Two-tank Simulation Development

* 1. Model of Example System

An example system was chosen for the development and testing of the methods presented in this report. The fundamental model of this system was derived and the system was modelled in Simulink. A description of this system and the generation of the model is provided here.

### Goals

A model of a system containing two tanks with heating coils is to be developed. The dynamic behaviour of all variables in the system is to be modelled so that data generated from this model can be used in the development and testing of data-based monitoring methods and data-based methods of inferring connectivity between variables.

### Information

The chosen example system consists of two tanks. A diagram of the system is shown in Figure 1.

The outlet flow from both tanks is proportional to the square root of the level in each tank. The outlet from the first tank flows into the second tank. Each tank has its own supply of cold water with a control valve to control the level of each tank. Each tank also exchanges heat with a steam line. The temperature in the tanks is controlled using the control valves on the steam lines.

The main variables of interest are the flow rate of the inlet streams to the tanks, F1 and F2, the flow rates of the steam in the heating coils in both tanks, F3 and F4, the levels of both tanks, L1 and L2, and the temperatures of both tanks, T1 and T2. F1 and F2 are used as manipulated variables (MVs) to control L1 and L2 respectively. F3 and F4 are used as MVs to control T1 and T2 respectively. The controllers used are simple proportional integral derivative (PID) controllers that change the values of the MVs according to the deviation of the controlled variables (CVs) from their set-points (SPs).



Figure 1: Diagram of two-tank example system

The measurements obtained from real-life processes such as this would typically display significant amounts of random noise generated by sensors or just normal fluctuations in the values of the properties being measured.

The steady state values for variables in the process are given in Table 1. The steady state values for the CVs, L1, L2, T1 and T­2, are also their set-point values.

Table 1: Steady state values for two-tank model

|  |  |  |
| --- | --- | --- |
| Variable | Value | Units |
| L1 | 2.00 | [m] |
| L2 | 3.00 | [m] |
| T1 | 50.0 | [°C] |
| T2 | 50.0 | [°C] |
| T1,in | 25.0 | [°C] |
| T2,in | 25.0 | [°C] |
| T3 | 100 | [°C] |
| T4 | 100 | [°C] |
| F1 | 0.181 | [m3/min] |
| F2 | 0.0408 | [m3/min] |
| F3 | 0.5 | [m3/min] |
| F4 | 0.04 | [m3/min] |
| F1out | 0.191 | [m3/min] |
| F2out | 0.222 | [m3/min] |

The values of the parameters used in the model are given in Table 2. The value for the proportionality constant relating the underflow to the level, KL, was determined by substituting steady-state values into Equation 4 and solving for KL (at steady state the differential term is 0). The valve constant for each control valve was simply chosen so that the steady state value of the flow rate being controlled by the valve corresponded to a valve position of 50%.

Table 2: Parameters used in model of two-tank system

|  |  |  |  |
| --- | --- | --- | --- |
| Parameter | Description | Value | Units |
| aHeat | Constant for heat transfer coefficient calculation | 1.41(105) | [cal/(min°C)] |
| b | Constant for heat transfer coefficient calculation | 0.5 | [ ] |
| Cp | Heat capacity of water | 1 | [cal/(g°C)] |
| ρ | Density of water | 106 | [g/m3] |
| A1 | Cross-sectional area of tank 1 | 1 | [m2] |
| A2 | Cross-sectional area of tank 2 | 1 | [m2] |
| kL | Level constant | 0.128 | [m3/min/m0.5] |
| kv1 | Valve 1's constant | 0.00363 | [m3/min/%open] |
| kv2 | Valve 2's constant | 0.000815 | [m3/min/%open] |
| kv3 | Valve 3's constant | 0.01 | [m3/min/%open] |
| kv4 | Valve 4's constant | 0.000769 | [m3/min/%open] |

### Formulation of model

#### Mass balance

A generalised mass balance on a tank is given by:

Equation : Generalised mass balance

The mass balance of the first tank therefore requires determination of the accumulation term, which is the change in volume of the tank with time. There is one stream flowing in, F1, and one stream flowing out, F1,out. The heating steam flow, F3, flows into and out of the tank in heating coils and does not mix at all with the material in the tank, therefore it does not appear in the mass balance (it will appear in the energy balance since it exchanges heat with the material in the tank). Therefore the mass balance is given by:

Equation

The volume of the tank is given by the level multiplied by the cross sectional area. The level varies, so it remains within the derivative term, but the area can be taken out. The flow rate out of the tank is dependent on the level of the tank. The flow rate is related to the pressure driving force ((Marlin, 2000)), i.e. the static pressure exerted by the liquid. This relationship can be approximated by:

Equation ((Marlin, 2000))

Substitution into Equation 2 results in the mass balance for tank 1:

Equation : Tank 1 mass balance

For the second tank the mass balance is similar, except that the underflow from tank 1 also flows into it. The resulting mass balance for tank 2 is given by:

Equation : Tank 2 mass balance

#### Energy balance

Since there is not shaft work in the process, the energy balance is given by:

Equation

The change in internal energy, U, with time is given by:

Equation

The enthalpy of stream i, Hi, is given by:

Equation

Substituting these equations into the energy balance (assuming a value of 0 for Tref) gives the following equation for the first tank:

Equation

Under the assumption of perfect mixing in the tank the temperature of the stream flowing out of the tank is equal to the temperature in the tank. The temperature of the feed stream is designated T1,in.

Q represents the heat transferred to the liquid in the tank from the liquid in the heating coils. An energy balance on the liquid in the heating coils gives:

Equation

F3 is the flow rate of the heating steam, T3 is the temperature at which the steam enters the coils and Tout is the temperature at which it exits F3. Rearranging so that Tout is the subject of the equation gives:

Equation

The heat transferred can be determined using the overall heat transfer coefficient, UA. Assuming that the inner film resistance dominates the heat transfer through the coils, and that the resistance of the tube walls and the outer film resistance are negligible. An empirical equation relating the heat transfer coefficient to the flow rate of liquid was reported in Marlin as:

Equation

The heat transfer from the tubes is then given by the heat transfer coefficient multiplied by an approximation of the mean difference of the temperature in the tank and the temperature in the coils.

Equation

Combination of Equation 11 and Equation 13 to eliminate Tout, gives:

Equation

Substitution into Equation 9 results in the complete energy balance for tank 1:

Equation : Energy balance for tank 1

For tank 2 the energy balance is similar, except that the energy entering the system from the outlet stream of tank 1 has to be included. The flow rate of water into the tank is F2, entering at a temperature of T2,in. The flow rate of steam is F4, entering at a temperature of T4. The temperature of the tank is T2.

Equation : Energy balance for tank 2