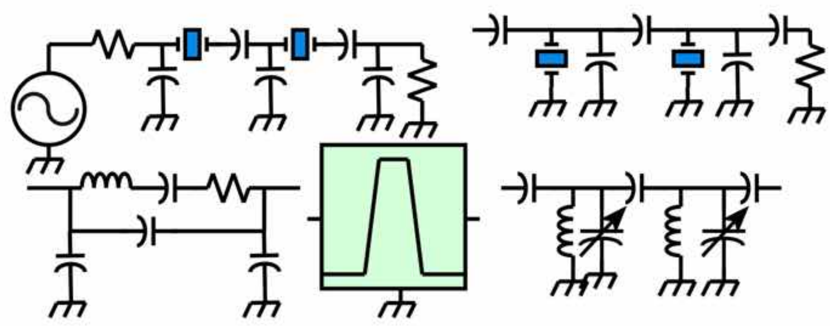

Designing Narrow-Bandwidth Ladder Filters

Wes Hayward, w7zoi, April 7, 2019.

Abstract: The performance of radio equipment depends upon the filters included in the radio. Especially critical are the narrow width bandpass circuits. This note summarizes the design methods for filters using either LC or quartz crystal resonators. Series and parallel LC circuits are included, as are the crystal LSB and USB ladders. The filter designs are then illustrated with computer simulations.

> A personal aside: I've spent a lot of time and effort in recent years with LC bandpass filters and lower sideband (LSB) ladder crystal filters. I've not really considered the upper sideband ladder. My goal was to examine this circuit, beginning with the classic paper by Dishal. Rather than merely presenting this work, it seemed to make sense to generalize and to compare and contrast the upper and lower sideband ladders. Some of the simulations illustrate the way coupling and loading might be determined.
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> My approach to crystal filter design has used a perturbation approach wherein the crystal is approximated by a LC resonator. The general discussion includes the design of the LC filters as well as LSB crystal filters with the perturbation scheme. Some approximations may be used in the filter designs and are always described. However, the analysis, or simulations use no approximations. They are exact, within the constraints of the simple linear models.
>
> None of the work presented here is new. Rather, the note is merely presented as a collected illustration of some of the methods I've found useful.

Some basic filter characteristics and terms.

What is the meaning of *narrow* in RF filters? A 1 or 2% bandwidth LC filter might be narrow. A 200 Hz wide crystal filter with .004% bandwidth might also be "narrow." Hence, a meaningful definition for *narrow* must include the resonators used to build the filters. A resonator is characterized by an unloaded Q, specified as Q_u. A bandwidth B_{min} is related to Q_u, $B_{min}=f/Q_u$ where f is the resonant frequency. B_{min} and f have the same units. It is impossible to build a single resonator passive filter that is narrower than B_{min}. Filters with more than one resonator are further restricted. An estimated filter minimum will be $2B_{min}$ for a circuit with 2 resonators. Hence, LC resonators with Q_u of 250 at 5 MHz could be used to build a double tuned circuit with a bandwidth of 40 kHz. Loss will be high for these *narrowest* filters, with loss decreasing as design bandwidth increases. Q_u of 100,000 and beyond is common for crystals, so filters with lower bandwidths are possible. The same B_{min} constraints apply.

This work is based upon so-called ***modern network theory*** first appearing in the 1950s. Earlier, filters were designed using ***image parameter*** concepts. Modern network theory has the following characteristics:

1. Designs are low pass filters. These prototypes are characterized by their transfer functions and by a cutoff frequency.
2. The low pass prototype filters are usually described by well defined polynomial transfer functions. These expressions describe an output voltage for a specified input voltage.

3. The low pass prototypes can be transformed to allow arbitrary cutoff frequency and terminations.
4. The prototypes can also be transformed to design high pass, bandpass, and band stop circuits.
5. The low pass prototype filters usually consist of a table of normalized component values. The number of components in the filter is commensurate with the order of the filter, which is the order of the polynomial that is the low pass transfer function. Higher order leads to greater stop band attenuation.
6. The low pass prototype filter can be used to examine the way energy is shared with the terminations and between elements in the prototype. This analysis produces an alternative description of the filter consisting of the Q of end sections and coupling coefficients describing the interface between elements. This new parameter set are end section normalized Q values and normalized coupling coefficients. This new parameter set can be used to design a very wide variety of filter types. Calculation of the "k and q" parameter set is described in an appendix.

The purpose of this paper is to summarize the mathematics and perhaps impart some intuition. Equations are presented with example data. The numeric data will allow the reader to do the calculations as the text is read. Calculations use fundamental units of Hz, Farad, Henry, etc, although we may often use practical components in text, such as capacitors in pF. We assume that the reader will seamlessly make these transformations.

There are no advanced mathematics here. It's just algebra, although often with imaginary (complex) elements.

After filters are designed, they are analyzed with computer programs. Some of the analysis is with programs we have written while others use SPICE. This report does not include measured results. An appendix presents more numeric detail for readers wishing to see such data. Generally, we obtain excellent correspondence between filter simulation and physical measurements.

Two filter orders (N=2 and N=4) are considered for each of the four resonator types. The reader should then be able to extend the presentation to any order. The key to this flexibility is found in the ***central design rules*** in section 2 below.

Contents.

The paper is divided into the following sections:

1. Basic Resonator Circuits.

This section presents some schematic diagrams and some single stage filters. There are no mathematics here, although the math to be presented later can be adapted to analyze these simple filters.

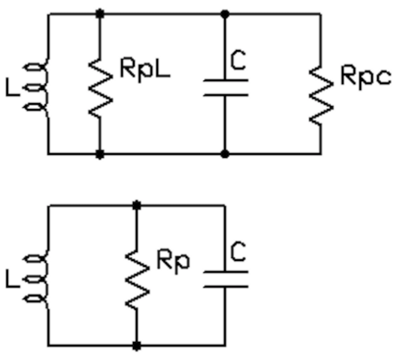

Fig 1-1.

A parallel tuned circuit places an inductor in parallel with a capacitor. Both elements have their own loss, each represented here by a parallel resistance. More often than not, we combine the two to form a single resonator parallel resistance, Rp, shown in the second part of Fig 1-1.

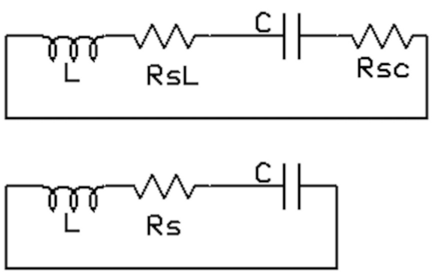

Fig 1-2.

A series tuned circuit is show in Fig 1-2. Again, the series loss resistances that model the individual elements are combined in a single resistance, Rs, that models the complete series tuned circuit shown in the lower part of Fig 1-2.

Fig 1-3 models the way a filter might be used in a system.

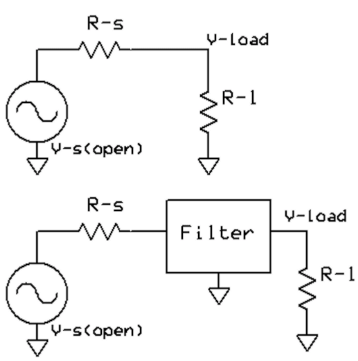

Fig 1-3.

The upper part of Fig 1-3 shows a source with internal resistance R-s followed by a load R-l. The source has an open circuit voltage of V-s. A filter circuit is inserted between the source and load, shown in the lower part of the figure.

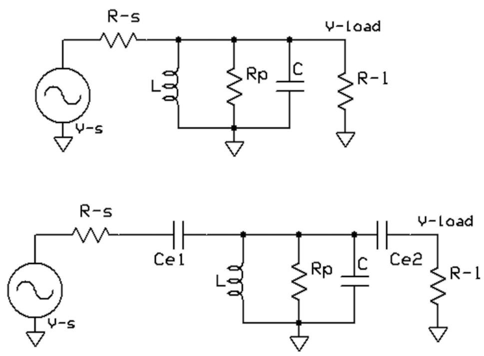

Fig 1-4.

Two variations are shown in Fig 1-4 for using a **parallel** tuned circuit as a filter. In the upper case, the reactive L and C must be carefully chosen with respect to the source and load. L and C must resonate at the filter center frequency. But the L (and hence, also the C) will be chosen to interact with the source and load resistances as well as the Rp which models resonator loss to establish a useful bandwidth. Consider an example. Source and load R are both set at 50 Ohms. A small inductor of 1 μH resonates at 5 MHz with 1013.2 pF. Assume, for the moment, that loss element Rp is very high and can be ignored. The source and load are both 50 Ohms, so their parallel value is 25 Ohms. The inductive reactance of L at 5 MHz is 31.4 Ohms. The loaded Q of this circuit is then 25/31.4, which is less than 1. The heavy loading comes from the terminations. This is a very wide filter, but still useful in some situations. This filter bandwidth will be slightly more than the center frequency.

The second part of Fig 1-4 shows a variation where series end capacitors Ce1 and Ce2 are placed in series with the source and load resistances. A careful capacitor choice produces whatever loading we wish. This will then turn the filter into something with a much narrower bandwidth. But the end capacitors will detune the original resonator from the initial value, so resonator C must be altered to maintain the original frequency.

If we allow the resonator loss resistance, Rp, to take on practical values, we will see that this single parallel tuned circuit filter will have an insertion loss. This can be calculated with methods that will follow.

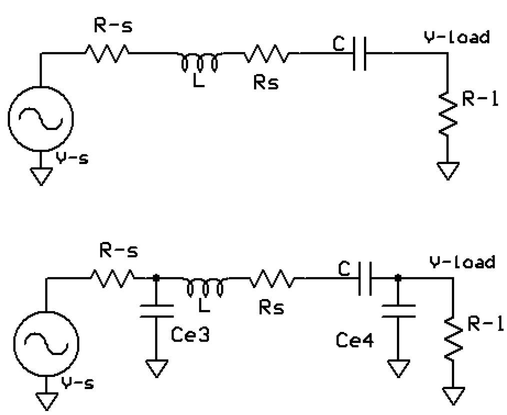

Fig 1-5.

The **series** tuned circuit form of a N=1 filter is shown in Fig 1-5. (N is the number of resonators.) The upper half shows the source and load resistances in series with the reactive elements and series loss modeling resistance R-s. In this variation, the higher the loading resistances R-s and R-l become, the lower the Q and the wider the circuit bandwidth. The lower half of Fig 1-5 shows the common modification where shunt capacitors, Ce3 and Ce4, transform the usual terminations to “look like” lower values when considering loads for the resonator. The equations for this transformation are presented later in Section 3.

2. The Central Design Rules for Coupled-Resonator Filters.

Modern network theory was mentioned earlier. The low pass circuits that are the basis for our coupled-resonator filters are all derived from characteristic polynomials. For example, a Butterworth polynomial describes a low pass that is maximally flat at the origin, frequency=0 for a low pass. (By maximally flat, we mean that the frequency derivative of the transfer function is zero at the origin.) Another popular polynomial is that of Chebyshev, a shape that results from an approximation that asks for the lowest possible response in the stopband, but allows an equal ripple within the passband. Filter enthusiasts refer to this as a "brick wall" shape. These details are discussed in the references. There are many other interesting and useful polynomials that are used. Once a polynomial has been chosen, it will define the shape of the filter response. It will also define a set of normalized low pass component values.

The low pass prototype component values can be used directly to design low pass, high pass, and even some bandpass filters. But most narrow band filter designs use an alternative set of filter design parameters, the so called normalized end section q and coupling coefficients. For example, a Butterworth bandpass filter with four resonators (order N=4) would have the following parameters:

$q_1 = 0.7654$, $q_4=0.7654$, $k_{12}=0.841$, $k_{23}=0.541$, $k_{34}=0.841$.

Every filter shape and order will have a corresponding set of parameters. We emphasize that these are not different filters from the normalized low pass parameters, but are just a new set of values that are derived from the low pass elements.

So, what do these parameters represent?

First, these parameters are *normalized*. This means that the most common low pass filter that is the basis for a bandpass is a circuit with a 1 radian per second cutoff and one ohm terminations. A N=4 Butterworth low pass would have the normalized values of C1=0.7654 Farad, L2=1.85 Henry, C3=1.85 Farad, and L4=0.7654 Henry. These normalized component values are converted to the k and q values shown above. (See Appendix B.) Or the normalized L and C low pass values can be denormalized to form low pass or high pass filters.

Denormalizing the k and q occurs with respect to a parameter called *filter Q*, or Q_f. Assume that we wanted to build a filter with a 5 MHz center frequency and a bandwidth of 0.2 MHz. Q_f is then 5/0.2=25. The other often used parameter is *nodal capacitance*, C_0. This is the capacitance value for the LC resonator forming the basis of the filter. The details of the denormalization will emerge in the later sections.

The normalized q is a value related to an input or output resonator. Once the parameter is denormalized, a loaded Q for the resonator is established. This is the Q of that resonator when loaded by the termination at the related end. Recall Fig 1-3 where an arbitrary filter was terminated at both input and output by source and load resistances. The end section Q for the first resonator is the result of loading by R-s, the source.

The normalized coupling coefficients have a similar role. The parameter k_{12}, when denormalized, will specify the element that couples energy from the first resonator to the second.

The central rule for filter design is then: **A filter shape is established when the end section Q values and the couplings between resonators are established. Passband shape does not depend upon how we realize the coupling and loading. Many different circuits may be used, but once the denormalized K and Q values are established, the filter shape close to the center frequency is determined.** This rule will be illustrated in the sections that follow.

3. Terminations that Load Resonators, Series to Parallel and Back…

The central rules stipulate that we must establish a specific end section Q to realize a desired filter shape. This section will derive the important equations that are used in later designs to obtain desired loading values. The equations presented are essentially transformations between parallel and series circuits. 5 MHz examples are used. Some capacitor values will be specified, but these are merely used for the numerical calculations. The important *outputs* of this section are the formulas. The initial parameters for examples are:

$$f = 5 \cdot 10^{6}, \omega = 2\pi f, C = 10^{-10}, R = 50, c = 500 \cdot 10^{-12}$$

Two capacitor values, C and c, are specified. We will use C to deal with a ***series*** circuit of R and C. Later, a ***parallel*** R and c will be analyzed. We begin with the impedance of the series RC of Fig 3-1.

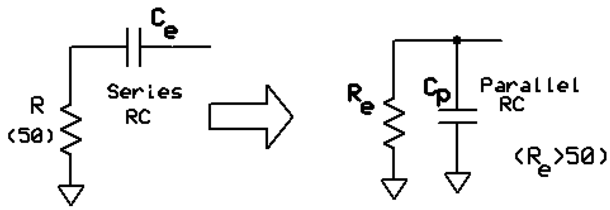

Fig 3-1. C_e=100 pF at 5 MHz and R =50**.**

$$Z = R - \frac{i}{\omega C}$$

$$Z = 50 - 318.31i$$ Eq. 3-1.

Lower case i is the square root of minus 1. This equation expands to yield

$$Z = \frac{(R\omega C - i)}{\omega C}, Y = \frac{1}{Z}, Y = \frac{\omega C}{(R\omega C - i)}$$ Eq. 3-2.

