lecture 10:

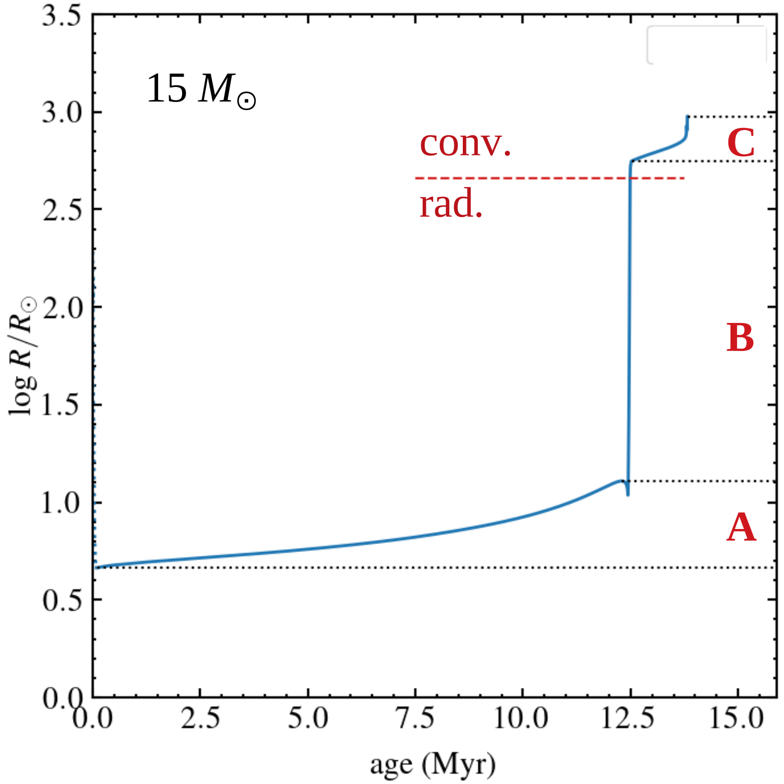
mass transfer

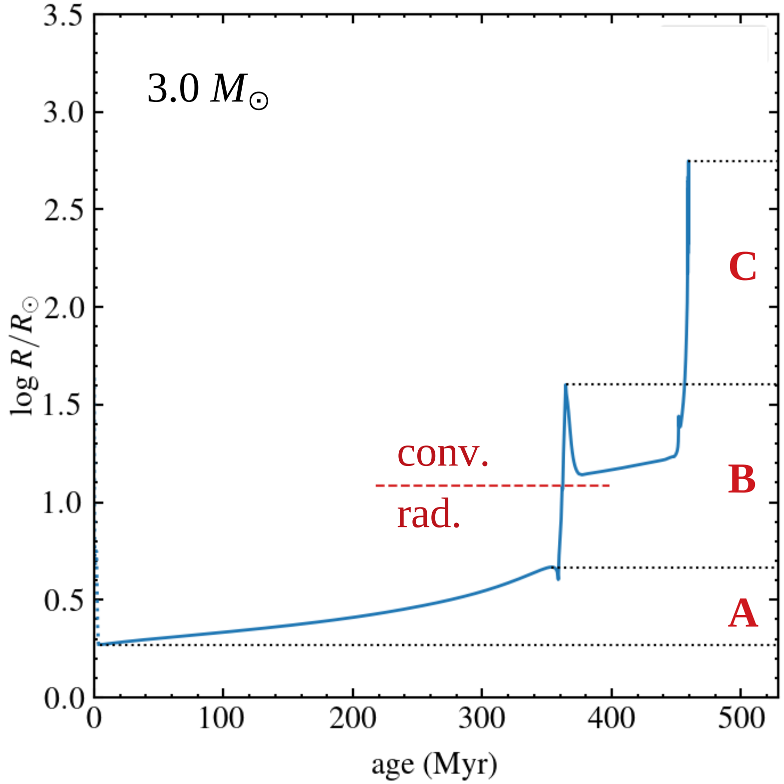
mass transfer

- define: primary star (mass M_1) as initially more massive than secondary (mass M_2):

$$M_{1\mathrm{i}} > M_{2\mathrm{i}} \quad \Rightarrow \quad \text{initial mass ratio } q_\mathrm{i} = M_{2\mathrm{i}}/M_{1\mathrm{i}} < 1$$

- the more massive star evolves faster, and will be the first to expand and fill its Roche lobe $\Rightarrow$ first phase of mass transfer from *1 to *2





mass transfer

- define: primary star (mass M_1) as initially more massive than secondary (mass M_2):

  $$M_{1i} > M_{2i} \quad \Rightarrow \quad \text{initial mass ratio } q_i = M_{2i}/M_{1i} < 1$$

- the more massive star evolves faster, and will be the first to expand and fill its Roche lobe $\Rightarrow$ first phase of mass transfer from *1 to *2

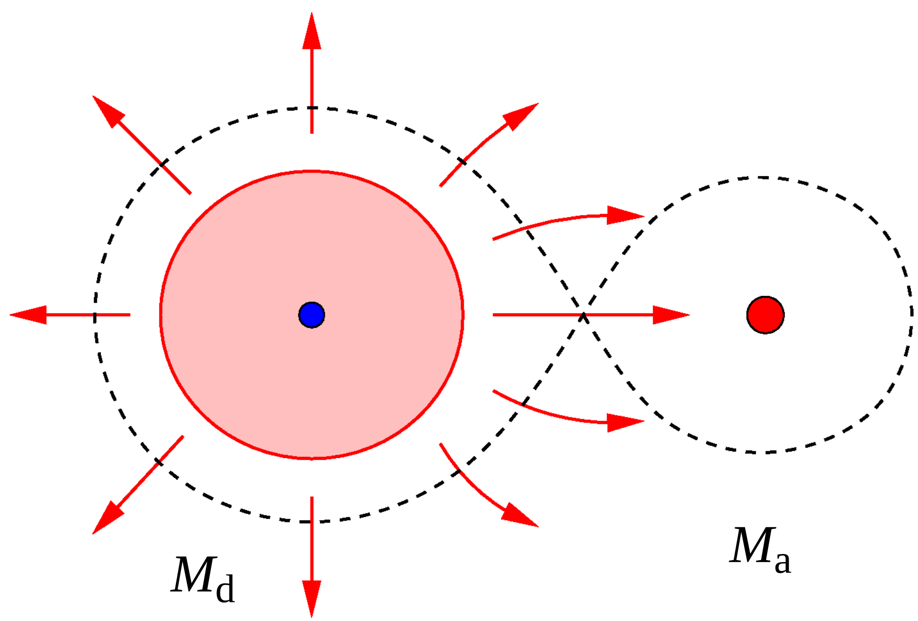
- during a phase of mass transfer:

  define mass-losing star as the mass donor (mass M_d), the other as the mass gainer or accretor (mass M_a)

  – initially $M_d = M_1$ and $M_a = M_2$
  – during binary evolution, further mass transfer can go back and forth between *1 and *2 $\Rightarrow$ stars can switch roles as donor and accretor

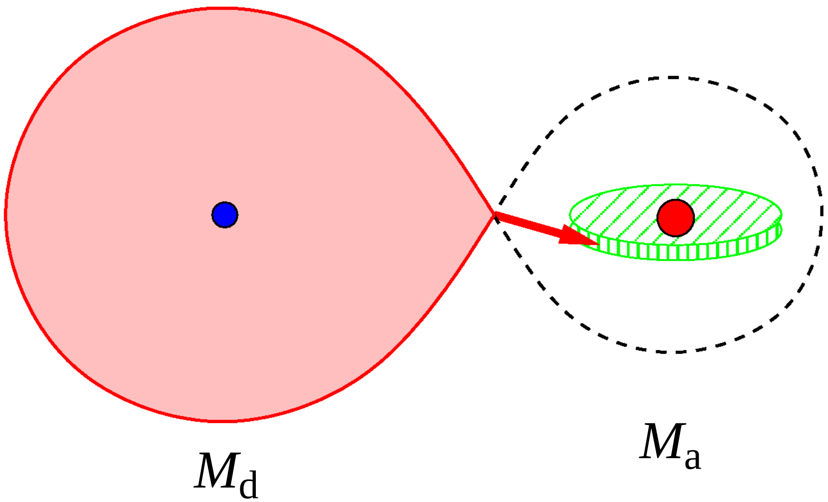
mass transfer

- in detached binaries, mass transfer can occur if one star has a strong stellar wind
  - part of the mass lost in the wind can be captured by the other star: wind accretion
  - this is usually rather inefficient, $|\Delta M_{\rm a} / \Delta M_{\rm d,wind}| \ll 1$, unless the wind is very dense and slow



mass transfer

- in semi-detached binaries, mass transfer occurs by **Roche-lobe overflow** (RLOF) through the L_1 point:
  - mass transfer stream enters companion's Roche lobe, and has insufficient energy to escape
  - RLOF can *in principle* be very efficient, $|\Delta M_\mathrm{a} \,/\, \Delta M_\mathrm{d,RLOF}| \approx 1$
  - however, this depends on timescale and stability of RLOF, and on how the companion deals with matter transferred to it

