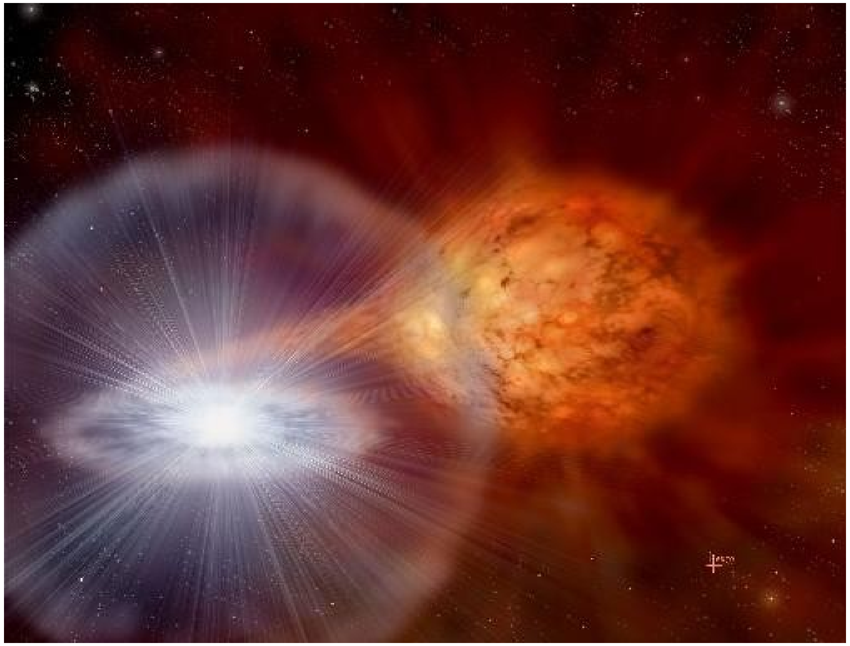
***ASBE part II*: binary evolution**



- how does the evolution of stars in binary systems differ from that of single stars?
- which kind of interaction processes play a role in binary stars, and how do they affect their evolution?
- how do observed types of binary systems fit into this evolution picture?

lecture 8:

binary star statistics, orbits and structure

binary and multiple systems

- incidence of binaries and multiples in stellar populations increases with stellar mass

multiplicity fraction:
fraction of multiple systems (including binaries)

fraction of triples and higher-order systems

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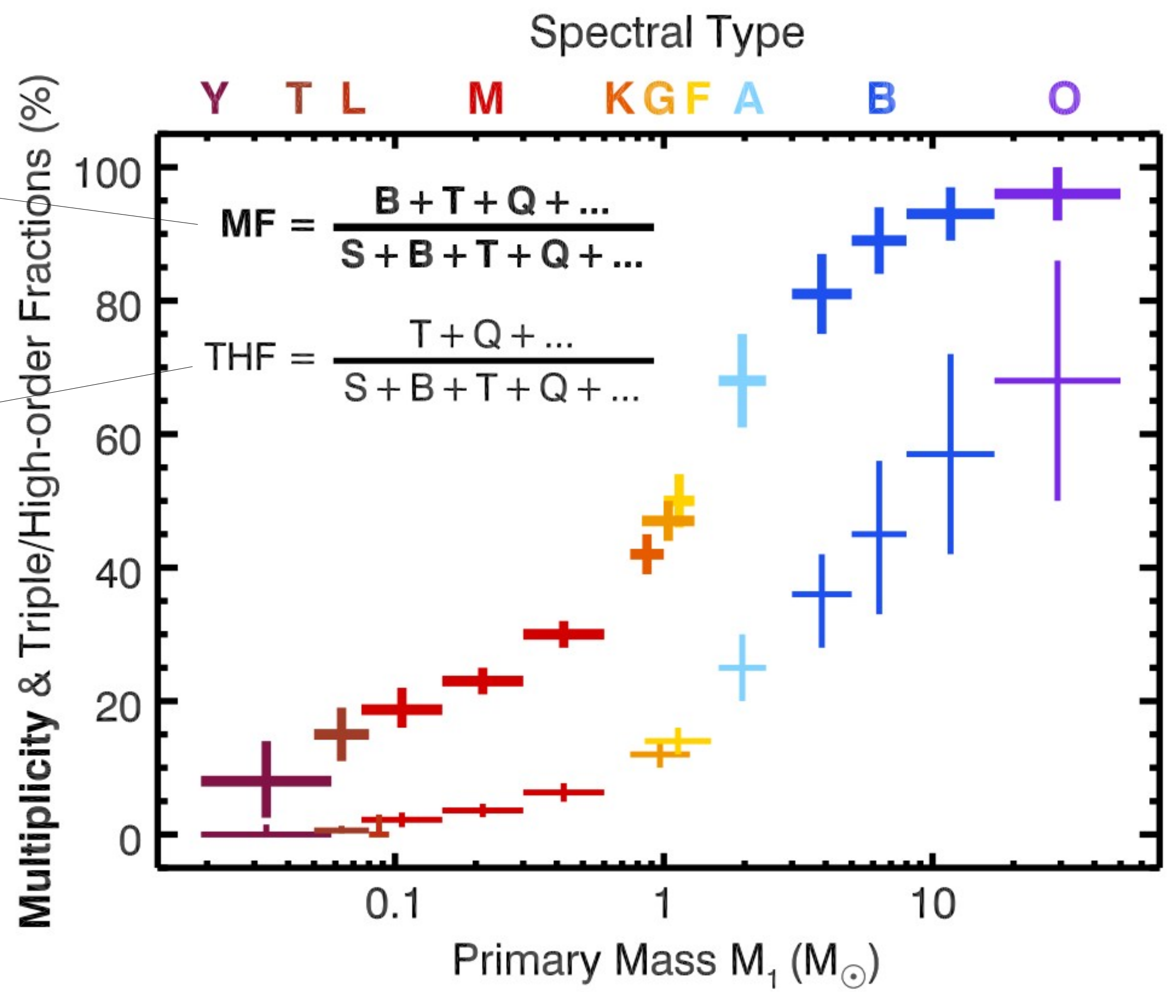

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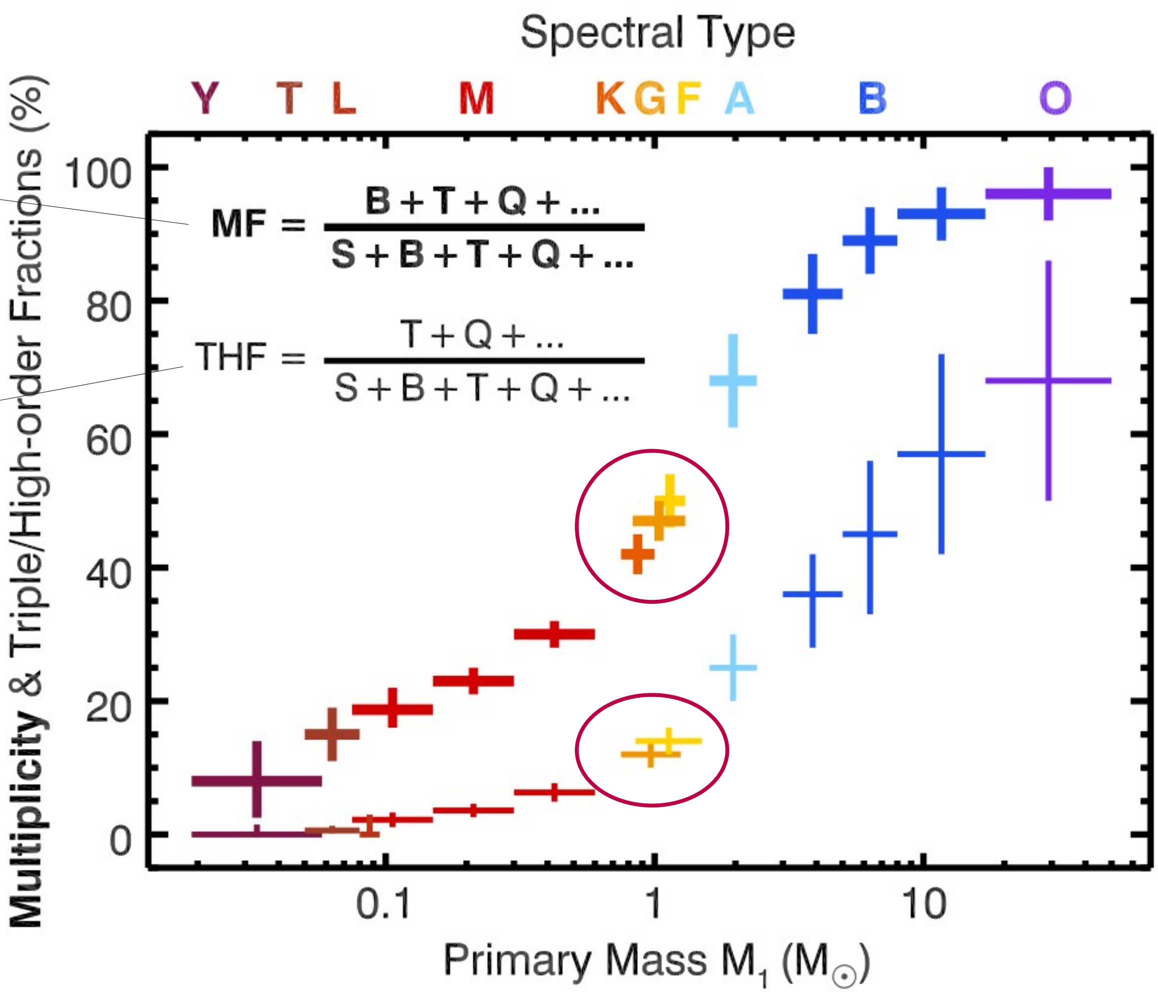

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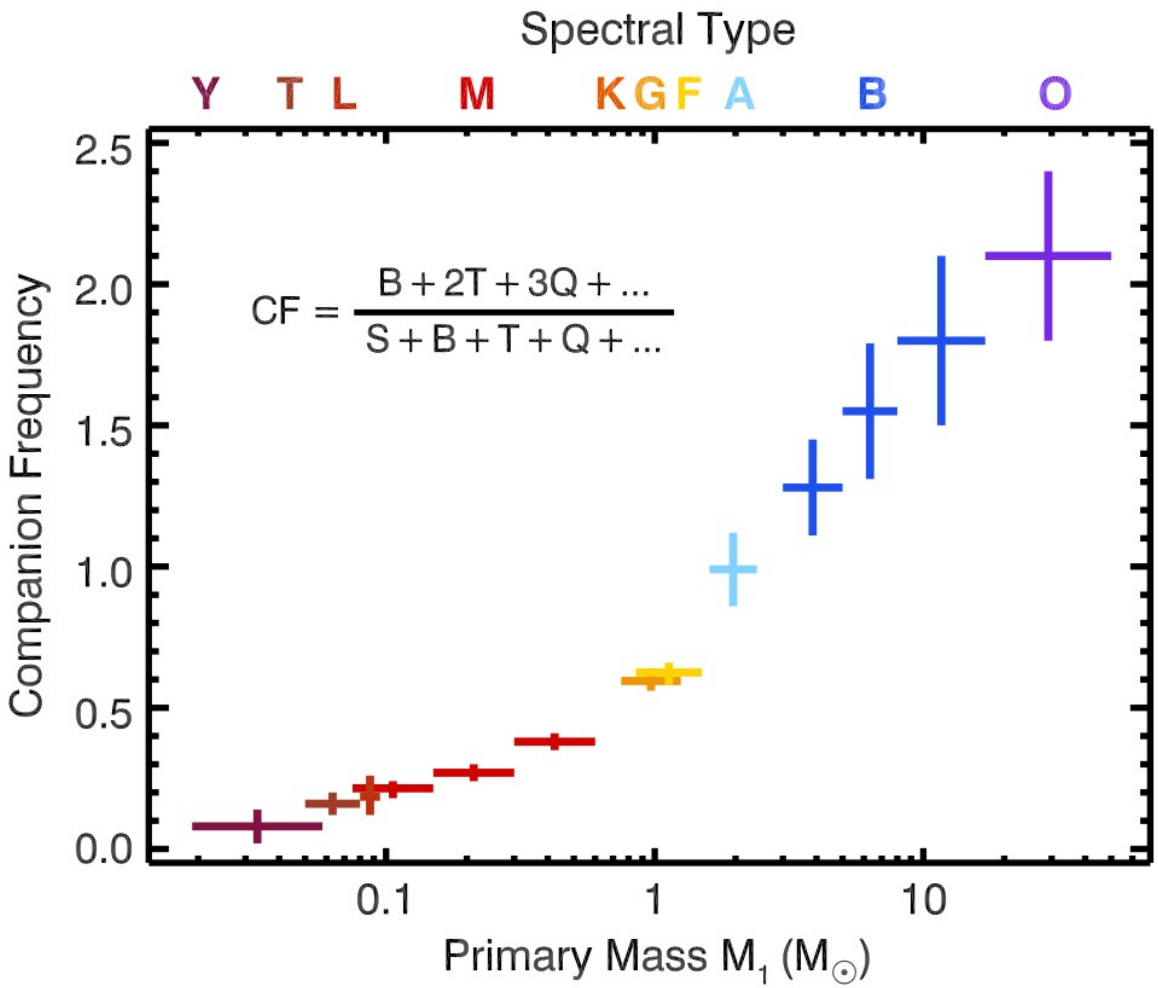
- solar-type stars: ≈55% single, ≈34% binaries, ≈8% triples, ≈3% higher-order

binary and multiple systems

- incidence of binaries and multiples in stellar populations increases with stellar mass

companion frequency: average number of companions per star



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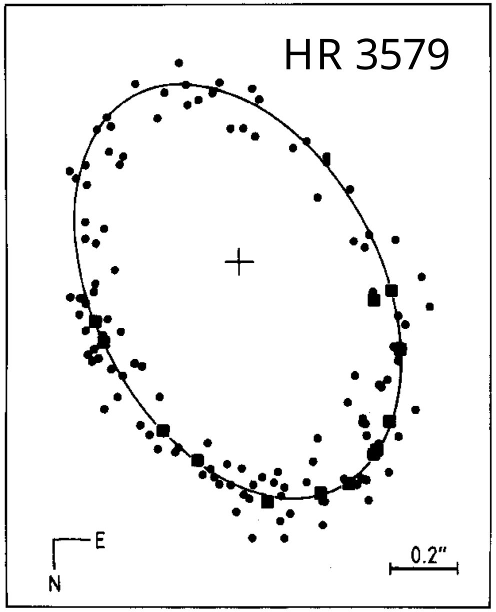
binary orbital dynamics