We have converted the impedance Z to an admittance, Y. Continuing, Y is resolved into real and imaginary parts, G and B,

$$Y = \frac{\omega C\,(R\omega C + i)}{(R\omega C - i)\,(R\omega C + i)} = \frac{R\omega^2 C^2 + i\omega C}{(R\omega C)^2 + 1} = G + iB$$ Eq. 3-3.

G is a conductance while B is a susceptance. Further expansion yields

$$G = \frac{(R\omega^2 C^2)}{(R\omega C)^2 + 1}, B = \frac{\omega C}{(R\omega C)^2 + 1}$$ Eq. 3-4.

G may be inverted to obtain an equivalent parallel resistance. If we were to write the susceptance of a parallel capacitor, it would be B= ω C_p where C_p is the parallel capacitance and ω is the angular frequency. Hence, C_p = B/ ω. The resulting equations are

$$R_p = \frac{(R\omega C)^2 + 1}{(R\omega^2 C^2)}, C_p = \frac{C}{(R\omega C)^2 + 1}$$ Eq. 3-5.

Recall that R=50 and C was 100 pF, resulting in 5 MHz calculated results

$$R_p = 2076.42$$

$$C_p = 9.7592e\text{-}11$$

Eq. 3-6.

The series capacitor has readily created a resistance much higher than the original 50 Ohms. The equivalent parallel capacitance C_p is close to, but still less than the original 100 pF.

We rewrite Eq 3-5 by defining the parallel resistance to be R_e. The subscript e emphasizes that this is an ***end*** resistance.

$$R_e = \frac{(R\omega C)^2 + 1}{(R\omega^2 C^2)}, X = \frac{1}{\omega C}$$

Eq. 3-7.

Note that Eq. 3-7 includes a definition for the reactance of the series capacitor. This becomes useful, leading to

$$RR_e = \frac{\frac{R^2}{X^2} + 1}{\frac{1}{X^2}}$$

$$RR_e = R^2 + X^2$$

Eq. 3-8.

This heads to the simple form.

$$X = \sqrt{RR_e - R^2}$$

Eq. 3-9.

Extracting the value of the capacitor produces

$$C_{end} = \frac{1}{\omega\sqrt{RR_{end} - R^2}}$$

Eq. 3-10.

For example, say that we needed a resistance of 3000 Ohms across the resonator to achieve the desired end section Q, but we had R=50 at the other end. The required capacitor from Eq. 3-10 is then

$$R_{end} = 3000$$

$$C_{end} = 8.28808e\text{-}11$$

Eq. 3-11.

We also need to know the equivalent parallel capacitance across the newly generated 3000 Ohms. This is given by Eq. 3-5, repeated here

$$C_{par} = \frac{C_{end}}{(R\omega C_{end})^2 + 1}$$

$$C_{par} = 8.14995e\text{-}11$$

Eq. 3-12.

The shunt 81.5 pF is close to, but not quite equal to the series end capacitance, 82.9 pF.

In the analysis above we were able to create a large resistance load with a series capacitor. Next we consider the case of adding a parallel capacitor to a load to generate the equivalent of a lower resistance. This is shown in the next figure. Lower case c represents capacitors merely to distinguish from the earlier case.

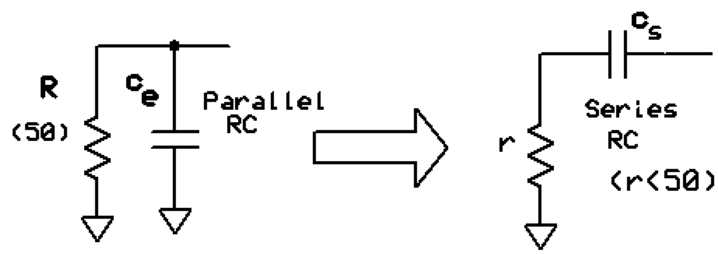

Fig 3-2. R=50 and c_e=c=500 pF for the numeric example.

We begin by writing the admittance for the parallel RC.

$$y = \frac{1}{R} + i\omega c$$

$$y = 0.02 + 0.015708i$$

Eq. 3-13.

But impedance is just Z=1/Y,

$$z = \frac{R}{1 + i\omega Rc} = \frac{R(1 - i\omega Rc)}{1 + (\omega Rc)^2}$$

Eq. 3-14.

The real part of z is just the end resistance that will load the end resonator,

$$r = \frac{R}{1 + (\omega Rc)^2}$$

Eq. 3-15.

But this is not the form we want. Rather, we wish to specify a value for r<R and then calculate the value of c that will yield that result. We solve Eq. 3-15 for c with the following sequence,

$$r + r(\omega Rc)^2 = R$$

Eq. 3-16.

$$\omega Rc = \sqrt{\frac{R - r}{r}}$$

Eq. 3-17.

$$c = \frac{1}{\omega R}\sqrt{\frac{R - r}{r}}$$

Eq. 3-18.

Cleaning things a little yields a "standard form,"

$$c_e = \frac{1}{\omega R_0}\sqrt{\frac{R_0 - r}{r}}$$

Eq. 3-19.

This value of parallel end capacitor gives us the required end resistance. We seek the equivalent series capacitance that will contribute to resonator tuning. We focus on the imaginary part of Eq. 3-14,

$$X = \frac{\omega R^2 c}{1 + (\omega R c)^2} = \frac{1}{\omega c_s}$$

Eq. 3-20.

$$\frac{\omega^2 R^2 c}{1 + (\omega R c)^2} = \frac{1}{c_s}$$

Eq. 3-21.

Solving for c_s while inserting an "e" subscript for later clarity yields

$$c_s = \frac{1 + (\omega R c_e)^2}{\omega^2 R^2 c_e}$$

Eq. 3-22.

Finally, consider an example where we wish to generate a 5 ohm load with a capacitor and 50 ohm termination.

$$r = 5, R_0 = 50$$

$$c_e \quad = 1.90986\text{e-}09$$

$$c_s \quad = 2.12207\text{e-}09$$

Eq. 3-23.

These transformations will be applied in the filter designs that follow.

4. A Double Tuned Circuit with Parallel Tuned LC Resonators.

Now that some of the preliminary details are out of the way, it's time to design a filter. Our first choice is a 200 kHz wide double tuned circuit at 5 MHz using parallel resonators. The schematic follows.

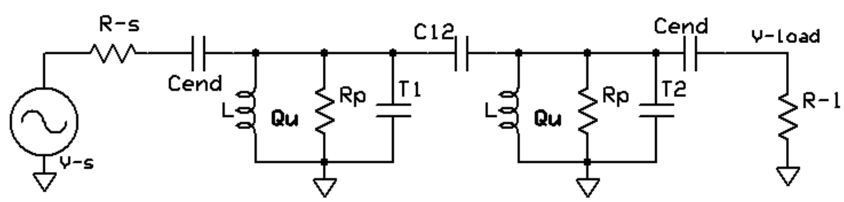

Fig 4-1.

The parameters for our filter are

$$k_{12} = \frac{1}{\sqrt{2}}, q = \sqrt{2}, f_c = 5\cdot 10^6, Q_u = 200, L = 3\cdot 10^{-6}, R_0 = 50, \text{bw} = 200\cdot 10^3$$

The Butterworth parameters are the k_{12} and q values shown while the center frequency is 5 MHz. We pick inductors of 3 µH with an unloaded Q of 200. The bandwidth was specified to be 200 kHz and the filter source and load will be 50 Ohms. The angular frequency is

$$\omega = 2\pi f_c$$ Eq. 4-1.

The nodal capacitance C_0 is the capacitance that resonates with the inductor that we have picked.

$$C_0 = \frac{1}{\omega^2 L}$$ Eq. 4-2

The filter Q is given by

$$Q_F = \frac{f_c}{\mathrm{bw}}$$ Eq. 4-3.

This filter Q is relatively low compared with the unloaded resonator Q, so this filter will be easily realized. A useful parameter is one we term the "normalized Q," signified by q_0 and defined as

$$q_o = \frac{Q_u}{Q_F}$$ Eq. 4-4.

If q_0 is more than 2, a double tuned circuit is usually practical. Later we will see that q_0 can be used to calculate insertion loss.

The end Q is

$$Q_{end} = qQ_F$$ Eq 4-5

Note that the normalized filter q is denormalized by multiplying by filter Q, Q_F. If the bandwidth was higher, Q_F would be lower and Q_{end} would be lower. Equation 4-5 is a first approximation and it does not account for the intrinsic loss in the resonator specified by Q_u. A more complete expression is

$$Q_e = \frac{1}{\left(\frac{1}{qQ_F} - \frac{1}{Q_u}\right)}$$ Eq. 4-6.

A coupling coefficient for a practical filter element is the normalized value divided by filter Q. The coupling capacitor is then the nodal capacitance multiplied by the coupling coefficient. Hence, the coupling capacitor is calculated with

$$C_{12} = C_0 \frac{k_{12}}{Q_F}$$ Eq. 4-7.

We have already calculated the denormalized end section Q = Q_e. This allows us to find the end resistance needed to properly load the resonator.

$$R_{pe} = Q_e \omega L$$ Eq. 4-8.

A series capacitor C_e is placed in series with the 50 ohm loads at each end of the filter to transform the low resistance to look like R_{pe} at the resonator. This calculation uses Eq. 3-10 derived above,

$$C_e = \frac{1}{\omega \sqrt{R_{pe} R_0 - R_0^2}}$$ Eq. 4-9.

The final detail is resonator tuning. We started with a nodal C0=338 pF, that has been diminished because of the ~10 pF capacitor coupling between resonators and by the 71 pF loading capacitors. We tune the resonators with

$$T_1 = C_0 - C_{12} - C_e$$ Eq. 4-10.

Note that this is still an approximation. A more refined equation was derived above (Eq. 3-12) if greater accuracy is needed. It would not make much difference in this case. Moreover, we would probably use trimmer capacitors as part of the resonators in a practical circuit, so a small approximation is allowed.

Finally, we calculate the filter insertion loss.

$$\mathrm{IL} = 20\log\left(\frac{q_o}{q_o - q}\right) \qquad \text{Eq. 4-11.}$$

The loss depends upon the normalized Q, q_0, as well as the q value, here the end Butterworth q. The insertion loss is in dB.

A computer simulation of this filter is shown below. This is an analysis using the *ladder* method, outlined in our IRFD text book. This is an analysis program that is independent of the design above.

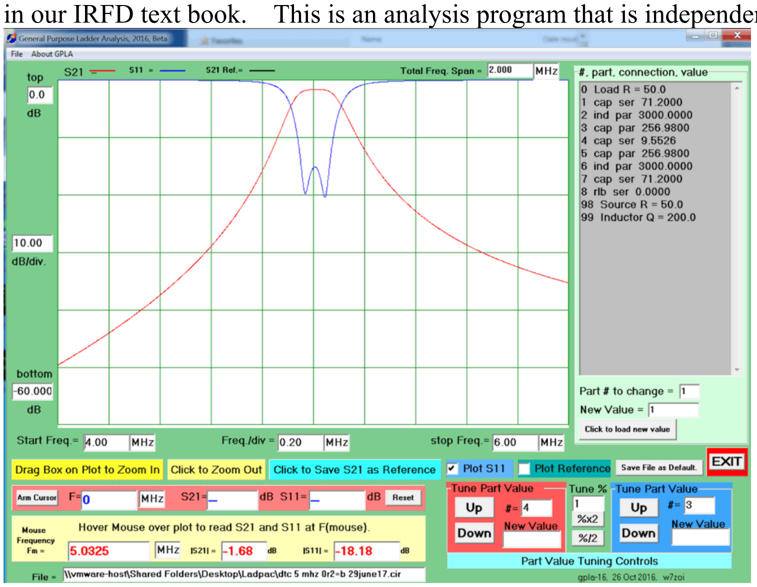

Fig 4-2. An analysis of the 5 MHz double tuned circuit with parallel resonators. Note that the insertion loss corresponds to the value from Eq. 4-11.

5. N=4 Bandpass Filter with Parallel LC Tuned Circuits.

The next filter is just like the earlier double tuned circuit, but now contains four parallel resonators. The same methods are used for end loading and for coupling as we used with the simpler double tuned circuit of section 4. The approximation used for tuning with the previous double tuned circuit is replaced with a better method. Again, we use Butterworth k and q parameters. The set-up data and equations are presented below. We picked a 6 µH inductor for this 5 MHz filter, but that is open to the designer.

$$f_c = 5, \mathrm{bw} = 0.2, \omega = 2\pi f_c 10^6, Q_u = 200, R_0 = 50, L = 6\cdot 10^{-6}$$

$$k_{12} = 0.841, k_{23} = 0.541, k_{34} = k_{12}, q = 0.7654$$

Next, we calculate the nodal capacitance and the filter Q:

$$C_0 = \frac{1}{\omega^2 L} \quad \text{Eq. 5-1.} \qquad Q_F = \frac{f_c}{\mathrm{bw}} \quad \text{Eq. 5-2.}$$

The end Q is calculated from the filter Q,

$$Q_{end} = \frac{1}{\left(\frac{1}{qQ_F} - \frac{1}{Q_u}\right)} \qquad \text{Eq. 5-3.}$$

This calculation includes both the normalized Butterworth end q and the unloaded resonator Q_u.

The next calculations are for the coupling capacitors.

$$C_{12} = C_0 \frac{k_{12}}{Q_F} \quad \text{Eq. 5-4.} \qquad C_{23} = C_0 \frac{k_{23}}{Q_F} \quad \text{Eq. 5-5.} \qquad C_{34} = C_0 \frac{k_{34}}{Q_F} \quad \text{Eq. 5-6.}$$

Note the symmetry with $k_{12}=k_{34}$, leading to $C_{12}=C_{34}$. This is a common occurrence with Butterworth and Chebyshev filters.

Having calculated Q_{end}, we can now continue with the end loading determinations.

$$R_{pe} = Q_{end}\omega L \quad \text{Eq. 5-7.} \qquad C_e = \frac{1}{\omega\sqrt{R_{pe}R_0 - R_0^2}} \quad \text{Eq. 5-8.}$$

This give us the capacitance value needed at the filter end to load resonators 1 and 4 with 50 Ohms. But the parallel capacitance that is presented to these resonators must be determined before tuning. The equation for this was derived in section 3. The parallel capacitance is given by

$$C_p = \frac{C_e}{1 + (R_0\omega C_e)^2} \quad \text{Eq. 5-9.}$$

This capacitance, 70.8 pF, is close to the end C value of 71.7 pF. This is the usual situation when C_e generates a large impedance transformation. In this case we go from 50 to almost 4000 Ohms. The capacitors are not as close to each other with a smaller transformation, which we might have with a wider bandwidth filter. We can now tune the filter.

$$T_1 = C_0 - C_{12} - C_p \quad \text{Eq. 5-10} \qquad T_2 = C_0 - C_{12} - C_{23} \quad \text{Eq. 5-11}$$

$$T_3 = T_2 \quad \text{Eq. 5-12} \qquad T_4 = T_1 \quad \text{Eq. 4-13}$$

We again take advantage of the symmetry that comes from the equally terminated Butterworth filter. Even without this symmetry, the formulas for T_3 and T_4 should be obvious.

Figure 5-1 below contains the transfer function for the filter we just designed. The schematic is also included. This is an LT-SPICE analysis rather than one using our “homebrew” software. Filter analysis is, after all, divorced from the filter design.

