > 0$, then we know it is not facing the camera, so we cull it, otherwise it stays. From camera e_L, if $(c - e_L) \cdot \vec{n} > 0$, then we know it's direction is not to the camera, so we cull it, otherwise it stays.

Problem 2

Projecting p with focal length -f, we get $p' = (\frac{-f}{p_z} p_x, \frac{-f}{p_z} p_y, -f)$

$q' = (\frac{-f}{q_z} q_x, \frac{-f}{q_z} q_y, -f)$

$m = (\frac{p_x + q_x}{2}, \frac{p_y + q_y}{2}, \frac{p_z + q_z}{2})$

So then $m' = (\frac{-2f}{p_z + q_z} \cdot \frac{p_x + q_x}{2}, \frac{-2f}{p_z + q_z} \cdot \frac{p_y + q_y}{2}, \frac{-2f}{p_z + q_z} \cdot \frac{p_z + q_z}{2}) = (\frac{-f(p_x + q_x)}{p_z + q_z}, \frac{-f(p_y + q_y)}{p_z + q_z}, -f)$

$0.5(p' + q') = (\frac{-f}{p_z} p_x, \frac{-f}{p_z} p_y, -f) + (\frac{-f}{q_z} q_x, \frac{-f}{q_z} q_y, -f) = 0.5(\frac{-2f(p_x + q_x)}{p_z + q_z}, \frac{-2f(p_y + q_y)}{p_z + q_z}, -2f)$

$= (\frac{-f(p_x + q_x)}{p_z + q_z}, \frac{-f(p_y + q_y)}{p_z + q_z}, -f) = m'$, so yes, they are mathematically equivalent when doing perspective projection.

If we do orthographic projection, $p' = (\alpha p_x, \alpha p_z, \alpha p_z)$

$q' = (\alpha q_x, \alpha q_z, \alpha q_z)$

$m = (\frac{p_x + q_x}{2}, \frac{p_y + q_y}{2}, \frac{p_z + q_z}{2})$

$m' = (\alpha \frac{p_x + q_x}{2}, \alpha \frac{p_y + q_y}{2}, \alpha \frac{p_z + q_z}{2})$

$0.5(p' + q') = 0.5(\alpha p_x + \alpha q_x, \alpha p_y + \alpha q_y, \alpha p_z + \alpha q_z) = (\alpha \frac{p_x + q_x}{2}, \alpha \frac{p_y + q_y}{2}, \alpha \frac{p_z + q_z}{2}) = m'$, so yes, even orthographic projection preserves these ratios.

Problem 3

a) If the surface is $p(t) =< 0, y(t), z(t) >$ where $y(t) = a\sqrt{t} + bsin(t)$ and $z(t) = ct$, then if we revolve the surface around the z axis, the parametric equation will be $p(u,v) =< y(u) \cdot cos(v), y(u) \cdot sin(v), z(u) >$ $=< (a\sqrt{u} + bsin(u)) \cdot cos(v), (a\sqrt{u} + bsin(u)) \cdot sin(v), cu >, u \in [0, 2\pi), v \in [0, 2\pi)$

b)We can represent a tangent plane with a normal vector and a point. To get the normal vector we do

$p_u(u,v) =< (\frac{a}{2\sqrt{u}} + bcos(u)) \cdot cos(v), (\frac{a}{2\sqrt{u}} + bcos(u)) \cdot sin(v), c >$

$p_v(u,v) =< -(a\sqrt{u} + bsin(u)) \cdot sin(v), (a\sqrt{u} + bsin(u)) \cdot cos(v), 0 >$

The normal vector will be

$n = p_u(u,v) \times p_v(u,v) =< -c(a\sqrt{u}+bsin(u)) \cdot cos(v), -c(a\sqrt{u}+bsin(u)) \cdot sin(v), ((\frac{a}{2\sqrt{u}}+bcos(u)) \cdot cos(v)) \cdot ((a\sqrt{u} + bsin(u)) \cdot cos(v)) - ((\frac{a}{2\sqrt{u}} + bcos(u)) \cdot sin(v)) \cdot (-(a\sqrt{u} + bsin(u)) \cdot sin(v)) >$