- *see lecture notes, sec. 15.1*

measuring binary stars

- visual binaries (VB):

  projected relative orbit of 2 stars on sky ⇒ measure P, e, i, $\alpha = a/d$



measuring binary stars

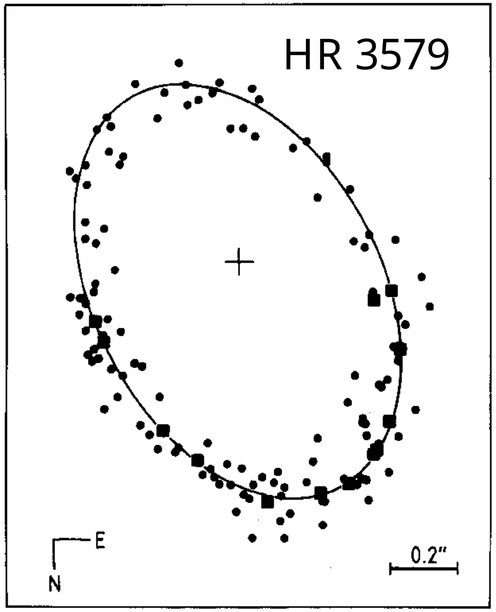
- visual binaries (VB):

  projected relative orbit of 2 stars on sky $\Rightarrow$ measure $P, e, i, \alpha = a/d$

  – using Kepler: $$\frac{GM}{d^3} = \left(\frac{2\pi}{P}\right)^2 \alpha^3$$

    if distance d is known $\Rightarrow$ total mass M

  – if both stellar orbits known, relative to centre of mass: α_1, α_2 $\Rightarrow$ mass ratio $q = M_1/M_2$



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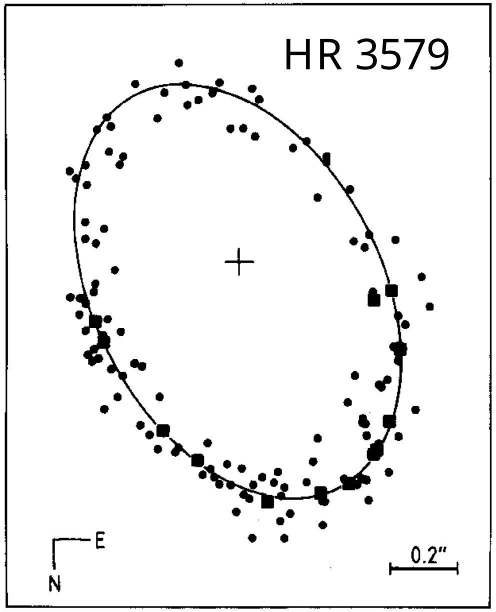


- astrometric binaries:

  orbit of the binary *photocentre* relative to fixed point on sky $\Rightarrow$ measure P, e, i, and $\alpha_{\text{photocentre}} = f(a/d, q, L_1/L_2)$

  - *Gaia* is able to do this for millions of binaries...

measuring binary stars

- spectrocopic binaries (SB):

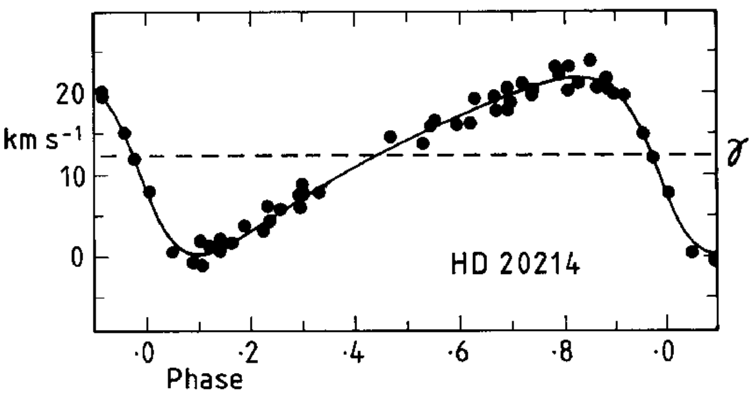
  radial velocity curve of brightest star (SB1)

  ⇒ measure P, e, and RV amplitude K_1:

  $$K_1 = 2\pi\, a_1 \sin i\, (1\text{–}e^2)^{-1/2} / P$$

  ⇒ mass function:

  $$f(M_2) = \frac{M_2^3 \sin^3 i}{(M_1 + M_2)^2} = K_1^3 \frac{P}{2\pi G} (1 - e^2)^{3/2}$$



measuring binary stars

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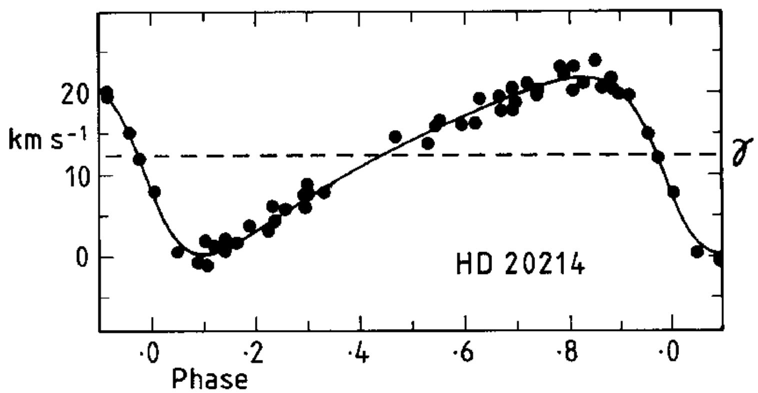

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- double-lined spectroscopic binaries (SB2):
  $f(M_1)$ and $f(M_2)$ ⇒ $q = M_1/M_2$ (but not individual masses, only $M_i \sin i$)

  - if visual/astrometric *and* spectroscopic orbit known ⇒ inclination i
    ⇒ M_1 and M_2, independent of distance

measuring binary stars

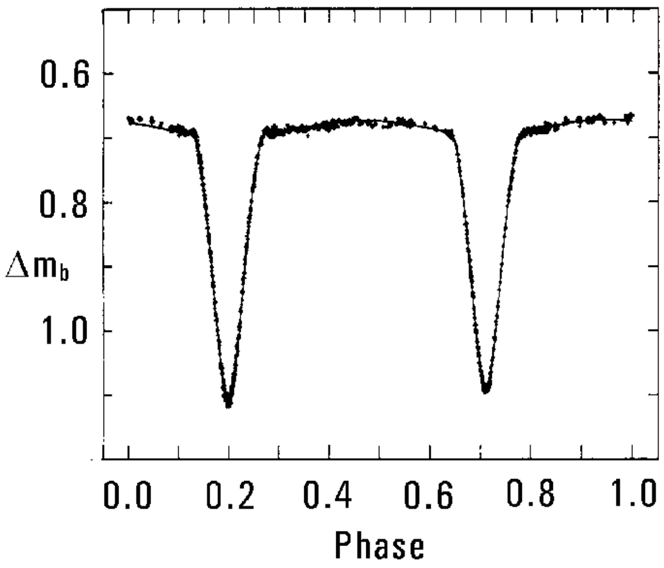
- eclipsing binaries (EB):

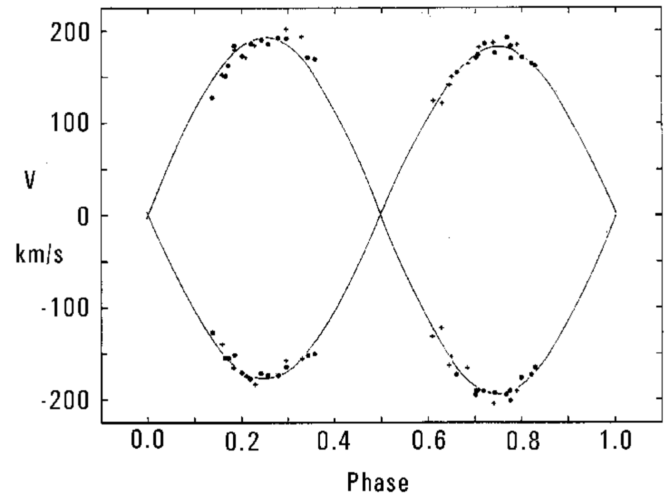
  observed lightcurve + accurate modelling of stellar atmospheres and distortions $\Rightarrow$

  measure P, e, i, R_1/a, R_2/a and $T_{\mathrm{eff},1}$, $T_{\mathrm{eff},2}$

- double-lined spectroscopic eclipsing binaries (ESB2):

  P, e, i, a, M_1, M_2, R_1, R_2, $T_{\mathrm{eff},1}$, $T_{\mathrm{eff},2}$ and d (!)





measuring binary stars

- eclipsing binaries (EB):

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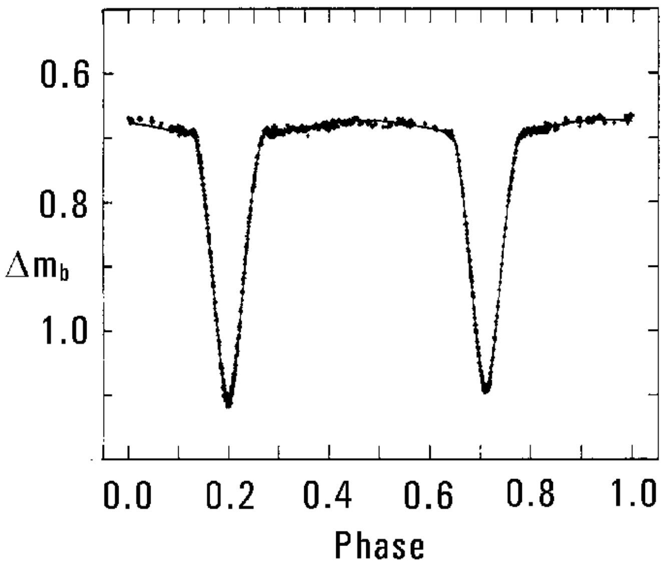
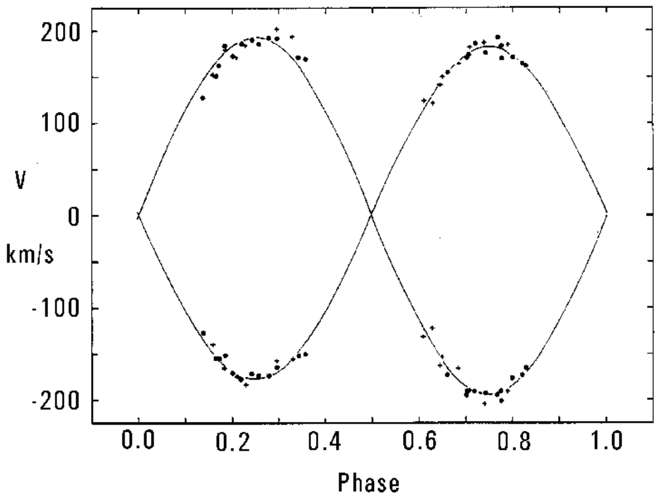
- double-lined spectroscopic eclipsing binaries (ESB2):

  P, e, i, a, M_1, M_2, R_1, R_2, $T_{\rm eff,1}$, $T_{\rm eff,2}$ and d (!)