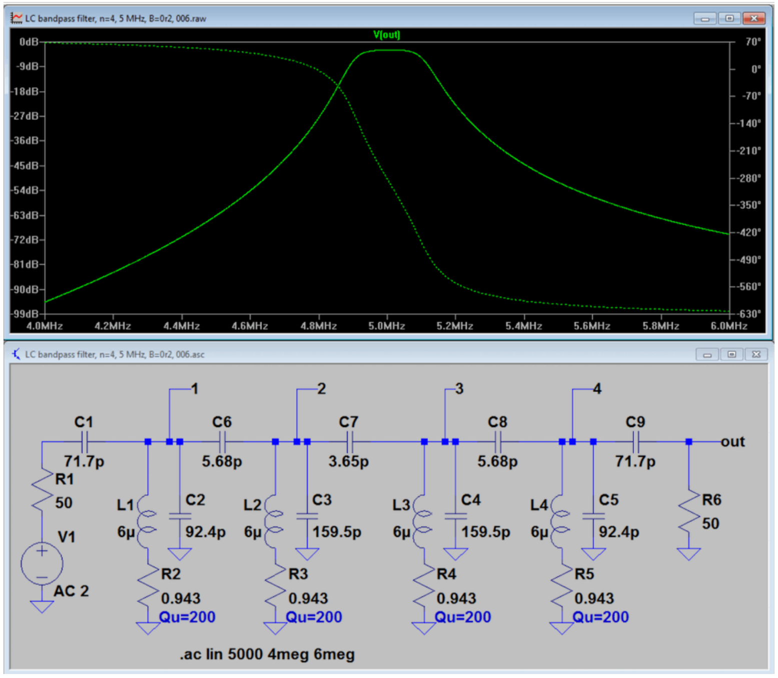

Fig 5-1. The classic Butterworth shape is evident. A simulation must be used to evaluate the insertion loss, for we lack a simple formula like we had with the double tuned circuit. Series resistors have been included with the inductors in the simulation to represent the unloaded Q of 200. Alternatively, parallel resistors could be used.

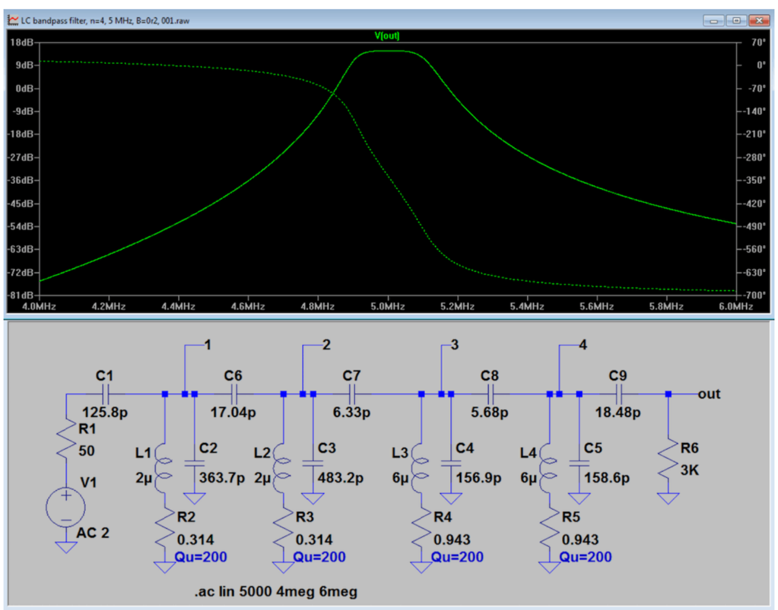

Fig 5-2. This filter is a special case, although this is not evident without careful inspection. There are two special features. First, the output resonator is terminated in 3K rather than 50 Ohms. Note that the filter performance is essentially unchanged, for the end section Q has been maintained. The second feature is the use of unequal inductors. Resonators 1 and 2 use 2 µH while #3 and #4 use 6 µH. The end calculations preserve end section Q. The coupling between resonators 2 and 3 is the unusual case owing to unequal inductors. The calculation is still straight forward. The nodal C values for each resonator are calculated. The geometric mean of the two C_0 values is evaluated and used to calculate the coupling capacitor, C7 in the above figure.

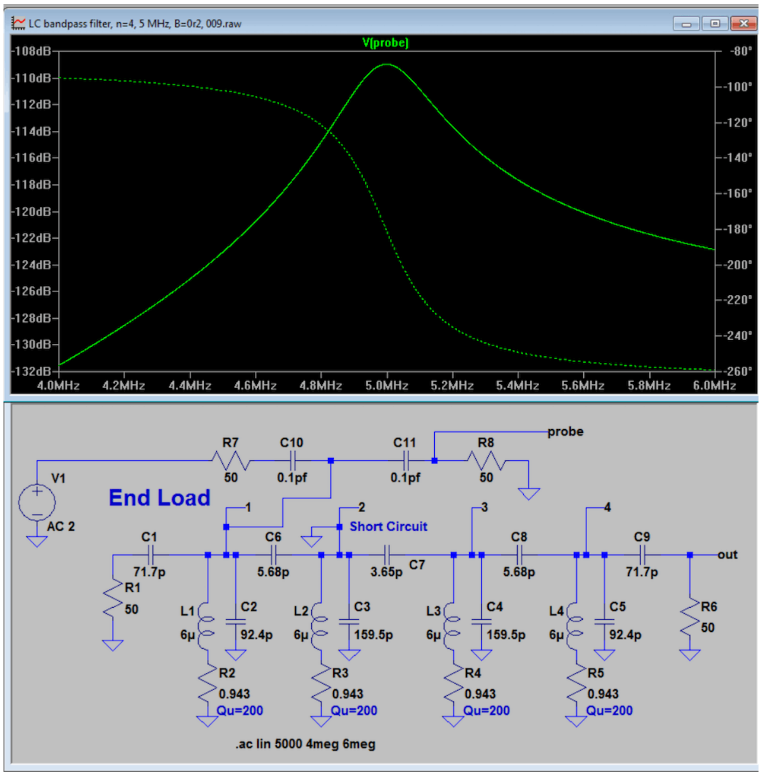

Fig 5-3. This circuit and plot shows how the same simulation, with just a modification or two, can be used to evaluate the end Q of a loaded resonator. Note that resonator 2 is short circuited. Hence, the coupling capacitor C6 in the above circuit continues to be part of the overall C to tune resonator 1, but all details related to resonators 2, 3, and 4 disappear. The end loading cap is attached to a termination, but the resonator is not excited with C_{end}. Two "probes" are attached only to the first resonator through VERY small capacitors, 0.1 pF. One probe comes from the RF source while the other goes to a detector. The sweep serves to measure the Q of the resonator. If we did not have the 50 ohm load and C_{end}, we would see a very narrow peak. This could be used to actually measure unloaded resonator Q. But the resonator is loaded to Q_{end} which is the value we would obtain from the swept response shown.

This computer simulation used a 50 ohm load and another 50 ohm detector, attached via very small capacitors, suggestive of the scheme we might use in a physical measurement. For a pure computer study, a current source could drive the resonator while the voltage is merely noted and plotted. Neither element will load the circuit. My personal preference is to emulate the physical measurement, but either method will provide the needed information.

The scheme is one that is often used for physical measurements. The very small capacitors in the simulation can be approximated with any manner of small "real world" capacitor. The scheme used depends upon the frequency. The key is to use identical "caps" at both probes and to guarantee that there is an insertion loss of at least 30 dB through the measurement setup. If these criterion are met, it should be a good measurement. Note that this "rule" can be tested with computer simulation. (This is NOT lore.)

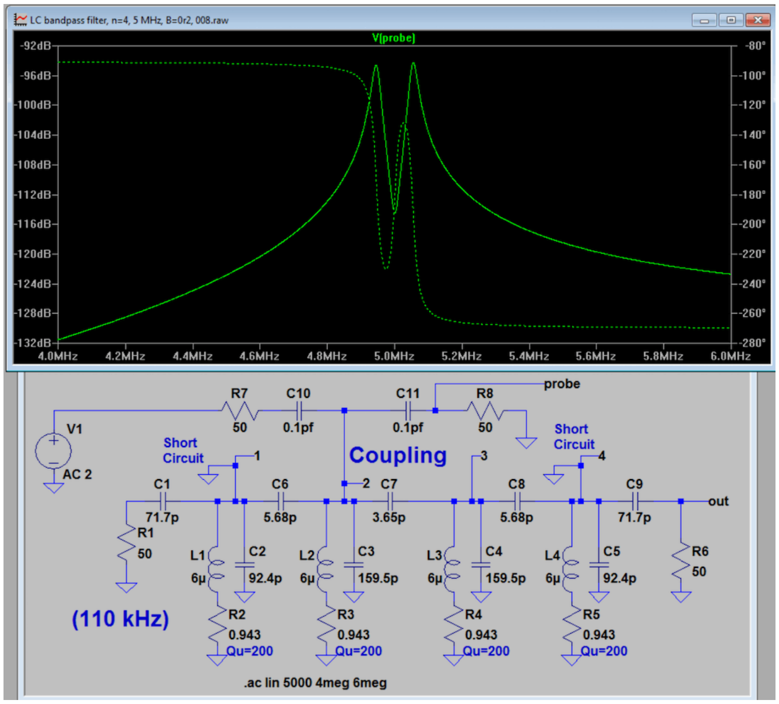

Fig 5-4. This simulation shows how we might measure coupling between resonators. The setup is just like that used above to measure the loaded Q of an end section. A resonator is excited with a probe and another is used to measure the voltage across the same tuned circuit. In this example, the two probes are attached to resonator #2. We have short circuited resonators #1 and #4, so the tuning of resonators 2 and 3 is preserved. Sweeping the system of two probes attached to #2 shows the coupling from 2 to 3, indicated by a double hump response. If coupling capacitor C7 is increased, the separation between peaks will increase. This behavior is specified by

$$\delta f_{23} = k_{23}\,\mathrm{bw} \qquad \text{Eq. 5-14}$$

The frequency spacing δf_{23} is related to the filter bandwidth and the normalized coupling coefficient, k_{23}. When we zoomed in to do a careful look at the peaks, we confirmed this calculated result. Equation 5-14 can be used to measure a coupling coefficient.

The methods presented here are very powerful, for they show how computer simulation can be used to study circuit behavior and fundamental concepts. But this is not a *measurement*. An actual physical measurement will often take less time, but provide more information. The physical examination includes the effects of stray L and C as well as loss. When dealing with filters, there are three levels to the problem: design or synthesis, analysis or simulation, and physical measurement. The best filter circuits should have good correspondence between all three. Don't skip any of the steps. Especially avoid the trap of assuming that a computer simulation can erase the need for physical measurement.

6. Double Tuned Circuit using Series Resonators

The next design example is for a DTC using series tuned circuits. The schematic is shown below.

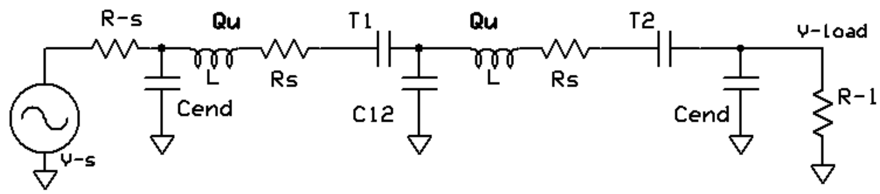

Fig 6-1.

The set up details are given with

$$f_c = 5, \mathrm{BW} = 0.2, \omega = 2\pi f_c 10^6, Q_u = 200, R_0 = 50, L = 7\cdot10^{-6}, k_{12} = \frac{1}{\sqrt{2}}, q = \sqrt{2}$$

We continue to design filters with a 200 kHz bandwidth at 5 MHz with unloaded resonator Q of 200. The same Butterworth coefficients are used. This example uses 7 μH inductors. The design begins with calculation of the nodal capacitance,

$$C_0 = \frac{1}{\omega^2 L}$$ Eq. 6-1.

Next, we calculate the coupling capacitor and the end section Q. The coupling cap is now a large valued shunt element.

$$C_{12} = \frac{C_0}{k_{12}\frac{\mathrm{BW}}{f_c}}$$ Eq. 6-2. $$Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{qf_c} - \frac{1}{Q_u}\right)}$$ Eq. 6-3. $$R_s = \omega\frac{L}{Q_e}$$ Eq. 6-4.

This low value series R is realized from a 50 ohm termination with a shunt end capacitor,

$$C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}}$$ Eq. 6-5.

While this value generates the transformation, this is not the series capacitor that results from a parallel to series conversion. The equivalent series value is

$$C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2}$$ Eq. 6-6. These equations were derived in Section 3.

This C_{es} value allows tuning of the first resonator with T1.

$$T_1 = \frac{1}{\left(\frac{1}{C_0} - \frac{1}{C_{es}} - \frac{1}{C_{12}}\right)}$$ Eq. 6-7.

The second resonator is identical to the first one.

The response of the double tuned circuit with a series LC is almost identical to the one using parallel circuits. The one with the series tuned circuits will drop off a little faster on the high frequency side. The two are compared below.

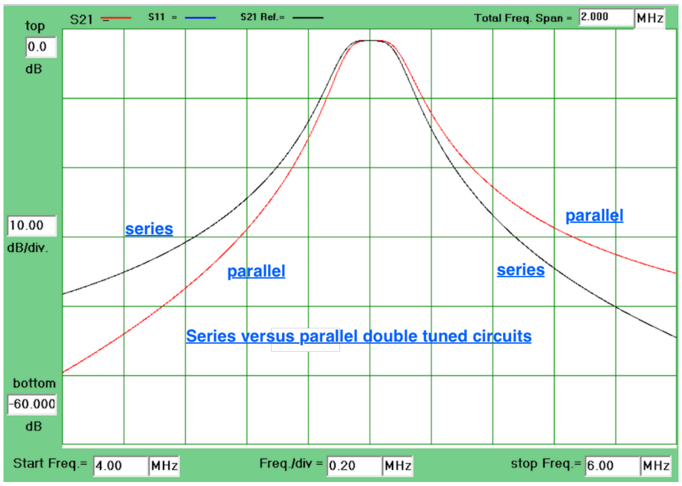

Fig 6-2. Comparison between DTCs using series versus parallel tuned circuits. Both filters have a 200 kHz bandwidth centered at 5 MHz with inductors having an unloaded Q of 200. The insertion loss is the same for both cases.

7. N=4 Bandpass Filter with Series LC Tuned Circuits.

Having designed a DTC with series resonators, we now consider a higher order circuit with four resonators. The "set-up" details follow. This example uses 12 µH inductors and retains the Butterworth shape.

$$f = 5, \mathrm{BW} = 0.2, \omega = 2\pi f 10^{6}, Q_u = 200, R_0 = 50, L = 12\cdot 10^{-6}$$

$$k_{12} = 0.841, k_{23} = 0.541, k_{34} = k_{12}, q = 0.7654$$

Design begins by calculating the nodal capacitance and then the three coupling values.

$$C_0 = \frac{1}{\omega^2 L}$$ Eq. 7-1

$$C_{12} = \frac{C_0}{k_{12}\frac{\mathrm{BW}}{f}}$$ Eq. 7-2.

$$C_{23} = \frac{C_0}{k_{23}\frac{\mathrm{BW}}{f}}$$ Eq. 7-3

$$C_{34} = \frac{C_0}{k_{34}\frac{\mathrm{BW}}{f}}$$ Eq. 7-4

Next we calculate the end Q and the related series resistance.

$$Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{qf} - \frac{1}{Q_u}\right)}$$ Eq. 7-5.

$$R_s = \omega\frac{L}{Q_e}$$ Eq. 7-6.

