Stellar Evolution – Hints to exercises – Chapter 5

5.1 Radiation transport

(a) The mean free path of a photon is $\ell_{\rm ph} = 1/(\kappa\rho)$, where κ is the opacity coefficient. The photons escape in a random walk fashion, so the distance d travelled after N scatterings is $d^2 \approx N\ell_{\rm ph}^2$. In order to travel from the center to the surface of the Sun, $N \approx (R_\odot/\ell_{\rm ph})^2$ steps are needed.

To obtain an estimate of the time needed for this random-walk process, take 'typical' values $\kappa \approx 1\,{\rm cm}^2/{\rm g}$ and $\rho \approx \bar{\rho}_\odot = 1.4\,{\rm g\,cm}^{-3}$, so that $\ell_{ph} \approx 1$cm. The time t needed is $t = N\ell_{\rm ph}/c \approx R_\odot^2/(c\ell_{\rm ph}) \approx 1.5 \times 10^{11}{\rm s} \approx 10^4{\rm y}$.

(N.B. The actual time is much longer, because $\kappa \gg 1{\rm cm}^2/{\rm g}$ in the outer layers, while $\rho > \bar{\rho}$ in the centre. Therefore the average $\ell_{\rm ph}$ is much shorter than 1 cm, and $t \approx 10^7{\rm yrs} \approx \tau_{\rm KH}$).

(b) $\ell_{ph} \approx 1$cm and $dT/dr \approx (T_{\rm eff} - T_c)/R_\odot \approx 10^{-4}$ K/cm.

(c) The total flux emitted in all directions by a blackbody is $F = \sigma T^4$, where $T \approx 10^7 K$ and $\sigma = 5.67 \times 10^{-5}$ erg/K^{4}cm^{2}s.

(d) $F \propto T^4 \Rightarrow \dfrac{\Delta F}{F} \propto 4\dfrac{\Delta T}{T}$, where $\Delta T/T \approx 10^{-11}$.

The net outward flux $\Delta F = 4F\Delta T/T \approx 4\sigma\,(10^7)^4 \times 10^{-11}$ erg/cm^{2}s. The corresponding luminosity comes from multiplying by the surface of sphere with radius $r \sim R_\odot/10$: $L = 4\pi r^2\,\Delta F \approx 1.4 \times 10^{34}$ erg/s. This is within one order of magnitude of the solar surface luminosity.

(e) The mean free path for photons becomes very long near the surface, where they escape (we can see them on Earth). Then LTE is no longer valid.

5.2 Opacity

(a) Correlate Fig. 5.2 with Section 5.3.1. Electron scattering gives constant κ at high T and low ρ. Free-free and bound-free absorption give $\kappa \propto T^{-3.5}$ between 10^4 K and $\sim 10^7$ K, depending on ρ. The H^- ion gives $\kappa \propto T^9$ between 3000 K and 10^4 K. Molecules are important for $T < 4000$ K and dust absorption dominates at $T < 1500$ K.

(b) Draw the relations (5.30), (5.32), (5.33) and (5.34) in Fig.5.2a, putting in the right values for X, Z and ρ.

(c) Use $\ell_{\rm ph} = 1/(\kappa\rho)$ and estimate $\log\rho$ and $\log\kappa$ from the location of the $1\,M_\odot$ model in Fig. 5.2b:

$T = 10^7$ K:	$\log\rho \approx 2.0$	$\log\kappa \approx 0.5$	$\ell \approx 3 \times 10^{-2}$ cm
$T = 10^5$ K:	$\log\rho \approx -2.5$	$\log\kappa \approx 4.5$	$\ell \approx 10^{-2}$ cm
$T = 10^4$ K:	$\log\rho \approx -6.0$	$\log\kappa \approx 2.0$	$\ell \approx 10^4$ cm

(d) Take the derivative of eq. (5.21) to obtain $\partial U_\nu/\partial T$ and divide by $\kappa_\nu = \kappa_0\nu^{-\alpha}$. This quantity should be integrated over ν according to eq. (5.24). Hint: substitute $x = h\nu/kT$.

5.3 Mass-luminosity relation for stars in radiative equilibrium

Hand-in exercise.

5.4 Conceptual questions about convection

Hand-in exercise.

5.5 **Applying Schwarzschild's criterion**

Assume $\nabla_{\rm ad} = \nabla_{\rm ad,ideal} = 0.4$. If $\nabla_{\rm rad} > 0.4$ energy transport is by convection, otherwise by radiation.

$$\nabla_{\rm rad} = \frac{3}{16\pi acG}\frac{\kappa lP}{mT^4}$$

(a) Low mass stars have low temperatures and therefore high opacities, leading to $\nabla_{\rm rad} > 0.4$ everywhere.

(b) Use the ideal gas law to calculate P and use the numbers from the table.

5.6 **Eddington Luminosity**

(a) See Section 5.4.

The outward acceleration $a_{\rm rad}$ due to the radiation pressure gradient is

$$a_{\rm rad} = \frac{1}{\rho}\frac{dP_{\rm rad}}{dr} == \frac{4aT^3}{3\rho}\frac{dT}{dr}$$

which cannot be larger than the gravitational acceleration Gm^2/r. Assuming radiation trasnport, dT/dr is given by eq. (5.16), which after substitution gives

$$l < \frac{4\pi cGm}{\kappa}.$$

(b) Starting with $L/L_{\rm Edd}$, use the equation for the radiative temperature gradient to substitute L, then use $4aT^3 dT/dr = dP_{\rm rad}/dr$, then use $P_{\rm rad} = P(1-\beta)$ and get rid of dP/dr using the HE equation.

(c) Rewrite $\nabla_{\rm rad}$ as

$$\nabla_{\rm rad} = \frac{\kappa l}{4\pi cGm}\frac{P}{\frac{4}{3}aT^4}$$

Now simplify the expression by introducing $l_{\rm Edd}$ and $P_{\rm rad}$. Then substitute into the Schwarzschild criterion.

(d) On the boundary of the core there is a transition from convective to radiative energy transport. So, on the boundary the Schwarzschild criterion can be written as an equality:

$$\frac{l_{\rm core}}{l_{\rm Edd,core}} = 4(1-\beta)\nabla_{\rm ad}.$$

Outside the core there is no energy generation, so $l_{\rm core} = L$. From (b) you know that $L = (1-\beta)L_{\rm Edd}$. From (a) you know that $l_{\rm Edd,core} \propto M_{\rm core}$ and also $L_{\rm Edd} \propto M$. Now combine all this knowledge.

$$\left.\begin{array}{rcl} l_{\rm core} = L & = & (1-\beta)L_{\rm Edd} \\ l_{\rm Edd,core} & = & \dfrac{M_{\rm core}}{M}L_{\rm Edd} \end{array}\right\} \Rightarrow \frac{l_{\rm core}}{l_{\rm Edd,core}} = (1-\beta)\frac{M}{M_{\rm core}}$$

$$\Rightarrow 4(1-\beta)\nabla_{ad} = (1-\beta)\frac{M}{M_{\rm core}}$$

and there you are.