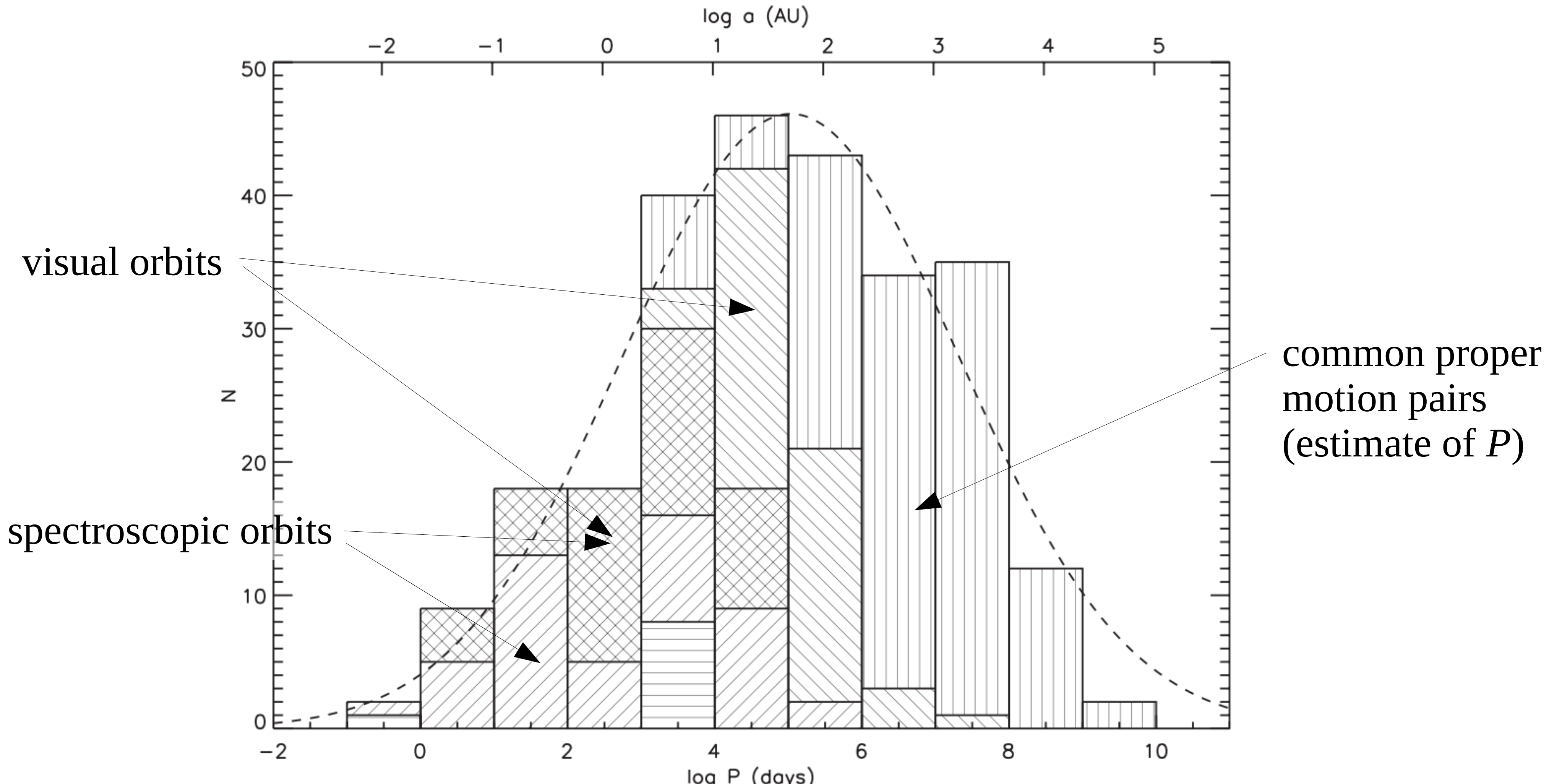
- binary systems are the only *direct* source of information on stellar masses and (in some cases) provide independent distance measurements





binary statistics

- complete sample of solar-type stars within ~25 pc:
  distribution of orbital periods



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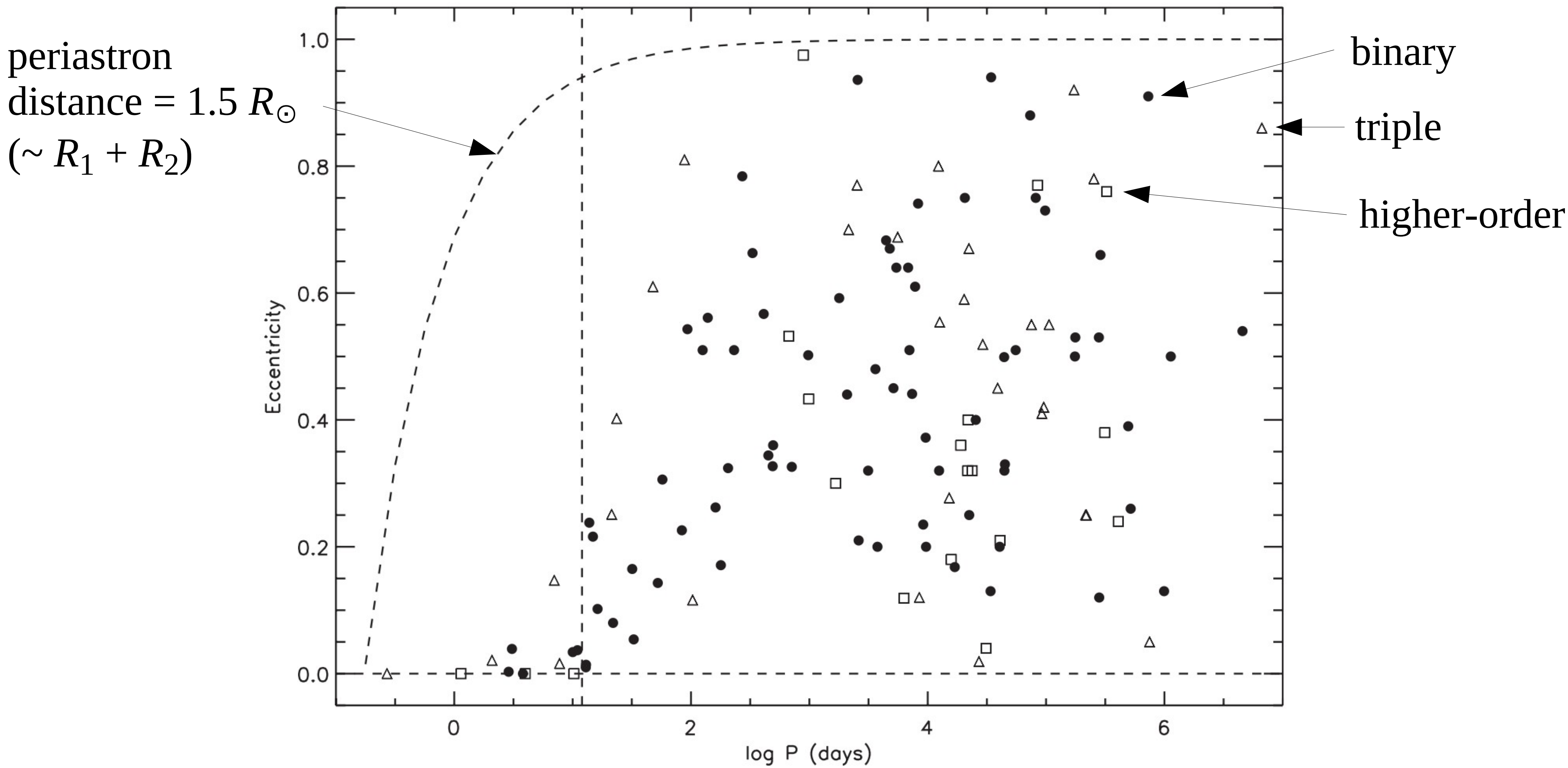
binary statistics

- complete sample of solar-type stars within ~25 pc: orbital periods and eccentricities



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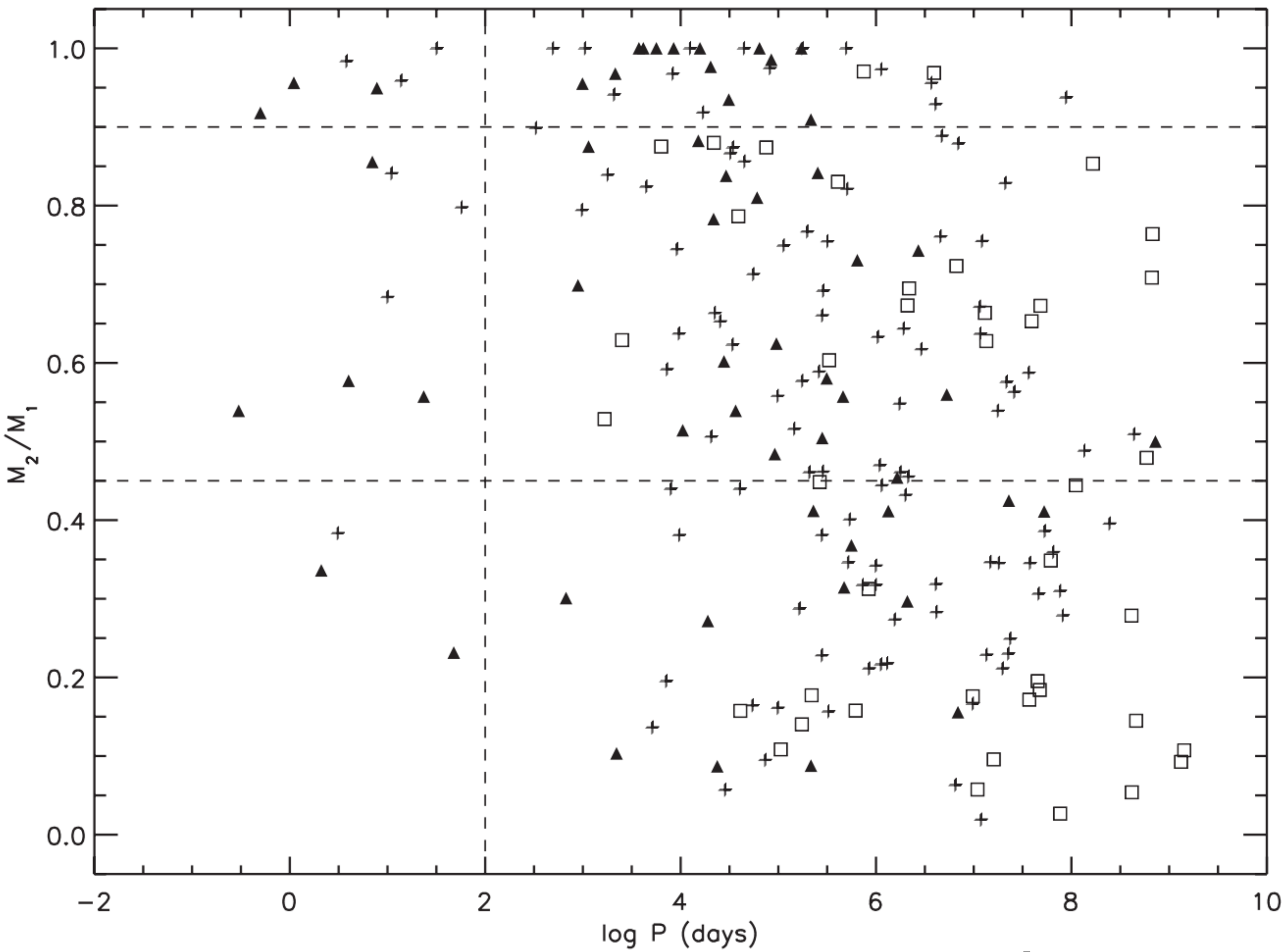
binary statistics