The end resistance is generated from a 50 ohm termination with a shunt end capacitance, with the related series capacitance needed to calculate tuning.

$$C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}}$$ Eq. 7-7.

$$C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2}$$ Eq. 7-8.

We can now tune the filter.

$$T_1 = \frac{1}{\frac{1}{C_0} - \frac{1}{C_{es}} - \frac{1}{C_{12}}} \quad \text{Eq. 7-9.}$$

$$T_2 = \frac{1}{\frac{1}{C_0} - \frac{1}{C_{23}} - \frac{1}{C_{12}}} \quad \text{Eq. 7-10.}$$

$$T_3 = \frac{1}{\frac{1}{C_0} - \frac{1}{C_{23}} - \frac{1}{C_{34}}} \quad \text{Eq. 7-11.}$$

$$T_4 = \frac{1}{\frac{1}{C_0} - \frac{1}{C_{es}} - \frac{1}{C_{34}}} \quad \text{Eq. 7-12.}$$

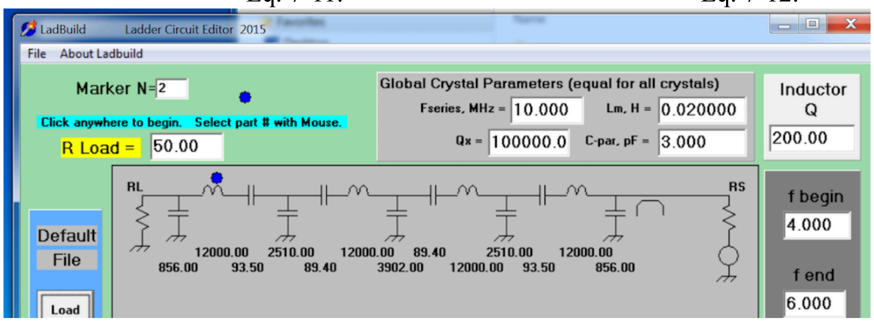

Fig 7-1. Schematic diagram for the finished design. This is a partial screen of a schematic editing computer program used to generate the file for the plot that follows. The simulated response for this filter is shown below. (Ignore the crystal parameters in Fig 7-1, for this filter uses no crystals.)

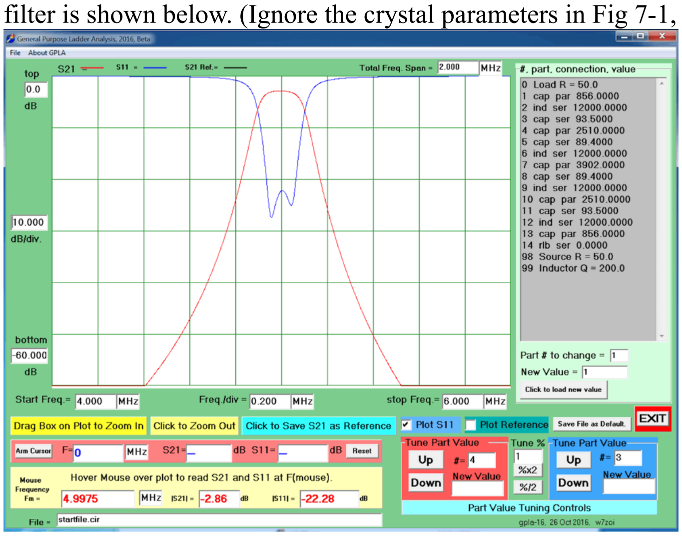

Fig 7-2.

8. Crystals, Crystal Filters, a N=2 Lower Sideband Ladder

We continue the quest to design filters, but now move to circuits using the quartz crystal. Fig 8-1 shows the familiar model for a quartz crystal. The crystal model has a very high motional inductance, L_m, that is tuned to a series resonant frequency, f_s, with a motional capacitance, C_m. The loss is modeled with an *equivalent series resistance*, ESR. The series resonator is bridged by a parallel capacitance, C_0.

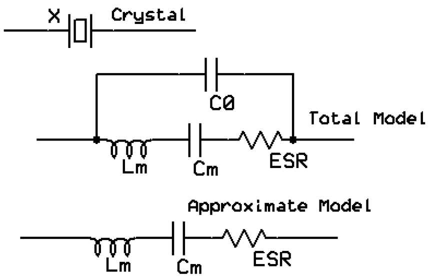

Fig 8-1. The approximate model is the series resonator portion of the more complete model. This model will be used for filter designs.

The model components can be inferred from a network analyzer sweep. If the crystal is treated as a series element, an analyzer sweep will produce an amplitude response as shown in Fig 8-2.

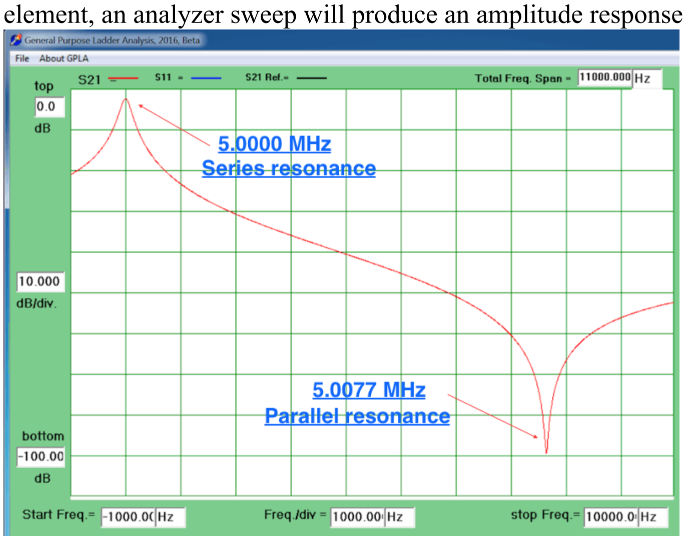

Fig 8-2. Network analyzer sweep of a crystal as a series element.

The crystal used for the sweep of Fig 8-2 has the parameters shown below. The inductor units are Henry. The motional capacitance, just over .01 pF, is the value that tunes the series arm of the model to the series resonant frequency of 5 MHz. It is important to calculate C_m with precision, for this will determine the filter response when simulated. C_m for this crystal is 10.132 fF, or .010132 pF.

$$f_s = 5 \cdot 10^6, L_m = 0.1, C_0 = 3.3 \cdot 10^{-12}, Q_u = 100000, \omega = 2\pi f_s$$

The parallel resonant frequency is give by

$$f_p = f_s \sqrt{1 + \frac{C_m}{C_0}}$$

Eq. 8-1, 7.7 kHz above series for this example.

Our first crystal filter will be a 300 Hz wide Butterworth with the N=2 parameters of k_{12}=0.7071 and q=1.414. We use a 200 ohm termination. This filter is of a form called the ***Lower Sideband Ladder***. This filter has a symmetric shape when the bandwidth is narrow. However, as bandwidth increases, the filter will tend to pass frequencies below parallel crystal resonance with poor low frequency attenuation, but excellent stopband attenuation above parallel resonance. The LSB Ladder circuit uses the crystals as series elements. The motional capacitance, central to filter designs, is given by

$$C_m = \frac{1}{\omega^2 L_m}$$ Eq 8-2

$$Q_F = \frac{f_s}{\mathrm{BW}}$$ Eq 8.3.

$$C_{12} = \frac{C_m Q_F}{k_{12}}$$ Eq. 8-4.

$$Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{qf_s} - \frac{1}{Q_u}\right)}$$ Eq. 8-5.

$$R_s = \omega \frac{L_m}{Q_e}$$ Eq. 8-6.

$$C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s \omega^2 R_0^2}}$$ Eq. 8-7.

C_{end} is a shunt capacitance. This combines with the terminating resistance to form an equivalent series RC with a capacitance of C_{es} and resistance R_s.

$$C_{es} = \frac{C_{end}^2 \omega^2 R_0^2 + 1}{C_{end} \omega^2 R_0^2}$$ Eq. 8-8.

The total capacitance in one node will allow us to calculate that resonant frequency of that node. This then determines the filter center frequency.

$$C_{net} = \frac{1}{\frac{1}{C_{es}} + \frac{1}{C_{12}} + \frac{1}{C_m}}$$ Eq. 8-9.

The resonant frequency becomes

$$f_1 = \frac{1}{2\pi \sqrt{L_m C_{net}}}$$ Eq. 8-10.

This is 190 Hz above the crystal series resonance.

The filter is designed, so it's time for analysis.

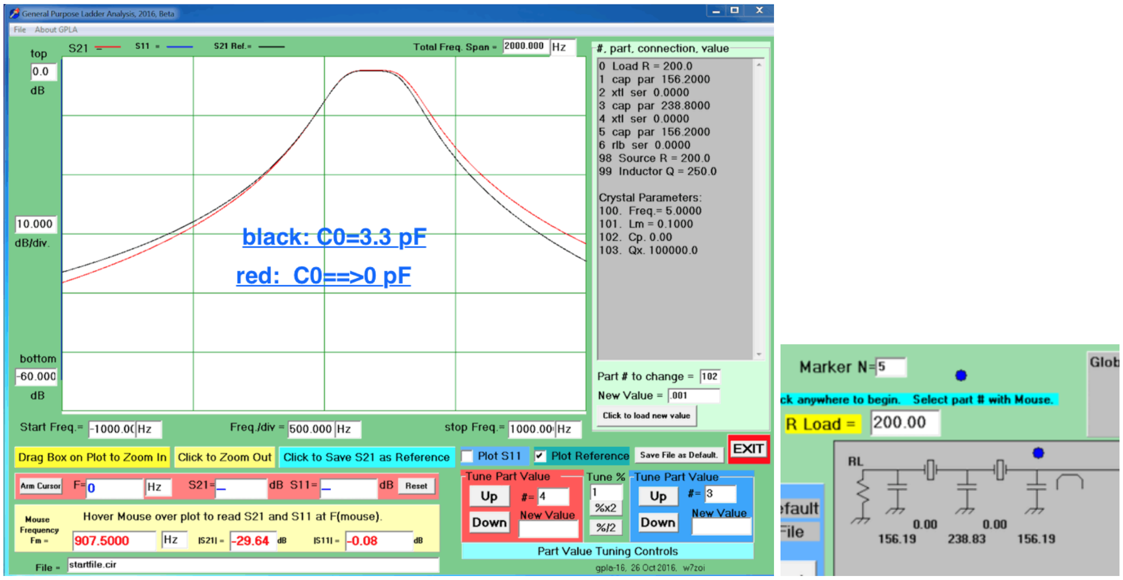

Fig 8-3. Two plots are shown. The red one is the filter with pure series resonators. That is, C_0 is set to zero. This was the element used for the design. The black trace shows the response when the 3.3 pF parallel capacitors in the more complete crystal model is included in the analysis. There is a small, but detectable difference. The schematic is shown to the right of the plot.

The bandwidth was changed from 300 Hz to 1 kHz in the next design,.

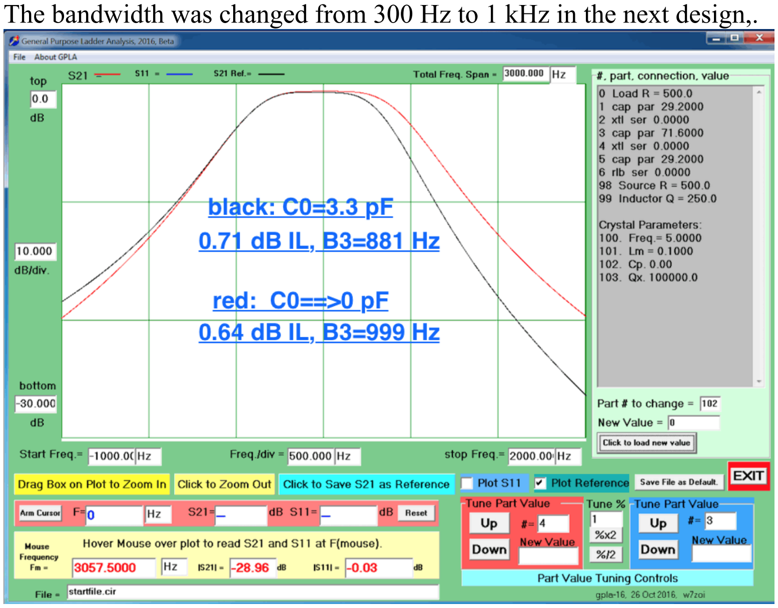

Fig 8-4. The two plots for the 1 kHz wide filter are similar to the previous figure showing the response with and without crystal parallel capacitance. The parallel C now has a greater impact on the results.

The final double tuned circuit crystal filter has a bandwidth of 2500 Hz, but still with a 5 MHz center.

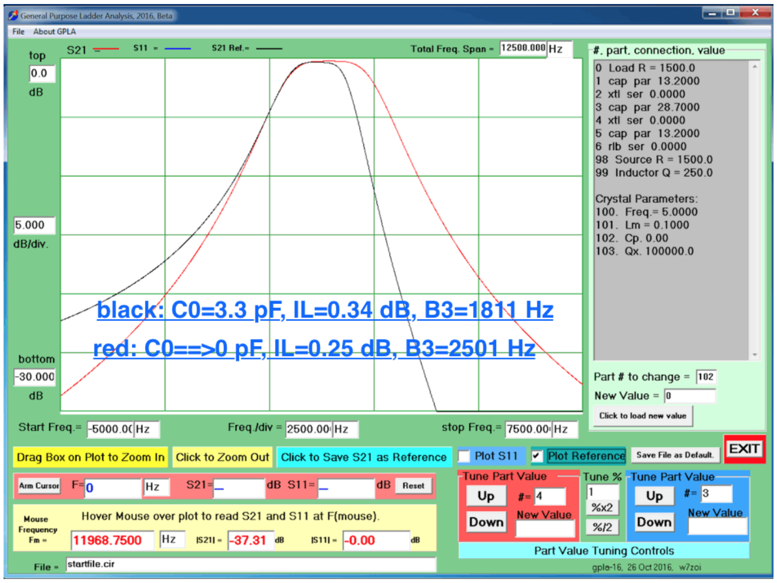

Fig 8-5. The 2500 Hz BW filter becomes much narrower when the C0 is included in the analysis. Moreover, the characteristic *lower sideband* ladder shape is becoming more apparent. The high frequency skirt is becoming steeper while that on the lower frequency edge is degraded.