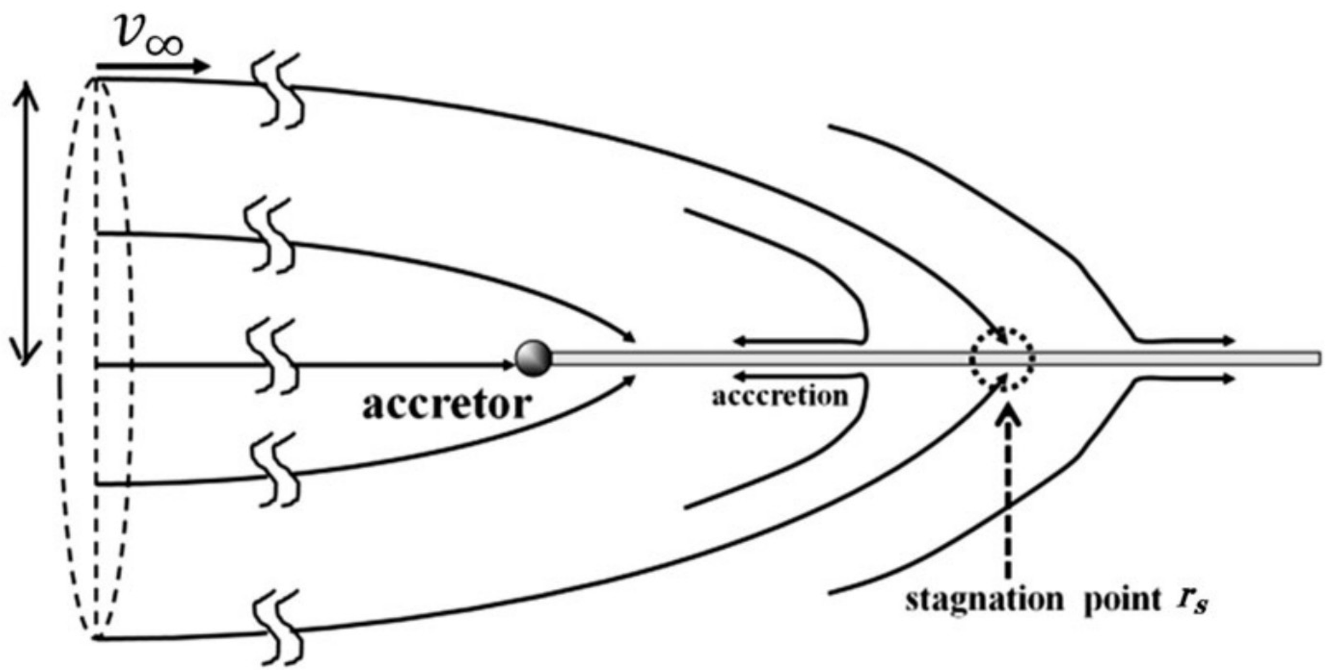


wind accretion

- usually described as Bondi-Hoyle-Lyttleton accretion

accretion
radius

$$R_{\rm acc} = \frac{2GM_{\rm a}}{v_\infty^2}$$



wind accretion

- usually described as Bondi-Hoyle-Lyttleton accretion
  - this is appropriate for *isotropic wind outflow* and $v_{\rm wind} \gg v_{\rm orb}$ ⇒ small accretion efficiency

$$\dot{M}_{\rm acc} \approx \dot{M}_{\rm d} \left(\frac{q}{1+q}\right)^2 \left(\frac{v_{\rm orb}}{v_{\rm wind}}\right)^4 \qquad (q = M_{\rm a}/M_{\rm d})$$

- BHL accretion applies to fast winds, such as from hot luminous stars
- for donor stars with slow and dense winds (e.g. AGB or RSG stars in binaries), the flow can be very different from BHL accretion ⇒

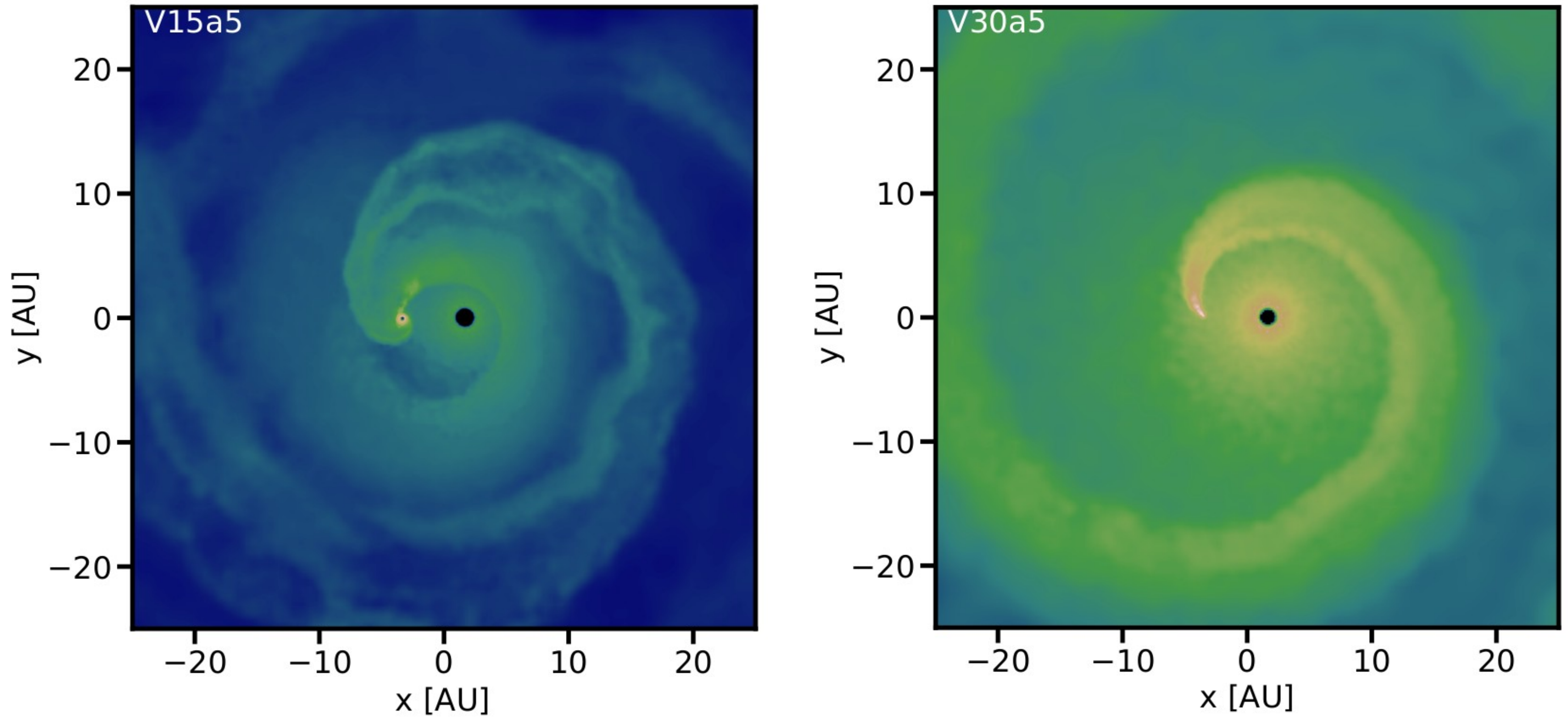
  substantially larger accretion efficiency possible

wind accretion

- hydrodynamical simulations of wind mass transfer from a 3 $M_\odot$ AGB star to a 1.5 $M_\odot$ companion (Saladino et al. 2018)

v_w = 15 km/s ($\approx 0.5\, v_{orb}$)
dM_{acc}/dt > BHL

v_w = 30 km/s ($\approx v_{orb}$)
$dM_{acc}/dt \approx$ BHL

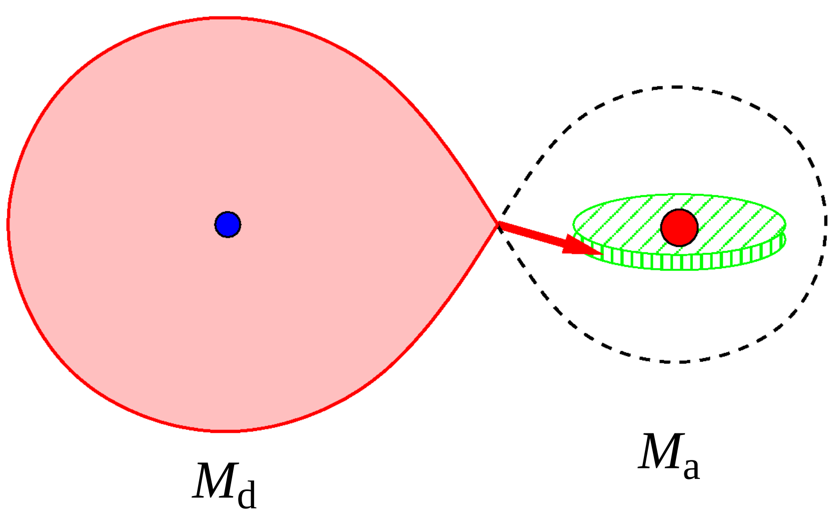


Roche-lobe overflow

- previous lecture: tidal interaction is strongly dependent on ratio *R/a*
- tides will often circularize the orbit ($e = 0$) and synchronize the donor's rotation with the orbit ($\Omega_d = \omega$), before radius approaches Roche lobe

  ⇒ commonly assumed that the *Roche geometry* applies

- for now just consider the *donor star*, and assume the accretor is an inert, passive object – we will consider its response later



rate of Roche-lobe overflow

- mass transfer rate via stream through L_1:

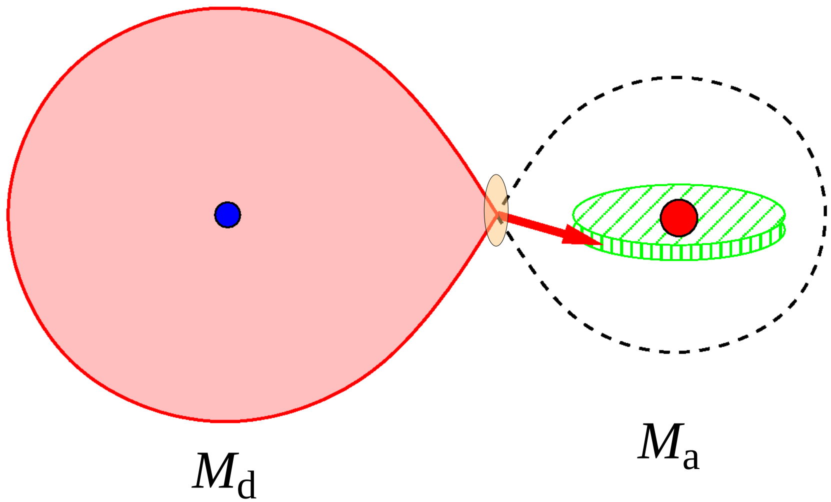
$$\mathrm{d}M_\mathrm{d}/\mathrm{d}t \approx -(\rho\, v)_{L1} \,.\, S$$

cross-section of "nozzle" around L_1

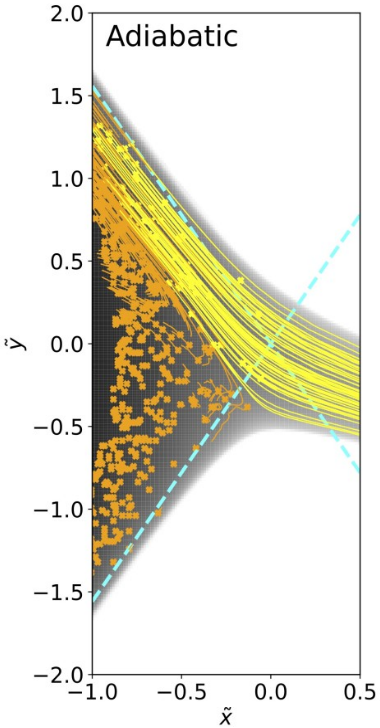
- this implies:

$$\mathrm{d}M_\mathrm{d}/\mathrm{d}t \sim -M_\mathrm{d} \,/\, P \,.\, (\Delta R \,/\, R)^3$$

(see lecture notes, sec. 17.2)



example from a recent hydro simulation: (Ryu+ 2025)



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(see lecture notes, sec. 17.2)

- consequences:
  - mass transfer rate is very sensitive to radius excess $\Delta R/R = (R_\mathrm{d} - R_\mathrm{L})/R$
  - mass transfer on nuclear or thermal timescale ($\mathrm{d}M_\mathrm{d}/\mathrm{d}t \ll -M_\mathrm{d}\,/\,P$) usually requires $\Delta R/R < 0.01$

  $\Rightarrow$ can make the approximation $R \approx R_\mathrm{L}$ during *stable* RLOF

orbital evolution

- to understand how mass transfer proceeds, we need to understand how the orbit evolves in response to mass loss and mass transfer
- consider the orbital angular momentum:

$$J = M_1 M_2 \sqrt{\frac{Ga(1-e^2)}{M_1 + M_2}}$$

(see lecture notes, section 17.1)

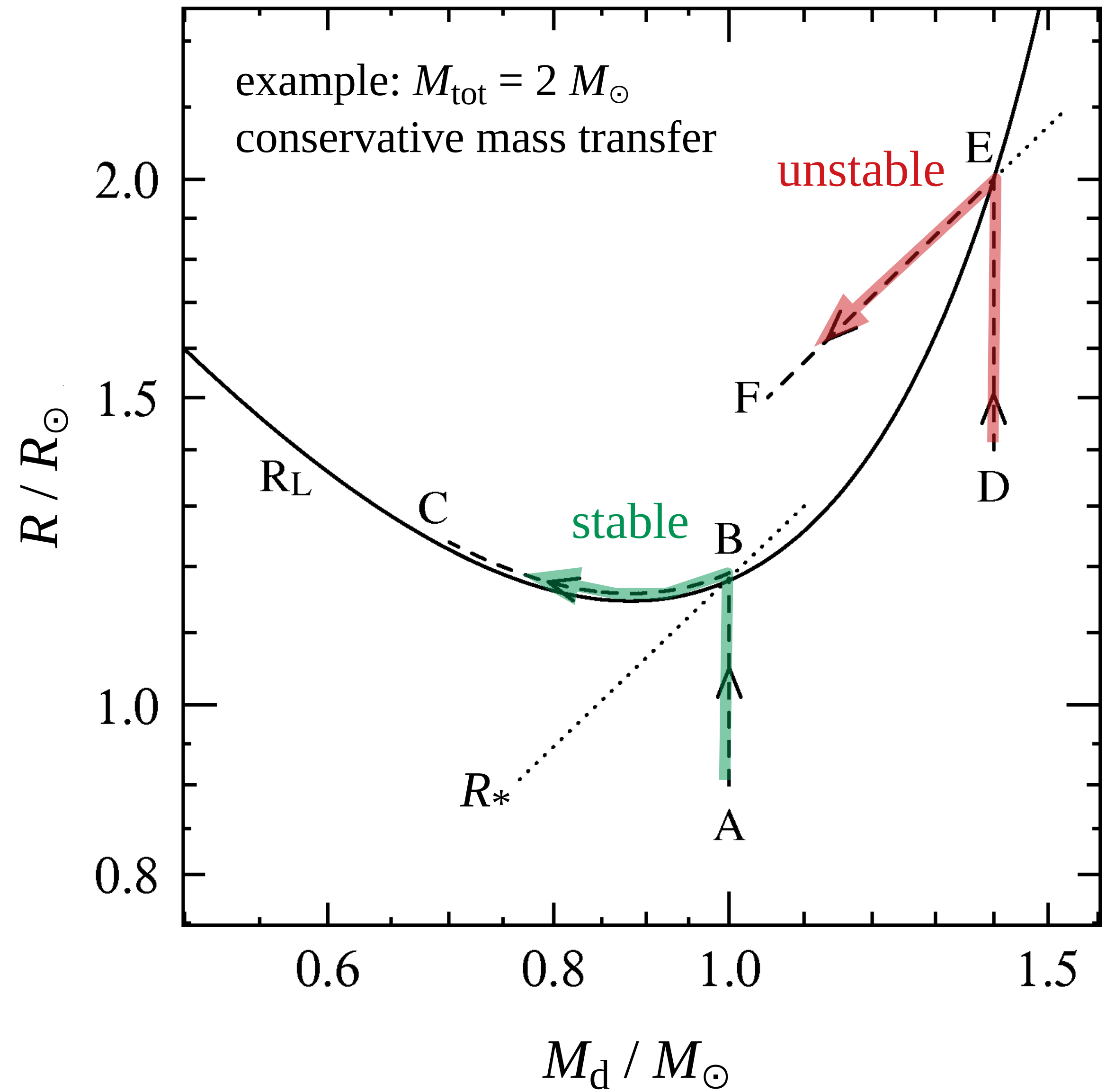
- approximations made:
  - ignore spin angular momentum (more precisely: ignore effect of *spin-orbit coupling* on J_{orb})
  - orbit is already circularised by tidal interactions ($e = 0$)

stability of mass transfer

stability of mass transfer depends on:

- the response of the stellar radius (R_d) to mass loss
- response of the Roche-lobe radius (R_L) to mass loss

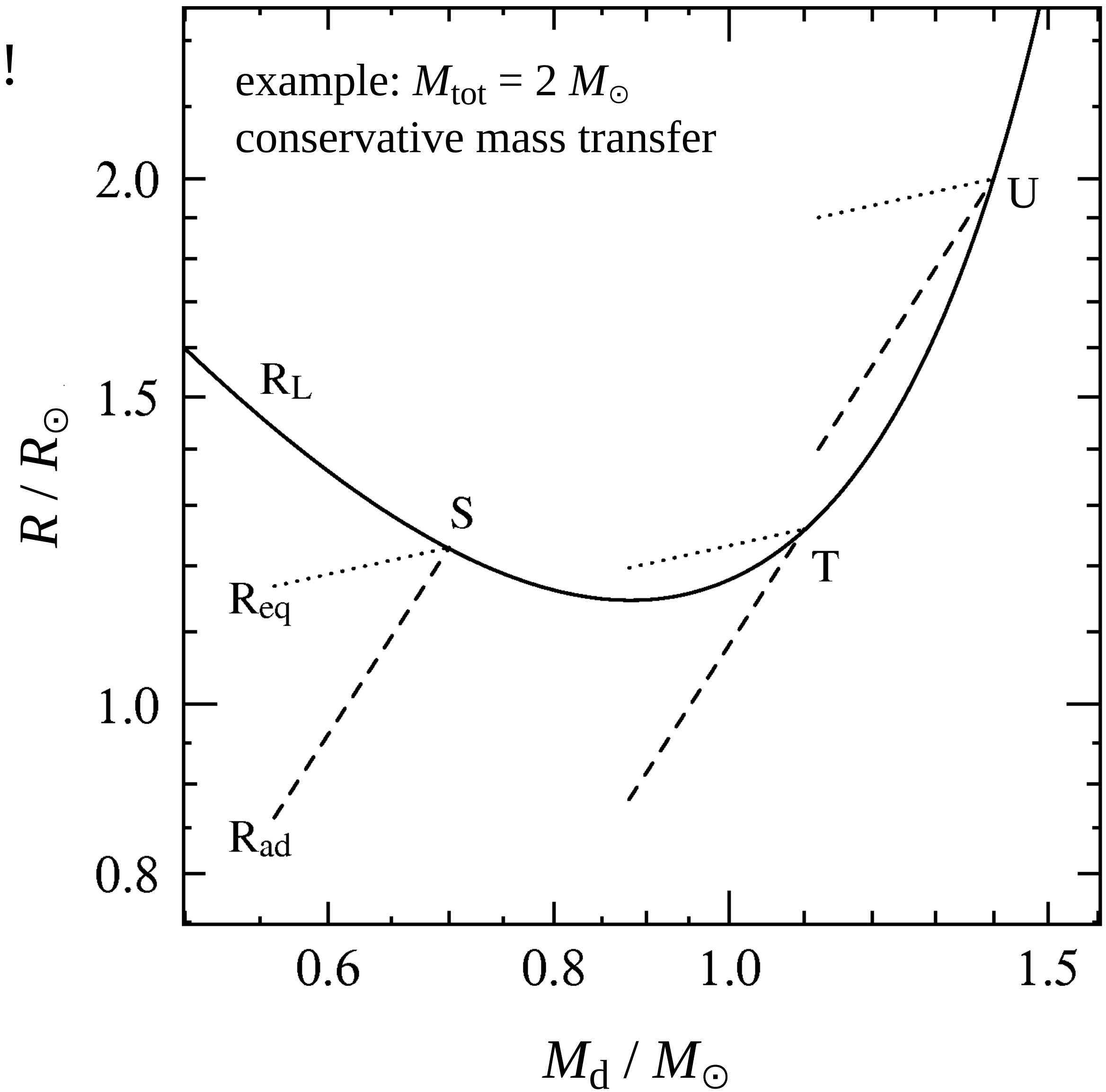
⇒ consider mass-radius diagram:



stability of mass transfer

but... stellar radius can respond on two very different timescales!

