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Homework 4 Solutions
Massive Stars & Binaries

Problem 1

Here we're examining the amount of energy production that is exclusively due to the fusion of hydrogen, helium, and metals in high mass stellar models. We're looking at stars with 15, 20, 30, 40, and 60 $M_\odot$. All of our models are evolved up to the end of core carbon burning.

Part (a)

There are a number of ways to visualize this information, one is shown below via several Hertzsprung-Russell diagrams.

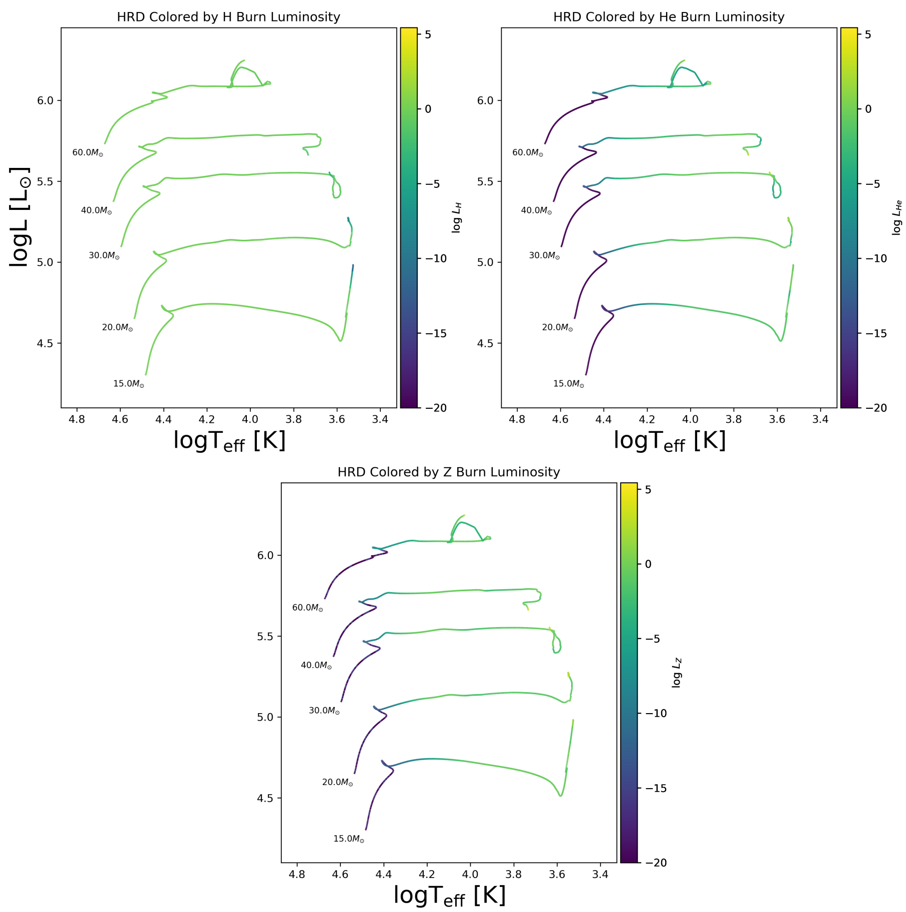

Figure 1: Problem 1 (a); HRDs of 15, 20, 30, 40, and 60 $M_\odot$ colored by their luminosity due to **left:** hydrogen fusion, **middle:** helium fusion, **right:** the fusion of metals.

In Fig. 1, we see that hydrogen fusion luminosity dominates during the main sequence (MS), as expected. After stars move along to their post MS phases, helium fusion begins to contribute on a similar level, along with energy production via the fusion of metals. Another way to examine this is shown below in Fig. 2, where points are colored according to the dominant source of energy production, with black marking where there is not one dominant source.