9. A N=4 Lower Sideband Ladder Crystal Filter

The next example is a LSB Ladder crystal filter. We continue our family of 5 MHz circuits with a filter bandwidth of 1 kHz. Most of the design is like the double tuned circuit, but some tuning details emerge with the higher order. Crystal details and Butterworth parameters are

$$f_s = 5\cdot 10^6, L_m = 0.1, C_0 = 3.3\cdot 10^{-12}, Q_u = 100000, \omega = 2\pi f_s, \mathrm{BW} = 1000$$

The k and q values for this Butterworth filter are

$$k_{12} = 0.841, k_{23} = 0.541, k_{34} = k_{12}, q = 0.7654$$

$$C_m = \frac{1}{\omega^2 L_m} \quad \text{Eq. 9-1} \qquad Q_F = \frac{f_s}{\mathrm{BW}} \quad \text{Eq. 9-2}$$

$$C_{12} = \frac{C_m Q_F}{k_{12}} \quad \text{Eq. 9-3.} \qquad C_{23} = \frac{C_m Q_F}{k_{23}} \quad \text{Eq. 9-4} \qquad C_{34} = \frac{C_m Q_F}{k_{34}} \quad \text{Eq. 9-5.}$$

The coupling capacitors are now defined, leaving the ends to be specified.

$$Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{q f_s} - \frac{1}{Q_u}\right)} \quad \text{Eq. 9-6.} \qquad R_s = \omega \frac{L_m}{Q_e} \quad \text{Eq. 9-7.} \qquad C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s \omega^2 R_0^2}} \quad \text{Eq. 9-8.}$$

All of the shunt capacitors are now specified. This is shown in part A of the schematic, Fig 9-1 below. But there is more to do. Using the previously examined equations, the series C resulting from our parallel to series transformation is

$$C_{es} = \frac{C_{end}^2 \omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2} \quad \text{Eq. 9-9.} \qquad C_{net} = \frac{1}{\frac{1}{C_{es}} + \frac{1}{C_{12}} + \frac{1}{C_m}} \quad \text{Eq. 9-10} \qquad C_1 = C_{net} \quad \text{Eq 9-11.}$$

Equations 9 and 10 give us the series capacitance and the net C of the first mesh. These allow calculation of the resonant frequency of mesh #1. This is shown in Fig 9-1B. We have defined C_1 as the net C related to mesh #1. When calculating a frequency of a mesh, all other meshes are open-circuited. The result is then

$$f_1 = \frac{1}{2\pi\sqrt{L_m C_1}} \quad \text{Eq. 9-12.} \qquad f_4 = f_1 \quad \text{Eq. 9-13.}$$

The first and last meshes are resonant 745 Hz above crystal f_s.

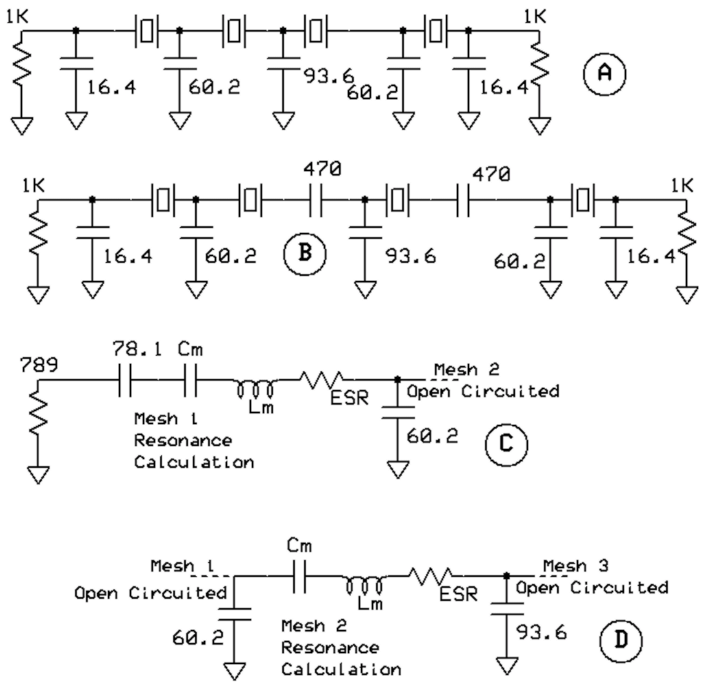

Fig. 9-1.

The next step is to evaluate the resonant frequency of mesh #2, shown in Fig 9-1D. We define the total mesh capacitance as C_2. Mesh frequency can then be calculated.

$$C_2 = \frac{1}{\frac{1}{C_m} + \frac{1}{C_{12}} + \frac{1}{C_{23}}} \quad \text{Eq. 9-14.} \qquad f_2 = \frac{1}{2\pi\sqrt{L_m C_2}} \quad \text{Eq. 9-15.}$$

The usual Butterworth symmetries apply, so all mesh frequencies are known. We compare them all and note that the highest is mesh 1 (and 4). Meshes 2 and 3 are just a bit lower. The filter will be properly tuned when all meshes are resonant at the same frequency when they are isolated from the others.

$$f_h = f_1 \quad \text{Eq. 9-16.} \qquad \omega_h = 2\pi f_h \quad \text{Eq. 9-17.}$$

With the highest frequency specified, we can calculate the total capacitance needed in any other mesh in order to tune that mesh. In this specific example, we need only apply this for mesh 2. Mesh 3 will then be identical and nothing needs to be done with #1 or #4, for they are already at the high frequency. In a higher order filter with N>4, the equation must be applied to all meshes that are not "highest."

$$C_h = \frac{1}{\omega_h^2 L_m} \quad \text{Eq. 9-18.} \qquad T_2 = \frac{1}{\frac{1}{C_h} - \frac{1}{C_m} - \frac{1}{C_{12}} - \frac{1}{C_{23}}} \quad \text{Eq. 9-19.}$$

The tuning is now done and our final circuit is the filter shown in Fig 9-1B where 470 pF capacitors have been inserted into meshes 2 and 3. The schematic is shown in the figure below.

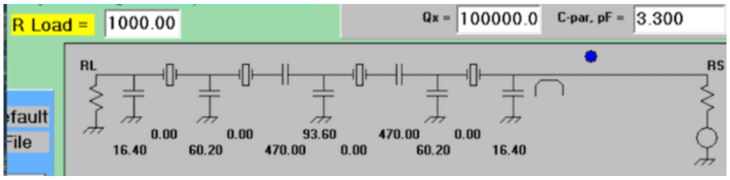

Fig 9-2

The response for this filter is presented in Fig 9-3.

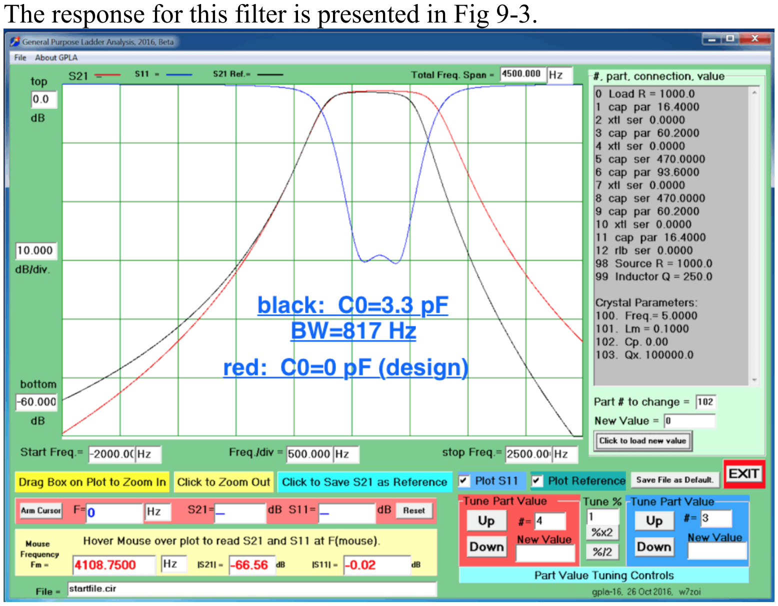

Fig 9-3.

We see the same behavior that we saw with the N=2 filters. We design as if the crystals were pure series resonators, but then simulate the filter with crystals that include C0. The desired Butterworth shape is obtained, but the bandwidth is narrower than desired. In this case, the bandwidth is lower by a factor of 1000/817=1.224. If we design a filter with bandwidth 1224 Hz, hoping to get 1000 Hz, we may do better. That is exactly what we did and it nearly worked. The new filter, shown below, had a bandwidth of 980 Hz, an insertion loss of 1.13 dB, and a Butterworth shape.

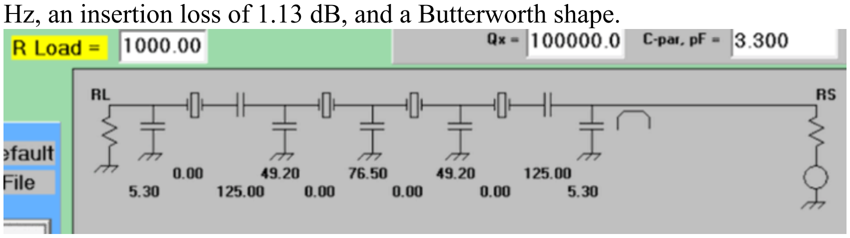

Fig 9-4. The new filter has different shunt capacitors throughout. Moreover, the tuning changed. The highest frequency meshes are now #2 and #3, so capacitance had to be inserted in meshes #1 and #4 to achieve tuning.

Sidebar

The lower sideband ladder topology is extremely powerful, but we often want a better way to do the design than the iterative scheme applied above. Repeated iteration may converge nicely and should be considered. Several years ago we tried an alternative scheme. We started with an **ideal** series resonator using the motional inductance that fit the crystals to be used in our filter. L_M had been measured using the now well known G3UUR method. The ideal series tuned circuit was mathematically evaluated to obtain the reactance slope, the rate of change of reactance with frequency. This slope is a measure of the inductance in the resonator.

Then we calculated the reactance as a function of frequency for an actual crystal, a series resonator **that includes the parallel capacitor**. This allows calculation of an effective inductance based upon the rate of change of reactance. The function that came out of the differentiation is itself frequency dependent. We found that we got the best bandwidth results when the effective inductance function was evaluated at the high frequency end of the desired passband. This method was the basis for many of our crystal filter design programs distributed with publications. The method had been described in QEX, in a reference included in the bibliography.

10. An Upper Sideband Ladder N=2 Crystal Filter.

Most crystal filters built or purchased by RF electronics experimenters have used the LSB ladder circuit. There is a second filter type, the so called Upper Sideband (USB) Ladder which uses crystals as parallel resonators. A schematic is shown below for a N=2 filter.

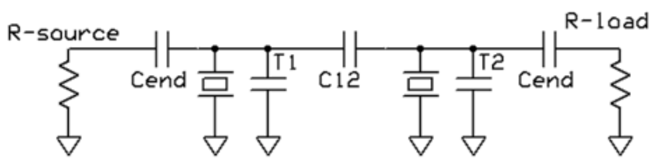

Fig 10-1. Crystal filter with crystals as parallel tuned circuits.

This filter is termed the USB Ladder, for the dominant passband is above a carrier frequency. The attenuation is high below that carrier, often at the price of attenuation above the passband. These shape details will be appreciated after some example filters have been designed and presented. The next figure shows the crystal model in the parallel configuration.

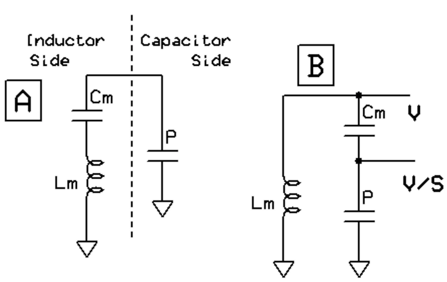

Fig 10-2. A crystal as part of a USB Ladder Filter.

Recall earlier Figure 8-2 that showed the response of a crystal configured as a series element. The dominant path through the crystal was at series resonance where the motional L and motional C resonated with each other. Parallel resonance occurs when the motional inductance resonates with the series combination of the motional and parallel capacitances. At frequencies between f_S and f_P, the series combination of L_M and C_M, will have an inductive reactance. Hence, a parallel tuned circuit can be built with a net parallel capacitance, P. This is Fig 10-2**A**. We are restricted to the frequency range between f_S and f_P. If we tried to operate at a resonant frequency below f_S, the series pair would look capacitive.

Operation close to or above parallel resonance would also bring us into a capacitive region. Crystal parallel capacitance C_0 is absorbed as part of P.

The left side of the figure, **A**, represents the way we will assemble the elements in our circuit. However, Fig 10-2**B** represents the familiar circuit topology where the inductor in a resonator is tuned with a capacitance, now the series combination of C_M and P. Note that P will usually be several pF, for it will include the crystal C0 and whatever other C we might add to build the filter. Motional C_M is very small, around .01 pF in our examples. If we have a voltage V across the internal resonator, as shown in Fig **10-2B**, the voltage at P will be reduced by a **factor S** where S is the capacitance ratio, (P+ C_M)/ C_M . With this correction in mind, we can now design our filter. We will use the close approximation of S=P/ C_M.

The USB Ladder uses slightly different methods than those used for the LSB Ladder. The familiar LSB circuit used the motional capacitance as the nodal capacitance. This was used to calculate coupling capacitors as well as the elements needed for end section loading. After these calculations, the various meshes in the filter were tuned by adding series C to one or more meshes. Tuning was proper when each mesh was at the same frequency with the others open circuited. The approach with the USB filter is different: Each ***node*** must now resonate at the filter center frequency when the other nodes are ***short*** circuited. Looking at the filter circuit, Fig 10-1, we see that capacitor P takes the place of nodal capacitor in an LC filter with parallel resonators. (The reader may want to go back and review the parallel resonator LC Double Tuned Circuit of Section 4.) Capacitor P will include C0, coupling capacitors to adjacent nodes, and the added capacitance T that might be needed to tune the resonator. The nodal C needed for resonance is determined and used to calculate the other parameters. Final tuning removes capacitance from the nodes to achieve resonance at each node.

Our first example, a double tuned circuit, will use 5 MHz crystals to build a 300 Hz wide filter. The same crystals used for some LSB examples will be used. The setup details are

$$f_s = 5\cdot 10^6, L_m = 0.1, C_0 = 3.3\cdot 10^{-12}, \mathrm{BW} = 300, f_c = 5.003\cdot 10^6, Q_u = 100000$$

$$k_{12} = \frac{1}{\sqrt{2}}, q = \sqrt{2}$$

We begin by calculating the crystal parallel resonant frequency from the series resonance,

$$f_p = f_s\sqrt{1 + \frac{C_m}{C_0}} \qquad \text{Eq. 10-1.}$$

This 5 MHz crystal is parallel resonant 7.7 kHz above the series resonance. A filter center frequency, f_C, is picked to be 3 kHz above series resonance. This is about half way between crystal resonances. The angular frequencies are calculated and used to evaluate motional capacitance and to calculate the nodal capacitance for the new resonators.

$$\omega_s = 2\pi f_s \quad \text{Eq 10-2.} \qquad C_m = \frac{1}{(\omega_s^2 L_m)} \quad \text{Eq 10-3.} \qquad \omega_c = 2\pi f_c \quad \text{Eq. 10-4}$$

With this data, we can now calculate the nodal capacitance. This is only a few pF, but is still very much larger than the motional capacitance within the complex resonator.

$$P = \frac{C_m}{(L_m C_m \omega_c^2 - 1)} \qquad \text{Eq 10-5}$$

The next crucial element is the correction factor S discussed above. The approximate form is

$$S = \frac{P}{C_m} \qquad \text{Eq. 10-6.}$$

We next calculate the details related to end loading.

$$Q_f = \frac{f_c}{\mathrm{BW}}$$ Eq. 10-7.

$$Q_e = \frac{1}{\left(\frac{1}{qQ_f} - \frac{1}{Q_u}\right)}$$ Eq. 10-8.

$$R_p = \frac{Q_e}{(\omega_c PS)}$$ Eq. 10-9.

