Stellar Evolution – Hints to exercises – Chapter 13

13.1 **Energy budget of core-collapse supernovae**

(a) $E_{\rm gr} \approx \dfrac{GM_c^2}{R_f} = 5.2 \times 10^{53}$ erg (since $R_i \approx R_{\rm WD} \gg R_f$)

(b) $E_{\rm kin} = \frac{1}{2} M_{\rm env} v^2$, with $M_{\rm env} = M_i - M_{\rm c} = 8.6\, M_\odot$) $\Rightarrow$ $v = 3.4 \times 10^8$ cm/s (3400 km/s)

(c) $E_{\rm ph} \approx L\,\Delta t = 4 \times 10^{48}$ erg

(d) Neutrinos take away the largest fraction of the energy released in the collapse. Number of neutrinos = $E_{\rm grav}/E_\nu$ (with E$_\nu$ the average energy of one neutrino) $\approx 6.5{\times}10^{58}$.

13.2 **Neutrino luminosity by Si burning**

(a) Number of ^{56}Fe nuclei: $N(\text{Fe}) = \dfrac{M_c}{m_{\rm Fe}} = 4.3 \times 10^{55}$.

$$L_{\nu,\rm Si} = \frac{N(\text{Fe}) \times E_{\rm nuc,Fe}}{t} = 4 \times 10^{45}\ \text{erg/s}.$$

The neutrinos in a supernova are emitted during about 10 seconds $\Rightarrow$

$$L_{\nu,\rm SN1987A} = \frac{E_{\rm grav}}{10\,\rm sec} = 5.2 \times 10^{52}\ \text{erg/s}.$$

(b) $L_\nu = 4\pi d^2 F_\nu$ (with F_ν the neutrino flux and d the distance between the object and the observer). If the neutrino flux measured on earth is equal for both cases,

$$\frac{L_{\rm SN1987A}}{L_{\rm Si}} = \frac{d^2_{\rm SN1987A}}{d^2_{\rm Si}} \ \Rightarrow\ d_{\rm Si} = 14\,\text{pc}.$$

13.3 **Carbon ignition of a white dwarf**

To compute the radius and gravitational potential energy of the white dwarf, use the fact that a WD close to the Chandrasekhar limit is approximately described by a $n = 3$ polytrope (relativistic degeneracy!).

Chapter 4, Table 4.1: $\rho_c = 54.2\,\bar\rho = 54.2\,\dfrac{M_{\rm WD}}{\frac{4\pi}{3} R^3_{\rm WD}}$,

with $M_{\rm WD} = M_{\rm Ch} \approx 1.4\,M_\odot$ and $\rho_c = 2 \times 10^9$ g/cm^3 $\Rightarrow$ $R_{\rm WD} = 2.6 \times 10^8$ cm

$$E_{\rm gr} = -\frac{3}{5-n}\frac{GM^2_{\rm WD}}{R_{\rm WD}} \approx -\frac{3}{2}\frac{GM^2_{\rm Ch}}{R_{\rm WD}} = -3.0 \times 10^{51}\ \text{erg}$$

To compute $E_{\rm nuc}$, consider that:

- Fusion of ^{12}C into ^{56}Ni releases $Q_1 = [m(^{56}\text{Ni}) - \frac{56}{12}m(^{12}\text{C})]\,c^2$ per ^{56}Ni nucleus.
- Fusion of ^{16}O into ^{56}Ni releases $Q_2 = [m(^{56}\text{Ni}) - \frac{56}{16}m(^{16}\text{O})]\,c^2$ per ^{56}Ni nucleus.
- Total number of ^{56}Ni nuclei created is $M_{\rm Ch}/m(^{56}\text{Ni})$.

With equal mass fractions of C and O:

$$E_{\rm nuc} = \frac{M_{\rm Ch}}{m(^{56}{\rm Ni})} \times 0.5(Q_1 + Q_2) = 2.2 \times 10^{51}\ {\rm erg}.$$

Hence we conclude that $E_{\rm nuc} < |E_{\rm gr}|$.

However, keep in mind that the *total* binding energy of the WD is $E_{\rm tot} = E_{\rm gr} + E_{\rm int}$. In this case $E_{\rm int}$ is provided by the Fermi energies of the degenerate electrons in the white dwarf. If this is taken into account, then $|E_{\rm tot}| \ll |E_{\rm gr}|$ and $E_{\rm nuc} > |E_{\rm tot}|$, i.e. enough energy is generated to unbind the WD.