Stellar Evolution – Hints to exercises – Chapter 11

11.1 **Core mass luminosity relation for AGB stars**

(a) Take the time derivative of the Paczynski relation

$$\frac{\dot{L}}{L_\odot} = 5.9\times 10^4 \left(\frac{\dot{M}_c}{M_\odot}\right) = 5.9\times 10^{-7}\mathrm{yr}^{-1}\left(\frac{L}{L_\odot}\right) \quad \Rightarrow$$

$$\int_{L(t_0)}^{L(t)} \left(\frac{L}{L_\odot}\right)^{-1} \mathrm{d}\left(\frac{L}{L_\odot}\right) = \int_{t_0}^{t} 5.9\times 10^{-7}\mathrm{yr}^{-1}\mathrm{d}t \quad \Rightarrow$$

$$\ln\left(\frac{L(t)}{L_\odot}\right) = \ln\left(\frac{L(t_0)}{L_\odot}\right) + 5.9\times 10^{-7}(t-t_0)/\mathrm{yr} \quad \Rightarrow$$

$$\frac{L(t)}{L_\odot} = 10^3\ \exp\left(\frac{t-t_0}{1.69\times 10^6\ \mathrm{yr}}\right)$$

(b) $L = 4\pi R^2\sigma T_{\mathrm{eff}}^4$

$$R(t) = \left[\frac{L(t)}{4\pi\sigma T_{\mathrm{eff}}^4}\right]^{1/2} = 120\,R_\odot\ \exp\left(\frac{t-t_0}{3.38\times 10^6\ \mathrm{yr}}\right)$$

(c) Rewrite Paczynski relation and insert the expression for $L(t)$.

$$\left(\frac{\dot{M}_c}{M_\odot}\right) = 5.9\times 10^{-7}\mathrm{yr}^{-1}\left(\frac{M_c}{M_\odot} - 0.52\right) \Rightarrow$$

$$\int_{M_c(t_0)}^{M_c(t)} \left(\frac{M_c}{M_\odot} - 0.52\right)^{-1} \mathrm{d}\left(\frac{M_c}{M_\odot}\right) = \int_{t_0}^{t} 5.9\times 10^{-7}\mathrm{yr}^{-1}\,\mathrm{d}t \Rightarrow$$

Paczynski relation: t_0: $L/L_\odot = 10^3 \Rightarrow M_c/M_\odot = 0.537$

$$\frac{M_c(t)}{M_\odot} = 0.52 + 0.017\exp\left(\frac{t-t_0}{1.69\times 10^6\ \mathrm{yr}}\right)$$

11.2 **Mass loss of AGB stars**

(a) Insert $L(t)$ and $R(t)$ from the previous question into the Reimers relation, keeping η as a free parameter:

$$\frac{\dot{M}}{M_\odot} = -4\times 10^{-13}\mathrm{yr}^{-1}\times\eta\times 10^3\times 120\times\exp\left[\left(1+\frac{1}{2}\right)\frac{t-t_0}{1.69\times 10^6\ \mathrm{yr}}\right]\left(\frac{M}{M_\odot}\right)^{-1}$$

Now rewrite and use the hint

$$-4.8 \times 10^{-8}\,\mathrm{yr}^{-1}\,\eta\,\exp\left[\frac{3(t-t_0)}{2 \times 1.69 \times 10^6\,\mathrm{yr}}\right] = \left(\frac{\dot{M}}{M_\odot}\right)\left(\frac{M}{M_\odot}\right) = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[\left(\frac{M}{M_\odot}\right)^2\right]$$

Rewrite and integrate this expression, then take the square root to obtain $M(t)$

$$\frac{M(t)}{M_\odot} = \left\{\left(\frac{M_0}{M_\odot}\right)^2 - 0.108\,\eta\left[\exp\left(\frac{t-t_0}{1.13 \times 10^6\,\mathrm{yr}}\right) - 1\right]\right\}^{1/2}$$

(b) Answer these questions for example by plotting $M_c(t)$ and $M(t)$ and estimating the answer from your plot. For $\eta = 3$: $\Delta t \approx 2.82 \times 10^6$ yrs, $L \approx 5.31 \times 10^3 L_\odot$, $M_{\mathrm{WD}} \approx 0.610\,M_\odot$.

(c) For $\eta = 1$: $\Delta t \approx 3.96 \times 10^6$ yrs, $L \approx 1.04 \times 10^4 L_\odot$, $M_{\mathrm{WD}} \approx 0.697\,M_\odot$.
For $\eta = 9$: $\Delta t \approx 1.76 \times 10^6$ yrs, $L \approx 2.83 \times 10^3 L_\odot$, $M_{\mathrm{WD}} \approx 0.568\,M_\odot$.