

# Symmetric-key Corruption Detection : When XOR-MACs Meet Combinatorial Group Testing

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XOR-GTM este o clasa de MAC (Message Authentication Code) ce verifica integritatea mesajului dar gaseste si partea mesajului care a fost corupta. Poate fi vazut de asemenea si ca o aplicare a CGT (Combinatorial Group Testing) pentru message authentication.

Incercarile similare pentru acest model au limitari in comunicatie, dar XOR-GTM are un cost de comunicatie mult mai mic, oferind aceeasi capacitate de detectie a coruptiei.

DirectGTM : alegerea matricii de test se face independent de MACk si o matrice d-disjuncta este sugerata sa fie folosita in loc de H, deci costul de comunicatie poate fi redus doar prin gasirea unei matrici d-disjuncte small-row.

Pentru a imbunatati costul de comunicatie, cand authenticatorul ia taguri MAC bazate pe matricea de test H, verificatorul poate folosi orice submatrice a row span-ului lui H pe post de matrice de test virtuala.

Implementarea va presupune 2 cazuri : XOR-GTM cu matrice de test de dimensiune  $t \times m$ , functiile de hash, de tag, verificare a integritatii, detectie a partii corupte si XOR-GTM-PPI ce foloseste  $H^R$  o matrice circulara cu functiile de hash, tag, detectie a partii corupte.

Pseudocod cand este folosita o matrice H de test de dimensiune  $t \times m$ :

<b>Algorithm</b> <b>XOR-GTM</b> [ $F_K, G_{K'}$ ].tag( $M$ ) <pre> 1. <math>S \leftarrow \text{XOR-GTM}[F_K].\text{hash}(M)</math> 2. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 3.   <math>S[i] \leftarrow 0^n</math> 4.   <b>for</b> <math>j = 1</math> <b>to</b> <math>m</math> <b>do</b> 5.     <math>Z \leftarrow F_K^j(M[j])</math> 6.     <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 7.       <b>if</b> <math>H_{i,j} = 1</math> 8.         <math>S[i] \leftarrow S[i] \oplus Z</math> 9.   <math>S \leftarrow (S[1], \dots, S[t])</math> 10. <b>return</b> <math>S</math></pre>	<b>Algorithm</b> <b>XOR-GTM</b> [ $F_K, G_{K'}$ ].verify( $M', T'$ ) <pre> 1. <math>\widehat{T} \leftarrow \text{XOR-GTM}[F_K, G_{K'}].\text{tag}(M')</math> 2. <b>if</b> <math>\widehat{T} = T'</math> <b>return</b> <math>\top</math> 3. <b>else return</b> <math>\perp</math></pre>	<b>Algorithm</b> <b>XOR-GTM</b> [ $F_K, G_{K'}$ ].detect( $M', T'$ ) <pre> // <math>R_i = \{i\}</math> for <math>i \in [t]</math> 1. <math>\mathcal{P} \leftarrow [m]</math> 2. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 3.   <math>S' \leftarrow G_{K'}^{i-1}(T'[i])</math> 4.   <math>\widehat{S} \leftarrow \text{XOR-GTM}[F_K].\text{hash}(M')</math> 5.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 6.     <math>\widehat{S}^R[i] \leftarrow \bigoplus_{j \in R_i} \widehat{S}[j]</math> 7.     <math>(S')^R[i] \leftarrow \bigoplus_{j \in R_i} S'[j]</math> 8.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 9.     <b>if</b> <math>\widehat{S}^R[i] = (S')^R[i]</math> <b>then</b> <math>B[i] \leftarrow \top</math> 10.    <b>else</b> <math>B[i] \leftarrow \perp</math> 11. <b>return</b> <math>\mathcal{P}</math></pre>
<b>Algorithm</b> <b>XOR-GTM</b> [ $F_K$ ].hash( $M$ ) <pre> 1. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 2.   <math>S[i] \leftarrow 0^n</math> 3.   <b>for</b> <math>j = 1</math> <b>to</b> <math>m</math> <b>do</b> 4.     <math>Z \leftarrow F_K^j(M[j])</math> 5.     <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 6.       <b>if</b> <math>H_{i,j} = 1</math> 7.         <math>S[i] \leftarrow S[i] \oplus Z</math> 8.   <math>S \leftarrow (S[1], \dots, S[t])</math> 9. <b>return</b> <math>S</math></pre>	<b>Algorithm</b> <b>XOR-GTM</b> [ $F_K, G_{K'}$ ].verify-S( $M', T'$ ) <pre> 1. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 2.   <math>S' \leftarrow G_{K'}^{i-1}(T'[i])</math> 3.   <math>\widehat{S} \leftarrow \text{XOR-GTM}[F_K].\text{hash}(M')</math> 4.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 5.     <math>\widehat{S}^R[i] \leftarrow \bigoplus_{j \in R_i} \widehat{S}[j]</math> 6.     <math>(S')^R[i] \leftarrow \bigoplus_{j \in R_i} S'[j]</math> 7.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 8.     <b>if</b> <math>\widehat{S}^R[i] = (S')^R[i]</math> <b>then</b> <math>B[i] \leftarrow \top</math> 9.     <b>else</b> <math>B[i] \leftarrow \perp</math> 10. <math>B \leftarrow (B[1], B[2], \dots, B[v])</math> 11. <b>return</b> <math>B</math></pre>	