- star will first try to restore hydrostatic equilibrium on its dynamical timescale (almost adiabatically) $\rightarrow R_{ad}$
- then it will attempt to restore thermal equilibrium on the (slower) thermal timescale $\rightarrow R_{eq}$

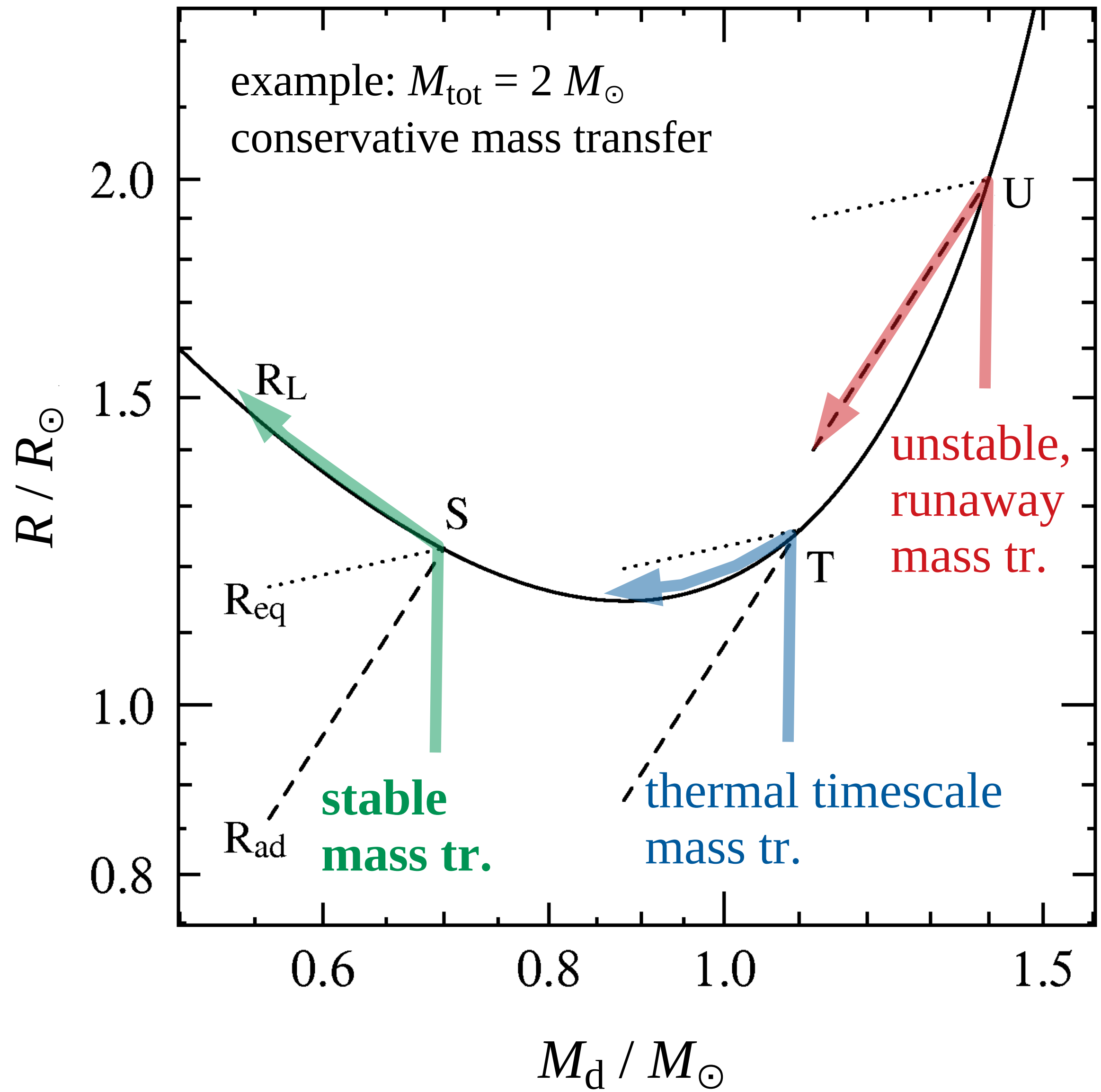


stability of mass transfer

this results in 3 possibilities or regimes of mass transfer:

- **S: stable mass transfer**: (nuclear timescale)

  driven by evolutionary expansion of donor ($dM/dt \approx -M_d/\tau_{nuc}$) or by orbital angular momentum loss

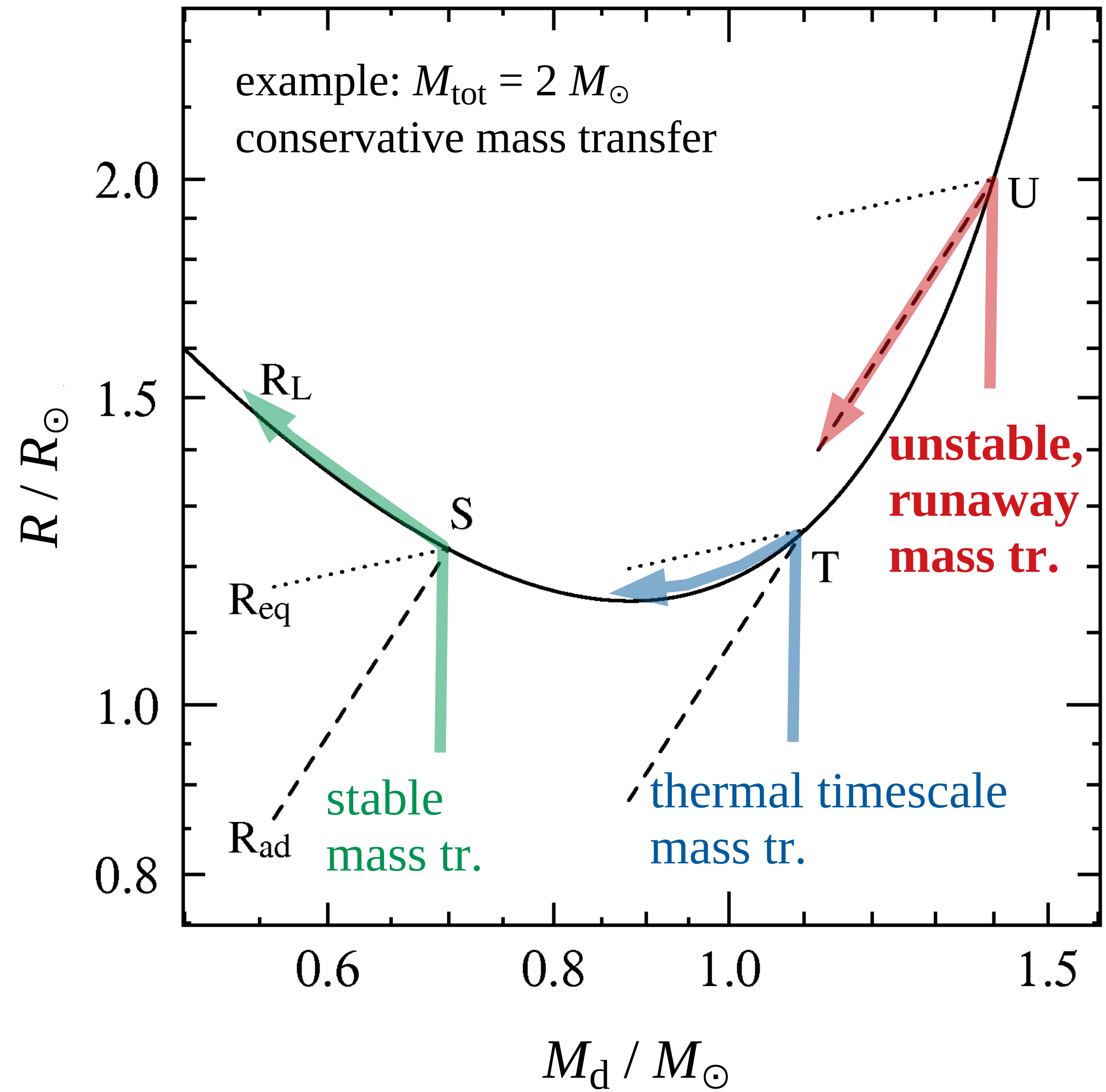


stability of mass transfer

this results in 3 possibilities or regimes of mass transfer:

- S: stable mass transfer (nuclear timescale)
- **U: unstable mass transfer** (dynamical timescale)

  runaway mass transfer rate ($dM/dt \gg M_d/\tau_{KH}$), probably leading to a common envelope



