The Simulation Research of Multi-model Adaptive KF and Target Tracking

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Abstract. The Kalman Filter (KF) has been applied to various fields in engineering. Since R. E. Kalman proposed this theory, many extended Kalman filtering theories have been developed to solve different problems in various situations, such as Extended Kalman Filter (EKF), Unscented Kalman Filter (UKF), Cubature Kalman Filter (CKF), and interactive multiple model (IMM) Kalman filter, etc. This paper will use the scenario of target tracking to perform simulation verification study of IMM Kalman filter.

Keywords: Kalman Filter · Target tracking · Adaptive · Multi-model

1 Fundamentals of Kalman Filter

1.1 The Background of KF

Kalman Filter was proposed in 1960 by R. E. Kalman [1]. It has great advantages over Wiener Filter, which needs the data of all past time. Kalman introduced the concept of state space into the stochastic estimation theory, and regarded the signal process as the output of a linear system under the corruption of white noise. The state equation is used to describe this input-output relationship. The system state equation, observation equation are used in the estimation process, including process noise and observation noise. Their statistical characteristics form a filtering algorithm. Since the information used is all quantities in time domain, Kalman filtering can not only estimate stationary one-dimensional random process but estimate multi-dimensional random process as well. At the same time, Kalman's algorithm is recursive, which is convenient for real time application on the computer, thus overcoming the weakness of the classic Wiener filtering method.

1.2 Mathematic Principles

Before the specified principle is presented, we need to define some parameters that will be used in the following equations.

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A represents the state transition matrix. B means the input gain matrix. w represents White-Gaussian noise, means the process noise. v means the measurement noise. So the discrete-time state equations are described as follows:

$$X_k = AX_{k-1} + Bu_{k-1} + w_k \tag{1-1}$$

$$Z_k = HX_k + v_k \tag{1-2}$$

The upper presents the state transition step, while the later shows the observation.

The Kalman Filter iteration equations are as follows:

$$\hat{x}_k^- = A\hat{x}_{k-1} + Bu_k \tag{1-3}$$

$$P_k^- = AP_{k-1}A^T + Q \tag{1-4}$$

The following step will mixed the observation, giving out a fusion data that represents the next period state and covariance.

$$\hat{z}_k = z_k - H\hat{x}_k^- \tag{1-5}$$

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{1-6}$$

$$\hat{x}_k = \hat{x}_k^- + K_k\hat{z}_k \tag{1-7}$$

$$P_k = (I - K_kH)P_k^- \tag{1-8}$$

Obviously, the Kalman Filter consists of the above-mentioned 5 equations. Given the origin data, such as x_0 and P_0, by the assistance from observations z_k, it's easy to predict future state with the minimum mean square error performance criterion.

1.3 Application

The earliest implementation of the Kalman Filter was Stanley Schmidt. During a visit to NASA's Ames Research Center, Kalman discovered that Schmidt's method was very useful for resolving the Apollo program's orbit prediction. Later, the navigation computer of Apollo spacecraft used this filter.

Nowadays, Kalman Filter has been deployed to nearly every aspect of our lives, including the following major fields

(1) Navigation guidance, target positioning and tracking;
(2) Communication & signal processing, digital image processing;
(3) Fault diagnosis and detection.

2 Adaptive Multi-model Algorithm

In this section, we will investigate the background and math principles of adaptive multi-model Kalman filter algorithm, which is a type of extended KF algorithm, here we call it interactive multiple model (IMM) [2–6]. This is the best Filter algorithm in solving robust active target tracking problems.

2.1 Backgrounds

In reality, there is a practical need to track a moving target, especially for strong maneuvering targets (not only uniform linear motion, but curved motion and accelerated motion as well). The motion mode is very complicated. It is difficult for a single model to express the target trajectory, thus using different models in different stages is a good solution.

The IMM algorithm uses multiple Kalman Filter for parallel processing, each filter corresponds to a different state model, and different state space models describe different target operating models, so each filter has a different estimation result for the target state. The basic idea of the IMM algorithm is that at each moment, assuming that a certain model is valid at the present moment, it is obtained by mixing the state of all filters at the previous moment to estimate the value of the initial conditions of the filter matched with this specific model. And then the formal filtering steps are implemented in parallel for each model. Finally, the model probability is updated based on the model matching likelihood function, and all the filter corrections are combined with state estimation.