The next formula specifies the coupling capacitor between resonators. The formula mirrors that used with LC parallel tuned circuits except for the correction factor S. This S factor has the effect of increasing the coupling C by almost 1000 times over that of an LC. This is, after some thought, reasonable. The physical resonator consists of the very small motional C and the much larger nodal capacitance P. The voltage across P will be reduced by S, so the coupling capacitance must be increase by S to produce the same coupled current into an adjacent shorted note. The same argument is applied to achieve the parallel resistance, R_P, that loads the end resonators. R_P is the resistance that must parallel an end resonator to establish the needed end Q. First, the coupling capacitance is calculated.

$$C_{12} = \frac{k_{12}}{Q_f} PS$$ Eq. 10-10.

In spite of the large correction factor S, the coupling capacitance is only 0.3 pF. The correction factor has similar impact upon the end capacitors in calculating R_P. First we pick an end termination.

$$R_0 = 500$$

$$C_{end} = \frac{1}{\omega_c \sqrt{R_p R_0 - R_0^2}}$$ Eq. 10-11.

A parallel capacitance is also calculated that is the C presented to the end resonator. This is needed to calculate tuning values.

$$C_p = \frac{C_{end}}{(R_0 \omega_c C_{end})^2 + 1}$$ Eq. 10-12

We now have the end capacitor that couples to the 500 Ohm load. The final step is to calculate the capacitance that must be added to each node to establish the final resonant frequency. We use C_{end} in this calculation even though a parallel equivalent would be proper. $\mathbf{C_p}$ and $\mathbf{C_{end}}$ are very close in value.

$$T_1 = P - C_0 - C_{12} - C_{end}$$ Eq. 10-13.

$$T_2 = T_1$$ Eq. 10-14.

The final filter circuit is

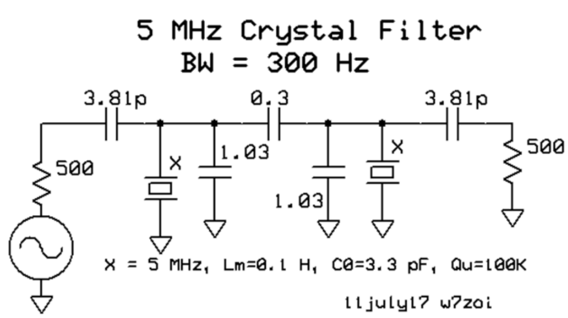

Fig 10-3.

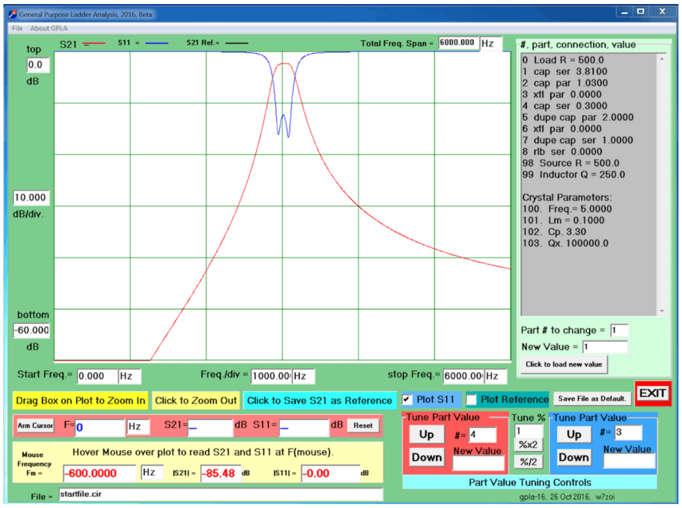

Fig 10-4. Response for a 300 Hz wide USB Ladder Crystal Filter at 5 MHz.

The bandwidth was increased to 1 kHz with this filter, resulting in the following plot.

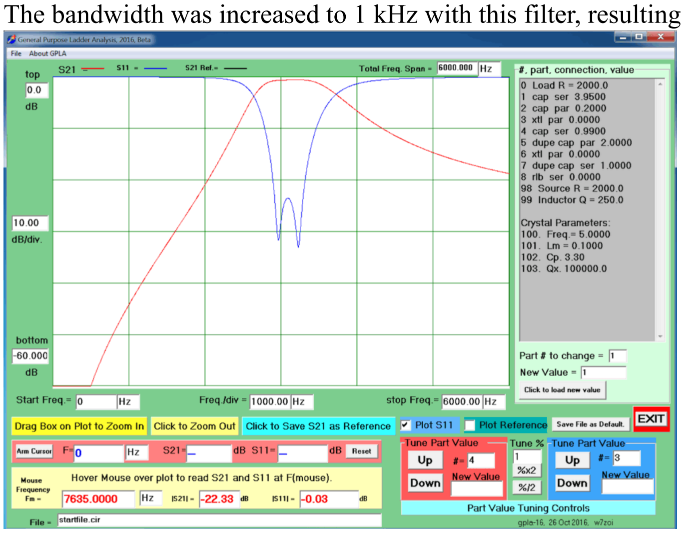

Fig 10-5. Response for a 1 kHz bandwidth filter. The upper sideband shape now becomes evident.

The N=2 Lower Sideband Ladder crystal filter was presented in section 8. The filters designed there had a nearly symmetrical shape. There was an expected *lower sideband* emphasis, but it was slight. This is not true in the filter of Fig 10-5. Study of the component values offers a hint. (Values are shown in the gray box on the right side of the figure.) The coupling capacitor attached to the termination is larger than the others by a considerable margin. This is the proper coupling to set the end section Q, but it also restricts the range available for filter center frequency. The shape is distorted by the behavior of the "inductor" arm of the composite resonator where the reactance changes significantly with frequency, even over the bandwidth of the filter. If the arm consisting of L_m and C_m was a pure inductor, it would have virtually constant reactance over the bandwidth. The question remains: What can be done to achieve better filter response symmetry?

The LSB Ladder was designed with a constraint of a minimum termination value. If the termination was not above a threshold, the filter could not be designed with the simple circuit desired. The USB has greater flexibility, but is still constrained. Note the very high RF resistance value of Eq. 10-9. If we **increase the termination value, R_0**, the coupling capacitor drops and the filter can be designed for a higher center frequency. This effect is illustrated with a slightly altered design. We expanded the bandwidth from 300 to 400 Hz, but kept the filter at 5 MHz with the same crystals. However, the termination is increased to 2000 Ohms. Four different filter center frequencies are then used: 1, 2, 3, and 5 kHz above crystal series resonance.

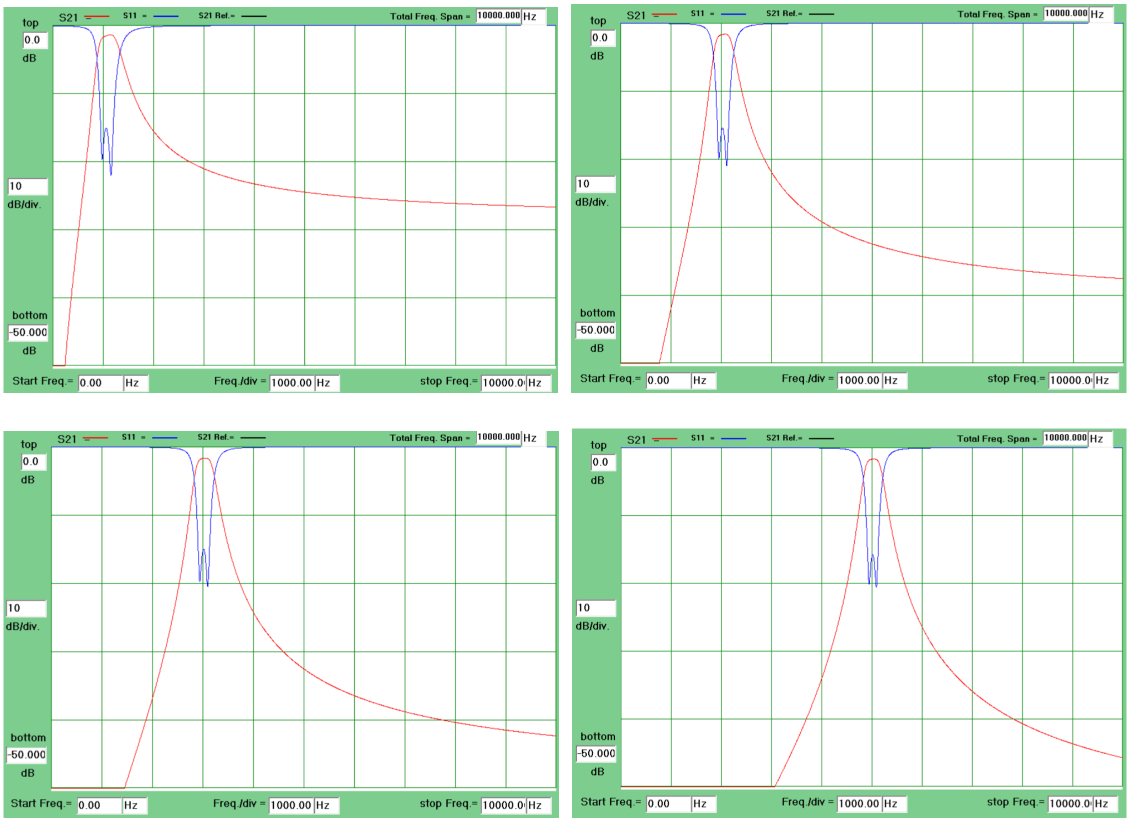

Fig 10-6. Crystal filters with two crystals, with increased termination resistance, which extends the center frequency range and allows construction of filters with improved response symmetry.

11. An Upper Sideband Ladder Crystal Filter with N=4 at 10 MHz.

The filters considered so far have all been at 5 MHz. We now switch to 10 MHz to further investigate some of the behavior of the USB Ladder topology. The initial data for a 1 kHz bandwidth filter, including the Butterworth k and q data, are presented.

$$f_s = 10\cdot 10^6, L_m = 0.02, C_0 = 3.3\cdot 10^{-12}, f_c = 10.006\cdot 10^6, Q_u = 100000, \mathrm{BW} = 1\cdot 10^3$$

$$k_{12} = 0.841, k_{23} = 0.541, k_{34} = k_{12}, q = 0.7654$$

We begin the design with a calculation of the parallel resonant frequency.

$$f_p = f_s\sqrt{1 + \frac{C_m}{C_0}} \qquad \text{Eq. 11-1.}$$

The parallel resonance is 19 kHz above the series for this crystal, leaving more room than was available with the 5 MHz crystals. We pick an initial filter center frequency 6 kHz above series. The filter schematic is shown below, followed by the angular frequencies.

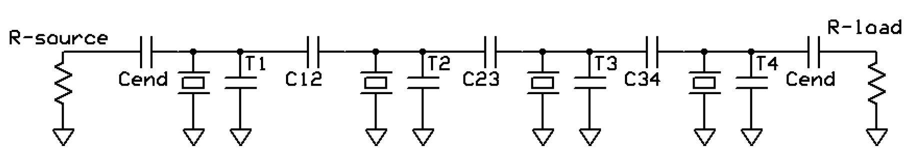

Fig 11-1. All crystals are assumed to be identical.

$$\omega_s = 2\pi f_s$$ Eq 11-2. $$C_m = \frac{1}{(\omega_s^2 L_m)}$$ Eq 11-3. $$\omega_c = 2\pi f_c$$ Eq 11-4.

The data is now available to calculate the nodal capacitance,

$$P = \frac{C_m}{(L_m C_m \omega_c^2 - 1)}$$ Eq. 11-5.

$$S = \frac{P}{C_m}$$ Eq. 11-6.

The resonator loading details follow.

$$Q_f = \frac{f_c}{\mathrm{BW}}$$ Eq. 11-7. $$R_p = \frac{Q_e}{(\omega_c PS)}$$ Eq 11-8.

The coupling capacitors are calculated next.

$$C_{12} = \frac{k_{12}}{Q_f} PS$$ Eq 11-9. $$C_{23} = \frac{k_{23}}{Q_f} PS$$ Eq. 11-10. $$C_{34} = C_{12}$$ Eq 11-11.

End loading details follow, including the capacitance reflected back to the resonator from the load when coupled to the load through a series C. (Note that C_e and C_P are nearly the same.) Set R_0=400.

$$Q_e = \frac{1}{\left(\frac{1}{qQ_f} - \frac{1}{Q_u}\right)}$$ Eq 11-12. $$C_e = \frac{1}{\omega_c \sqrt{R_p R_0 - R_0^2}}$$ Eq 11-13. $$C_p = \frac{C_e}{1 + (R_0 \omega_c C_e)^2}$$ Eq 11-14.

Tuning capacitors can now be calculated.

$$T_1 = P - C_0 - C_{12} - C_p$$ Eq 11-15.

$$T_2 = P - C_0 - C_{12} - C_{23}$$ Eq 11-16.

$$T_3 = P - C_0 - C_{34} - C_{23}$$ Eq 11-17.

$$T_4 = P - C_0 - C_{34} - C_p$$ Eq 11-18.

We now consider a variety of initial conditions.

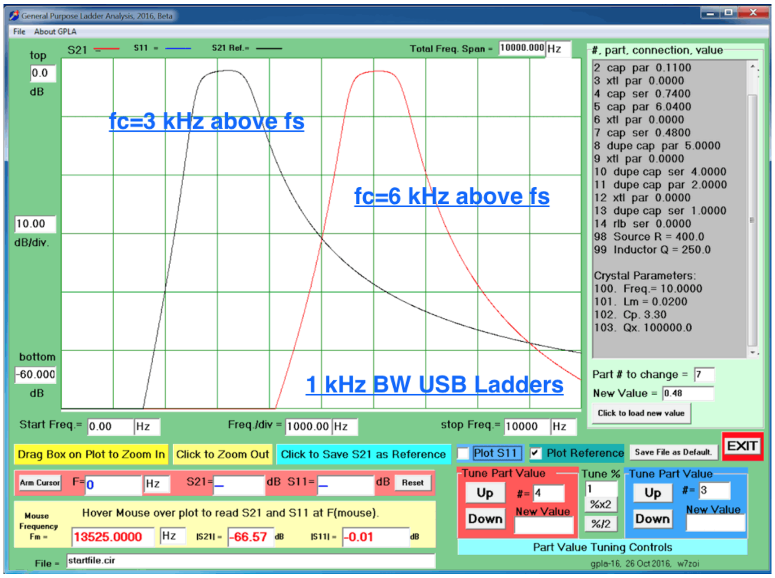

Fig 11-2. The design filter, 1 kHz bandwidth with center frequency 6 kHz above series resonance, is shown in red. The black response is the result of changing the center frequency to 3 kHz above series. The low frequency skirt gets steeper as f_C approaches F_S while the upper skirt becomes less steep.