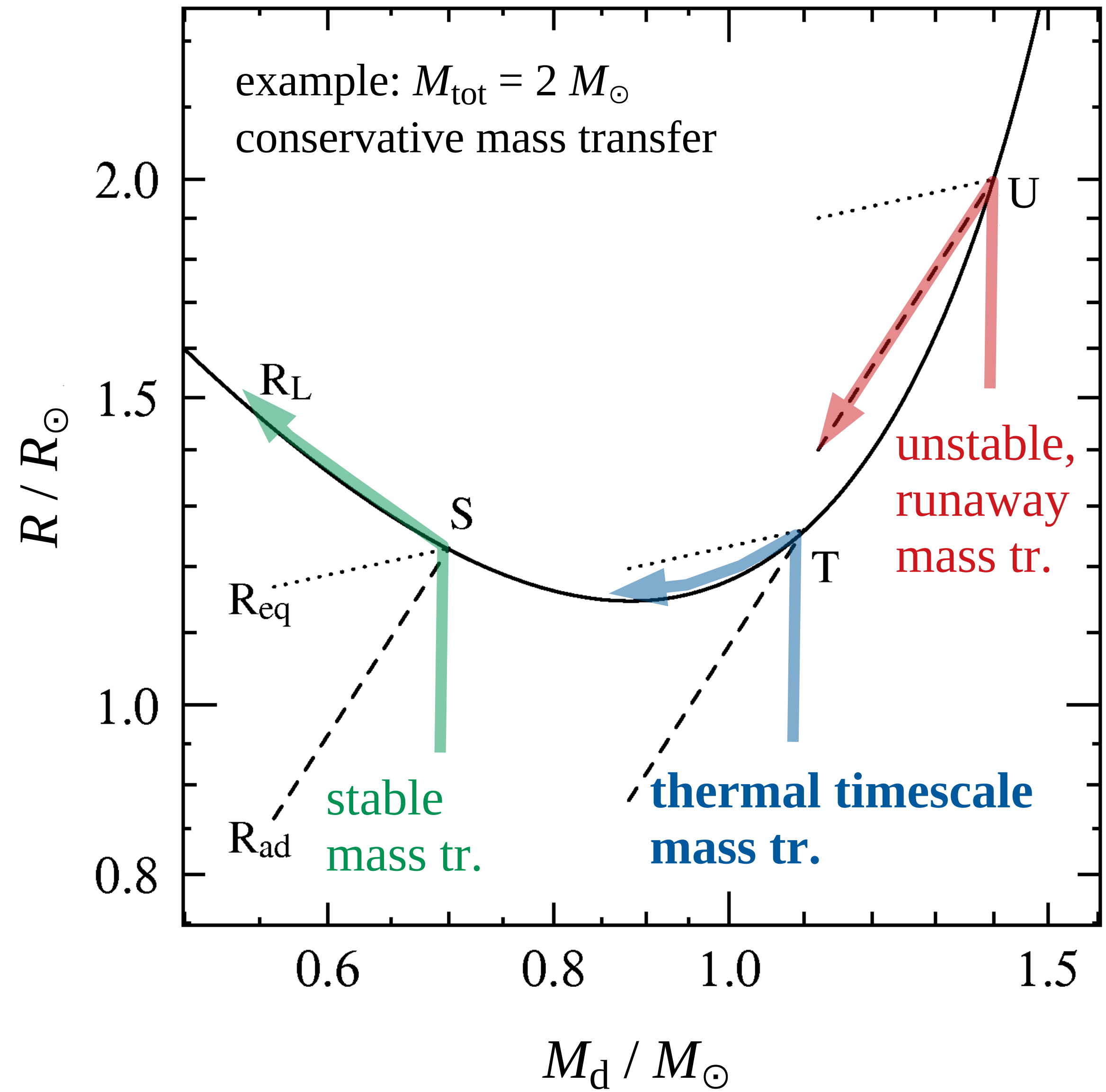
stability of mass transfer

this results in 3 possibilities or regimes of mass transfer:

- S: stable mass transfer (nuclear timescale)
- U: unstable mass transfer (dynamical timescale)
- **T: thermal-timescale mass transfer** (self-regulating)

  driven by thermal readjustment of donor

  $\Delta R/R$ adjusts itself such that $dM/dt \approx -M_d/\tau_{KH}$



stability of mass transfer

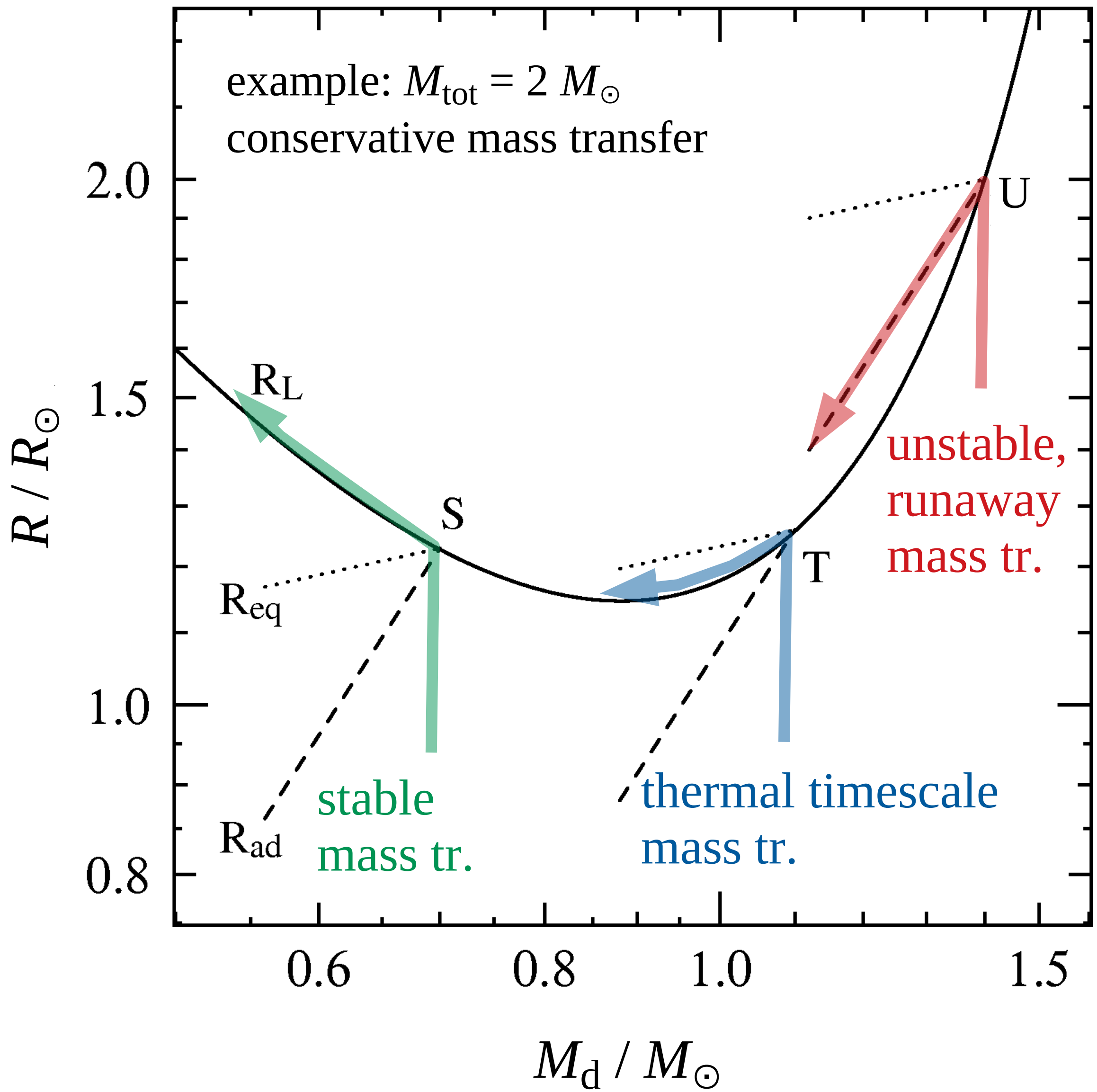
canonical approach: compare
mass-radius exponents

$$\zeta_L \equiv \frac{d\log R_L}{d\log M_d}$$

$$\zeta_{ad} \equiv \left(\frac{d\log R_d}{d\log M_d}\right)_{ad}$$

$$\zeta_{eq} \equiv \left(\frac{d\log R_d}{d\log M_d}\right)_{eq}$$

- Stable: $\zeta_L < \min(\zeta_{ad}, \zeta_{eq})$
- Unstable: $\zeta_L > \zeta_{ad}$
- Thermal: $\zeta_{eq} < \zeta_L < \zeta_{ad}$



