Stellar Evolution – Hints to exercises – Chapter 9

9.1 **Kippenhahn diagram of ZAMS**

Discuss your answers to this question with your fellow students or with the teaching assistant.

(a) Hint: Low-mass stars are dominated by bound-free and free-free absorption, for which κ is higher the lower the temperature.

(b) Hint: for $M > 1.3M_\odot$ the CNO-cycle dominates (see Section 9.2.2).

(c) See Section 9.2.2.

9.2 **Conceptual questions**

(a) See Section 9.1.1. Convection is so efficient that a fully convective star is capable of transporting any luminosity, independent of its structure – unlike a radiative star where L is strongly coupled to the T-gradient.

(b) See Section 9.1.1.

(c) See Sections 7.4.1 and 9.2.

(d) See Section 9.2.

9.3 **Central Temperature vs Mass**

First derive the homology relation for the radius, eq. (7.36), assuming that the stars are homogeneous and have constant opacity, i.e. by using (7.32) for the luminosity and equating this to the luminosity produced by nuclear reactions. Then substitute the radius into the homology relation (7.28) for the central temperature to find (7.38), i.e. $T_c \propto \mu^{7/(\nu+3)} M^{4/(\nu+3)}$. Take $\nu = 4$ for pp burning ($M \leq 1.3M_\odot$) and $\nu = 18$ for CNO burning ($M \geq 1.3M_\odot$).

To find the proportionality constant use solar values for pp burning and demand that T_c is a continous function to match the pp and CNO parts of the relation. In detail: for $M \leq 1.3M_\odot$, $T_c = T_{c,\odot}\,(M/M_\odot)^{4/7}$ and for $M \geq 1.3M_\odot$, $T_c = C\,(M/M_\odot)^{4/21}$. Find a numerical expression for C by demanding that $T_{\rm pp} = T_{\rm CNO}$ for $M = 1.3M_\odot$, which gives $C = 1.43 \times 10^7$ K.

$$T = \begin{cases} 1.30 \times 10^7\,(M/M_\odot)^{4/7}\ \mathrm{K} & \text{if } M \leq 1.3M_\odot, \\ 1.43 \times 10^7\,(M/M_\odot)^{4/21}\ \mathrm{K} & \text{if } M > 1.3M_\odot. \end{cases}$$

9.4 **Mass-luminosity relation**

Assuming energy transport by radiation gives $T \propto M\kappa l/(T^3R^4)$ (eq. 7.29). Substitute $\kappa \propto \rho T^{-7/2}$; eliminate ρ by $\rho \propto M/R^3$; and eliminate T using $T \propto \mu M/R$ (eq. 7.28). This altogether will give you eq. (7.33), i.e.

$$L \propto \mu^{7.5} M^{5.5} R^{-0.5}. \tag{1}$$

Now obtain a homology relation for R using $\mathrm{d}l/\mathrm{d}m = \epsilon \propto \rho T^4$, i.e. $L \propto M\rho T^4$, again eliminating T and ρ. This will give you $R \propto \mu^0 M^{3/7}$ (compare to eq. 7.36). Use this relation to express (1) in terms of M and μ only:

$$L \propto \mu^{7.5} M^{5.3}.$$