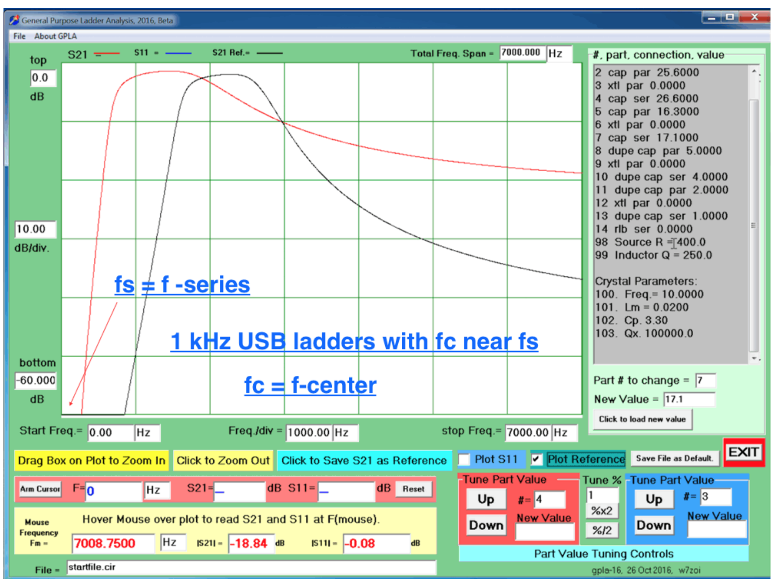

Fig 11-3. This is a more extreme example of moving center frequency toward the series resonance.

The next figure expands the bandwidth to 3 kHz, suitable for SSB applications. Performance is further degraded. Two center frequencies were used: 3 and 5 kHz above series crystal resonance.

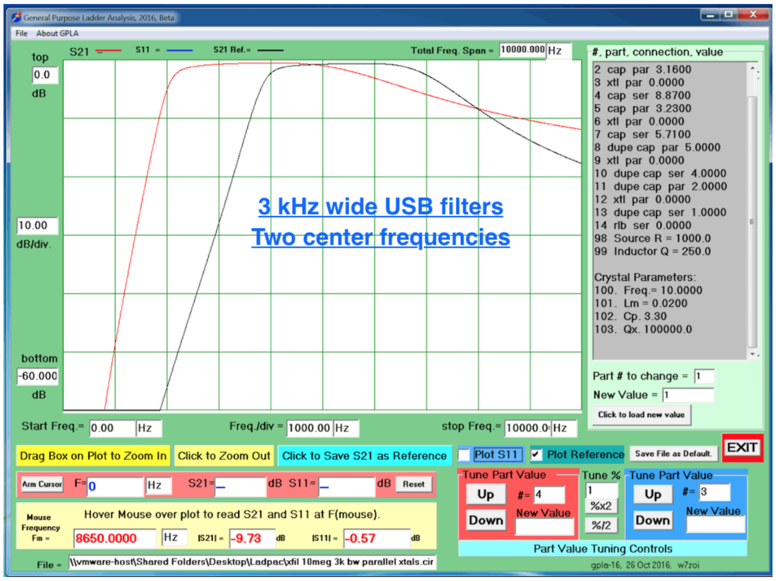

Fig 11-4.

The result was disappointing.   However, at this point we recall the results obtained with the N=2 USB crystal filter.   Increasing the terminating resistance values provided greater flexibility in the choice of center frequency while also improving response shape symmetry.   Increasing the termination from 400 to 2000 ohms will make a profound difference and provide greater flexibility

The response shown below illustrates this greater flexibility.   Another SSB filter was designed, this one with a much higher termination.   The center frequency was moved up to 8 kHz above crystal series resonance.   Performance is now much better.   A companion LSB ladder filter was designed, providing a comparison between the topologies. Both had a 3 kHz BW with the same 10 MHz crystals.   The LSB filter is a routine design with 800 Ohm terminations while the USB circuit was designed with a load of 8400 ohms.   The two filters are nearly mirror images of each other.   Moving the USB filter center frequency down would lead to filters nicely spaced around a single BFO, the requirement for ISB, or ***Independent Sideband*** reception.

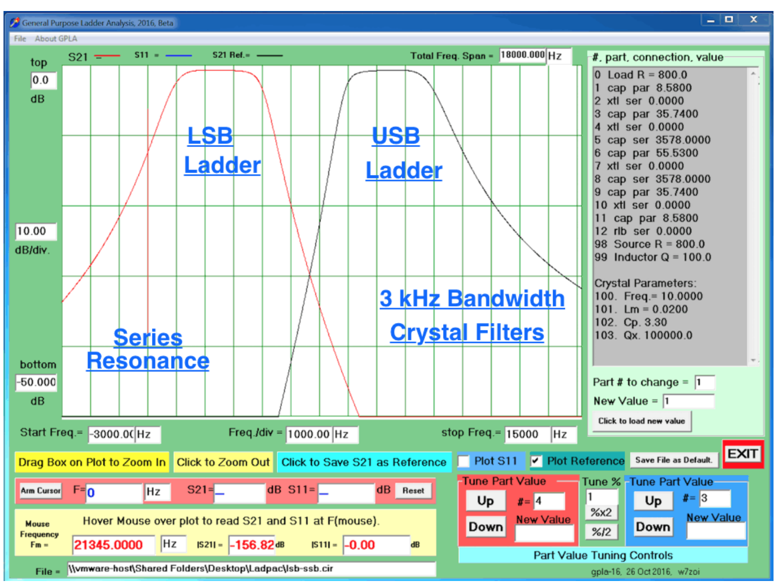

Fig 11-5.   Comparison between LSB and USB filters, 3 kHz bandwidth.

The schematic diagrams for both filters are shown below.

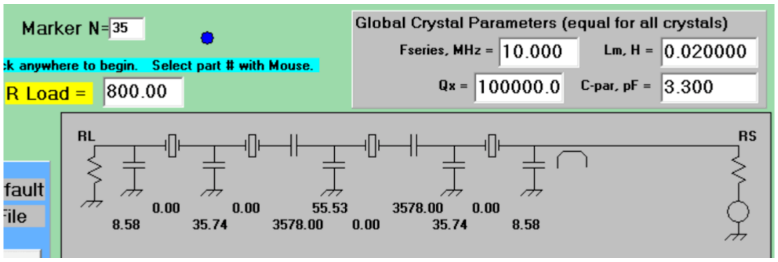

Fig 11-6. Lower Sideband Ladder, 3 kHz Bandwidth.

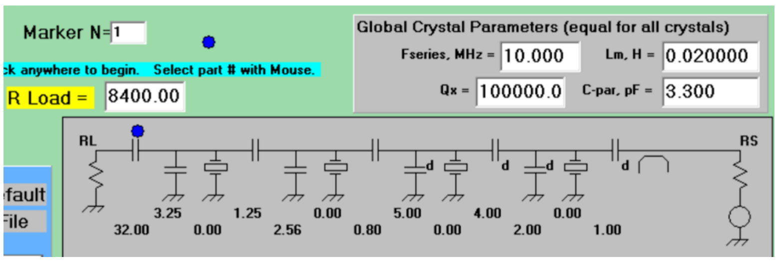

Fig 11-7. Upper Sideband Ladder, 3 kHz Bandwidth.

Consider the component values in both filters. First, the very large (almost 3600 pF) series tuning capacitors in the LSB ladder can probably be replaced with a short circuit with little change in performance. All other components in that filter are very manageable in value.

The USB filter is not as friendly. We see several very small valued parts, which are difficult. Not only are such parts rarely available in discrete form, but the low value exacerbates the impact of stray capacitance. The high termination that was picked is possible, but messy. The end 32 pF capacitors (components marked with "d" on the schematic are duplicates of others in the circuit) would normally cause tuning problems, but do not detune the filter owing to the high terminating impedance.

12. Crystal Notch Filters

Although unusual, the crystal notch filter can be a handy circuit for both communications and measurement applications. Simple filters of this sort are easily designed starting with Fig 12-1.

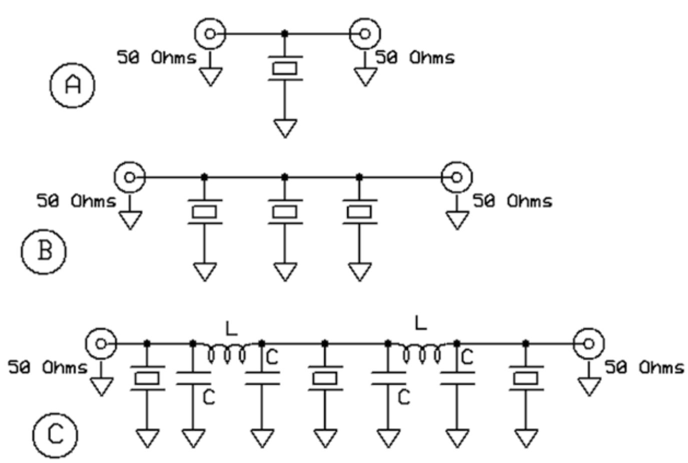

Fig 12-1. Evolution of a crystal notch ladder filter.

The simplest notch circuit places a crystal as a parallel notch element in a 50 Ohm system as shown in Fig 12-1A. The depth of the notch can be surprisingly good, depending upon the crystal unloaded Q. Indeed, this is the basis of a method for the measurement of crystal Q. If the parameters describing the 5 MHz crystal used in many other filters in this note are used here, a single crystal produces a notch with a 5.1 dB depth. The 5 MHz crystal assumed here has a motional L of 0.1 Henry, C0=3.3 pF, and Q_U=100,000. Additional crystals can be added in parallel with the original to produce a slight improvement. The circuit of Fig 12-1B produces a 10.6 dB notch depth with the same crystals. Several crystals in parallel serve to alter the equivalent circuit of the original, resulting in the behavior of one altered crystal.

Three crystals are decoupled from each other if they are separated with a circuit providing 90 degrees of phase difference. This is realized with a quarter wavelength of transmission line or with an equivalent CLC π network where the reactance each of the elements is the characteristic resistance terminating resistance of the ladder. This is illustrated in Fig 12-1C. The 50 ohm elements at 5 MHz would be C=637 pF and L=1.59 µH. This circuit produces a notch depth of 18 dB. Further enhancement results if the terminating resistance is altered as shown in Figures 12-2 and 12-3.

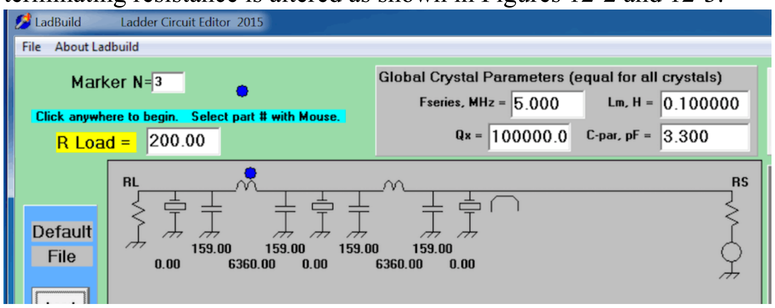

Fig 12-2. A notch filter with three crystals in a 200 Ohm environment.

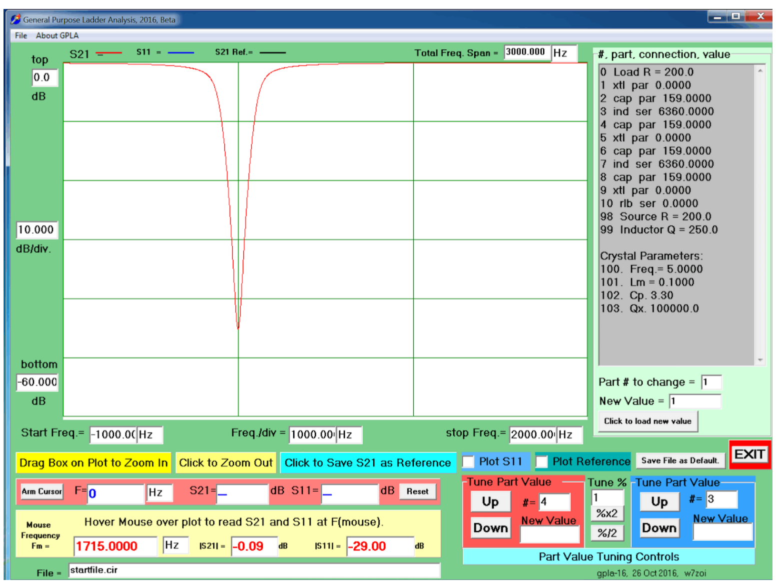

Fig 12-3. The response of the circuit in Fig 12-2. The notch depth is now 45 dB. Notice that the π network components change with the new termination. An 800 Ohm system yield a notch depth of 79 dB, still with just 3 crystals. Added crystals will produce even deeper notches with more phase shifting networks between them.

13. Bibliography.

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9. Hayward, Oscillator Noise Evaluation with a Crystal Notch Filter, QEX, July/August, 2008.

14. Appendix A: Equations with calculation results. The numbers correspond to sections in the main text. This appendix shows numeric results related to the examples.