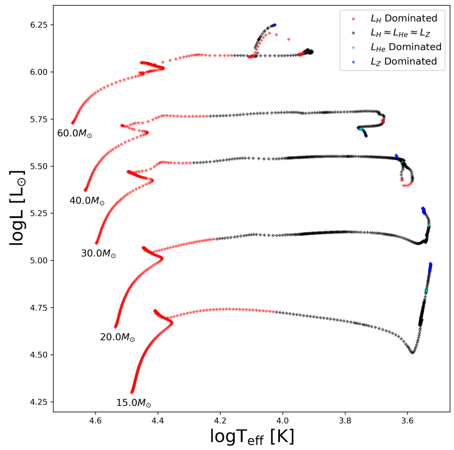

Figure 2: Problem 1 (a); HRD of 15, 20, 30, 40, and 60 $M_\odot$ colored by their dominant luminosity source. I.e., hydrogen fusion (red), or helium fusion (cyan), or the fusion of metals (blue). Points in black are where no single source dominantes. Dominance is asserted by a particular source having at least 10 times greater luminosity than both of the other sources.

From Fig. 2 it is apparent that metal and helium fusion only really become dominant sources briefly near the end of the evolution. For much of the Hertzsprung gap, the sources contribute relatively equally.

Problem 2

Now we're having a look at the effects of mass loss (according to the so-called "Dutch" scheme) on the evolution of our stars from the previous problem. High mass stars may be heavily affected by mass loss, in particular by "line driven" mass loss due to their substantial radiation outputs. The more substantial radiation pressure in these stars can carry away layers of the star in a stellar wind, depleting its mass and affecting its evolution.

Part (a)

First, let's have a look at how the tracks compare with and without mass loss. Below are HRDs of each of our masses.

We can see from Fig. 3 that models with mass loss are generally less luminous and cooler than those without mass loss. This makes sense since removing mass from the star effectively reduces the internal gravity acting on stellar layers, reducing central pressure, therefore temperature, therefore luminosity; the star effectively begins to evolve like a cooler, less luminous, lower mass star because it indeed has less mass. However, we see that above 30 $M_\odot$ in our models, late evolutionary behavior changes. Models with M > 40 $M_\odot$ start to become much hotter (although still less luminous) than models in absence of mass loss in later stages. Notably, the mass loss in these later stages is substantial, reaching the highest levels seen in our modeling. These stars become Wolf-Rayet stars (as we'll see below), and the temperatures increase substantially in part because much of the H-rich envelope has been stripped away.

Part (b)

Let's continue our analysis by looking at the surface element abundances in our models – in particular the helium, carbon, and nitrogen abundances. Shown below are the surface abundances for these elements as a function of time.

We can see from Fig. 4 that the surface of abundance of carbon shoots up near the end of the model, after experiencing heavy mass loss, suggesting that carbon features prominently in the surface composition of the star at these times. For other models, we can see that typically the presence of helium