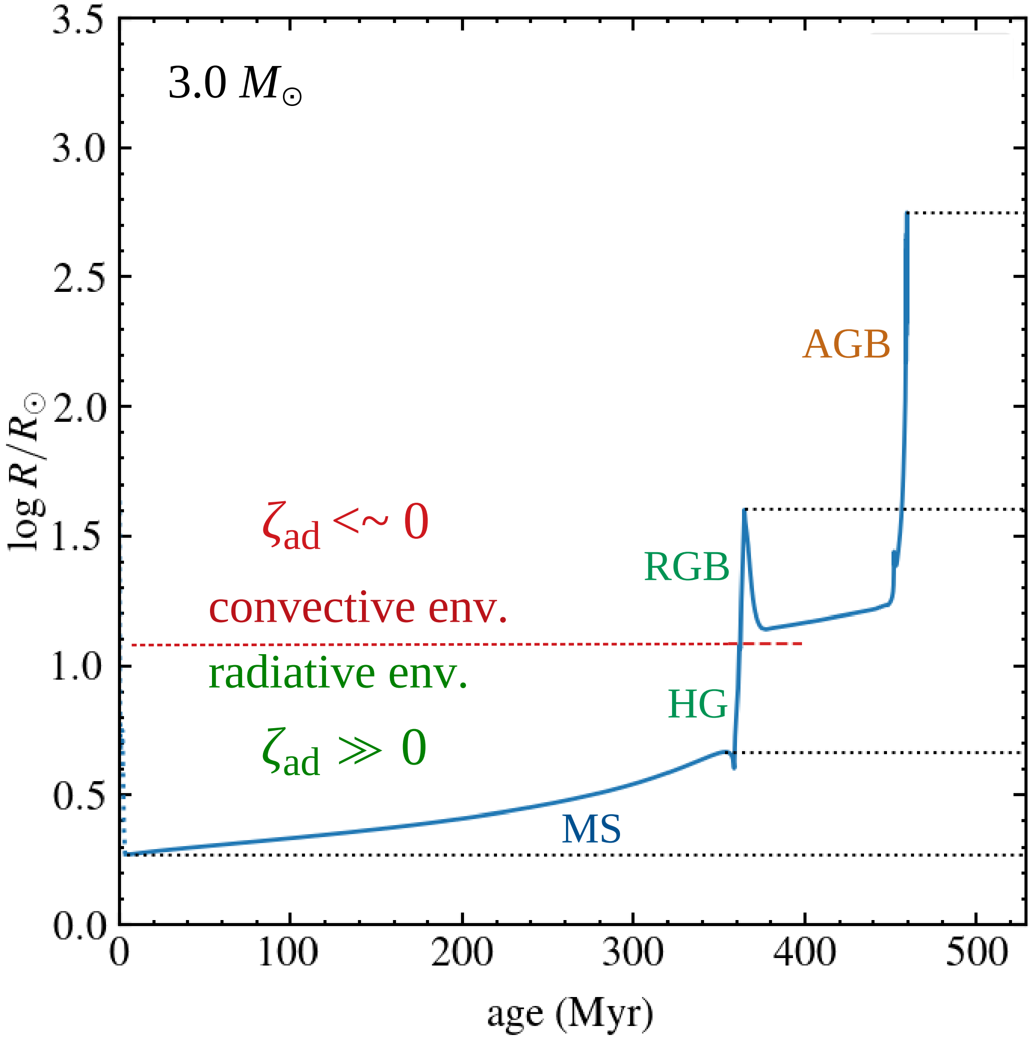
stability of mass transfer

- adiabatic stellar response to *rapid* mass loss (ζ_{ad}):
  - stars with radiative envelopes (upper MS, HG stars) shrink in response to rapid mass loss:

    $$\zeta_{ad} \gg 0$$

  - stars with convective envelopes (low-mass MS, red giants) expand, or keep a similar radius:

    $$\zeta_{ad} <\sim 0$$



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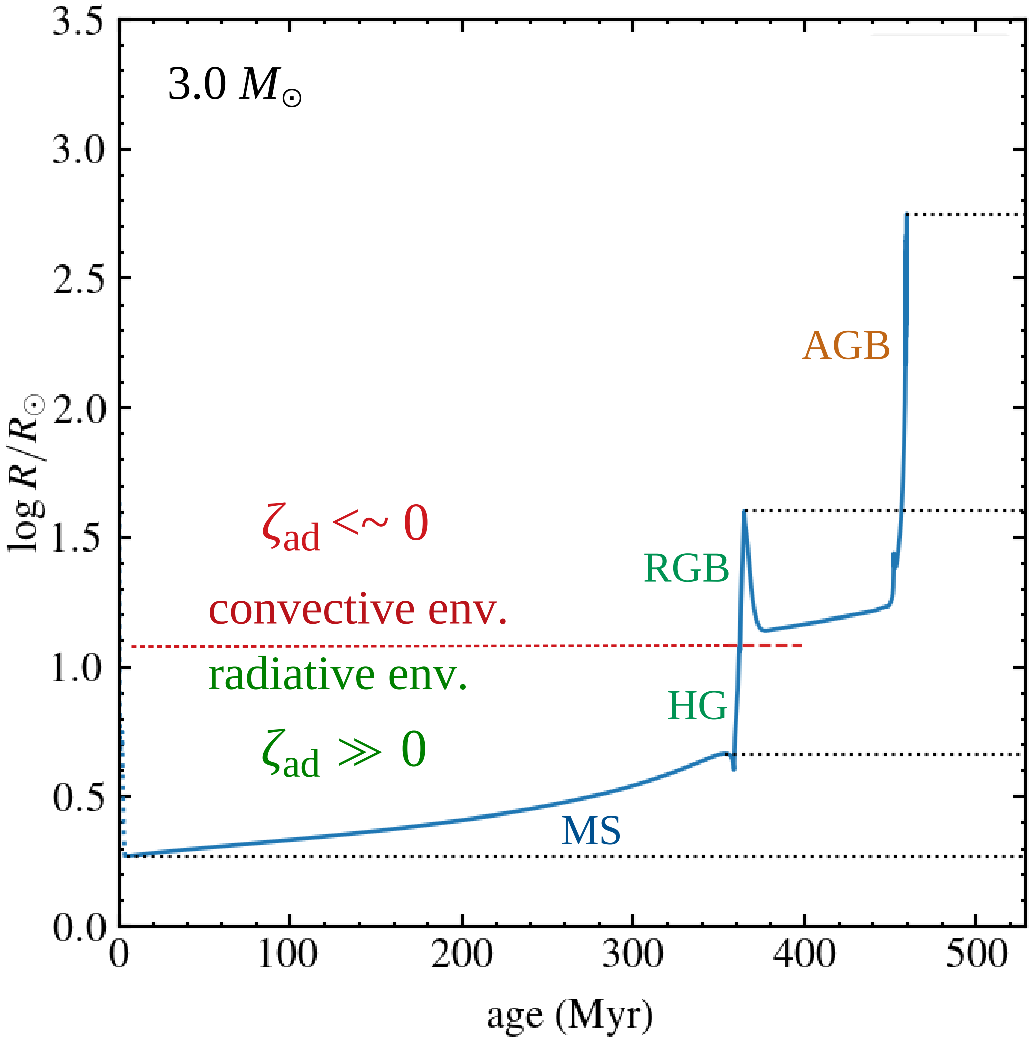
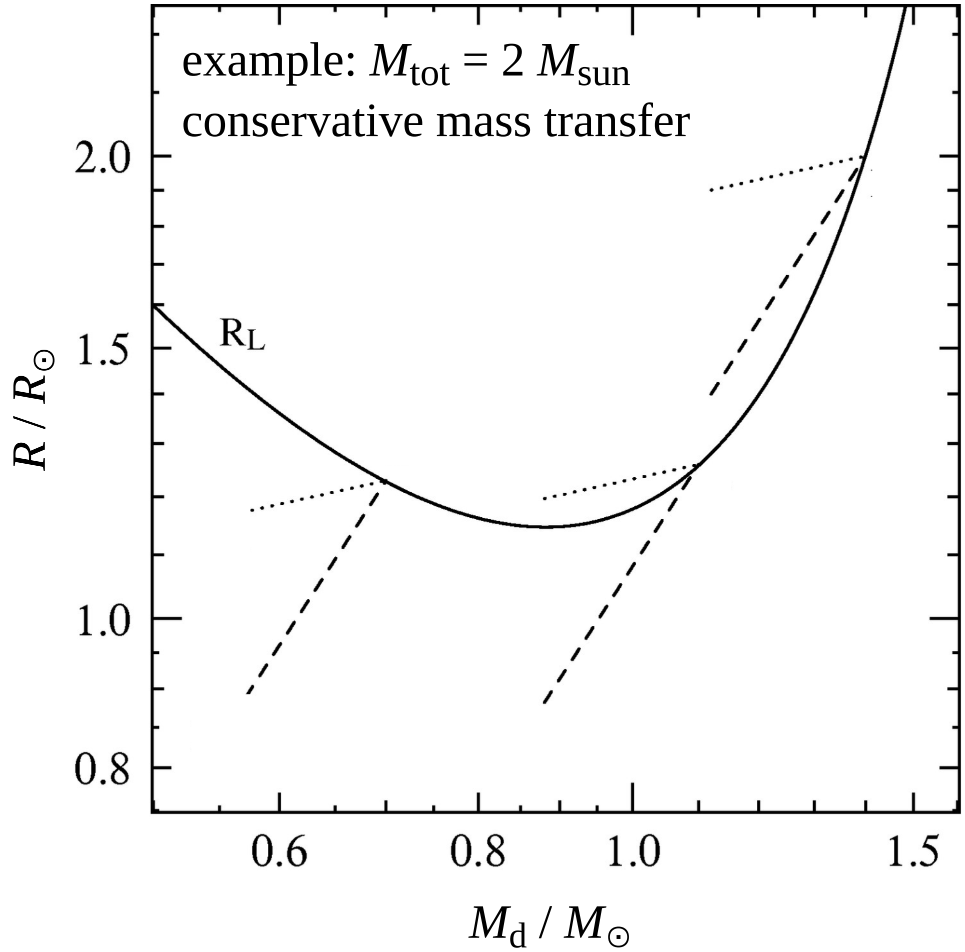
    example: fully convective star, n = 1.5 polytrope: $R \propto M^{-1/3}$

    $\Rightarrow \zeta_{ad} = -1/3$



stability of mass transfer

- Roche lobe response to mass loss (ζ_L) depends on binary mass ratio, and on mode of mass loss from system
- for *conservative* mass transfer: $\zeta_L = 2.13\,(M_d/M_a) - 1.67$



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  - stability condition ($\zeta_L < \zeta_{ad}$) translates into a critical mass ratio: RLOF is **stable** (nuclear/thermal timescale) when $M_a/M_d > q_{crit}$

- for donor stars with radiative envelopes: $q_{crit} \sim 0.25$
- for donor stars with convective envelopes: $q_{crit} \sim 0.8 - 1$

