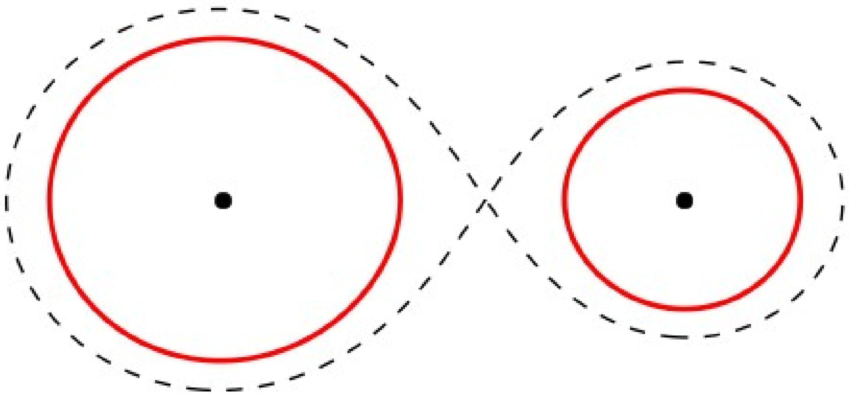
lecture 9:

tidal interactions

tidal interactions

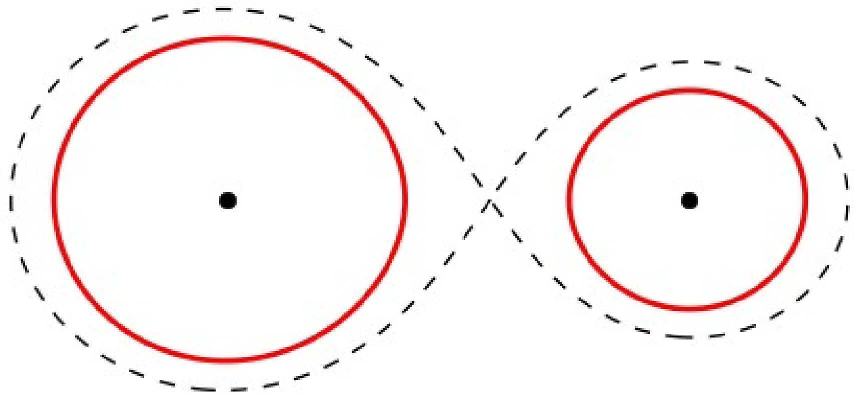
- the tidal influence of a close companion distorts an object from spherical symmetry
- tidal interaction is a slow, non-conservative process acting in close binary systems wherein kinetic energy is dissipated in the star(s)
- the total angular momentum of the system is conserved but angular momentum can be exchanged between the binary orbit and the spins of the stars



- tides are important in many astrophysical systems from moons to galaxies

tidal locking

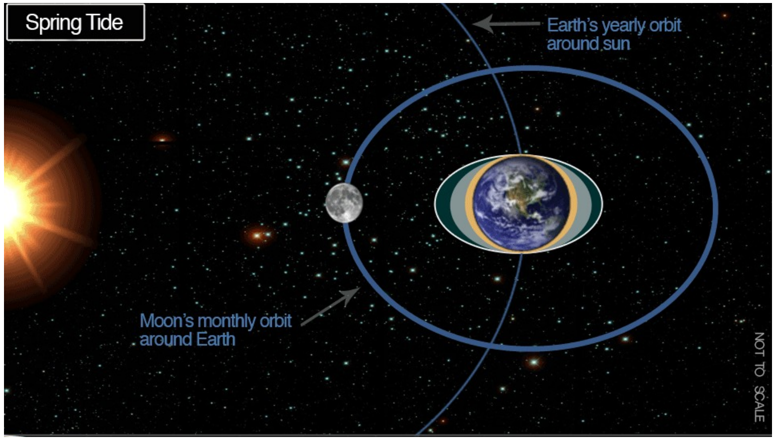
- over time, tidal dissipation tends to bring a system to a tidally locked configuration where:
    1. the orbits and spins are **synchronized** – the spin and orbital periods are the same
    2. the orbit is **circularized –** the eccentricity is zero, $e = 0$
    3. the spins are orbits are **aligned**

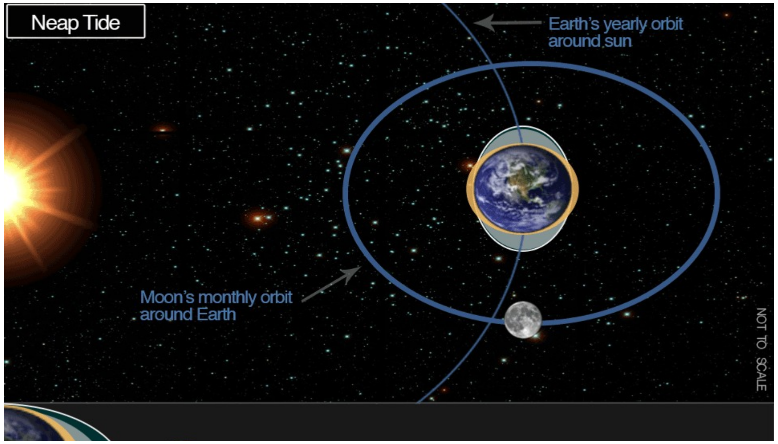


tides in the sun-earth-moon system

- several effects of tides can be seen on Earth:

  1. the oceans rise and fall twice a day on Earth – the amplitude depends on the orientation of the Sun and the Moon





even though the Sun is more massive, the Moon is much closer and thus dominates the ocean tide

Images: NOAA, ESA/NASA

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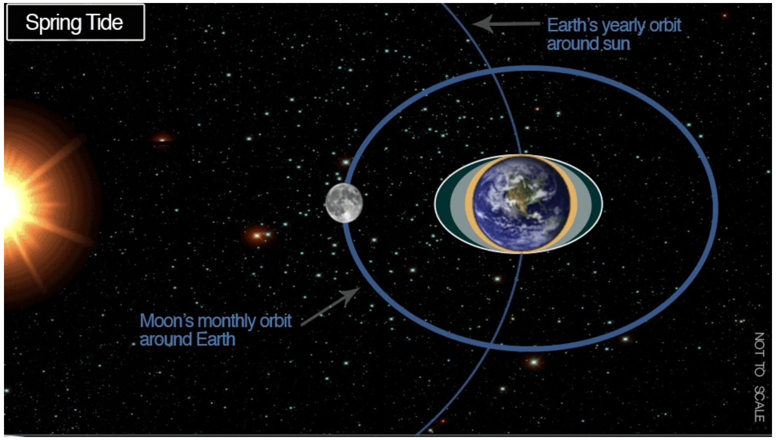
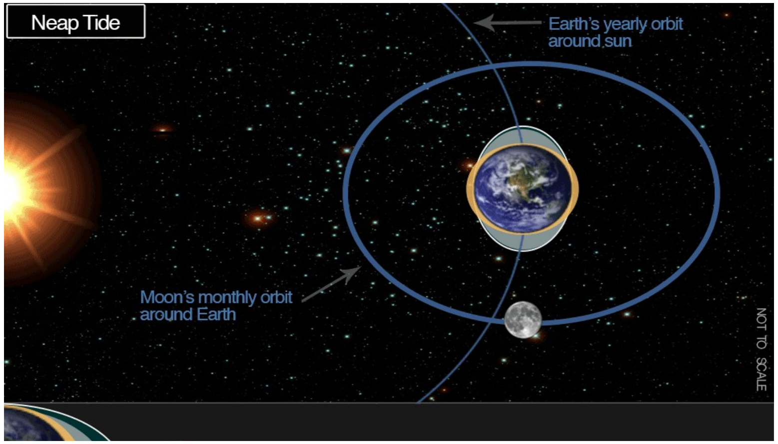




  even though the Sun is more massive, the Moon is much closer and thus dominates the ocean tide

  2. the Moon is tidally locked to Earth, so we always see the same face

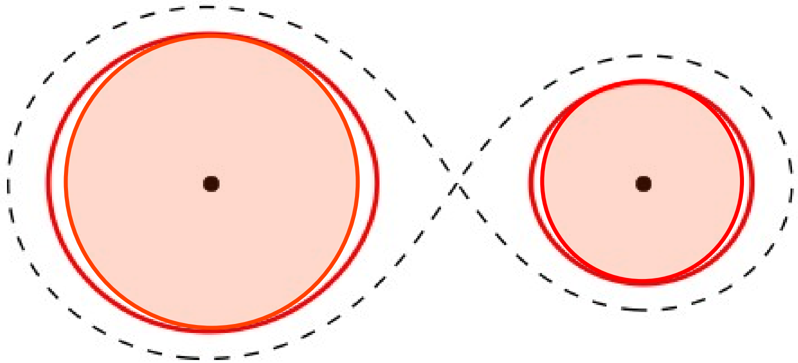
  3. the lunar orbit is almost circular, $e = 0.055$