2.2 Mathematical Equations

Under this algorithm, we assume that the target has r states, corresponding to r motion models(that is, r state transition equations), and set the target state equation represented by the j-th model as:

$$X_j(k+1) = \phi_j(k)X_j(k) + G_j(k)W_j(k) \tag{2-1}$$

And the observation equation is:

$$Z(k) = H(k)X(k) + V(k) \tag{2-2}$$

In (2-1) and (2-2), $W_j(k)$ and $V(k)$ are zero-mean white noise with covariance matrixes Q_j and R, respectively. The transfer between the models is determined by the Markov transition matrix, where the element p_{ij} represents the probability of the target moving from i-th motion model to j-th motion model. The probability transition is as follows:

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1r} \\ \cdots & \cdots & \cdots \\ p_{r1} & \cdots & p_{rr} \end{bmatrix} \tag{2-3}$$

For IMM, each iteration has the following four steps:

1) Interaction
   The predicted probability of model j, that is, the normalization constant is

$$\bar{c}_j = \sum_{i=1}^{r} p_{ij}\mu_i(k-1) \tag{2-4}$$

And the probability from model i to model j is

$$\mu_{ij}(k=1|k-1) = \sum_{i=1}^{r} p_{ij}\mu_i(k-1)/\bar{c}_j \tag{2-5}$$

The mixed state of model j is estimated as

$$\hat{X}_{0j}(k-1|k-1) = \sum_{i=1}^{r} \hat{X}_i(k-1|k-1)\mu_{ij}(k-1|k-1) \tag{2-6}$$

The mixed covariance of model j is estimated as

$$\begin{aligned} P_{0j}(k-1|k-1) = &\sum_{i=1}^{r} \mu_{ij}(k-1|k-1)P_i(k-1|k-1) \\ &+ [\hat{X}_i(k-1|k-1) - \hat{X}_{0j}(k-1|k-1)]\cdot \\ &[\hat{X}_i(k-1|k-1) - \hat{X}_{0j}(k-1|k-1)]^T \end{aligned} \tag{2-7}$$

In (2-7), $\mu_j(k-1)$ is the probability of model j at time $k-1$.

2) Filtering
The initial state estimate is

$$\hat{X}_j(k|k-1) = \phi(k-1)\hat{X}_{0j}(k-1|k-1) \tag{2-8}$$

The prediction error covariance is

$$P_j(k|k-1) = \phi_j P_{0j}(k|k-1)\phi_j^T + G_j Q_j G_j^T \tag{2-9}$$

The Kalman filter gain is as follows

$$K_j(k) = P_j(k|k-1)H^T[HP_j(k|k-1)H^T + R]^{-1} \tag{2-10}$$

The posterior state estimate is

$$\hat{X}_j(k|k) = \hat{X}_j(k|k-1) + K_j(k)[Z(k) - H(k)\hat{X}_j(k|k-1)] \tag{2-11}$$

And the posterior covariance is

$$P_j(k|k) = [I - K_j(k)H(k)]P_j(k|k-1) \tag{2-12}$$

3) Model Probability Updates
Use the likelihood function to update the model probability, we have

$$\Lambda_j(k) = \frac{1}{(2\pi)^{n/2}|S_j(k)|^{1/2}} exp\left\{-\frac{1}{2}\nu_j^T S_j^{-1}(k)\nu_j\right\} \tag{2-13}$$

In (2-13), the parameters are as follows:

$$\nu_j(k) = Z(k) - H(k)\hat{X}_j(k|k-1) \tag{2-14}$$

$$S_j(k) = H(k)P_j(k|k-1)H(k)^T + R(k) \tag{2-15}$$

So the probability of model j is:

$$\mu_j(k) = \Lambda_j(k)\bar{c}_j/c \tag{2-16}$$

where

$$c = \sum_{j=1}^{r} \Lambda_j(k)\bar{c}_j \tag{2-17}$$

4) Combination
Based on the model probability, the estimated results of each filter are weighted and combined to obtain the total state estimate and total covariance estimate.
The total estimation of state is:

$$\hat{X}(k|k) = \sum_{j=1}^{r} \hat{X}_j(k|k)\mu_j(k) \tag{2-18}$$

The total estimation of covariance is:

$$P(k|k) = \sum_{j=1}^{r} \mu_j(k)\{P_j(k|k) + [\hat{X}_j(k|k) - \hat{X}(k|k)][\hat{X}_j(k|k) - \hat{X}(k|k)]^T\} \tag{2-19}$$

In conclusion, the final output of the filter is the weighted average of multiple filter estimation results, and the weight is the probability that the model correctly describes the target maneuver at that moment, referred to as the model probability.

This algorithm has modular characteristics. When the movement law of the target is clear, the weighing algorithm can choose a correct model to describe the movement of the object more accurately. When the movement law of the target cannot be predicted, a more general model should be selected, that is, the model should be more robust.

3 Simulation of Target Tracking

In this paper, we will conduct two simulations based on MATLAB platform.

The first simulation is derived from the actual problem of car lane change, that is, the present trajectory is a serpentine curve. The specific scenario is as follows: Assume that the scanning period of the radar is 2 s, and the target moves downward at a constant speed from 0–400 s. Straight line movement, 400–600 s; the object produces a rightward acceleration of 0.075 m/s^2; 600–610 s, the acceleration disappears, the object maintains the existing speed and keeps moving at a constant speed; 610–660 s, the object produces a leftward acceleration of 3 m/s^2; 660–900 s, the acceleration is canceled, and the object has a lateral velocity of 0 at this moment, and continues to move downwards in a straight line at a constant speed.

Comparison: We can clearly figure out in Fig. 1, the estimation is more close to the observation, which floats around the actual trace, so the covariance is bigger. The accuracy and prediction cannot meet the requirements of target tracking.