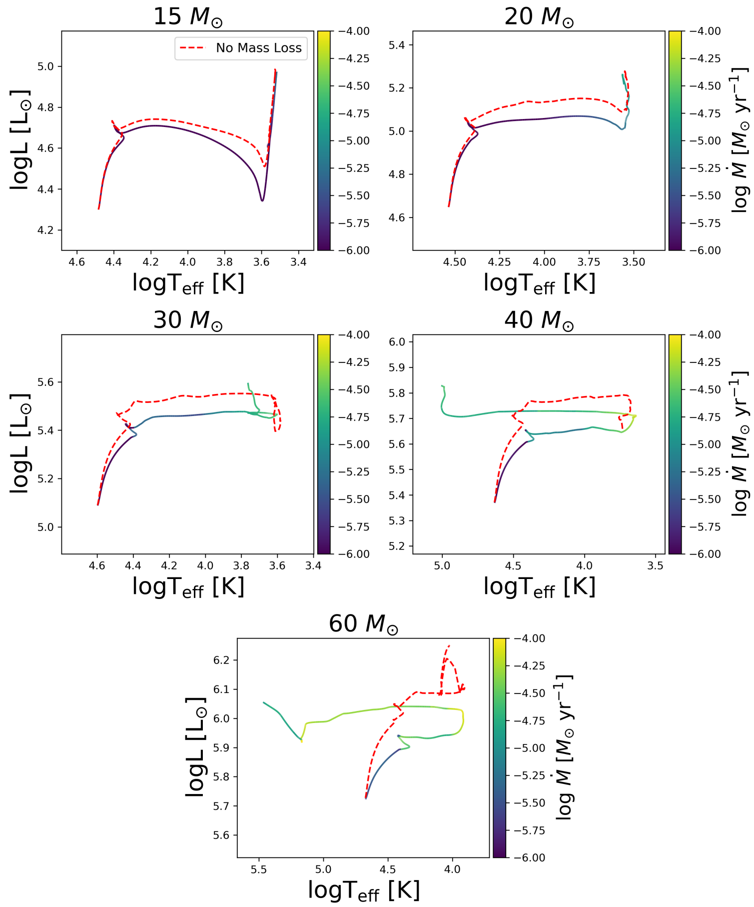

Figure 3: Problem 2 (a); HRDs of 15, 20, 30, 40, and 60 $M_\odot$. Red dashed lines mark models without mass loss, while colored lines are colored by the amount of mass loss that the model is experiencing. Mass loss here is according to the Dutch scheme.

rises alongside the abundance of nitrogen, while the abundance of carbon is depleted. Generally this trend for the other models happens to a greater degree with increasing initial mass. This fits with the picture that mass loss is shedding the outer layers of these stars, revealing regions nearer the helium burning shell outside the carbon core, which is also enriched in nitrogen as a by-product of triple alpha fusion. It is also in line with the carbon core being revealed in the 60 $M_\odot$ case, where mass loss is heaviest for the included models.

Wolf-Rayet (WR) stars are distinguished by having high levels of helium and nitrogen or carbon in their surface composition. It is believed that these stars represent O type stars that have undergone severe mass loss, revealing the inner layers of their structure, like our models here appear to show. Continuing our analysis, we can look at the HRDs of these models colored by surface hydrogen, helium abundance, and the ratios of nitrogen to carbon surface abundances. This information is displayed in Fig. 5 below.

WR stars are often identified by a relatively low surface hydrogen abundance ($X < 0.3$), and are sub-divided into two classes: WN (high surface N abundance) and WC (high C abundance). From Fig. 5, apparently we should expect to see WR stars in a region roughly of $\log T_{\rm eff} > 4.0$ [K] and $\log L > 5.5$ [$L_\odot$], or in the hatched region shown in Fig. 5. At the end of the 60 $M_\odot$ track, we see the N/C ratio drop, as the surface C abundance rises dramatically in this phase, as seen in Fig 4.

<u>Problem 3</u>

Binary stellar systems are common, especially for high mass stars. Here we're exploring the probability