Images: NOAA, ESA/NASA

tidal deformation

- consider a detached binary
- differential gravitational field of companion causes deformed shape: "tidal bulges"
- in Roche geometry (when $e = 0$, $\boldsymbol{\omega} = \boldsymbol{\Omega}_1 = \boldsymbol{\Omega}_2$) the equilibrium shape is determined by the Roche potential

tidal bulges are aligned with companion, and constant in time

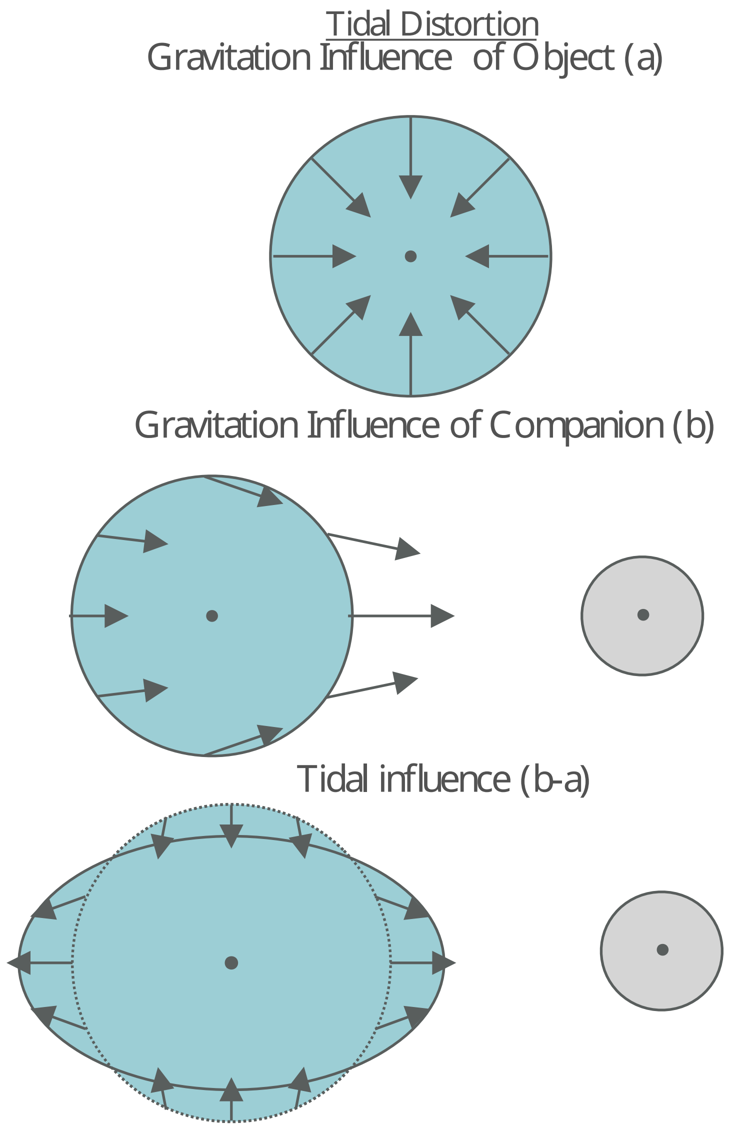


- when tidally locked, the bulges do not move w.r.t. the star and there is no dissipation

the shape of the tidal bulge

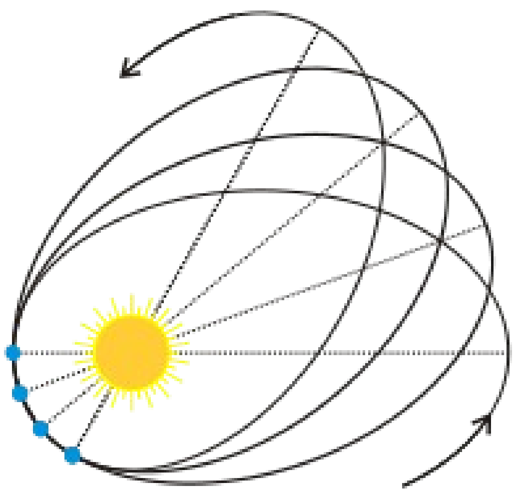
- to first order the shape of tidal bulge can be approximated as a spherical ellipsoid (Lagrangian polynomial)
- the deformed star has a non-zero quadrupole moment (spherical symmetry is broken)

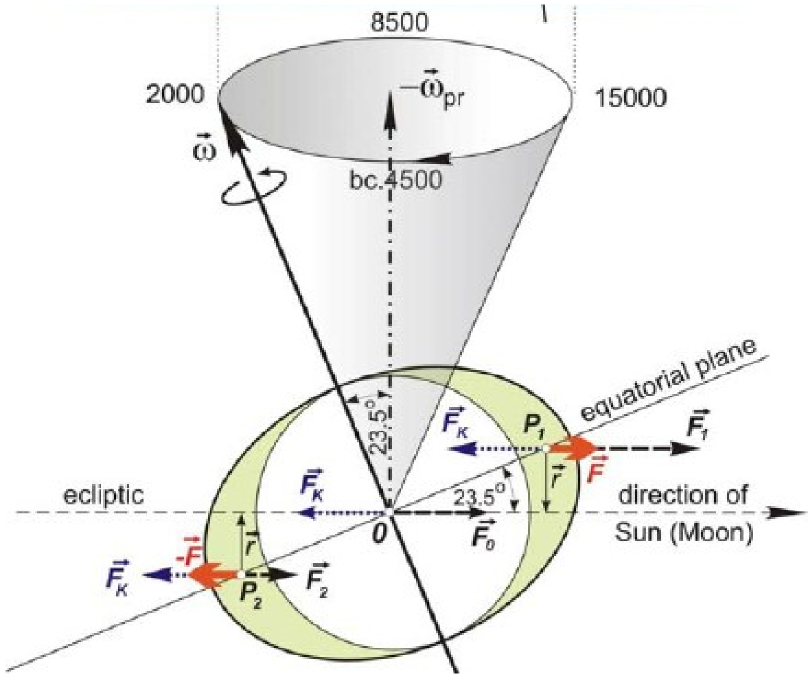
but the dipole moment is zero
(the two bulges are the same shape/mass)



apsidal motion, precession

- tidal deformation (a non-zero quadrupole moment) implies the effective gravitational force is no longer $\propto 1/r^2$
- this can lead to *apsidal motion* in an eccentric orbit (when e > 0), or to *precession* in an inclined orbit (when $\boldsymbol{\omega} \neq \boldsymbol{\Omega}_{1,2}$)





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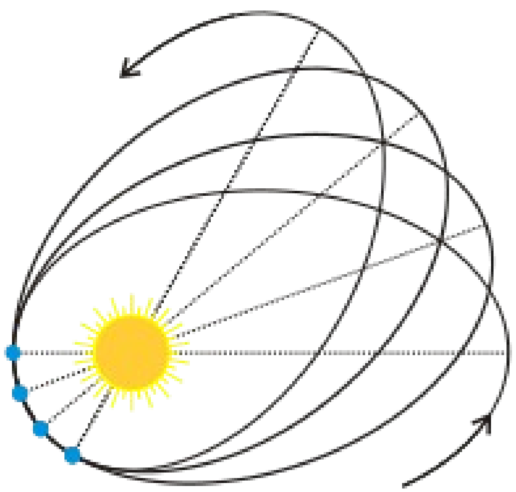
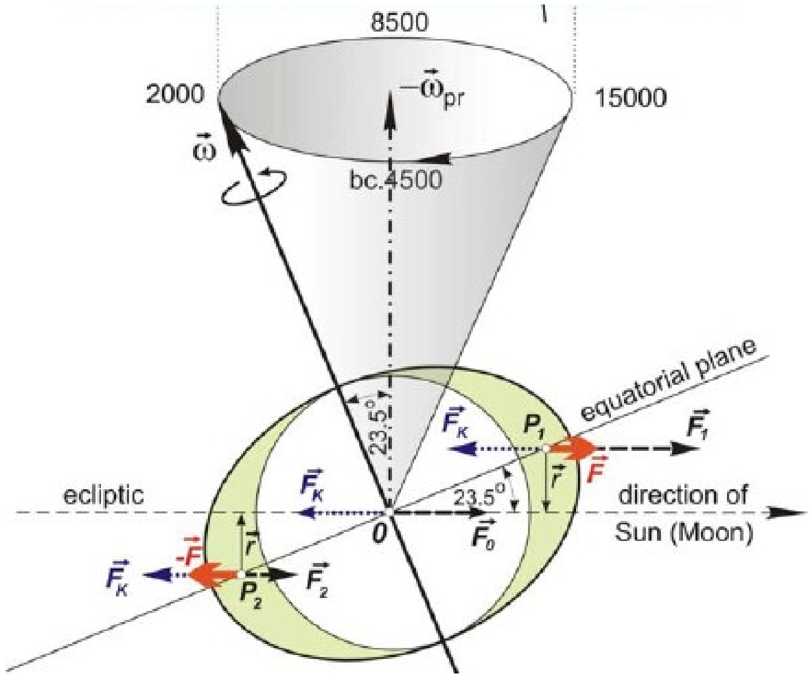