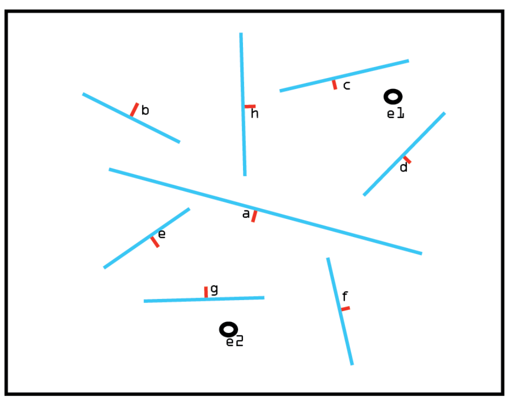
The point will be $x_0 = (a\sqrt{u} + bsin(u)) \cdot cos(v), y_0 = (a\sqrt{u} + bsin(u)) \cdot sin(v), z_0 = cu$ Then the tangent plane can be written as $n_x(x - x_0) + n_y(y - y_0) + n_z(z - z_0) = 0$

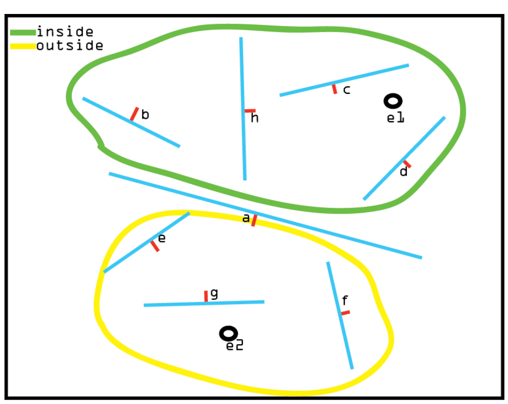
c) As we determine in part b, the normal vector is

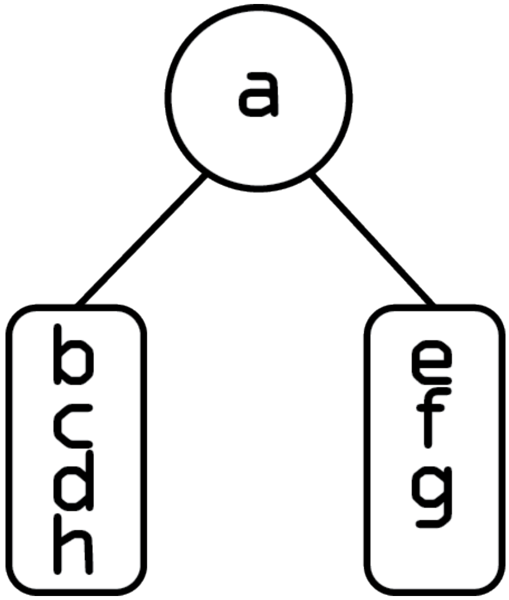
$n = p_u(u,v) \times p_v(u,v) =< -c(a\sqrt{u}+bsin(u)) \cdot cos(v), -c(a\sqrt{u}+bsin(u)) \cdot sin(v), ((\frac{a}{2\sqrt{u}}+bcos(u)) \cdot cos(v)) \cdot ((a\sqrt{u} + bsin(u)) \cdot cos(v)) - ((\frac{a}{2\sqrt{u}} + bcos(u)) \cdot sin(v)) \cdot (-(a\sqrt{u} + bsin(u)) \cdot sin(v)) >$

Problem 4

a) I did my best to recreate the diagram for purposed of determining which shapes were inside and outside Using a as the root node, we can split the other shapes into the inside and outside groups



