ARIMA ssf

We consider that the auto-regressive polynomial contains some unit roots. It can be factorized in a stationary polynomial (defined by the roots outside the unit circle) and in a non-stationary polynomial (defined by the roots on the unit circle), which is notated:

$$ \Delta\left(B)\right) = 1 + \delta_1 B + \cdots + \delta_d B^d $$

The state space form of an ARIMA model is similar to the state space form of an [ARMA model(arma_ssf.md)] except for its initialization.

Initialization

The initial conditions can be written as follows:

$$ \alpha_{-1} = \begin{pmatrix}1 \ 0 \ \vdots\ 0 \end{pmatrix} $$
$$ P_{*} = \Sigma \Omega \Sigma' $$
$$ B = \Lambda $$
$$ P_{\infty}= \Lambda \Lambda' $$

$$ \Omega $$ is the unconditional covariance of the state array of the stationary model. $$ \Sigma = \begin{pmatrix} 1 & 0 & \cdots & 0 \ \lambda_1 & 1 & \cdots & 0 \ \lambda_2 & \lambda_1 & \cdots & \vdots\ \vdots & \vdots & \vdots & \vdots \ \lambda_{r-1} & \lambda_{r-2} & \cdots & \lambda_{1} \end{pmatrix} $$

where $$ \lambda_{i} $$ are generated by $$ \frac{1}{\Delta\left(B\right)} $$
$$ \Lambda $$ is a r x d matrix; its first d rows form an identity matrix; other cells are defined by the recursive relationship: $$ \Lambda \left(i,j\right) = -\sum_{k=1}^d {\delta_k \Lambda \left(i-k,j\right)}$$