- these effects change the orientation of the orbit/spin vectors, but not the shape of the orbit (or the *evolution*)

time-varying tidal distortion

- in general, in a detached binary one can have:

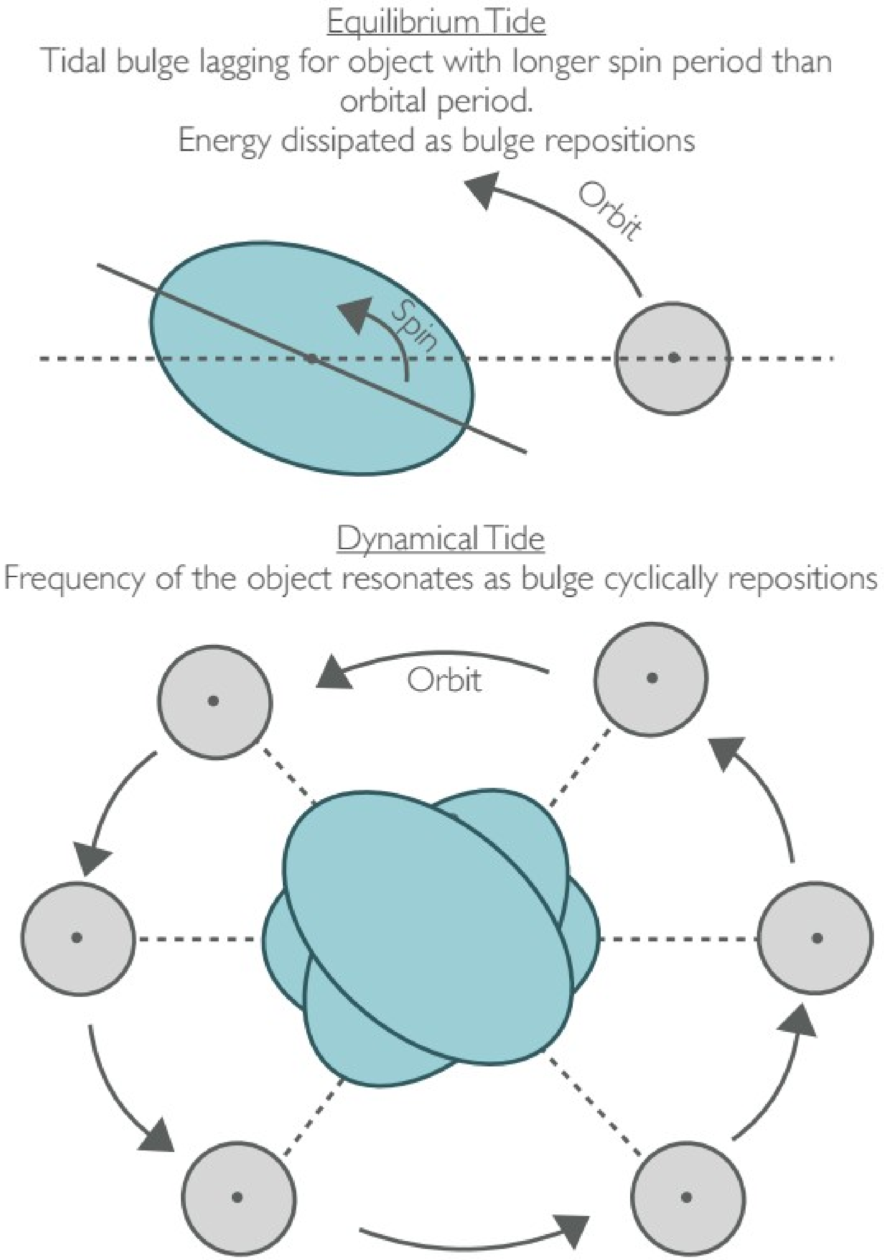
  $e > 0$, $\boldsymbol{\omega} \neq \boldsymbol{\Omega}_{1,2}$

  (either *non-synchronous* or *misaligned* rotation)

- any of these circumstances mean the tidal field is variable in time, with a period

$$\frac{1}{P_{tid}} = \left| \frac{1}{P_{orb}} - \frac{1}{P_{spin}} \right|$$

- when the tidal bulge is varying in time w.r.t. the star, dissipation can occur



tidal interaction

- as long as changes in structure due to varying tidal field are *adiabatic*, there is no dissipation of energy $\Rightarrow$
  $E_{\rm orb}$ and $J_{\rm orb}$ are both conserved (and therefore a and e)
- however if dissipation of tidal energy occurs $\Rightarrow$
  $J_{\rm tot} = J_{\rm orb} + J_{\rm rot}$ is conserved, but $E_{\rm tot}$ decreases

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$$J = M_1 M_2 \sqrt{\frac{Ga(1-e^2)}{M_1+M_2}} + I_1\Omega_1 + I_2\Omega_2$$

orbit spin

$$E = -\frac{GM_1M_2}{2a} + \tfrac{1}{2}I_1\Omega_1^2 + \tfrac{1}{2}I_2\Omega_2^2$$

- tidal dissipation leads to secular changes in a, e and $\Omega_{1,2}$

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orbit spin

let's assume *2 is a point mass

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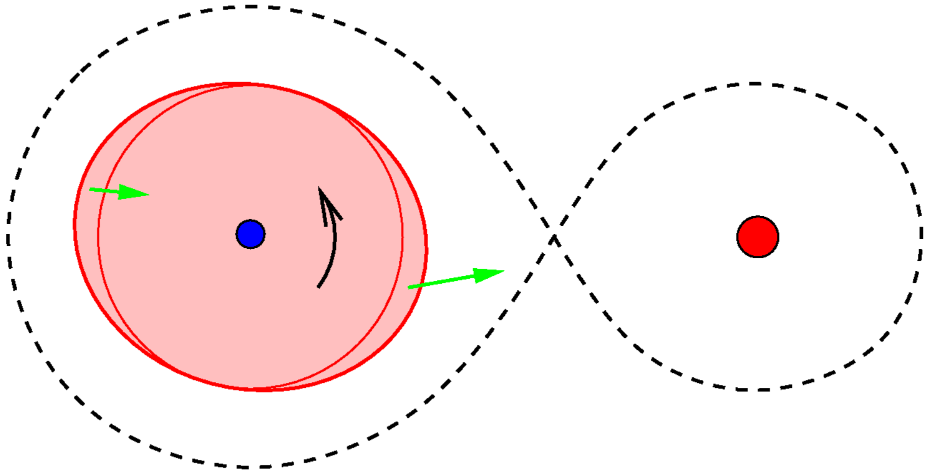
tidal interaction

- exchange of orbital and spin angular momentum: spin-orbit coupling
- example for the case $\omega > \Omega$:



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- resulting torque transfers J_{orb} into J_{rot} $\Rightarrow$ Ω increases, a decreases (or vice versa if $\omega < \Omega$)

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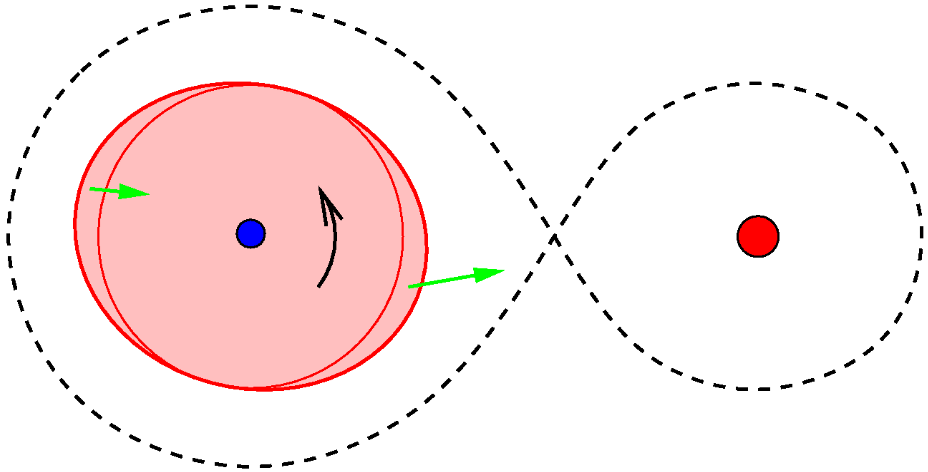


- tidal friction causes a time lag in tidal bulges, compared to their instantaneous equilibrium shapes
- resulting torque transfers $J_{\rm orb}$ into $J_{\rm rot}$ $\Rightarrow$ Ω increases, a decreases (or vice versa if $\omega < \Omega$)
- even when $J_{\rm orb} \approx$ constant in an eccentric orbit, orbital energy can be dissipated by tidal friction $\Rightarrow$ in this case e decreases

tidal equilibrium

- due to tidal dissipation, E_{tot} decreases (while J_{tot} is conserved) until an equilibrium situation is achieved

- Hut (1980) shows that there is only one possible stable equilibrium state:

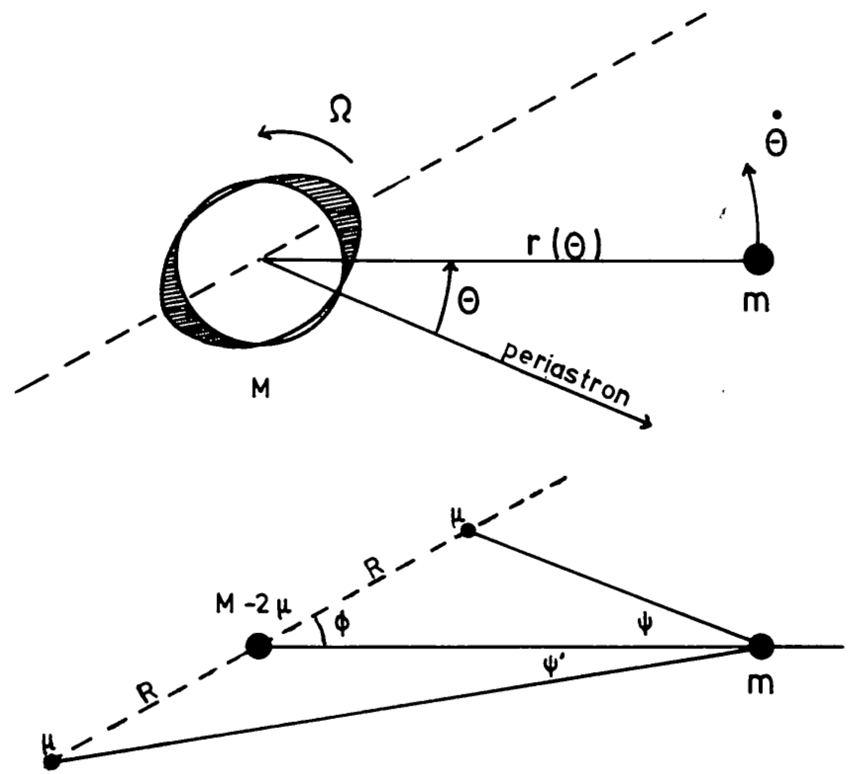
$e = 0$	(circular orbit)
$\omega = \Omega_1 = \Omega_2$	(co-rotation)
$i_1 = i_2 = 0$	(co-planarity)

- stable equilibrium is possible only if $J_{orb} > 3J_{rot}$
  - otherwise, tidal forces lead to a continual decay of the orbit, until stars collide (Darwin instability)

tidal evolution

- *how* do tides change the binary orbit and stellar spin as a function of time?
- a simple description of tidal evolution is the equilibrium tide model
(Darwin 1879, Alexander 1973, **Hut 1981**)
- assumptions:
  - a constant small *time lag*
    direction and amplitude)
    between the *actual* shape of a star
    equilibrium shape
  - tidal bulges approximated by two
    point masses

    $\mu \,\propto\, m_{\mathrm{comp}}\,(R/r)^3$



- this yields a tidal perturbing force $\propto\,(R/r)^5$ [relative to gravity]
that slowly changes a, e and Ω

tidal evolution

- results of the equilibrium tide model (Hut 1981) in the limit of $e^2 \ll 1$:

$$\dot{a} \approx -6\frac{k}{T}q(1+q)\left(\frac{R}{a}\right)^8 a\left(1-\frac{\Omega}{\omega}\right)$$

$$\dot{e} \approx -27\frac{k}{T}q(1+q)\left(\frac{R}{a}\right)^8 e\left(1-\frac{11}{18}\frac{\Omega}{\omega}\right)$$

$$\dot{\Omega} \approx 3\,\frac{k}{T}\frac{q^2}{r_{\mathrm{g}}^2}\left(\frac{R}{a}\right)^6(\omega-\Omega)$$

- consequences:
  - when $\Omega < \omega \;\Rightarrow\; a$ decreases, Ω increases, and vice versa
  - when $\Omega < 18/11\ \omega \;\Rightarrow\; e$ decreases (circularization)
  - however, *e* can increase when $\Omega > 18/11\ \omega$ (e.g. Earth-Moon system)

- all rates depend on high powers of *R*/*a* $\Rightarrow$ tidal interaction is only important when star fills a significant fraction of its Roche lobe

tidal evolution

- timescales for synchronization and circularization:

$$\tau_{\rm sync} = \left|\frac{\Omega - \omega}{\dot{\Omega}}\right| \approx \frac{r_{\rm g}^2}{3}\frac{T}{k\,q^2}\left(\frac{R}{a}\right)^{-6}$$

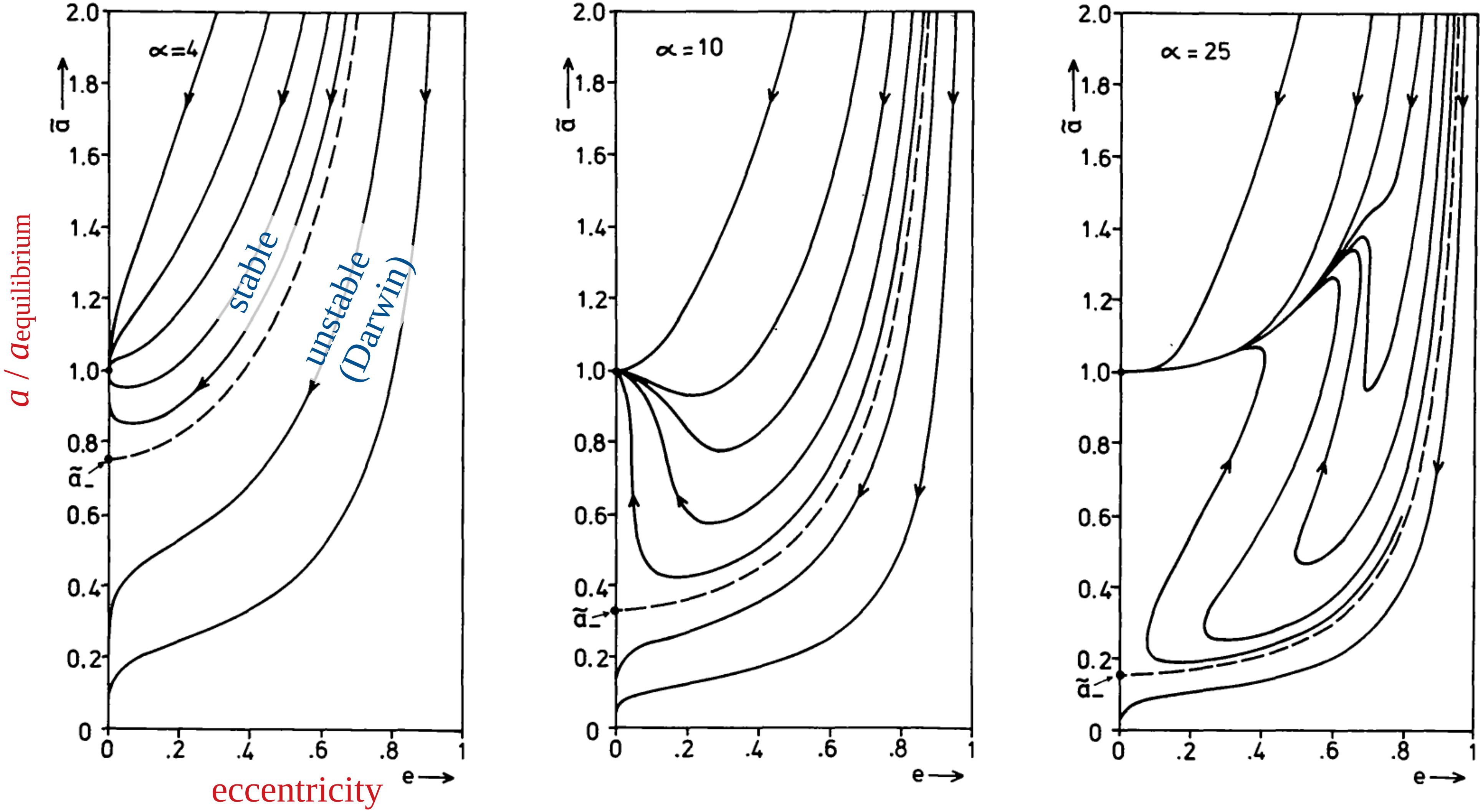
$$\tau_{\rm circ} = \left|\frac{e}{\dot{e}}\right| \approx \frac{2}{21}\frac{T}{k\,q(1+q)}\left(\frac{R}{a}\right)^{-8}$$

(assuming $\Omega \approx \omega$)