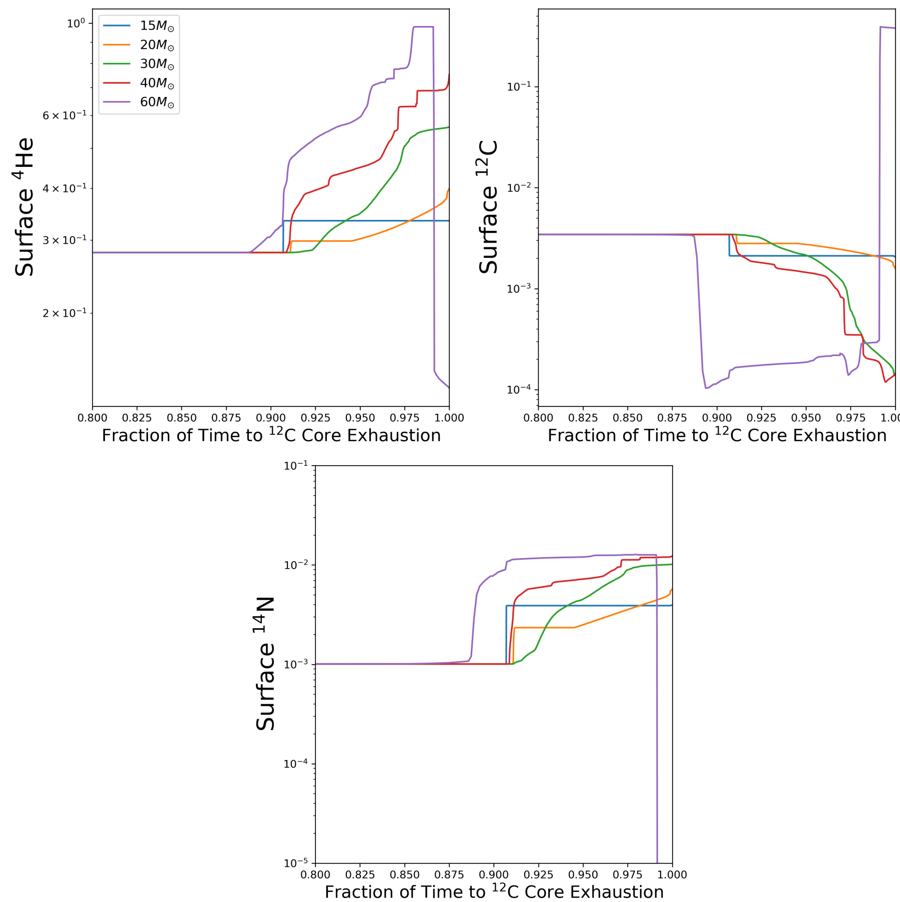

Figure 4: Problem 2 (b); Helium, carbon, and nitrogen surface abundances of 15, 20, 30, 40, and 60 $M_\odot$ `MESA` stellar models as a function of time to core carbon exhaustion. Only the final 80% of the lifetime is shown, as abundance levels remain static prior to this.

that our models computed in the previous problems might interact with a binary companion over its lifetime. Binary interactions can also lead to mass transfer and subsequent effects that may substantially alter the evolution of a star as well.

Part (a)

We are given equations for the Roche lobe radius, orbital period, and the probability distributions of the mass ratio and log orbital period for binary systems. With these equations, we may determine the probability that a binary interaction will occur. This is deemed to happen when the primary (higher or equal mass) star's radius expands beyond its Roche lobe radius, R_L. At this point, stellar material from the primary star is outside the spatial region where the primary star's gravity dominates, thus it becomes unbound and may flow to the binary companion, thereby transferring mass. There are a number of ways to calculate the probability of interaction, the chosen method here is Monte Carlo simulation, where a number of log orbital periods and mass ratios are drawn in the respective intervals [0.15, 3.5] days, and [0.1, 1.0]. The probability of interaction is calculated as the fraction of samples that satisfy the condition $R_1 > R_{L,1}$ is satisfied, where subscript 1 refers to the primary star, and R is the stellar radius.

Generally, the probability of a binary interaction rises for each of our models as time goes on. This makes sense, later on stars inflate after core hydrogen exhaustion, becoming giants. Thus, the probability that an interaction will occur is highest after hydrogen fusion. We also see from Fig. 6 that the probability of interaction generally has an overall increase as the initial mass of the model is increased; this makes