- complete sample of solar-type stars within ~25 pc:
  period versus mass ratio

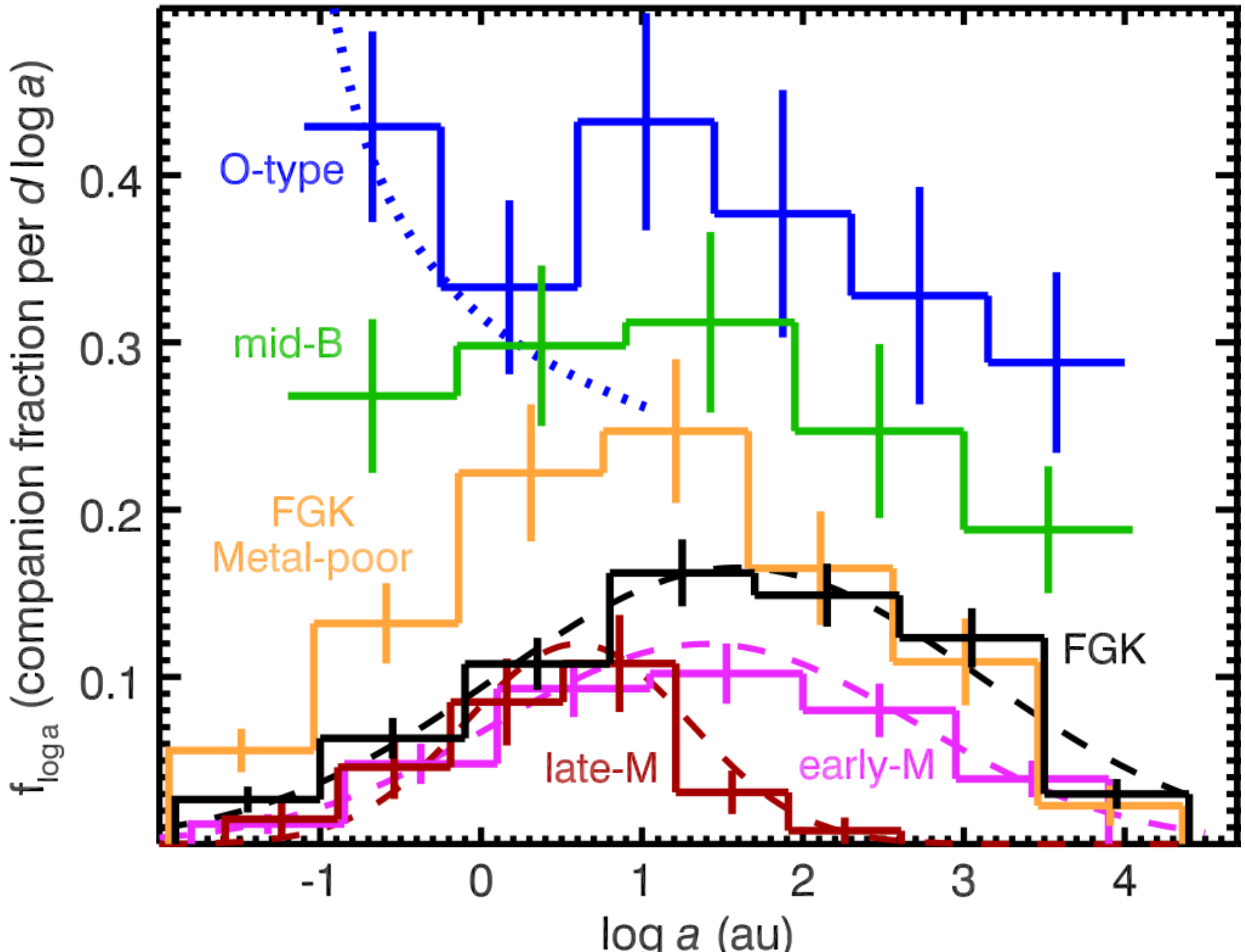


Raghavan+2010

binary statistics

- orbital period distributions for different populations: strongly dependent on mass (and metallicity)

companion fraction per log *a* interval



$> 15\ M_\odot$

$\sim 5–8\ M_\odot$

$0.8–1.2\ M_\odot$

$0.3–0.6\ M_\odot$

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the Roche geometry

- structure of a binary system is usually described in the *Roche geometry*:

  combined effect of gravity of the two stars and their orbital motion, in the co-rotating frame of the binary

- assumptions:
  - gravity of both stars is the same as for point masses
  - the orbit is circular ($e = 0$)
  - all matter co-rotates with the orbit, with angular frequency $\omega = 2\pi/P$ ($\Rightarrow$ stellar rotation is synchronized with orbit)

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- stellar structure determined by the Roche potential:

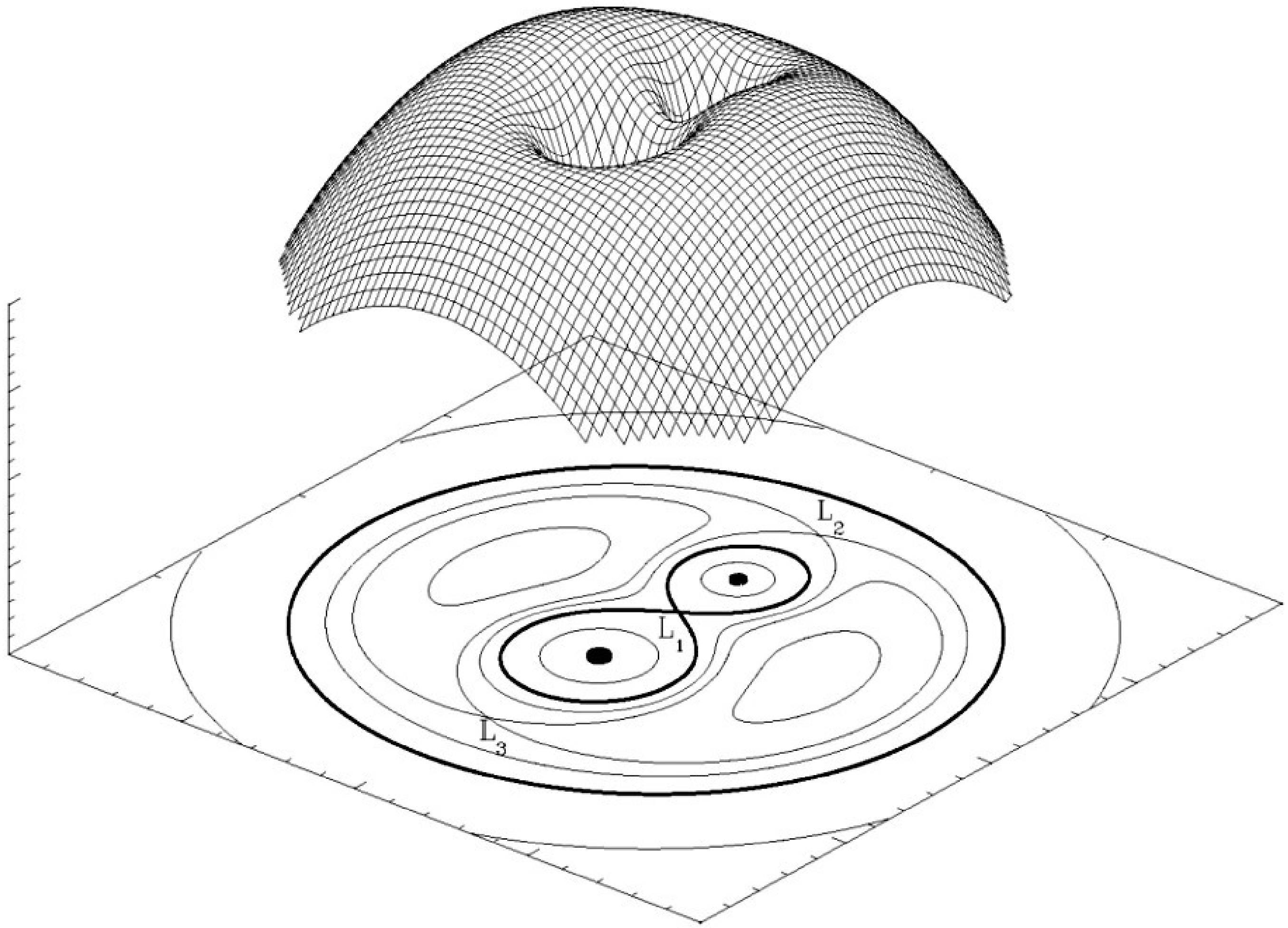
$$\Phi = -\frac{GM_1}{|\mathbf{r} - \mathbf{s}_1|} - \frac{GM_2}{|\mathbf{r} - \mathbf{s}_2|} - \tfrac{1}{2}\omega^2 r_\perp^2$$

positions of star 1 and 2

distance from orbital axis

the Roche potential

- the Roche potential in the plane of the orbit, for $q = M_2/M_1 = 0.5$



the Roche potential

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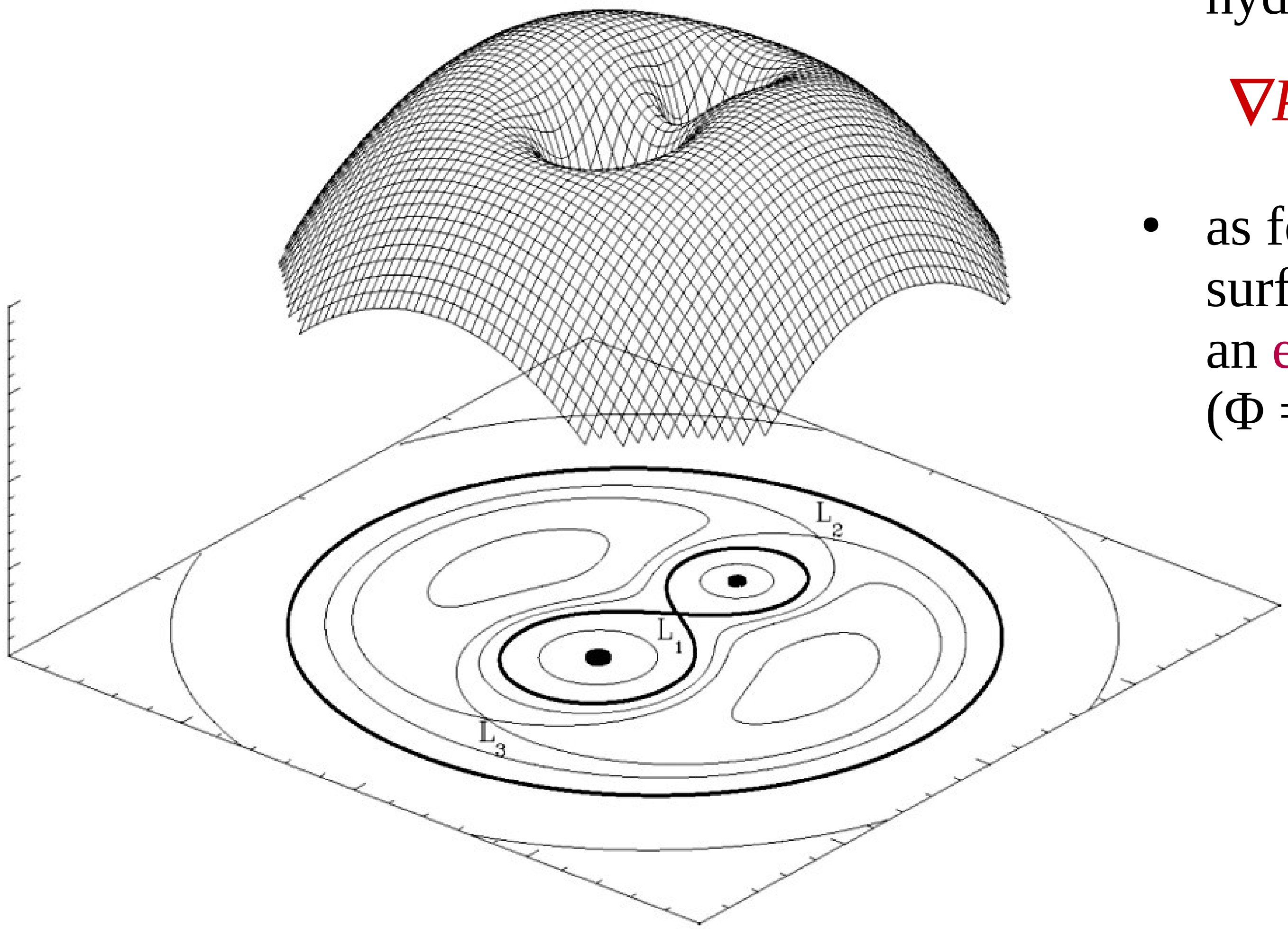