- consequences for an evolving binary (with increasing R/a):
  - synchronization is typically reached before circularization
  - if $e > 0$, then Ω can be synchronized with $\omega_{\rm periastron}$ (which is $> \omega$): "pseudo-synchronization"; after which orbit is circularized on longer timescale $\tau_{\rm circ}$

tidal evolution

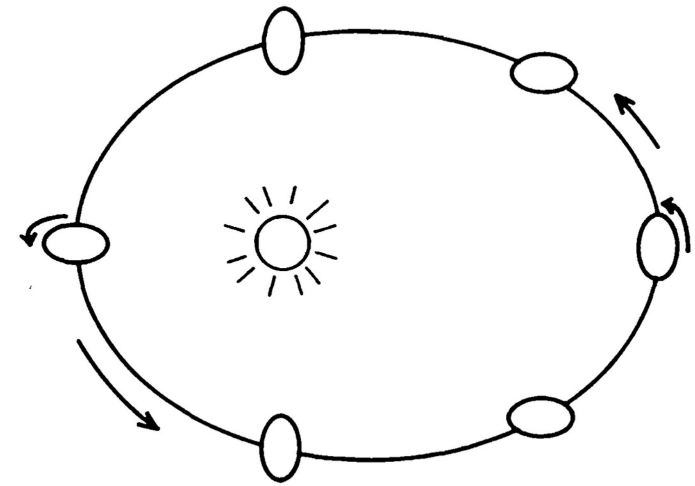
- examples of tidal evolution towards equilibrium (from Hut 1981):

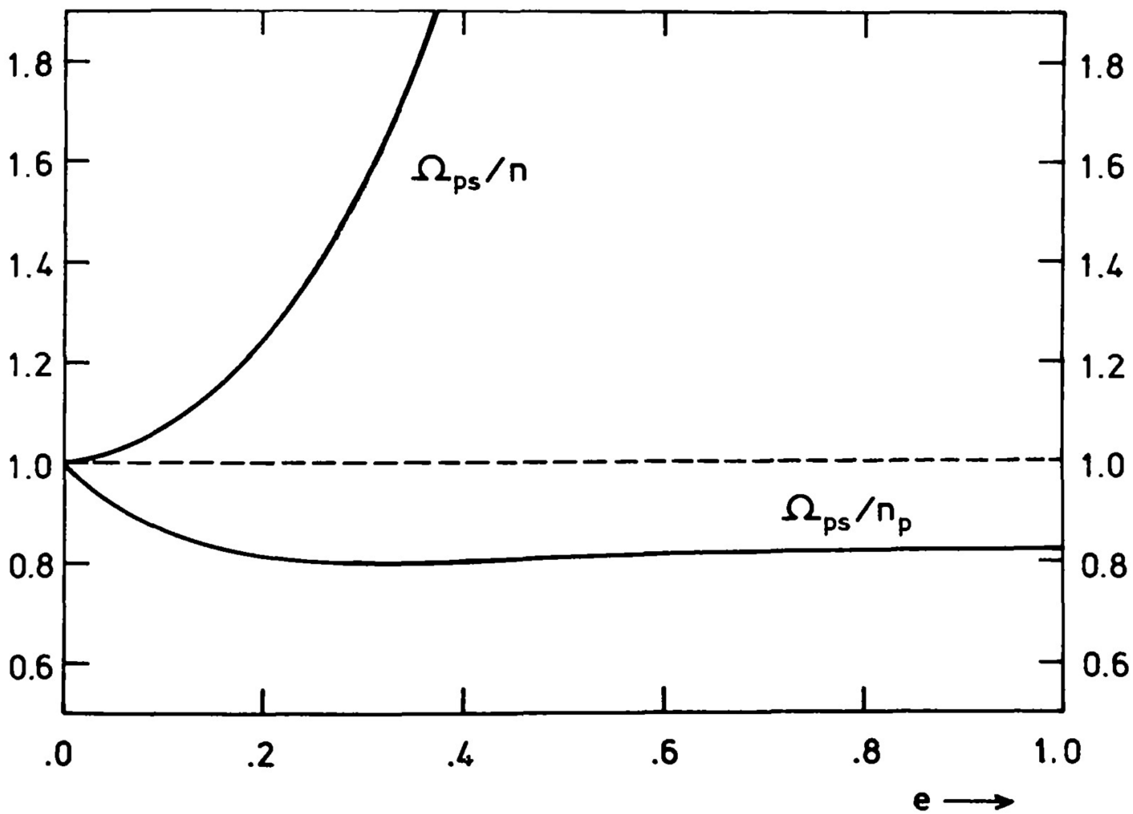


α = ratio of spin AM to orbital AM at equilibrium; proxy for changing the mass ratio q

pseudo-synchronization

- occurs in eccentric orbits
- tides are strongest at periastron:
  most of the synchronization occurs here





n is the mean orbital angular velocity ($2\pi/P$),
n_p is the orbital velocity at periastron (ω_{per})

Hut 1981

dissipation mechanisms

- tidal dissipation timescales are highly dependent on the structure of the star: for this we need to assume a dissipation mechanism
- stars with convective envelopes:
  - turbulence leads to efficient dissipation
  - well described by the *equilibrium tide* model, as long as the convective timescale is short compared to the tidal period
- stars with radiative envelopes, or high orbital eccentricities: should consider dynamical tides
  - variable tidal field forces stellar oscillations, which are damped by radiative energy losses
  - much less effective than turbulent dissipation, and very non-linear
  - possibility of resonances, and resonance locking
  - poorly described by the equilibrium tide model

dissipation in the equilibrium tide

- the dependence of the strength of the tidal dissipation on the structure of the star is contained in the factor k/T:

Eggleton 2006, Preece+ 2022

$$\frac{k}{T} = \frac{3}{t_{visc}} \frac{1}{(1-Q)^2}$$

$$\frac{1}{t_{visc}} = \frac{1}{MR^2} \int_0^M \nu \, \gamma \, dm = \frac{1}{3\,MR^2} \int_0^M wl \, \gamma \, dm$$

where: Q is the dimensionless quadrupole moment of the star,
ν is the convective viscosity ($\nu = w_{\mathrm{conv}} l_{\mathrm{mix}}/3$, from MLT),
γ is a scaling factor for the rate of strain tensor for the deformed star

all these quantities can be computed from a stellar structure model

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- this holds as long as the convective turnover time $\tau_{\rm conv}$ is short relative to the tidal period $P_{\rm tid}$

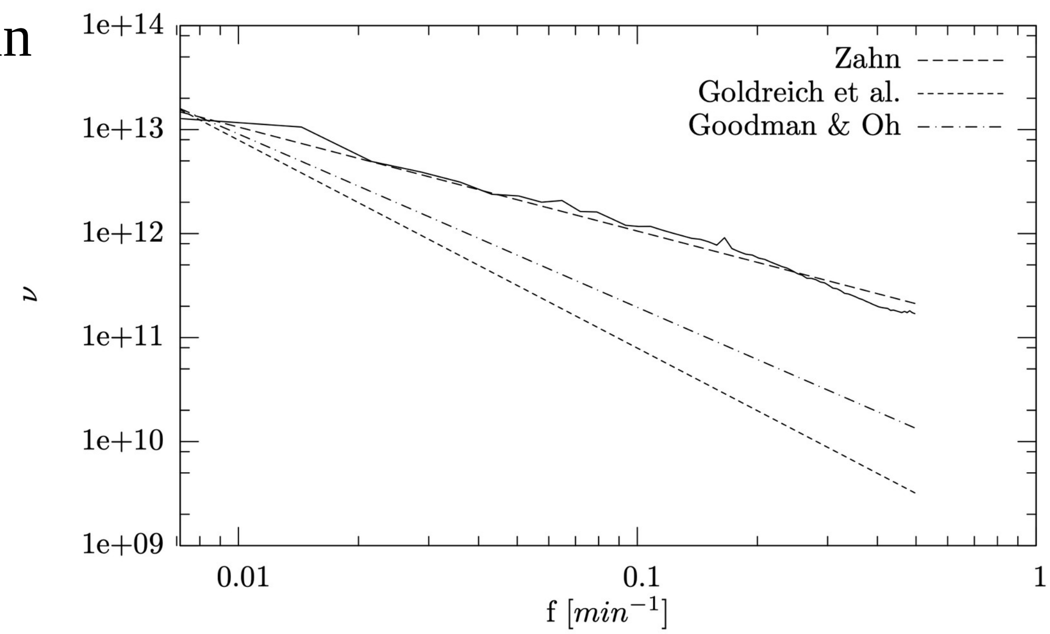
if $\tau_{\rm conv} > P_{\rm tid}$: enter the fast tide regime

fast equilibrium tides

- occur when the convective turnover time $\tau_{\rm conv}$ is long compared to the tidal period $P_{\rm tid}$
- tidal dissipation becomes less efficient by orders of magnitude $\Rightarrow$ use effective viscosity:

$$\nu_{eff} = \nu \left| \left(\frac{P_{tide}}{t_{conv}} \right)^n \right| \qquad \text{with} \qquad \frac{1}{P_{tid}} = \left| \frac{1}{P_{orb}} - \frac{1}{P_{spin}} \right|$$