A4. L/C DTC with parallel resonators.

$$\omega = 2\pi f_c \qquad C_0 = \frac{1}{\omega^2 L} \qquad Q_F = \frac{f_c}{\text{bw}} \qquad q_o = \frac{Q_u}{Q_F}$$

$$\omega = 3.14159e{+}07 \qquad C_0 = 3.37737e{-}10 \qquad Q_F = 25 \qquad q_o = 8$$

$$Q_{end} = qQ_F \qquad Q_e = \frac{1}{\left(\frac{1}{qQ_F} - \frac{1}{Q_u}\right)} \qquad C_{12} = C_0\frac{k_{12}}{Q_F}$$

$$Q_{end} = 35.3553 \qquad Q_e = 42.9474 \qquad C_{12} = 9.55265e{-}12$$

$$R_{pe} = Q_e\omega L \qquad C_e = \frac{1}{\omega\sqrt{R_{pe}R_0 - R_0^2}} \qquad T_1 = C_0 - C_{12} - C_e \qquad \text{IL} = 20\log\left(\frac{q_o}{q_o - q}\right)$$

$$R_{pe} = 4047.7 \qquad C_e = 7.11967e{-}11 \qquad T_1 = 2.56988e{-}10 \qquad \text{IL} = 1.68965$$

A5. L/C n=4 with parallel resonators.

$$C_0 = \frac{1}{\omega^2 L} \qquad Q_F = \frac{f_c}{\text{bw}} \qquad Q_{end} = \frac{1}{\left(\frac{1}{qQ_F} - \frac{1}{Q_u}\right)}$$

$$C_0 = 1.68869e{-}10 \qquad Q_F = 25 \qquad Q_{end} = 21.1594$$

$$C_{12} = C_0\frac{k_{12}}{Q_F} \qquad C_{23} = C_0\frac{k_{23}}{Q_F} \qquad C_{34} = C_0\frac{k_{34}}{Q_F}$$

$$C_{12} = 5.68074e{-}12 \qquad C_{23} = 3.65432e{-}12 \qquad C_{34} = 5.68074e{-}12$$

$$R_{pe} = Q_{end}\omega L \qquad C_e = \frac{1}{\omega\sqrt{R_{pe}R_0 - R_0^2}} \qquad C_p = \frac{C_e}{1 + (R_0\omega C_e)^2}$$

$R_{pe} = 3988.46$ $\quad C_e = 7.17302\text{e-}11$ $\quad C_p = 7.0831\text{e-}11$

$$T_1 = C_0 - C_{12} - C_p \qquad T_2 = C_0 - C_{12} - C_{23}$$

$T_1 = 9.23569\text{e-}11$ $\quad T_2 = 1.59534\text{e-}10$ $\quad T_3 = T_2$

$T_4 = T_1$ $\quad \delta f_{23} = k_{23}\mathrm{bw}$ $\quad \delta f_{23} = 0.1082$

A6. DTC L/C with series resonators.

$$C_0 = \frac{1}{\omega^2 L} \qquad C_{12} = \frac{C_0}{k_{12}\dfrac{\mathrm{BW}}{f_c}} \qquad Q_e = \frac{1}{\left(\dfrac{\mathrm{BW}}{qf_c} - \dfrac{1}{Q_u}\right)} \qquad R_s = \omega\frac{L}{Q_e}$$

$C_0 = 1.44745\text{e-}10$ $\quad C_{12} = 5.11749\text{e-}09$ $\quad Q_e = 42.9474$ $\quad R_s = 5.12048$

$$C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}} \qquad C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2} \qquad T_1 = \frac{1}{\left(\dfrac{1}{C_0} - \dfrac{1}{C_{es}} - \dfrac{1}{C_{12}}\right)}$$

$C_{end} = 1.88473\text{e-}09$ $\quad C_{es} = 2.09977\text{e-}09$ $\quad T_1 = 1.60332\text{e-}10$

A7. n=4 L/C with series resonators.

$$C_0 = \frac{1}{\omega^2 L} \qquad C_{12} = \frac{C_0}{k_{12}\dfrac{\mathrm{BW}}{f}} \qquad C_{23} = \frac{C_0}{k_{23}\dfrac{\mathrm{BW}}{f}} \qquad C_{34} = \frac{C_0}{k_{34}\dfrac{\mathrm{BW}}{f}}$$

$C_0 = 8.44343\text{e-}11$ $\quad C_{12} = 2.50994\text{e-}09$ $\quad C_{23} = 3.90177\text{e-}09$ $\quad C_{34} = 2.50994\text{e-}09$

$$Q_e = \frac{1}{\left(\dfrac{\mathrm{BW}}{qf} - \dfrac{1}{Q_u}\right)} \qquad R_s = \omega\frac{L}{Q_e} \qquad C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}} \qquad C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2}$$

$Q_e = 21.1594$ $\quad R_s = 17.8167$ $\quad C_{end} = 8.55622\text{e-}10$ $\quad C_{es} = 1.32929\text{e-}09$

$$T_1 = \frac{1}{\dfrac{1}{C_0} - \dfrac{1}{C_{es}} - \dfrac{1}{C_{12}}} \qquad T_2 = \frac{1}{\dfrac{1}{C_0} - \dfrac{1}{C_{23}} - \dfrac{1}{C_{12}}} \qquad T_3 = \frac{1}{\dfrac{1}{C_0} - \dfrac{1}{C_{23}} - \dfrac{1}{C_{34}}} \qquad T_4 = \frac{1}{\dfrac{1}{C_0} - \dfrac{1}{C_{es}} - \dfrac{1}{C_{34}}}$$

$T_1 = 9.35206\text{e-}11$ $\quad T_2 = 8.9375\text{e-}11$ $\quad T_3 = 8.9375\text{e-}11$ $\quad T_4 = 9.35206\text{e-}11$

A8. n=2 Crystal Filter, LSB Ladder, Series Crystals.

$$f_p = f_s\sqrt{1 + \frac{C_m}{C_0}} \qquad C_m = \frac{1}{\omega^2 L_m} \qquad Q_F = \frac{f_s}{\mathrm{BW}}$$

$$f_p = 5.00767\text{e+}06 \qquad C_m = 1.01321\text{e-}14 \qquad Q_F = 16666.7$$

$$C_{12} = \frac{C_m Q_F}{k_{12}} \qquad Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{q f_s} - \frac{1}{Q_u}\right)} \qquad R_s = \omega\frac{L_m}{Q_e}$$

$$C_{12} = 2.38816\text{e-}10 \qquad Q_e = 30839.1 \qquad R_s = 101.871$$

$$C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}} \qquad C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2} \qquad C_{net} = \frac{1}{\frac{1}{C_{es}} + \frac{1}{C_{12}} + \frac{1}{C_m}}$$

$$C_{end} = 1.56205\text{e-}10 \qquad C_{es} = 3.18366\text{e-}10 \qquad C_{net} = 1.01314\text{e-}14$$

$$f_1 = \frac{1}{2\pi\sqrt{L_m C_{net}}}$$

$$f_1 = 5.00019\text{e+}06$$

A9. n=4 LSB Ladder Crystal Filter, Series Crystals.

$$C_m = \frac{1}{\omega^2 L_m} \qquad Q_F = \frac{f_s}{\mathrm{BW}} \qquad C_{12} = \frac{C_m Q_F}{k_{12}} \qquad C_{23} = \frac{C_m Q_F}{k_{23}}$$

$$C_m = 1.01321\text{e-}14 \qquad Q_F = 5000 \qquad C_{12} = 6.02385\text{e-}11 \qquad C_{23} = 9.36425\text{e-}11$$

$$C_{34} = \frac{C_m Q_F}{k_{34}} \qquad Q_e = \frac{1}{\left(\frac{\mathrm{BW}}{q f_s} - \frac{1}{Q_u}\right)} \qquad R_s = \omega\frac{L_m}{Q_e} \qquad C_{end} = \sqrt{\frac{(R_0 - R_s)}{R_s\omega^2 R_0^2}}$$

$$C_{34} = 6.02385\text{e-}11 \qquad Q_e = 3979.29 \qquad R_s = 789.486 \qquad C_{end} = 1.64368\text{e-}11$$

$$C_{es} = \frac{C_{end}^2\omega^2 R_0^2 + 1}{C_{end}\omega^2 R_0^2} \qquad C_{net} = \frac{1}{\frac{1}{C_{es}} + \frac{1}{C_{12}} + \frac{1}{C_m}} \qquad C_1 = C_{net}$$

$C_{es} = 7.80796e\text{-}11$ $\quad C_{net} = 1.01291e\text{-}14$ $\quad C_1 = 1.01291e\text{-}14$

$$f_1 = \frac{1}{2\pi\sqrt{L_m C_1}} \qquad C_2 = \frac{1}{\frac{1}{C_m} + \frac{1}{C_{12}} + \frac{1}{C_{23}}} \qquad f_2 = \frac{1}{2\pi\sqrt{L_m C_2}}$$

$f_1 = 5.00074e{+}06$ $\quad f_4 = f_1$ $\quad f_2 = 5.00069e{+}06$

$f_1 - f_s = 744.861$ $\quad f_4 = 5.00074e{+}06$ $\quad C_2 = 1.01293e\text{-}14$ $\quad f_2 - f_s = 690.952$

$$f_h = f_1 \qquad \omega_h = 2\pi f_h \qquad C_h = \frac{1}{\omega_h^2 L_m} \qquad T_2 = \frac{1}{\frac{1}{C_h} - \frac{1}{C_m} - \frac{1}{C_{12}} - \frac{1}{C_{23}}}$$

$f_h = 5.00074e{+}06$ $\quad \omega_h = 3.14206e{+}07$ $\quad C_h = 1.01291e\text{-}14$ $\quad T_2 = 4.69808e\text{-}10$

A10. n=2 USB Ladder Crystal Filter, Parallel Crystals.

$$f_p = f_s\sqrt{1 + \frac{C_m}{C_0}} \qquad \omega_s = 2\pi f_s \qquad C_m = \frac{1}{(\omega_s^2 L_m)} \qquad \omega_c = 2\pi f_c$$

$f_p = 5.00767e{+}06$ $\quad \omega_s = 3.14159e{+}07$ $\quad C_m = 1.01321e\text{-}14$ $\quad \omega_c = 3.14348e{+}07$

$$P = \frac{C_m}{(L_m C_m \omega_c^2 - 1)} \qquad S = \frac{P}{C_m} \qquad Q_f = \frac{f_c}{\mathrm{BW}} \qquad Q_e = \frac{1}{\left(\frac{1}{qQ_f} - \frac{1}{Q_u}\right)}$$

$P = 8.4409e\text{-}12$ $\quad S = 833.083$ $\quad Q_f = 16676.7$ $\quad Q_e = 30863.3$

$$R_p = \frac{Q_e}{(\omega_c PS)} \qquad R_p = \frac{Q_e}{(\omega_c PS)} \qquad C_{end} = \frac{1}{\omega_c\sqrt{R_p R_0 - R_0^2}} \qquad C_p = \frac{C_{end}}{(R_0\omega_c C_{end})^2 + 1}$$

$R_p = 139622$ $\quad R_p = 139622$ $\quad C_{end} = 3.81423e\text{-}12$ $\quad C_p = 3.80057e\text{-}12$

$$T_1 = P - C_0 - C_{12} - C_{end} \qquad T_2 = T_1$$

$T_1 = 1.02851e\text{-}12$ $\quad T_2 = 1.02851e\text{-}12$

A11. N=4 USB Crystal Ladder, Parallel Crystals.

$$f_p = f_s\sqrt{1+\frac{C_m}{C_0}} \qquad \omega_s = 2\pi f_s \qquad C_m = \frac{1}{(\omega_s^2 L_m)} \qquad \omega_c = 2\pi f_c$$

f_p = 1.00192e+07 $\quad \omega_s$ = 6.28319e+07 $\quad C_m$ = 1.26651e-14 $\quad \omega_c$ = 6.28696e+07

$$P = \frac{C_m}{(L_m C_m \omega_c^2 - 1)} \qquad S = \frac{P}{C_m} \qquad Q_f = \frac{f_c}{\mathrm{BW}} \qquad R_p = \frac{Q_e}{(\omega_c P S)}$$

P = 1.05511e-11 $\quad S$ = 833.083 $\quad Q_f$ = 10006 $\quad R_p$ = 15008.1

$$C_{12} = \frac{k_{12}}{Q_f}PS \qquad C_{23} = \frac{k_{23}}{Q_f}PS \qquad C_{34} = C_{12}$$

C_{12} = 7.38793e-13 $\quad C_{23}$ = 4.75252e-13 $\quad C_{34}$ = 7.38793e-13

$$Q_e = \frac{1}{\left(\frac{1}{qQ_f} - \frac{1}{Q_u}\right)} \qquad C_e = \frac{1}{\omega_c\sqrt{R_p R_0 - R_0^2}} \qquad C_p = \frac{C_e}{1 + (R_0 \omega_c C_e)^2}$$

Q_e = 8293.78 $\quad C_e$ = 6.58011e-12 $\quad C_p$ = 6.40474e-12

$$T_1 = P - C_0 - C_{12} - C_p \qquad T_2 = P - C_0 - C_{12} - C_{23} \qquad T_3 = P - C_0 - C_{34} - C_{23}$$

T_1 = 1.07596e-13 $\quad T_2$ = 6.03708e-12 $\quad T_3$ = 6.03708e-12

$$T_4 = P - C_0 - C_{34} - C_p$$

T_4 = 1.07596e-13

Appendix B. g_m to k_m and q_m. Converting Normalized Low Pass Component Values to Normalized Coupling and End Loading Parameters.

Most bandpass filters are designed with normalized k and q parameters. As mentioned in the early part of this report, these parameters are related to the low pass prototype filters.

Here we present equations that will make this format transformation for two popular filter polynomials, the Butterworth and the Chebyshev.

The calculations to make this transformations are generally straight forward. There are, however, some subtle details that can complicate the calculations if they are not appreciated. The first has to do with the frequency used in defining the normalized filter components for Chebyshev filters. Classic equations, including those presented in reference 5 (IRFD, pp 62-65) calculate the components on the basis of a ripple cutoff frequency. Assume a Chebyshev low pass has a peak to peak ripple of 1 dB. The highest frequency in the passband where the attenuation is 1 dB is then the ripple cutoff frequency. But we want to design our filters using 3 dB cutoff. The two cutoff values are related by a factor W. Normalized component values, gm, based upon ripple cutoff are converted to those based upon a 3 dB cutoff merely by multiplying by W.

Butterworth filters are simpler than the Chebyshev, for the 3 dB defined parameters are the same as a ripple cutoff based set. In the equations to follow, merely set W=1 for a Butterworth low pass prototype filter. Similarly, when starting with prototype values that are already normalized to a 3 dB cutoff, such as any of the data in Zverev (ref. 1), merely set W=1 in the following equations.

The other subtle detail is unique to Chebyshev filters with an even order. These filters have a low pass prototype with n components where n is an even number and are terminated in a normalized source resistance that is not 1, which is the usual load resistance. Typical odd order filters are simpler with equal terminations. To add further mystery, even order Chebyshev filters end up with equal normalized end Q when the corresponding low pass prototypes have unequal terminations. (Not all filter shapes have equal normalized Q at the ends. An example of this is the transitional filters such as the Gaussian-to-6 dB.)

With these details in mind, a normalized coupling coefficient, k_m, is calculated with

$$k_m = \frac{1}{W} \frac{1}{\sqrt{g_m g_p}}$$

where

$$p = m + 1$$

Reiterating, the normalized frequency W is set to 1 for a Butterworth filter, or for a Chebyshev when the normalized components g_m and g_p are given as 3 dB based values. Note that k has the form of a resonant frequency between adjacent elements in a low pass prototype filter. An example might be the beginning end of a n=5 Chebyshev filter with a ripple of 0.1 dB. This filter has W=1.1347 using analysis presented below. If we consider the first and second element while using ripple cutoff based components, g_1=1.1468 and g_2=1.3712. The result is k=0.7028. While this is specified as k_1, we more often see this specified as k_{12}.

The end section normalized q values are

$$q_n = g_n W,\ q_1 = \frac{g_1 W}{r_s}$$

Note the simplicity; the end section q values equal the end component values when they are defined on a 3 dB basis. Using the previous example of a 5th order 0.1 dB Chebyshev, q_1=1.3013, which is also q_5. For this odd order Chebyshev, r_s=1.

The ratio of the 3 dB cutoff to the ripple cutoff begins with

$$E = \sqrt{10^{A/10} - 1}$$

where A is the peak to peak ripple in dB. E is the ripple as a power ratio. An intermediate variable Y is then

$$Y = \frac{1}{n} \ln\left(\frac{1}{E} + \sqrt{\left(\frac{1}{E^2} - 1\right)}\right)$$

where E is defined above and n is the filter order. This leads to Θ, which will become our frequency ratio.

$$\Theta = \frac{1}{2}\left(e^{Y} + e^{-Y}\right)$$

Θ is merely a dummy variable at this point. It is convenient to be able to set W= Θ for the Chebyshev filters.

The source resistance for even ordered Chebyshev filters is calculated with

$$d = \frac{A}{8.68589},\ x = \frac{1}{2} \ln\left(\frac{e^{d} + 1}{e^{d} - 1}\right)$$

$$s = \left(\frac{e^{x} - 1}{e^{x} + 1}\right)^2$$

x and d are essentially dummy variables leading to a normalized source resistance s. For the even order Chebyshev case, we set r_s=s in order to calculate q_1.