Fig. 1: XOR-GTM using  $t \times m$  test matrix  $H$  and extension rule  $R$  with  $v$  elements.

Pentru a reduce semnificativ memoria ocupata de matricea de test, XOR-GTM-PPI foloseste  $H^R$  – o matrice circulara.

<b>Algorithm</b> XOR-GTM[ $F_K, G_{K'}$ ].tag( $M$ ) <pre> 1. <math>S \leftarrow \text{XOR-GTM}[F_K].\text{hash}(M)</math> 2. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 3.   <math>T[i] \leftarrow G_{K'}^i(S[i])</math> 4.   <math>T \leftarrow (T[1], \dots, T[t])</math> 5. <b>return</b> <math>T</math></pre>	<b>Algorithm</b> XOR-GTM[ $F_K, G_{K'}$ ].detect( $M', T'$ ) <pre> // <math>c_i \in \{m\}</math>, <math>i \in [w]</math>: <math>H_1 = \{c_i + 1 : i \in [w]\}</math> 1. <math>\mathcal{P} \leftarrow [m]</math> 2. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 3.   <math>S' \leftarrow G_{K'}^{i-1}(T'[i])</math> 4.   <math>\widehat{S} \leftarrow \text{XOR-GTM}[F_K].\text{hash}(M')</math> 5.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 6.     <math>\widehat{S}^R[i] \leftarrow \bigoplus_{j \in R_i} \widehat{S}[j]</math> 7.     <math>(S')^R[i] \leftarrow \bigoplus_{j \in R_i} S'[j]</math> 8.   <b>for</b> <math>i = 1</math> <b>to</b> <math>v</math> <b>do</b> 9.     <b>if</b> <math>\widehat{S}^R[i] = (S')^R[i]</math> <b>then</b> <math>\mathcal{S} \leftarrow \{(c_j + i - 1) \bmod m + 1 : j \in [w]\}</math> 10.    <math>\mathcal{P} \leftarrow \mathcal{P} \setminus \mathcal{S}</math> 11. <b>return</b> <math>\mathcal{P}</math></pre>
<b>Algorithm</b> XOR-GTM[ $F_K, G_{K'}$ ].hash( $M$ ): <pre> // <math>H_i = H_i^R</math> for all <math>i \in [t]</math> // <math>b_i \in \{m\}</math>, <math>i \in [w]</math>: <math>H_{*,1}^R = \{b_i + 1 : i \in [w]\}</math> // <math>w = 2^s + 1</math> 1. <b>for</b> <math>i = 1</math> <b>to</b> <math>t</math> <b>do</b> 2.   <math>S[i] \leftarrow 0^n</math> 3.   <b>for</b> <math>j = 1</math> <b>to</b> <math>m</math> <b>do</b> 4.     <math>Z \leftarrow F_K^j(M[j])</math> 5.     <math>\mathcal{I} \leftarrow \{(b_k + (j-1)) \bmod m + 1 : k \in [w]\}</math> 6.     <b>for all</b> <math>i \in \mathcal{I}</math> <b>do</b> <math>S[i] \leftarrow S[i] \oplus Z</math> 7.   <math>S \leftarrow (S[1], \dots, S[t])</math> 8. <b>return</b> <math>S</math></pre>	

Fig. 3: XOR-GTM-PPI: XOR-GTM using projective-plane incidence matrix for  $H^R$ .