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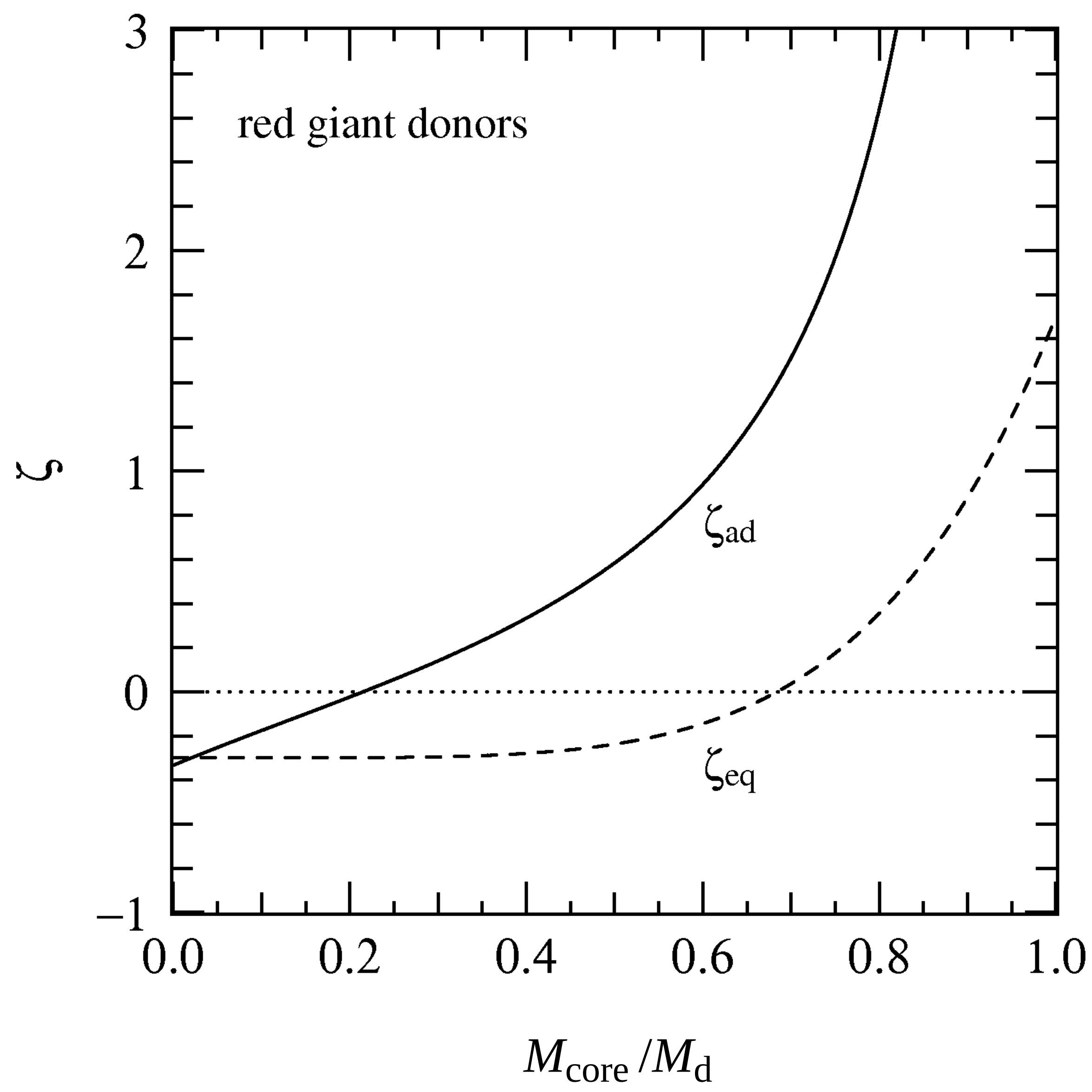
- for *non-conservative* mass transfer, ζ_L (and q_{crit}) may be smaller or larger than this, depending on amount of *angular momentum* lost
  - e.g. for isotropic re-emission by accretor, $\zeta_L < \zeta_{L,cons}$ $\Rightarrow$ stabilizing effect on mass transfer (lower q_{crit})

stability of mass transfer

- example: response to mass transfer of a red giant donor with convective envelope

  (approximated by a *condensed polytrope*)



stability of mass transfer

- example: response to mass transfer of a red giant donor with convective envelope

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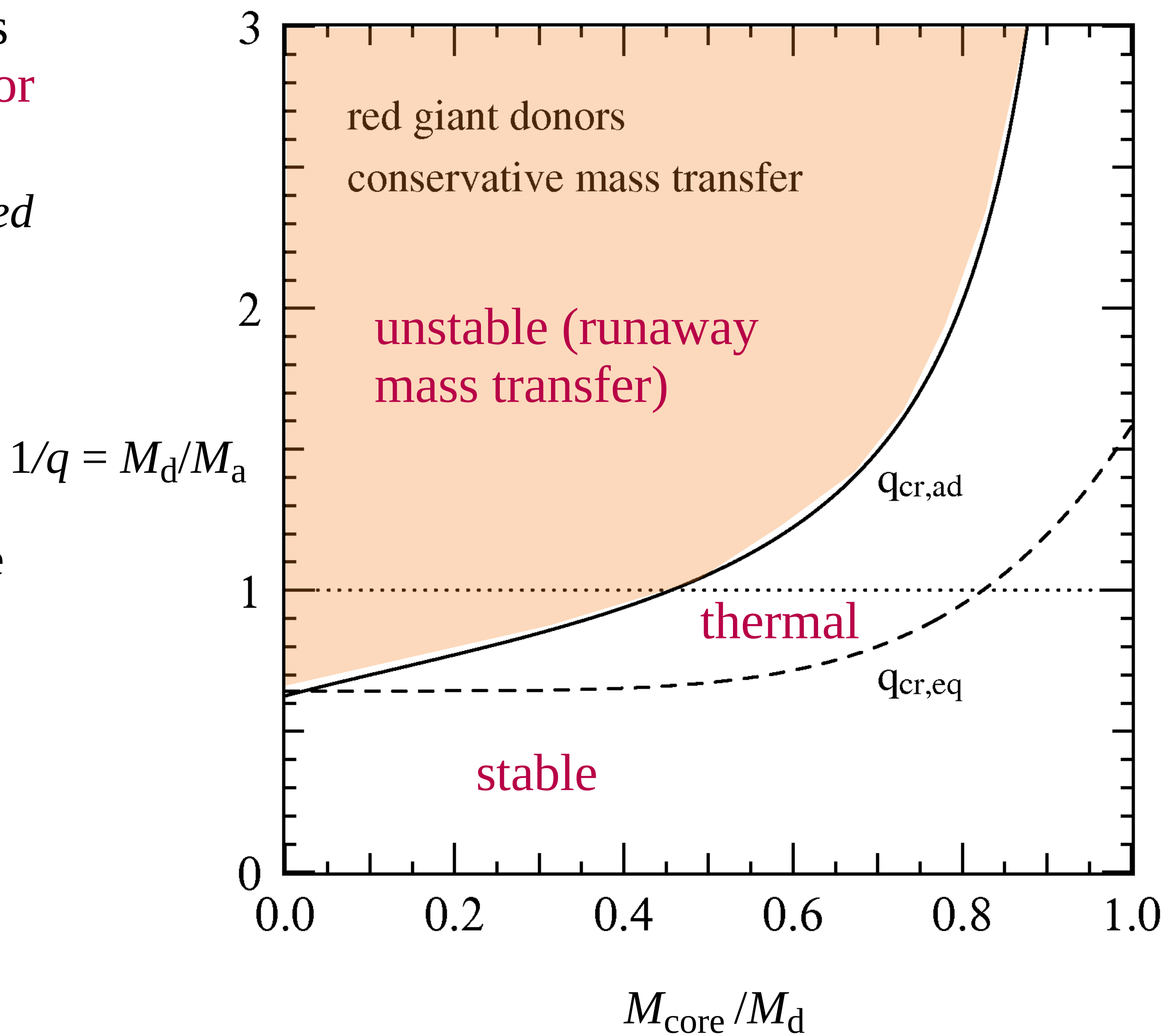
- this translates into a critical mass ratio for stability

  (shown here for conservative mass transfer)



stability of mass transfer

- example: response to mass transfer of a red giant donor with convective envelope

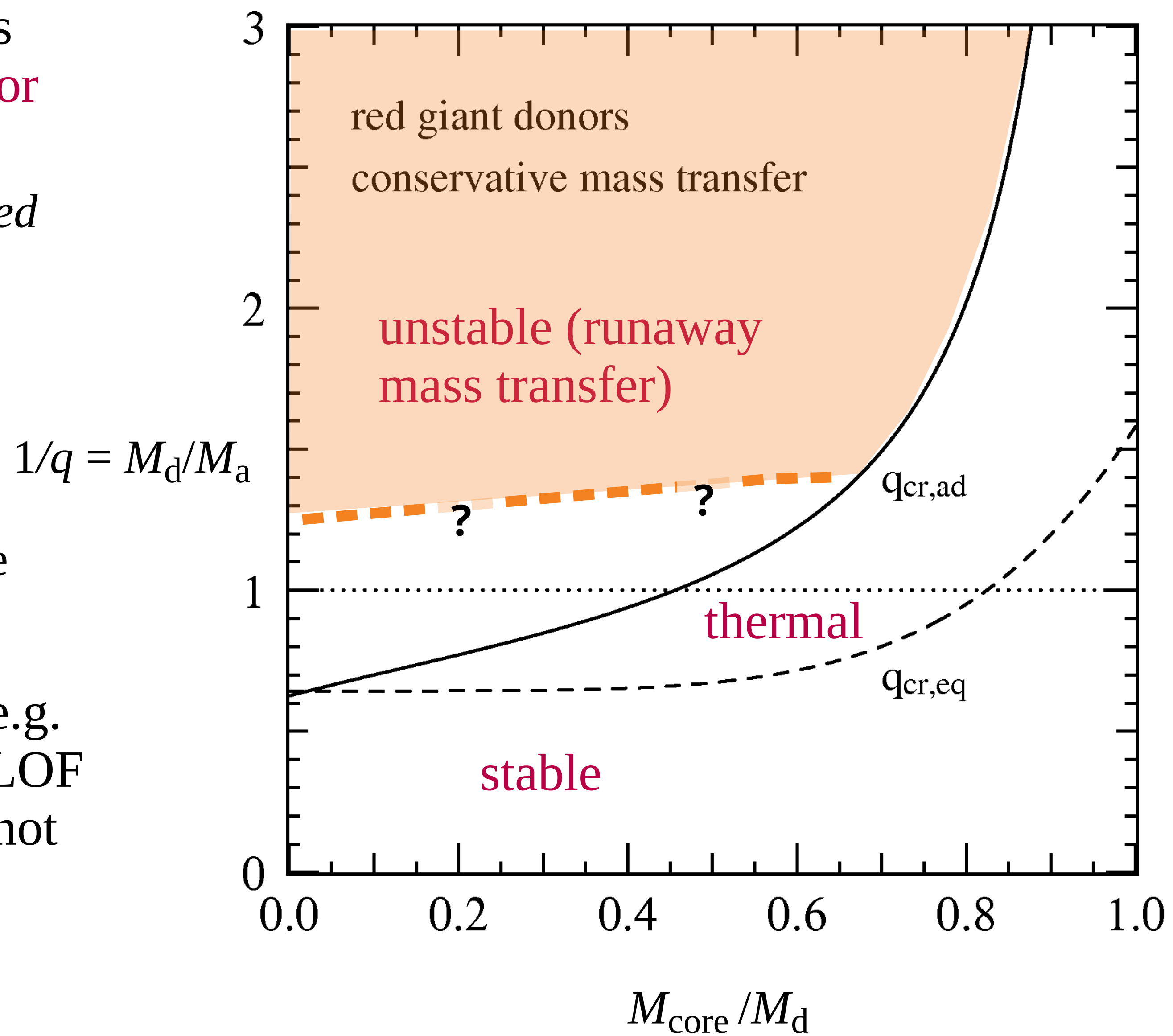
  (approximated by a *condensed polytrope*)