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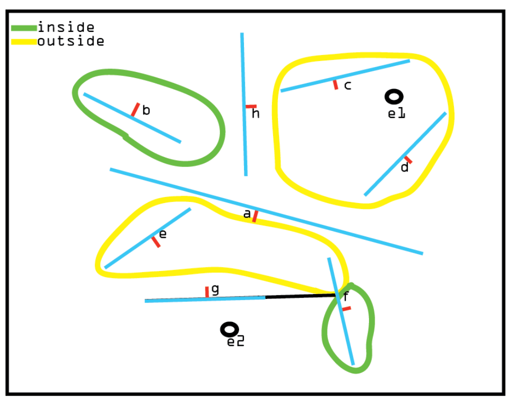
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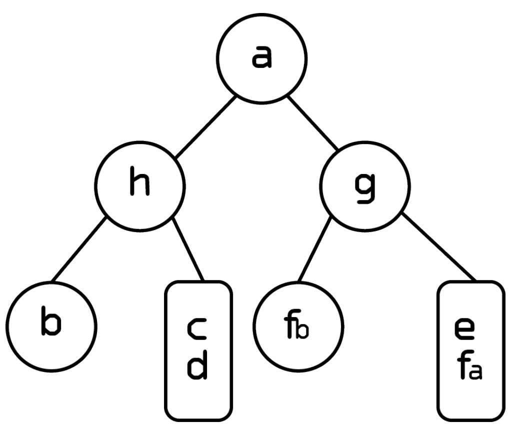
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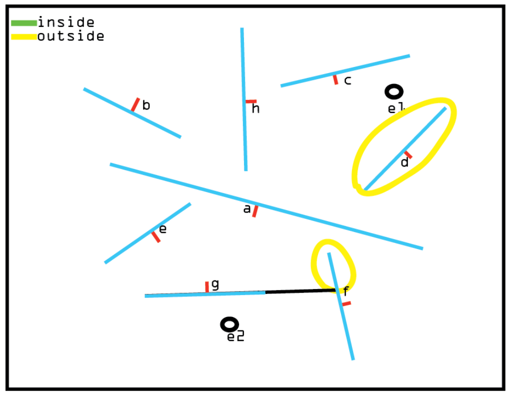
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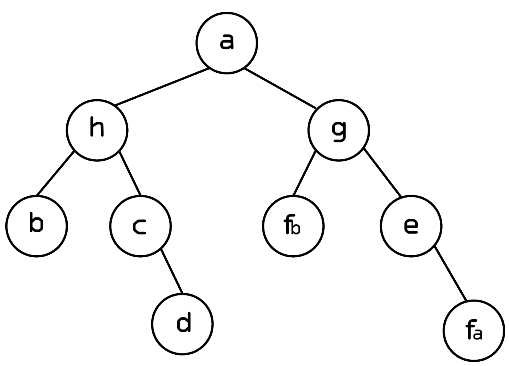
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b) For e_1, e_1 is inside a, so we draw everything outside a. e_1 is outside g, so we draw everything inside g. This means we draw f_b first. Then we draw g. e_1 is outisde e, so we draw everything inside e. There is nothing inside e, so we draw e, then f_a. Then we draw a. e_1 is outside h, so we draw everything inside h. So we draw b, then h. e_1 is outside c, so we draw everything inside c. There is nothing inside c, so we draw c, then d. So the order is (f_b, g, e, f_a, a, b, h, c, d).
For e_2, e_2 is outside a, so we draw everything inside a first. e_2 is inside h, so we draw everything outside h first. e_2 is outside c, so we draw everything inside c first. There is nothing inside c, so we draw c, then d, then h, then b, then a. e_2 is inside g, so we draw everything outside g first. e_2 is outside e, so we draw everything inside e first. There is nothing inside e, so we then draw e, then f_a, then g, then f_b. So the order is then (c, d, h, b, e, f_a, g, f_b)