- hydrostatic equilibrium:

$$\nabla P = -\rho \, \nabla \Phi$$

- as for rotating stars, stellar surface should coincide with an equipotential surface (Φ = constant)

the Roche potential

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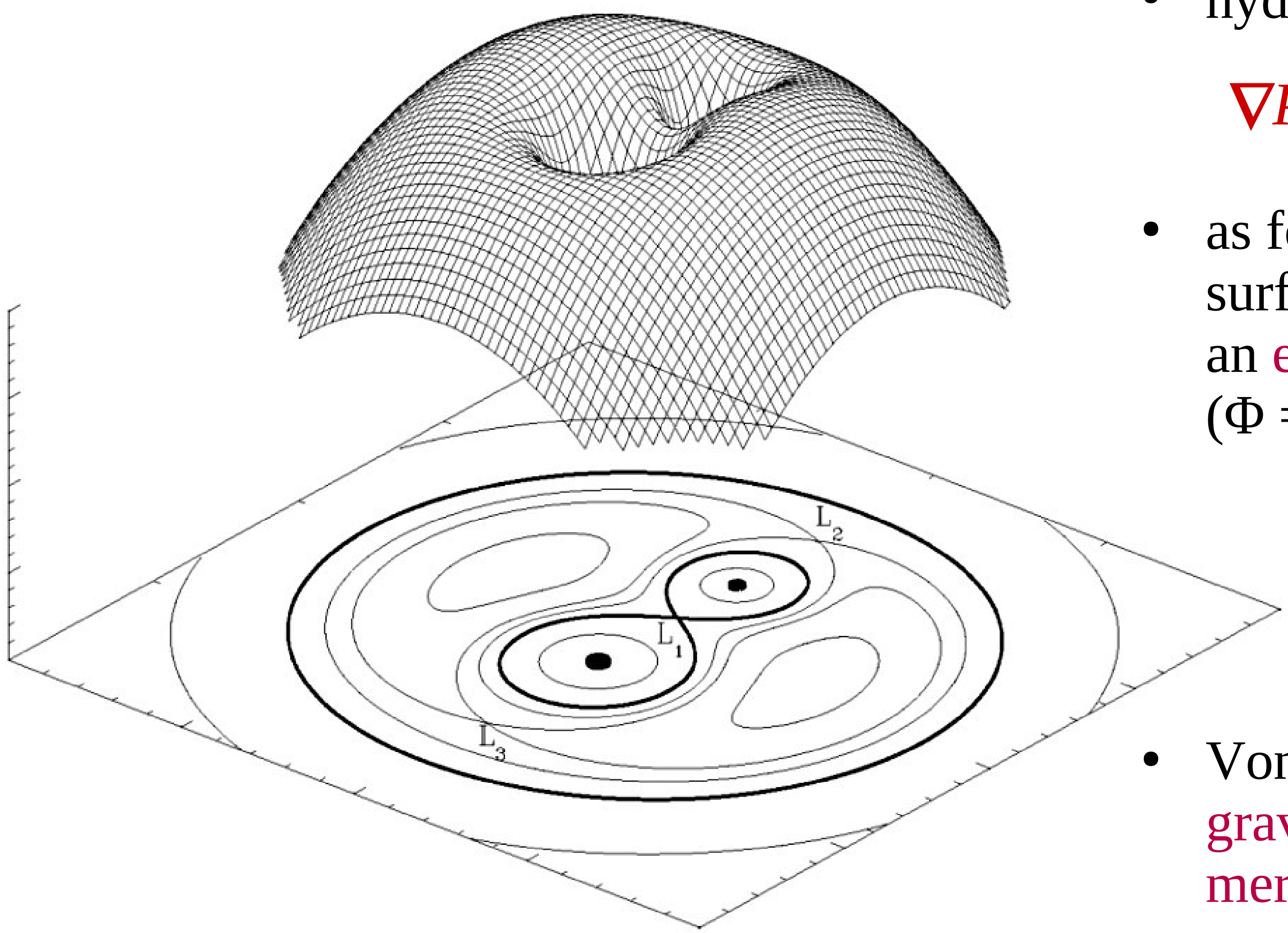


- hydrostatic equilibrium:

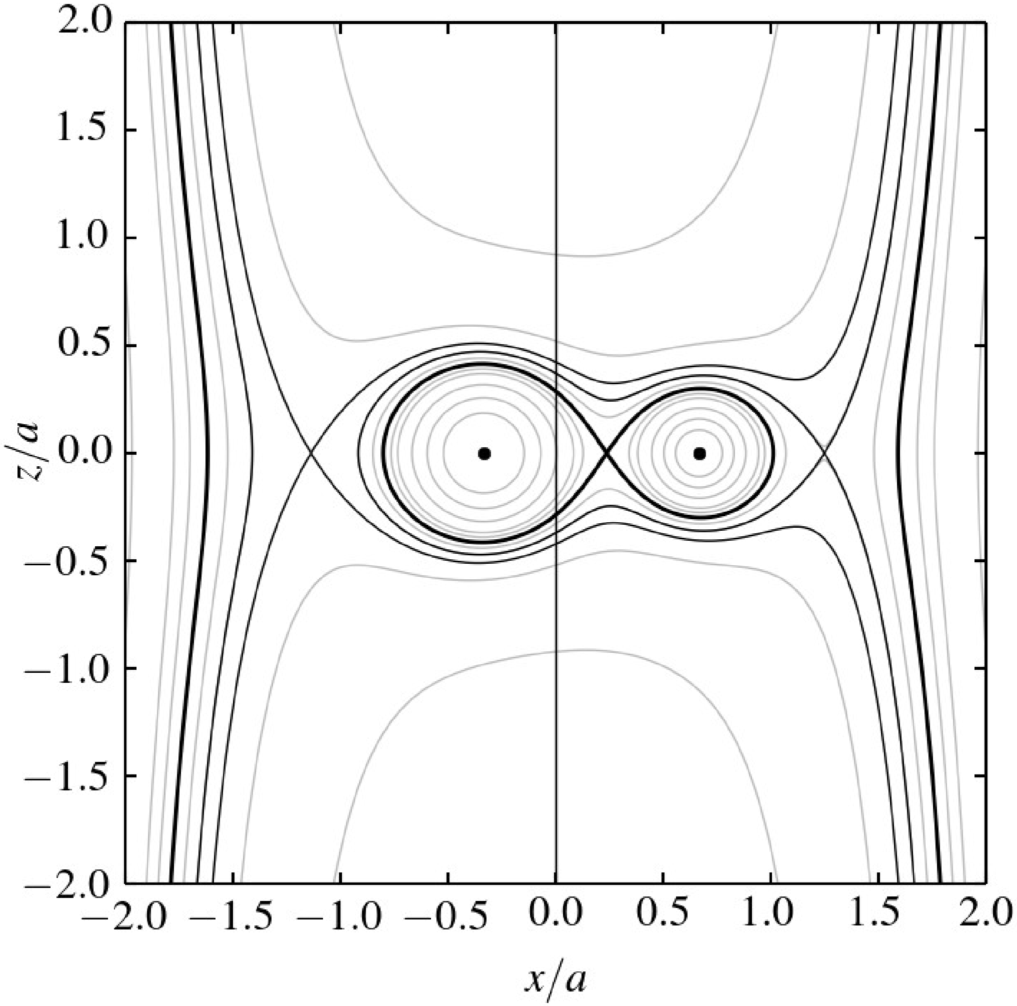
$$\nabla P = -\rho\, \nabla \Phi$$

- as for rotating stars, stellar surface should coincide with an equipotential surface (Φ = constant)

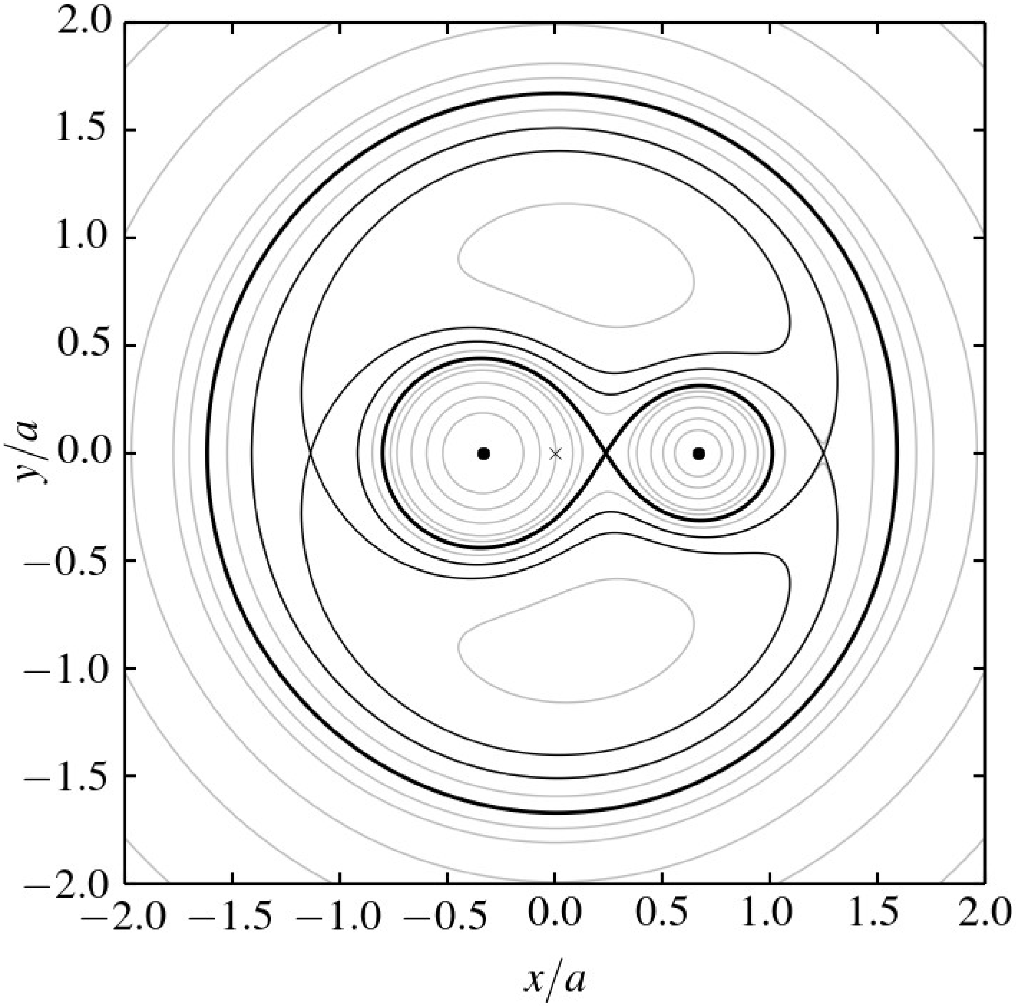
- Von Zeipel's theorem $\Rightarrow$ gravity darkening and meridional currents (but in non-axisymmetric way...)