On the contrary, in Fig. 2, we add bigger amplitude and frequency in the simulation, the algorithm can easily filter the noise. The output is smoother and close to the actual trace, which is what we want to see.

The second simulation is derived from the first simulation and further expand to a more complex scene. It simulates the situation where a motor vehicle moves forward at a constant speed and then turns to the right to achieve a U-turn. In order to further compare the advantages and weakness of the traditional KF and this IMM algorithm, our radar accuracy and scan interval have decreased this time. The scan period is 2 s, and the variance of the noise is 900. The initial position is preset to (2000 m, 0 m), from 2–400 s, the object moves upward at a constant speed, the speed is (0, 10 m/s). From 402–600 s, the acceleration is (0.075 m/s^2, 0.075 m/s^2); at 602–610 s, the acceleration disappears and keep moving at a constant speed. At 612–660 s, the acceleration is (-0.3 m/s^2, -0.3 m/s^2). Finally, at 662–900 s, the acceleration disappears and remains move at a constant speed until the end of the observation.

Comparison: In Fig. 3, Fig. 4, Fig. 5, it is clear that the former 20 filtering has a big error and standard deviation. According to Fig. 4, the initial trace suffers a great concussion thus cannot meet our requirements of tracking. However, the lateral starts to use the adaptive multi-model algorithm, the performance has a huge advantage over the former one. The result is beyond our expectation, totally meet the criterion and tracks the actual trace accurately as well as quickly.

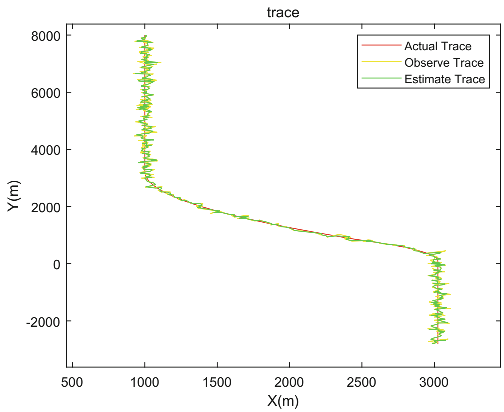

Fig. 1. The linear KF results

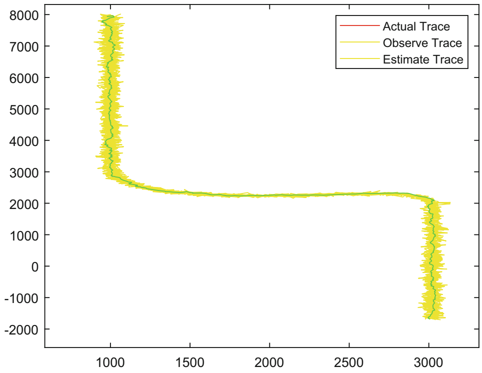

Fig. 2. The IMM and KF results

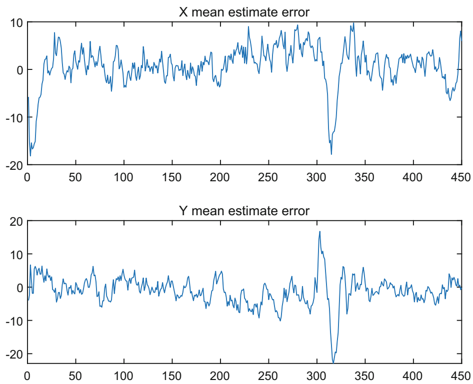

Fig. 3. Mean squared estimation error

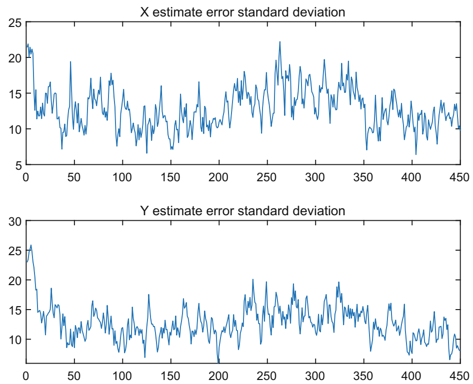

Fig. 4. Standard deviation of estimation error

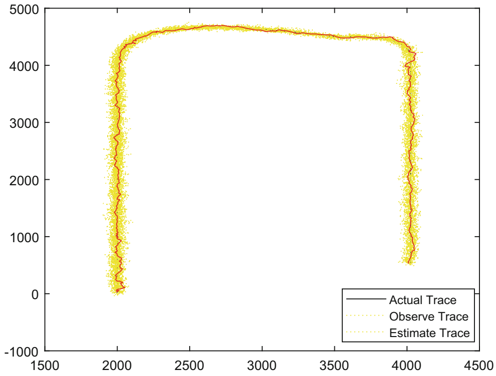

Fig. 5. KF and IMM results of U-turn target

4 Conclusion

According to the simulation results of IMM Kalman filter with application in target tracking, the multi-model adaptive KF algorithm has a great advantage compared with the traditional Kalman Filter.

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