- this translates into a critical mass ratio for stability

  (shown here for conservative mass transfer)

- N.B. recent calculations (e.g. with MESA) show that RLOF in "dynamical" regime is not necessarily unstable

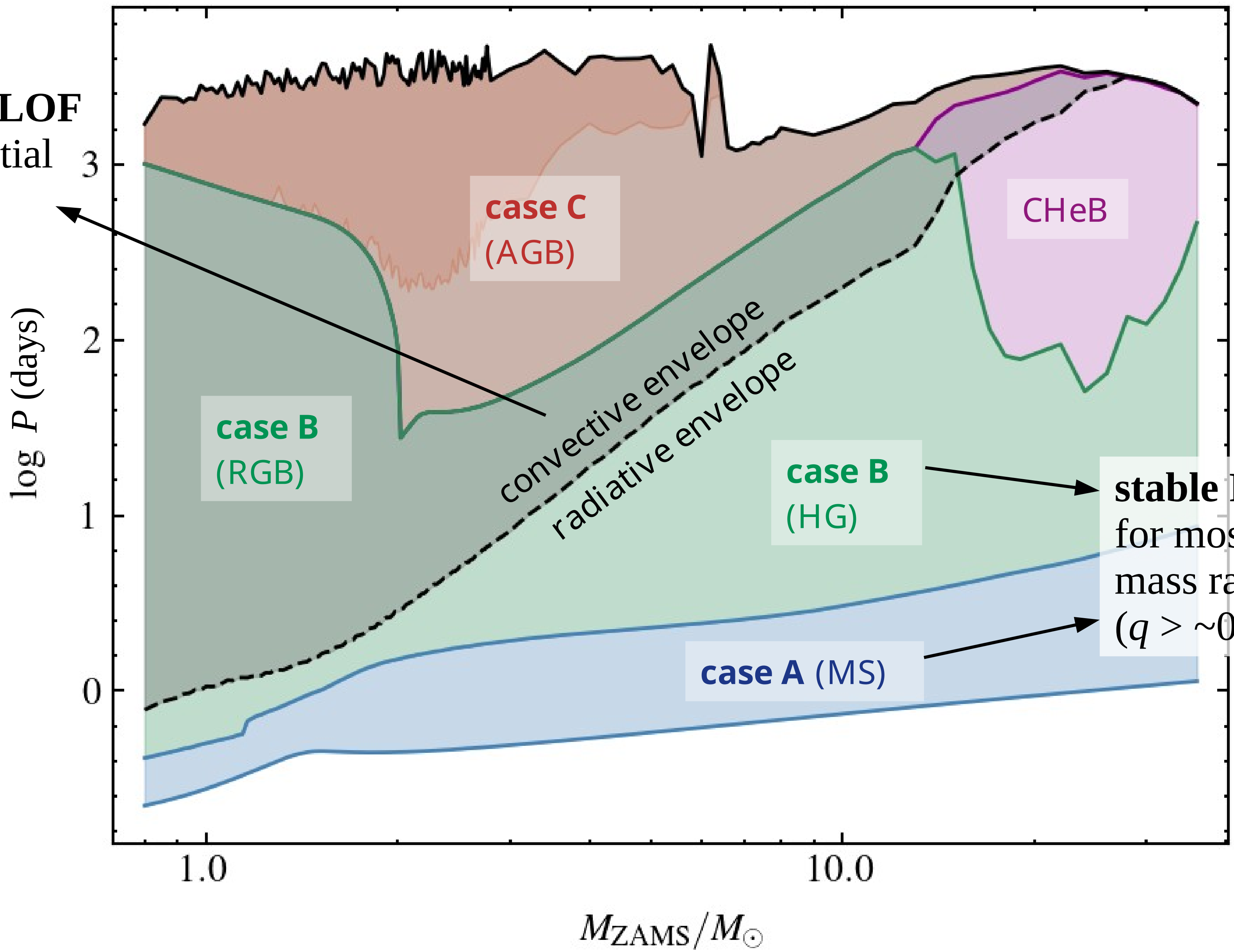
  e.g. see Temmink+ 2023



stability of mass transfer

dependence on primary mass and orbital period:

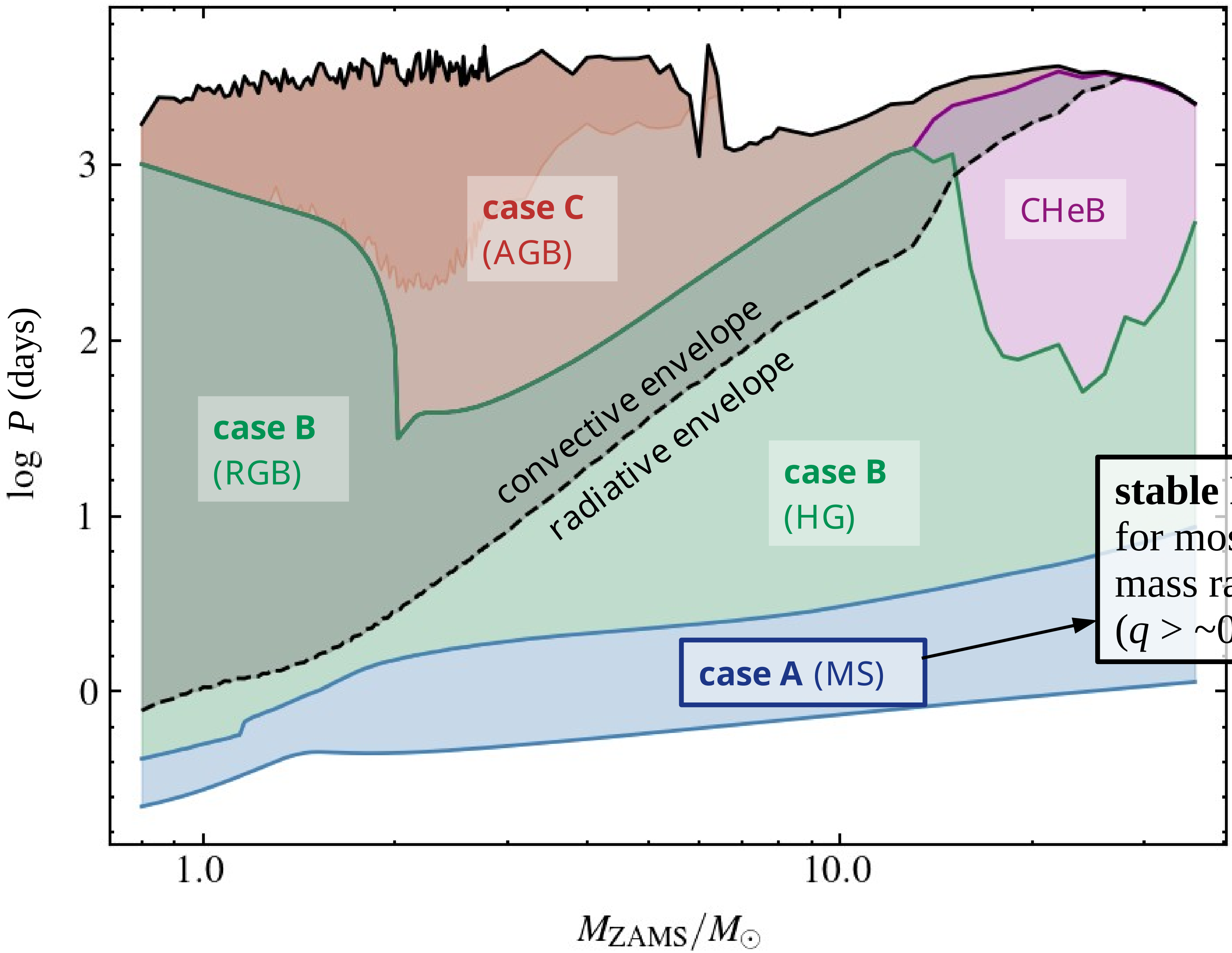
unstable RLOF
for most initial mass ratios
($q < 0.8$–1)



stable RLOF
for most initial mass ratios
($q >$ ~0.25)

stability of mass transfer

dependence on primary mass and orbital period:



stable RLOF for most initial mass ratios (q > ~0.25)