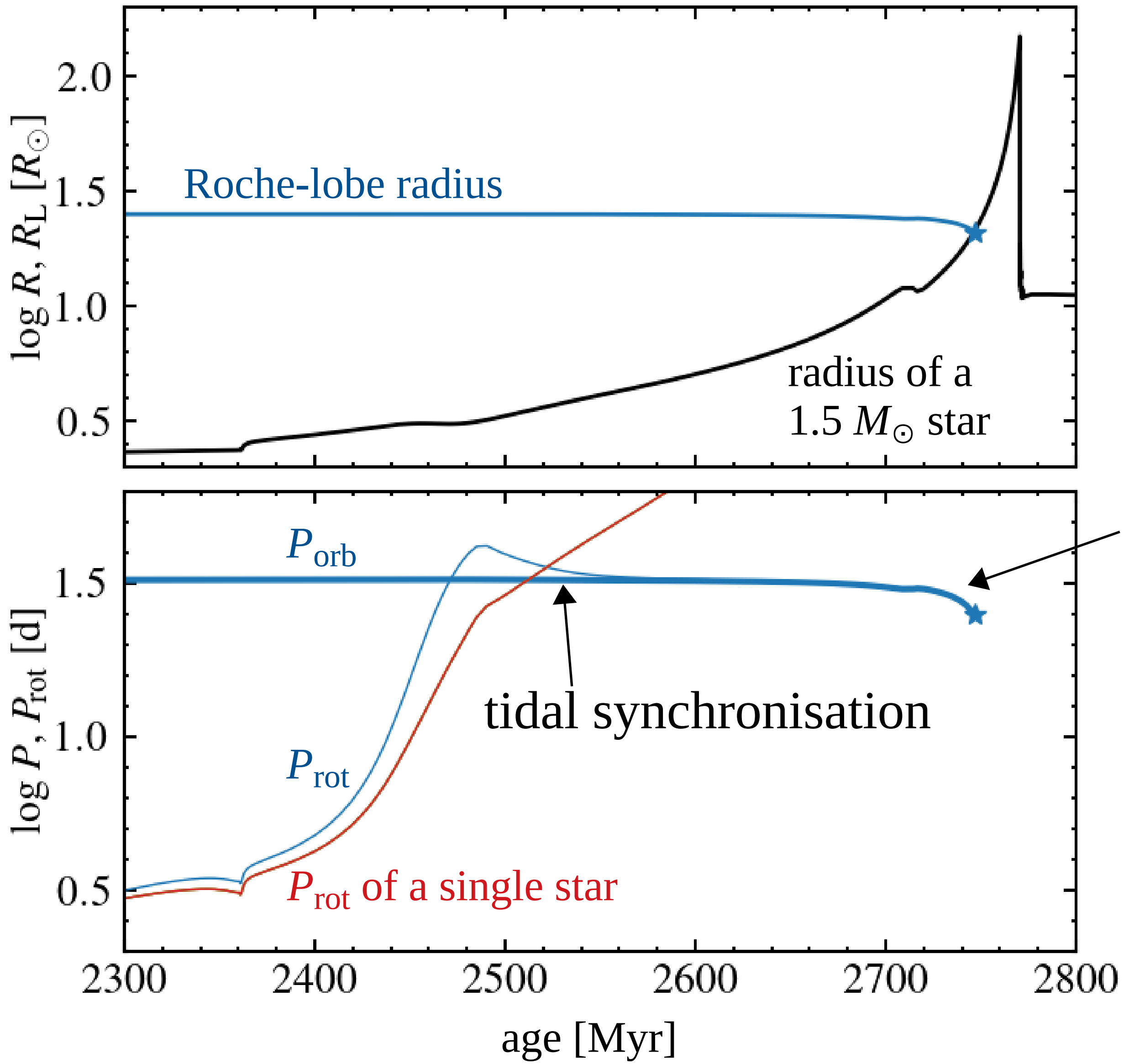
- the value of n is theoretically uncertain ("Achilles heel of tidal theory")
- typically use n = 2; however, n = 1 and n = 5/3 have also been proposed



Zahn 2008

tidal evolution

- example of tidal evolution in binary with a 1.5 $M_\odot$ star on RGB (e = 0):



transfer of $J_{orb} \rightarrow J_{rot}$
as star expands

tidal evolution

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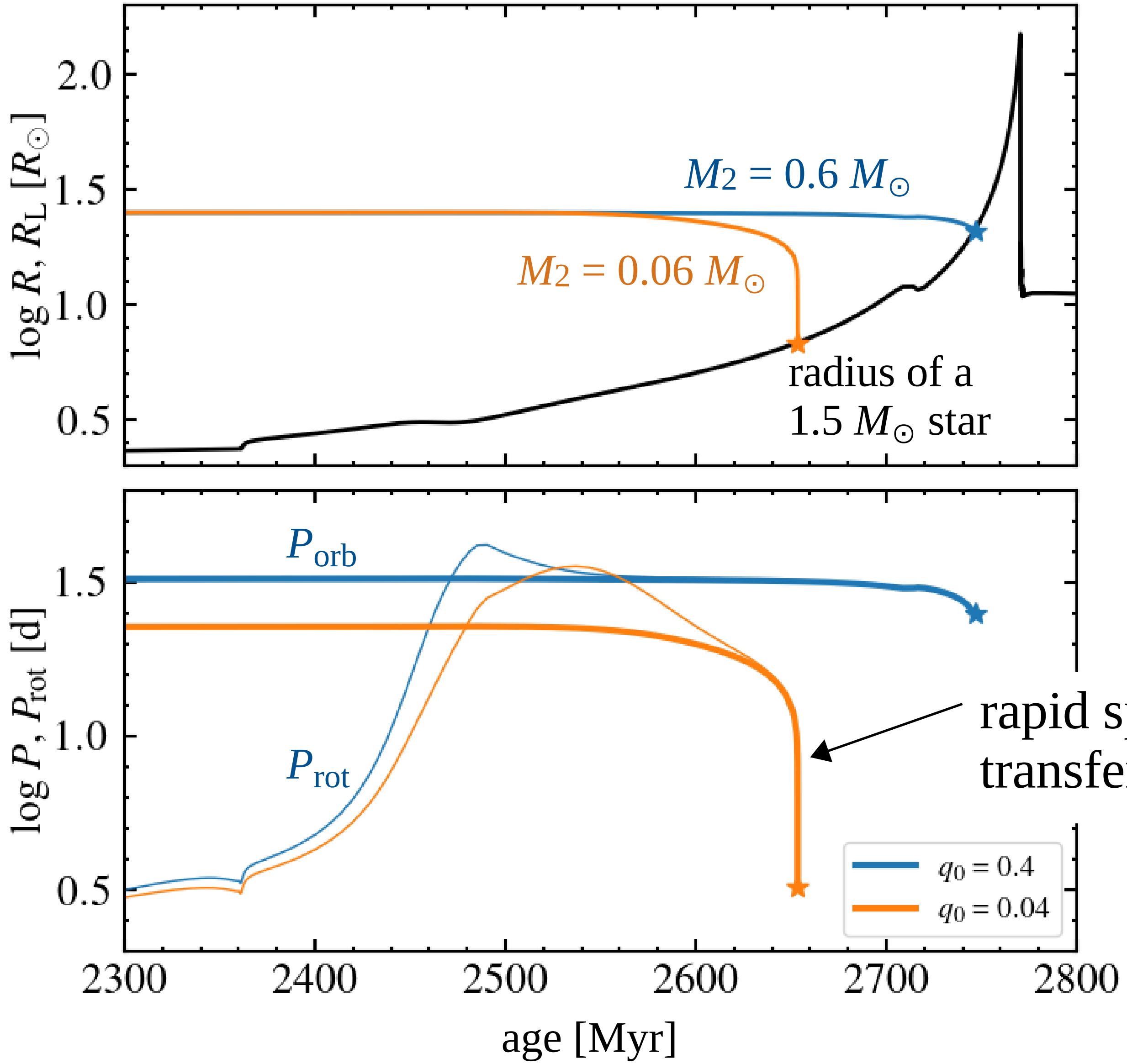


rapid spiral-in due to unstable transfer of $J_\mathrm{orb} \rightarrow J_\mathrm{rot}$

effect of smaller mass ratio:

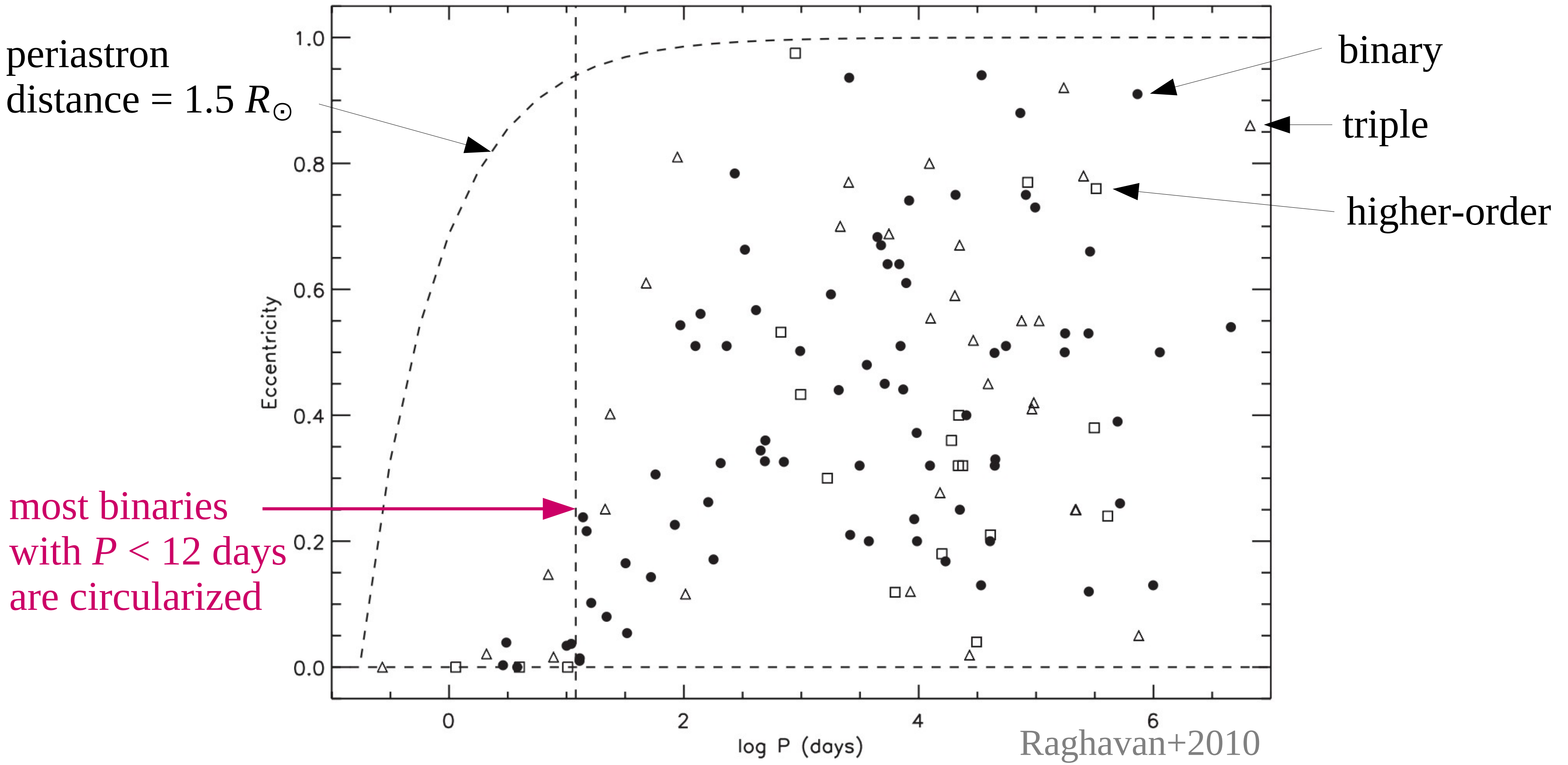
- weaker tides
- system can become **Darwin-unstable** when $J_\mathrm{orb} < 3J_\mathrm{rot}$

Darwin instability

- take a co-rotating star in a binary
- as the star evolves and expands, to conserve angular momentum and maintain co-rotation, AM must be transferred from the orbit to the spin
- consequently the orbit shrinks, and ω increases,
- this forces the star to spin up further, increasing spin AM relative to orbital AM
- if q is sufficiently small, $3J_{\rm rot}$ can become $> J_{\rm orb}$
- co-rotation cannot be maintained, and the stars keep spiralling together on the tidal dissipation timescale

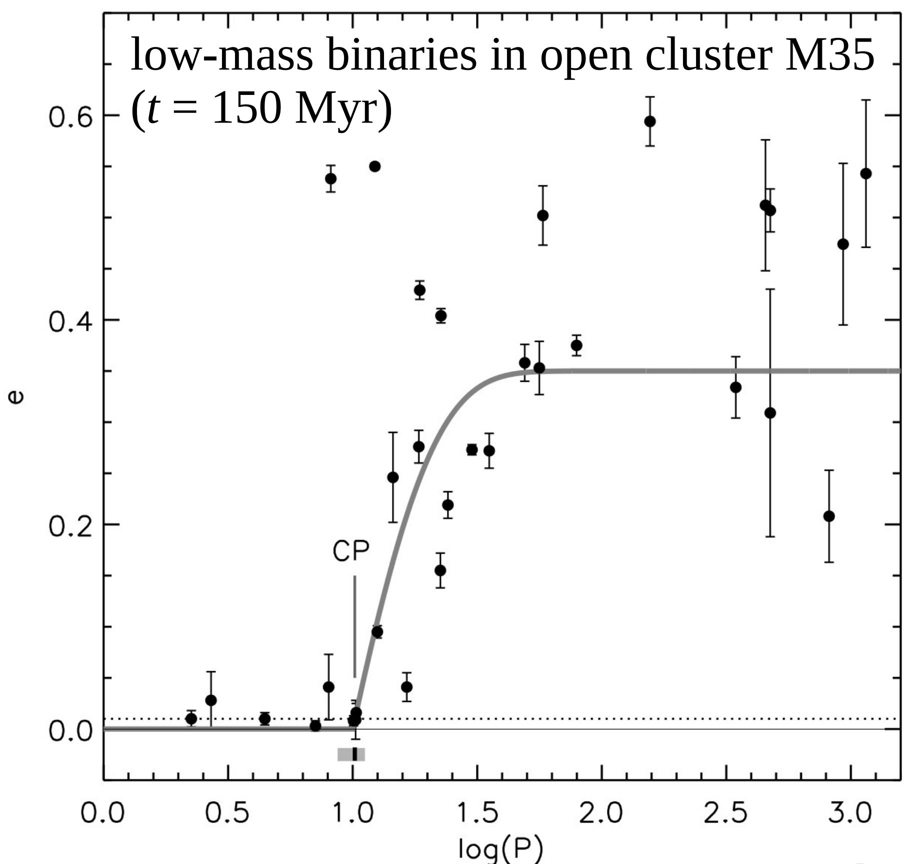
tidal circularisation

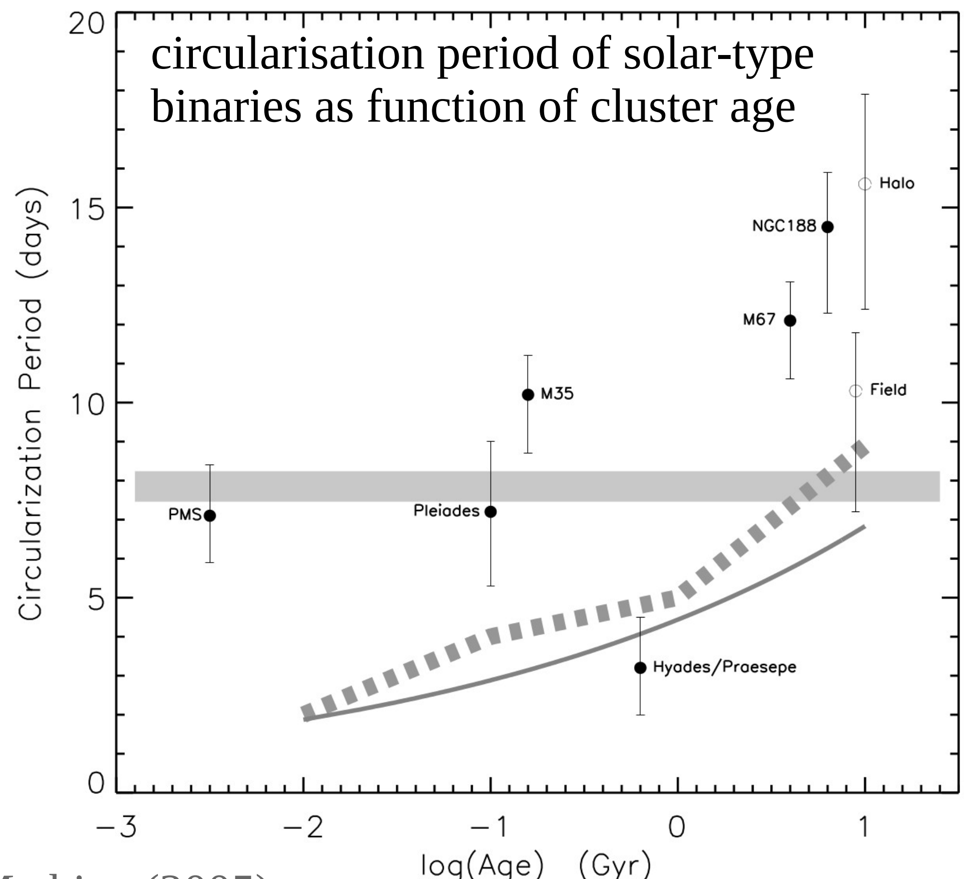
- orbital periods and eccentricities of solar-type binaries within ~25 pc of Sun
- these are low-mass MS stars with convective envelopes: expect equilibrium-tide model to hold



tidal circularisation

- use (P, e) of binaries in star clusters as tests of tidal circularisation
- circularisation in low-mass MS binaries occurs at longer periods than equilibrium tide model predicts ⇒ pre-MS circularisation?