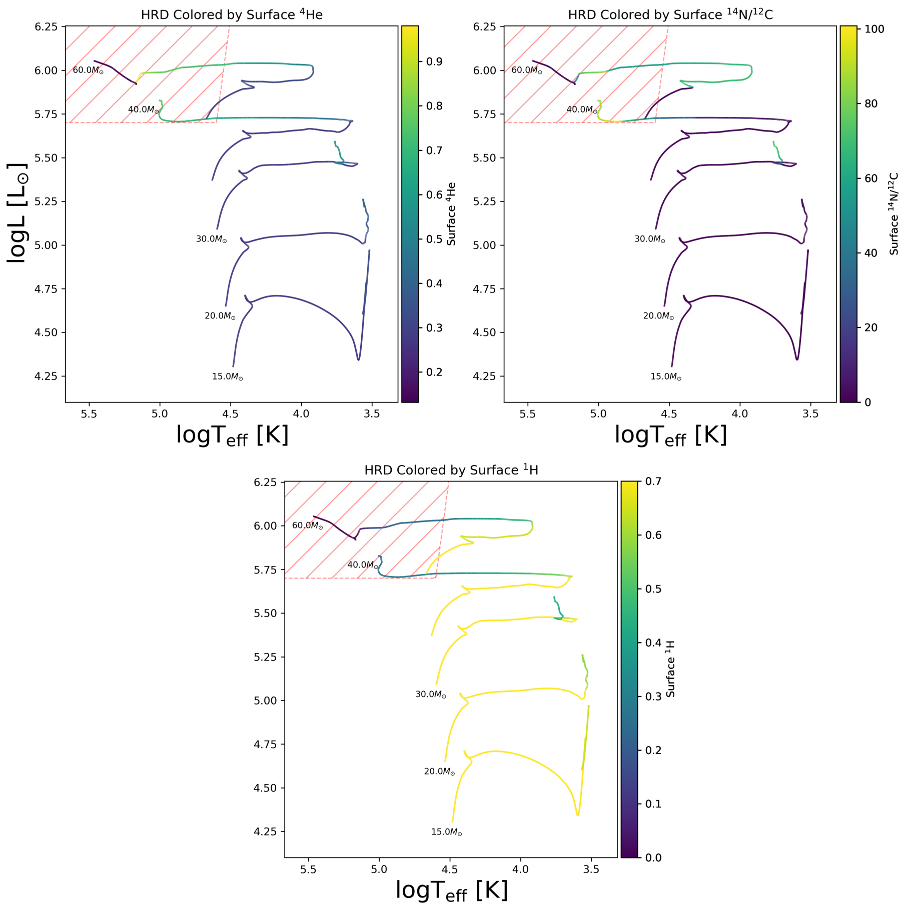

Figure 5: Problem 2 (b); HRDs of 15, 20, 30, 40, and 60 $M_\odot$ MESA stellar models, colored by surface helium (left), nitrogen to carbon ratio (middle), and hydrogen (right) abundances. The hatched region shows roughly where Wolf-Rayet stars might be expected.

sense too, since higher mass stars tend to be more tenuous and inflated objects, increasing the likelihood of a Roche lobe overflow for a greater range of binary companion masses.

In summary, the probability of binary interactions is low on the main sequence but is likely very high in post-main sequence evolutionary phases for massive stars.

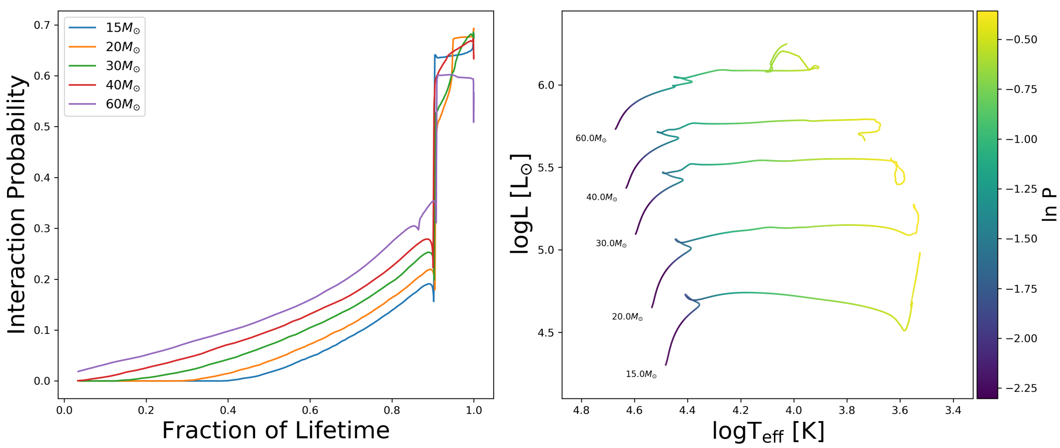

Figure 6: Problem 3 (a); Probability of interaction between binary stars for each of our models across their lifetime is shown at left; HRD with evolutionary tracks of 15, 20, 30, 40, and 60 $M_{\odot}$ `MESA` stellar models, colored by their log binary interaction probabilities shown at right. Probability has been calculated via a Monte Carlo simulation of 10,000 binary companion scenarios (i.e., sampling mass ratio q and log orbital period $logP$.)