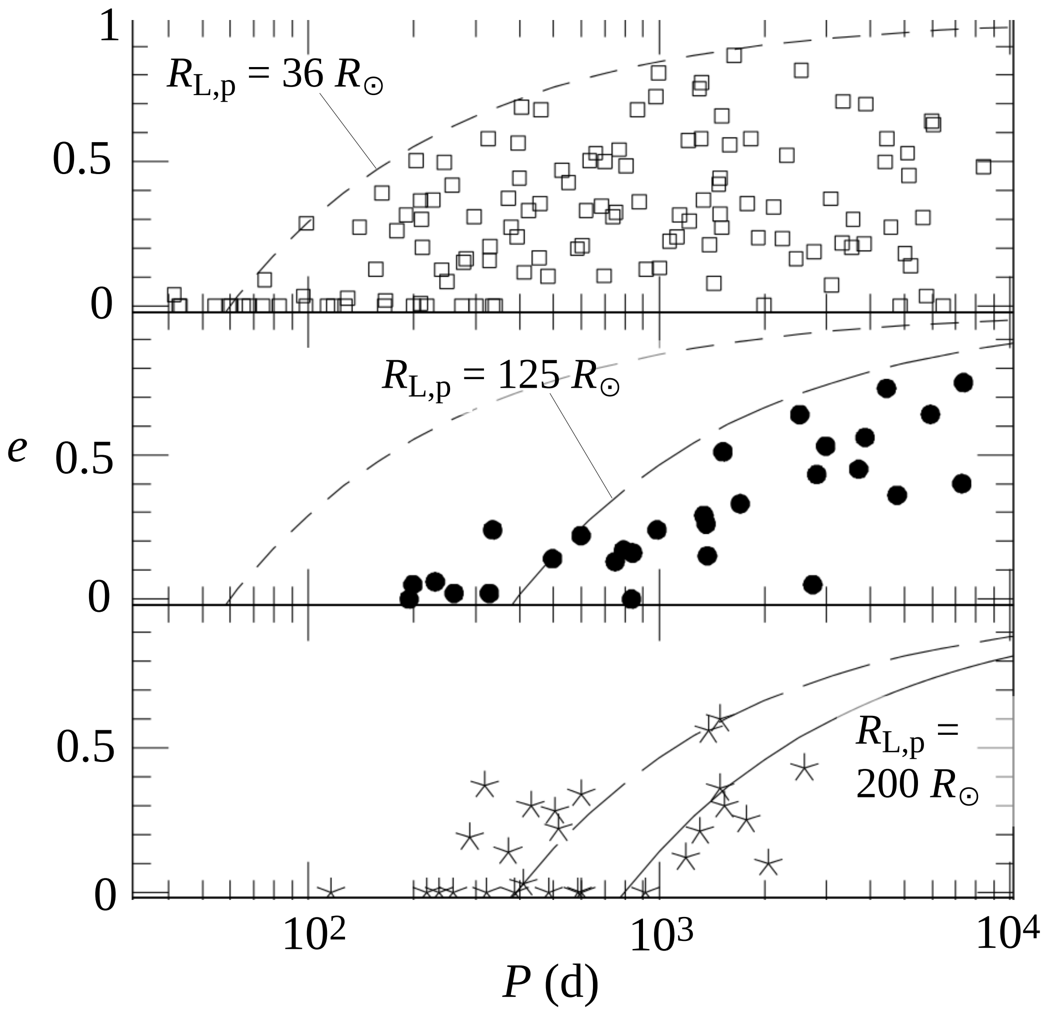


Meibom & Mathieu (2005)

tidal circularisation

- tidal circularisation in RG binaries: circularisation period increases with (typical) radius of the sample
- (P, e) values consistent with equilibrium tide model with convective damping

Verbunt & Phinney (1995)



K giants in open clusters
(relatively unevolved)

early M giants
(more evolved)

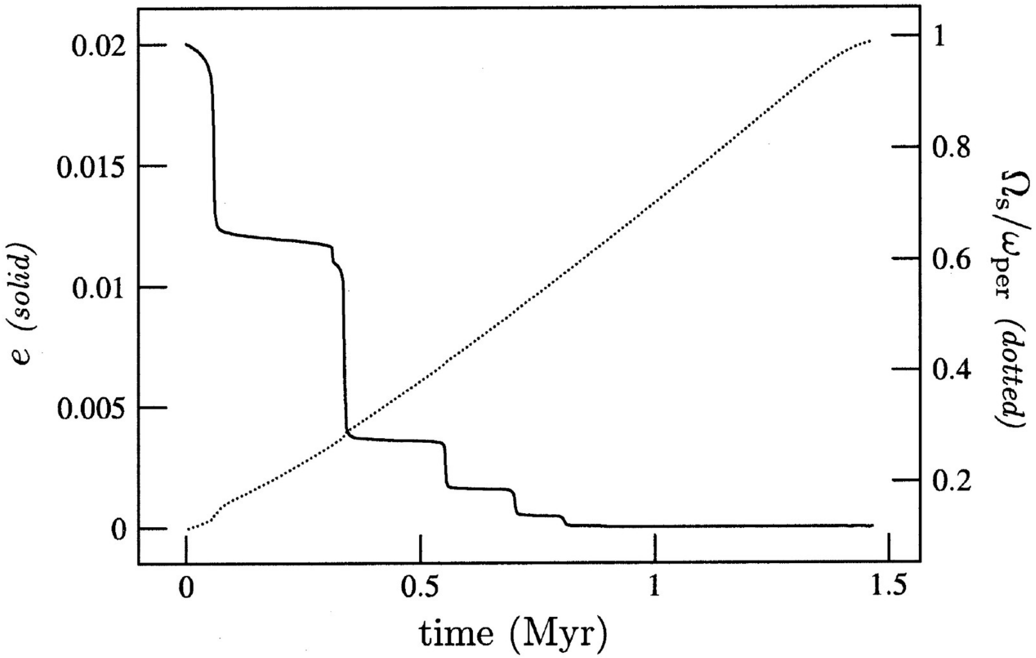
post-AGB stars:
have passed their maximum radii
(evidence for having experienced RLOF...)

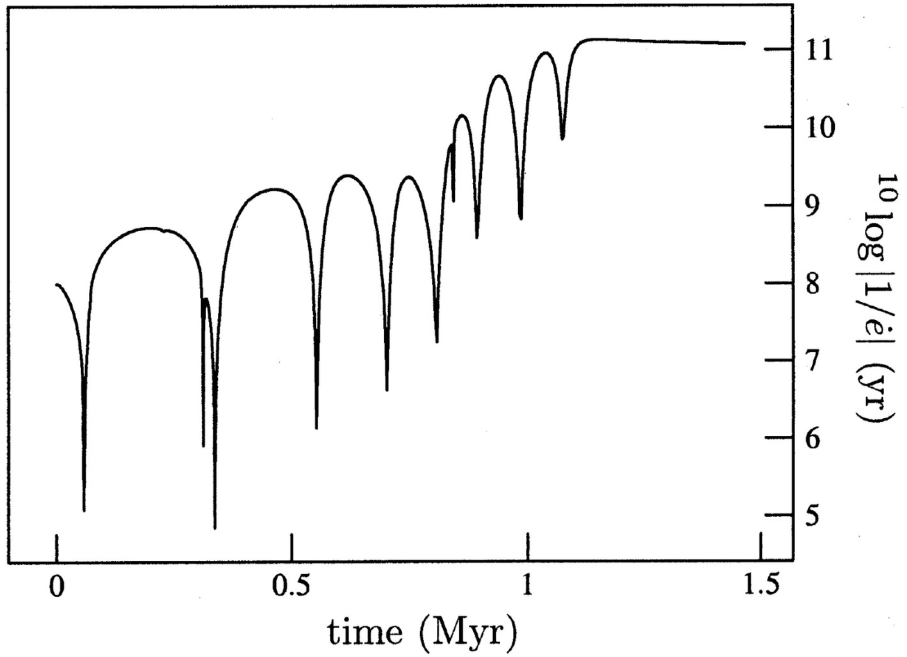
Jorissen et al (2009)

dynamical tides

- example of effect of dynamical tides on eccentricity evolution
- massive star in a P = 3 day orbit with e = 0.02

Witte & Savonije 1999





further reading

- *Evolutionary Processes in Binary and Multiple Star Systems*
  – P. Eggleton (2006, book)
- *Tidal Dissipation in Binary Systems*
  – J.P. Zahn (2008, review article)
  https://arxiv.org/pdf/0807.4870.pdf
- *Tidal Dissipation in Stars and Giant Planets*
  – G. Ogilvie (2014, review article)
  https://arxiv.org/pdf/1406.2207.pdf
- two introductory videos about oceanic tides (same concepts apply to stars):

  https://www.youtube.com/watch?v=erpJ_EhQ9KM

  https://www.youtube.com/watch?v=p4qJVjw-DWI

with thanks to Holly Preece for contributions to the lecture material