the Roche potential

- equipotential surfaces of the Roche potential, for $q = M_2/M_1 = 0.5$



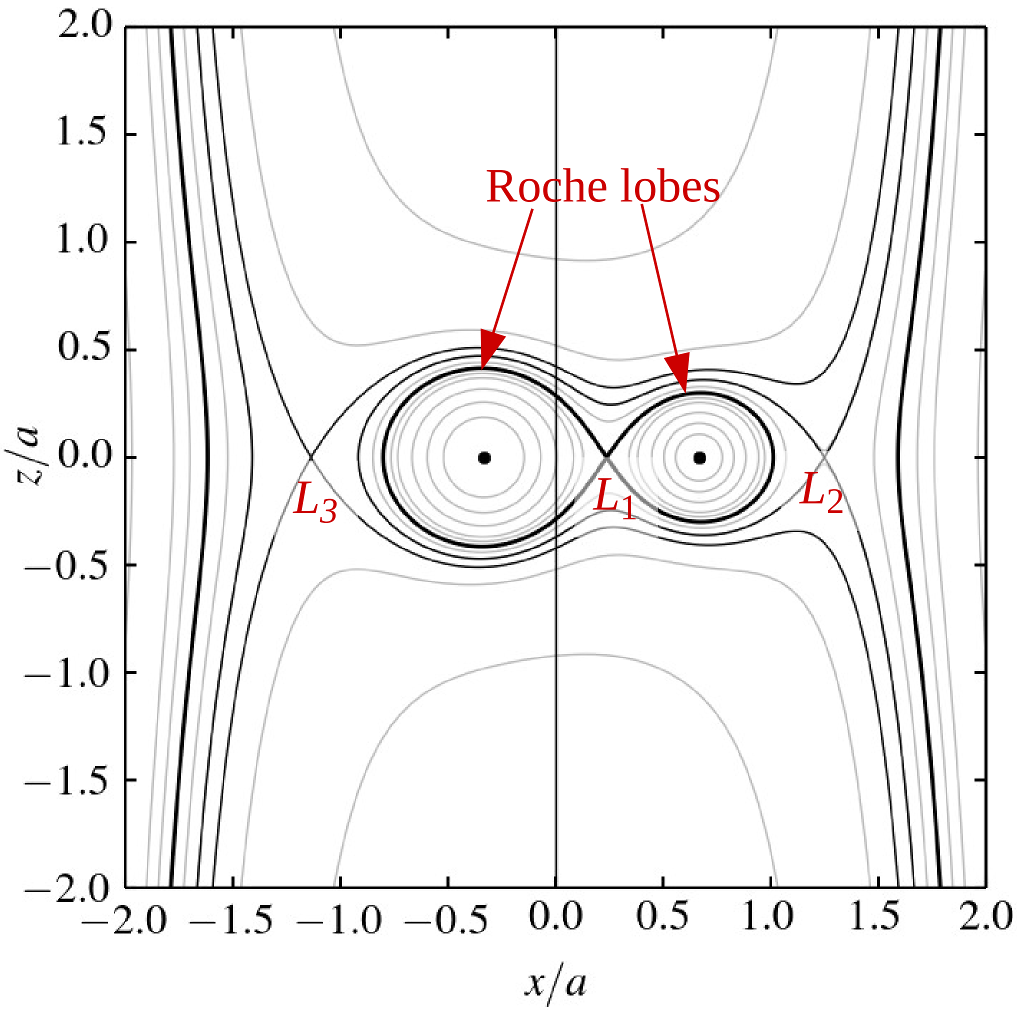
along the orbital axis



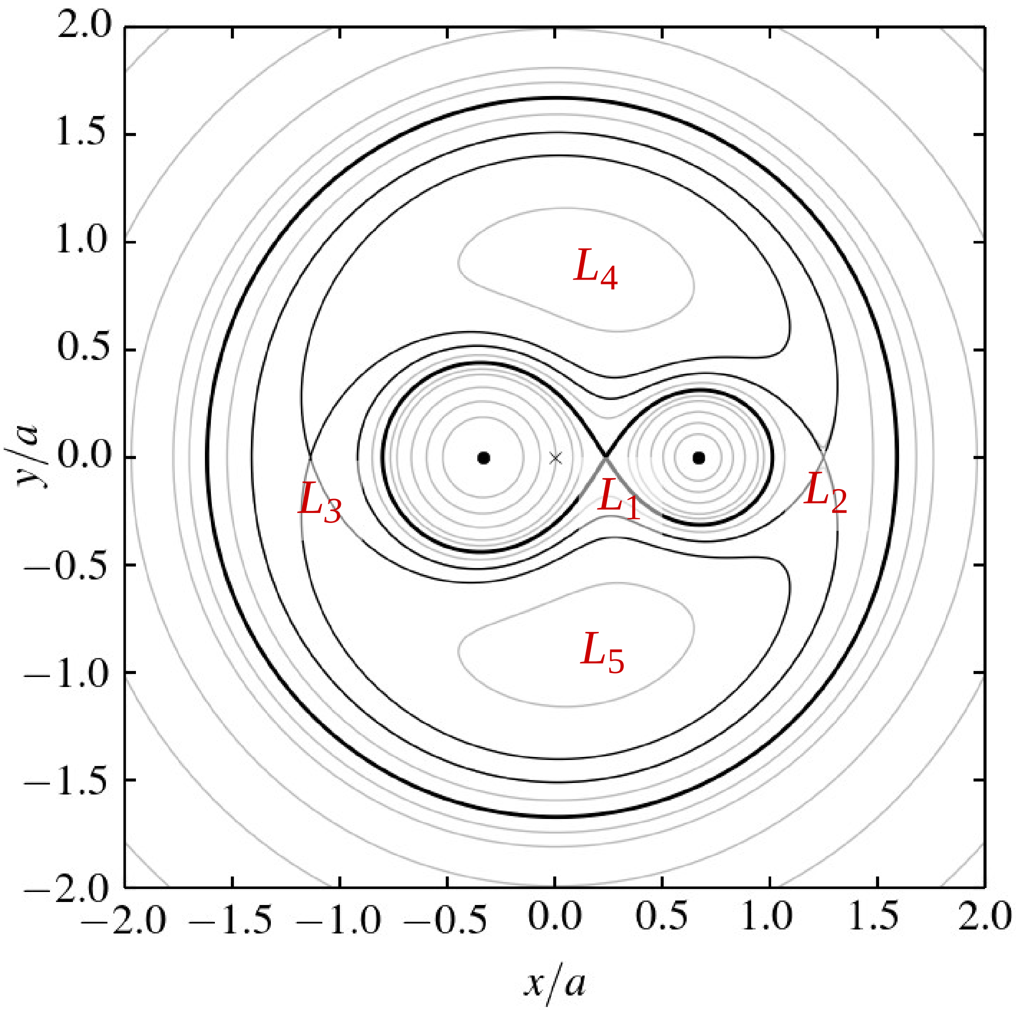
in the orbital plane

the Roche potential

- equipotential surfaces of the Roche potential, for $q = M_2/M_1 = 0.5$



along the orbital axis

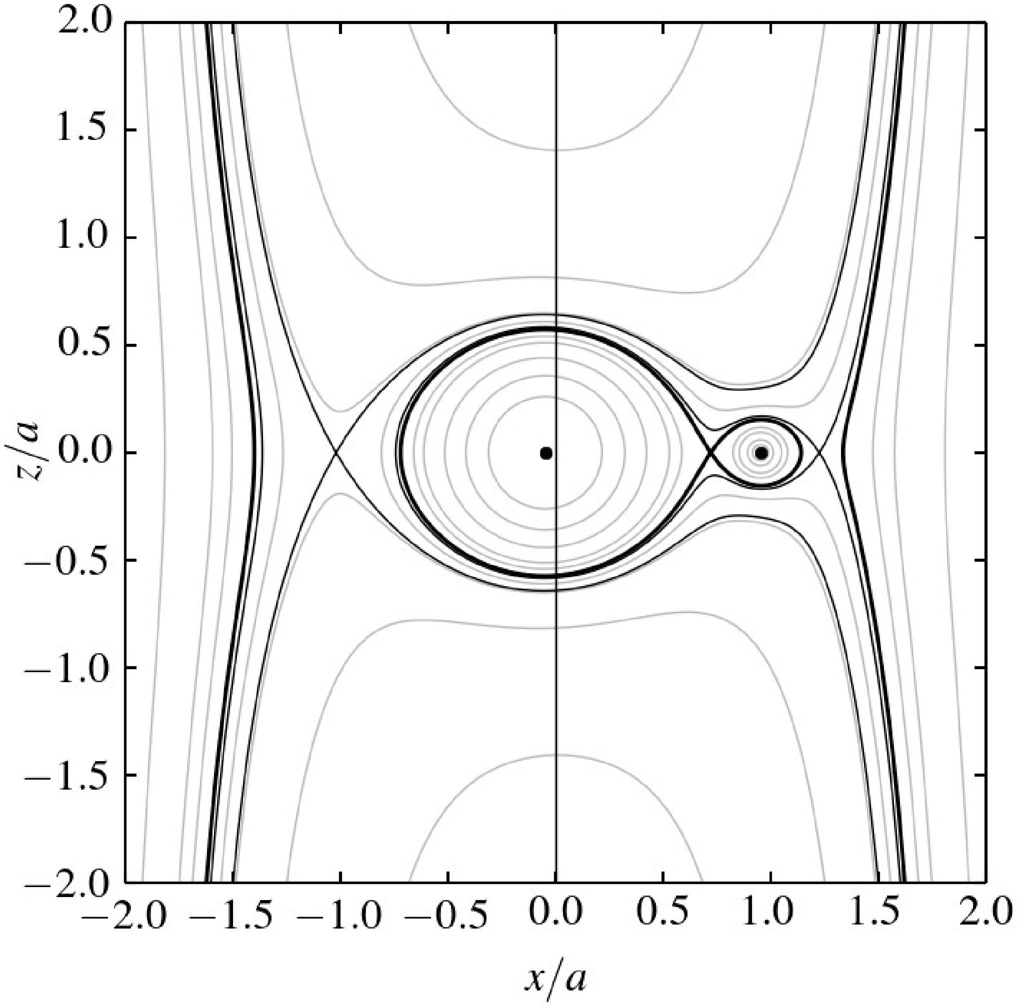


in the orbital plane

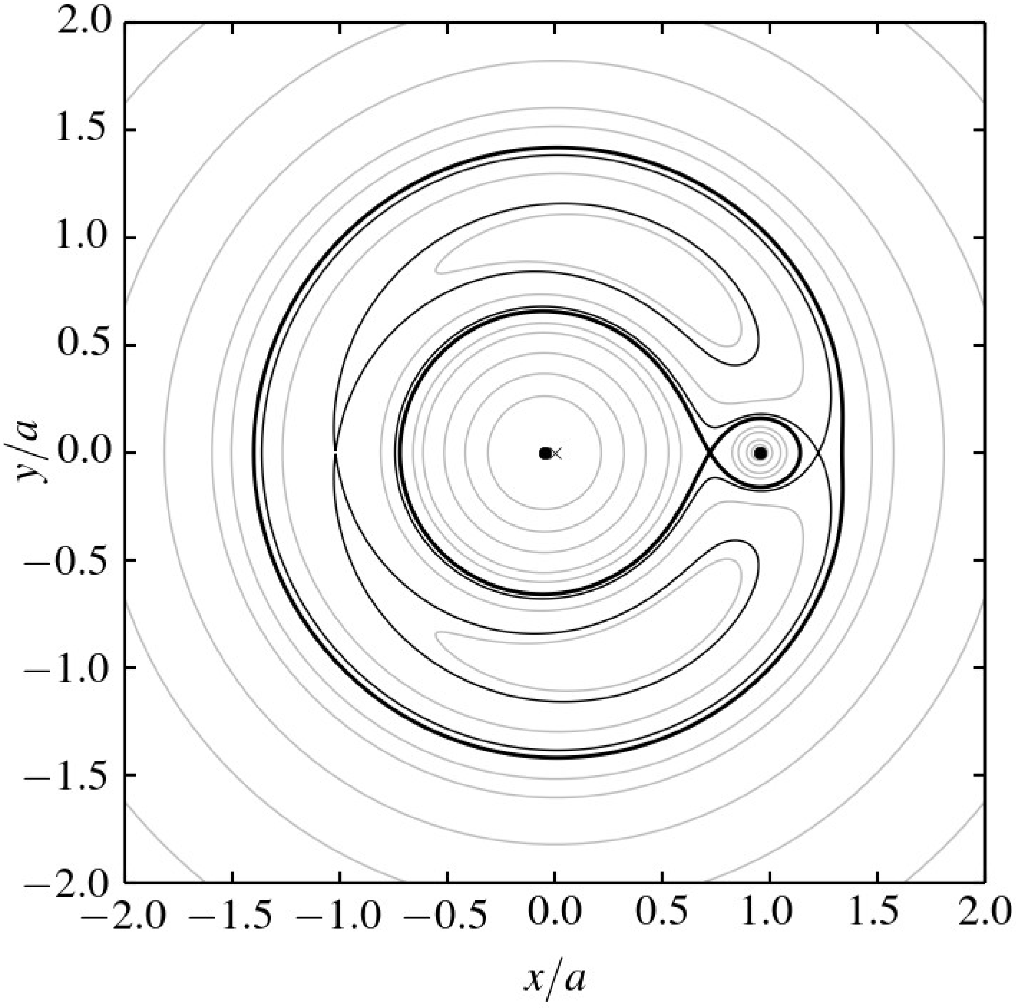
the Roche potential

- the shape of equipotential surfaces depends *only* on the mass ratio

  e.g. for $q = M_2/M_1 = 0.05$:

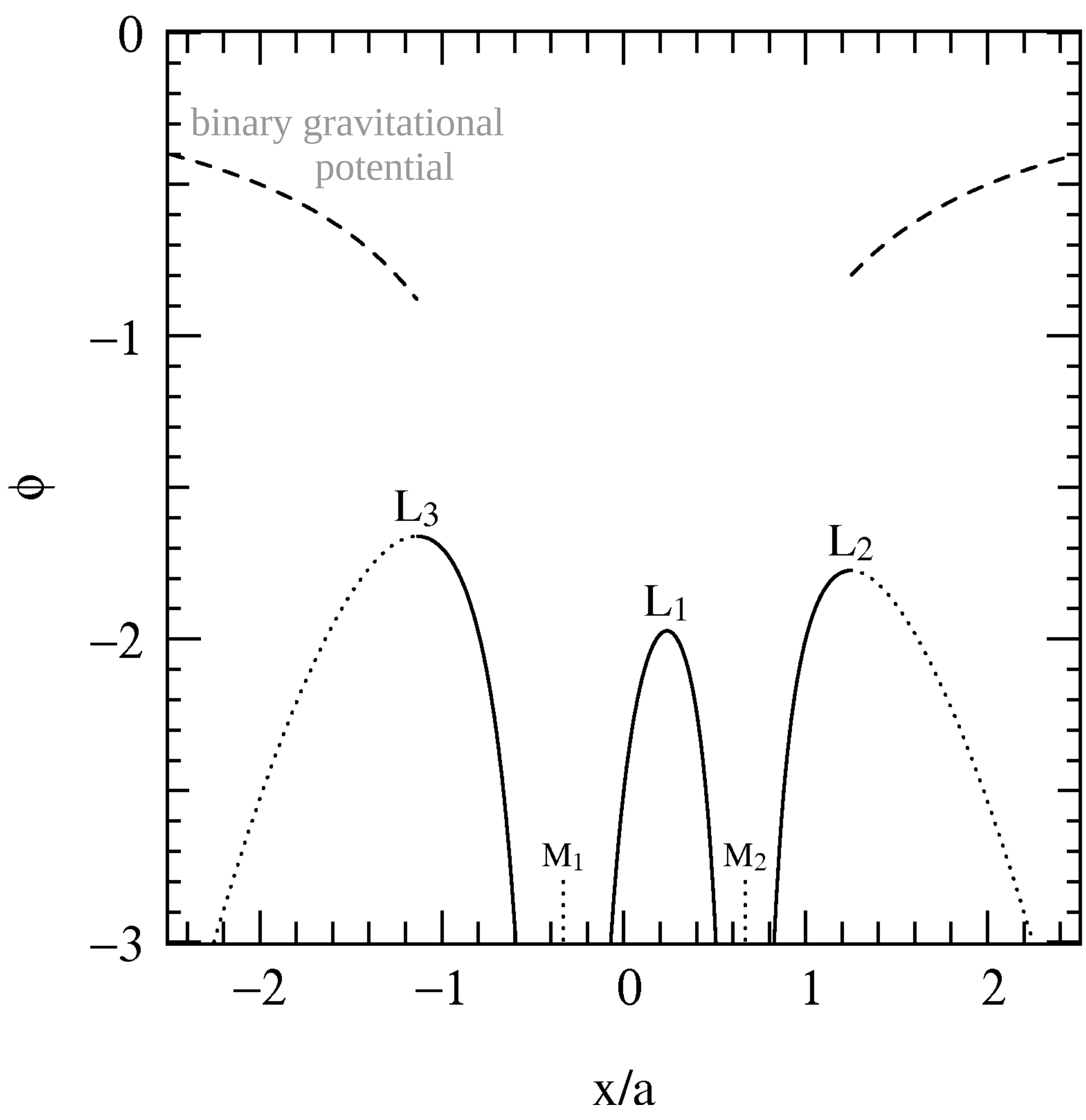


along the orbital axis



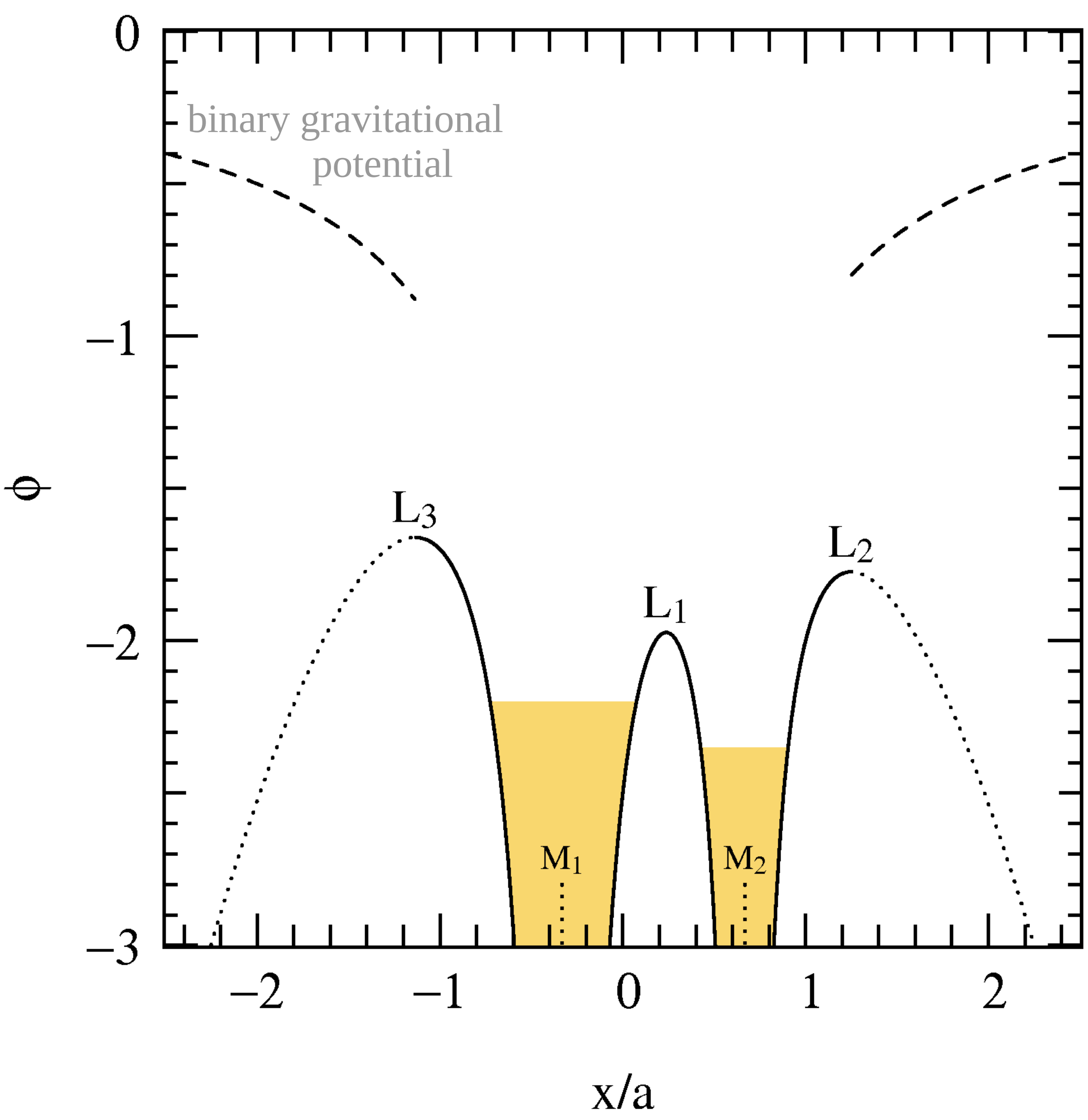
in the orbital plane

the Roche geometry



- shape of Roche potential along *x*-axis (connecting the stars) for $q = 0.5$

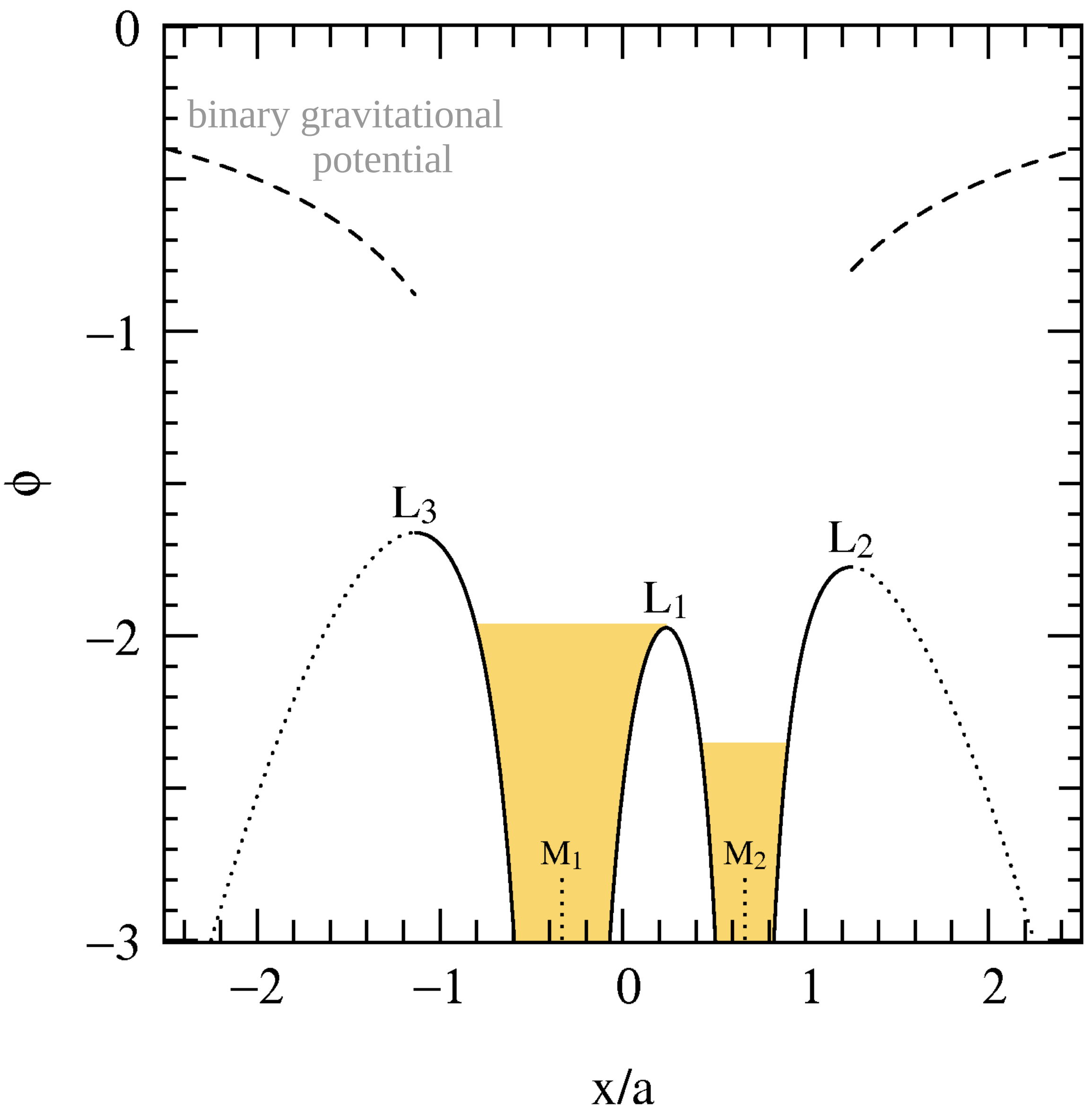
the Roche geometry



- shape of Roche potential along *x*-axis (connecting the stars) for q = 0.5
- hydrostatic equilibrium allows three possible configuratons:

  – a detached binary

the Roche geometry



- shape of Roche potential along *x*-axis (connecting the stars) for $q = 0.5$
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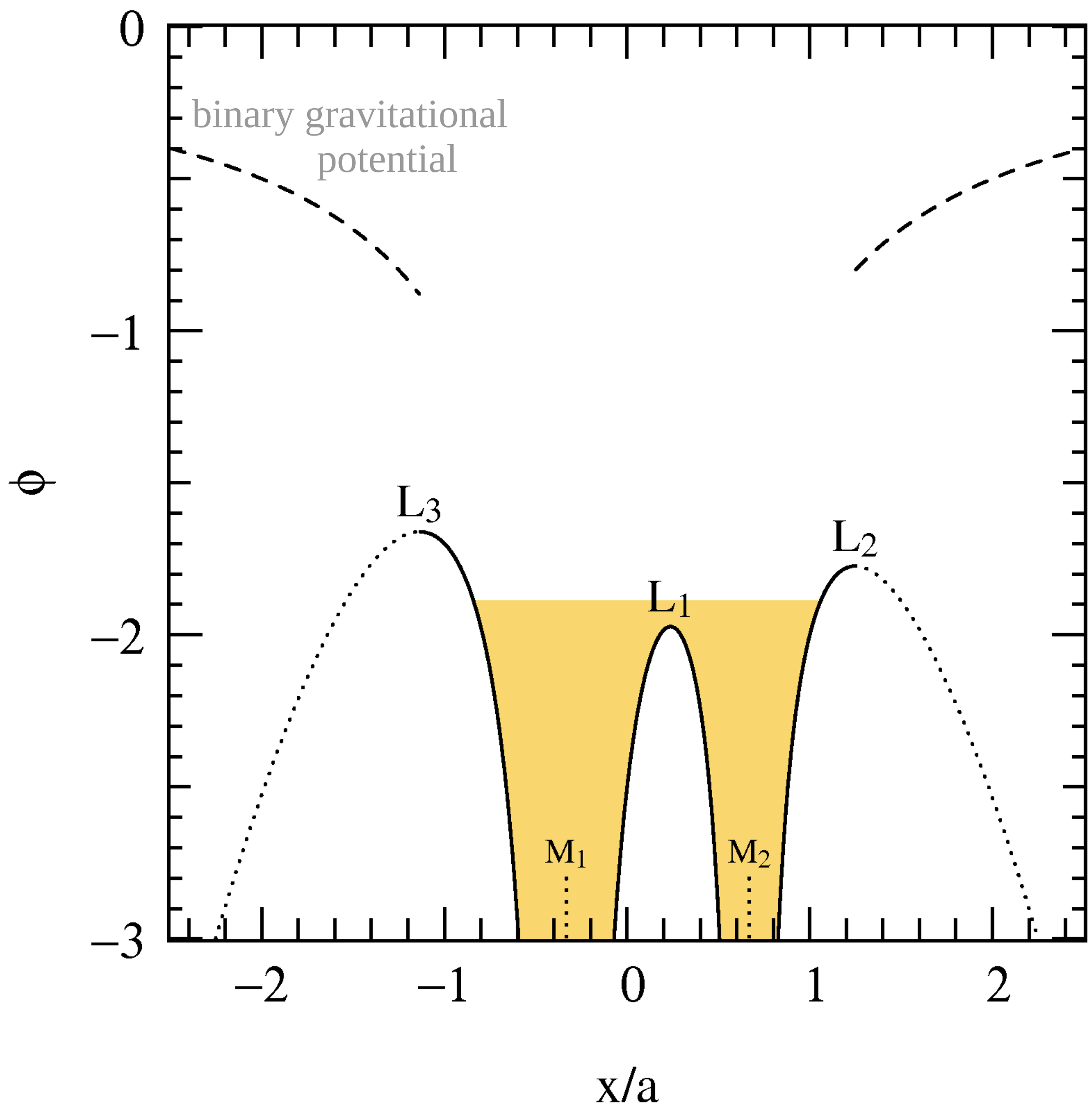
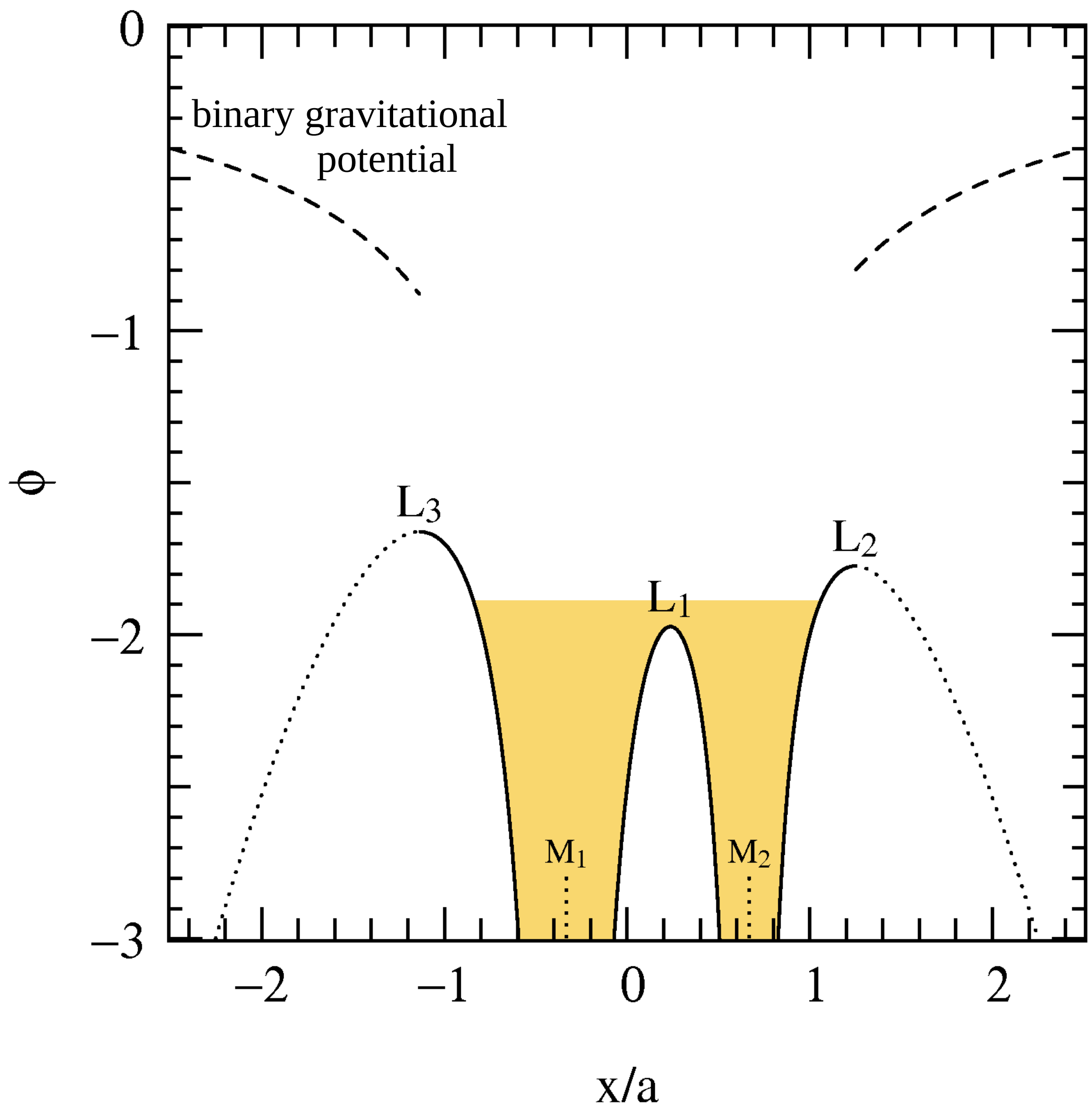
  – a semi-detached binary

the Roche geometry



- shape of Roche potential along *x*-axis (connecting the stars) for *q* = 0.5
- hydrostatic equilibrium allows three possible configuratons:
  - – a detached binary
  - – a semi-detached binary
  - – a contact binary

the Roche geometry



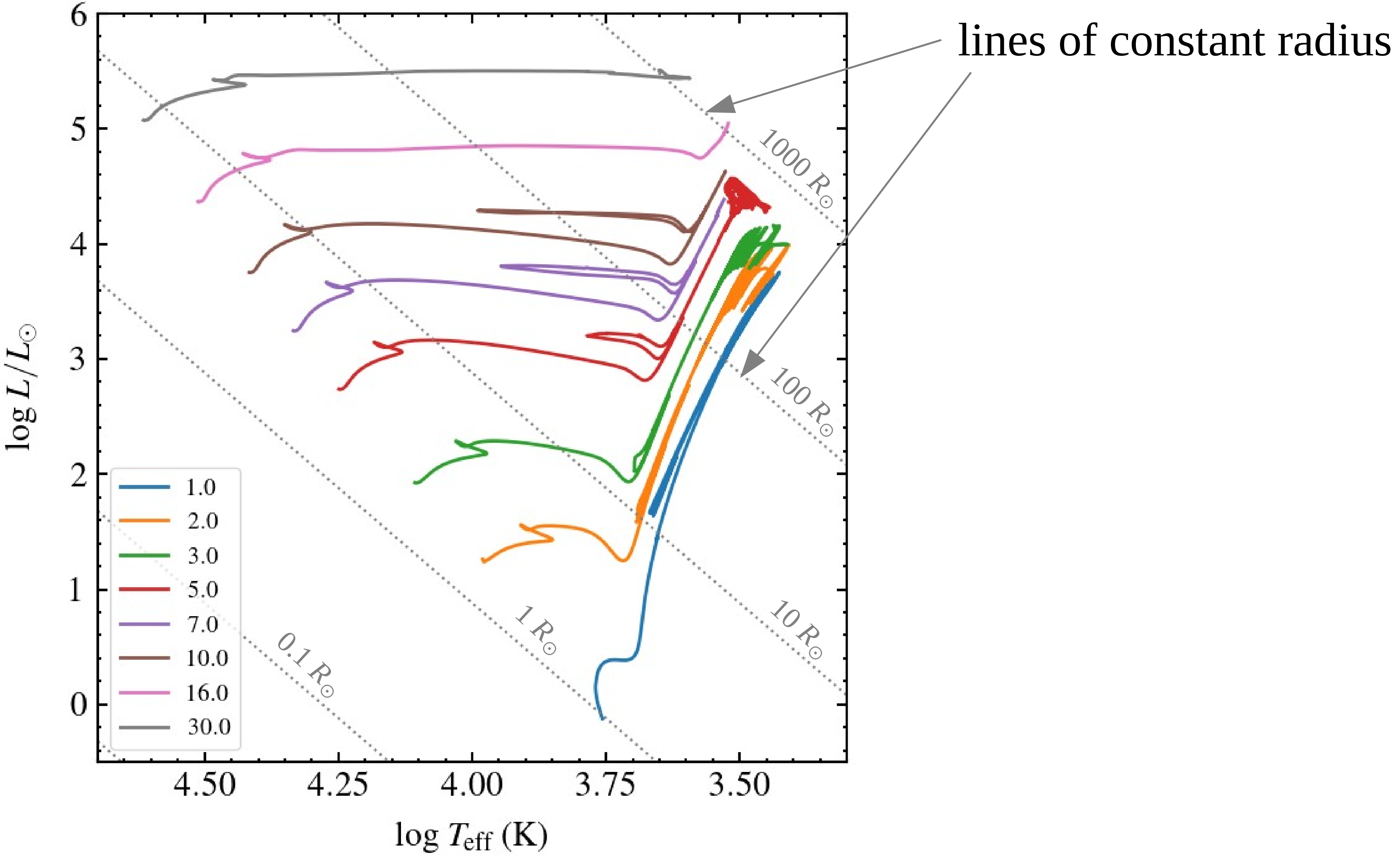
- shape of Roche potential along *x*-axis (connecting the stars) for $q = 0.5$
- hydrostatic equilibrium allows three possible configuratons:
  - – a detached binary
  - – a semi-detached binary
  - – a contact binary
- outside L_2/L_3, corotation cannot be maintained ⇒

  Roche geometry no longer applies, escaping gas is still bound...

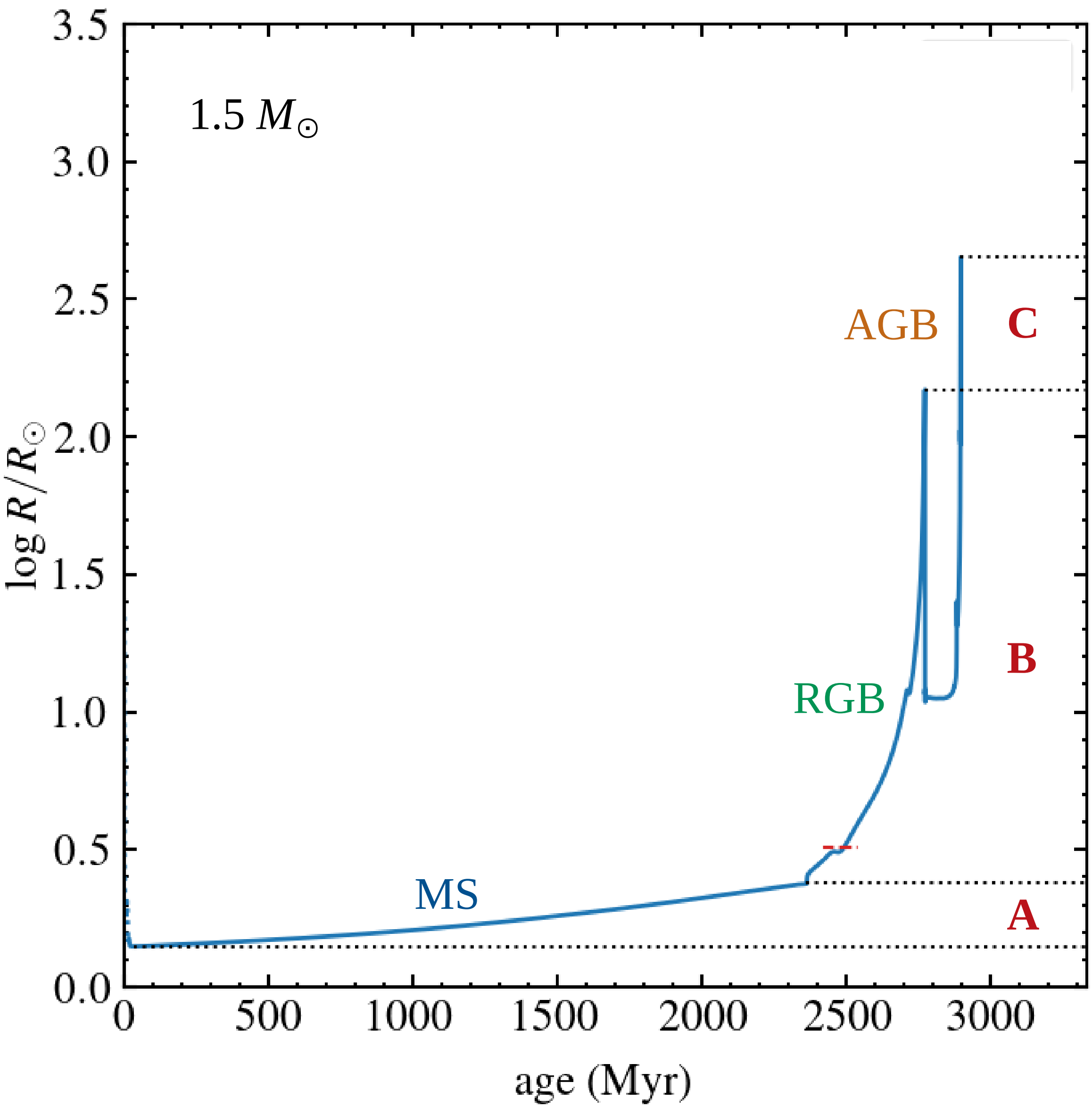
the Roche geometry

- assumptions:
  - gravity of both stars is the same as for point masses OK
  - the orbit is circular ($e = 0$) ??
  - all matter co-rotates with the orbit (stellar rotation is synchronized with orbit) ??
- last two assumptions may not generally hold, but are satisfied in case of strong tidal interaction (next lecture)

evolution of stellar radius

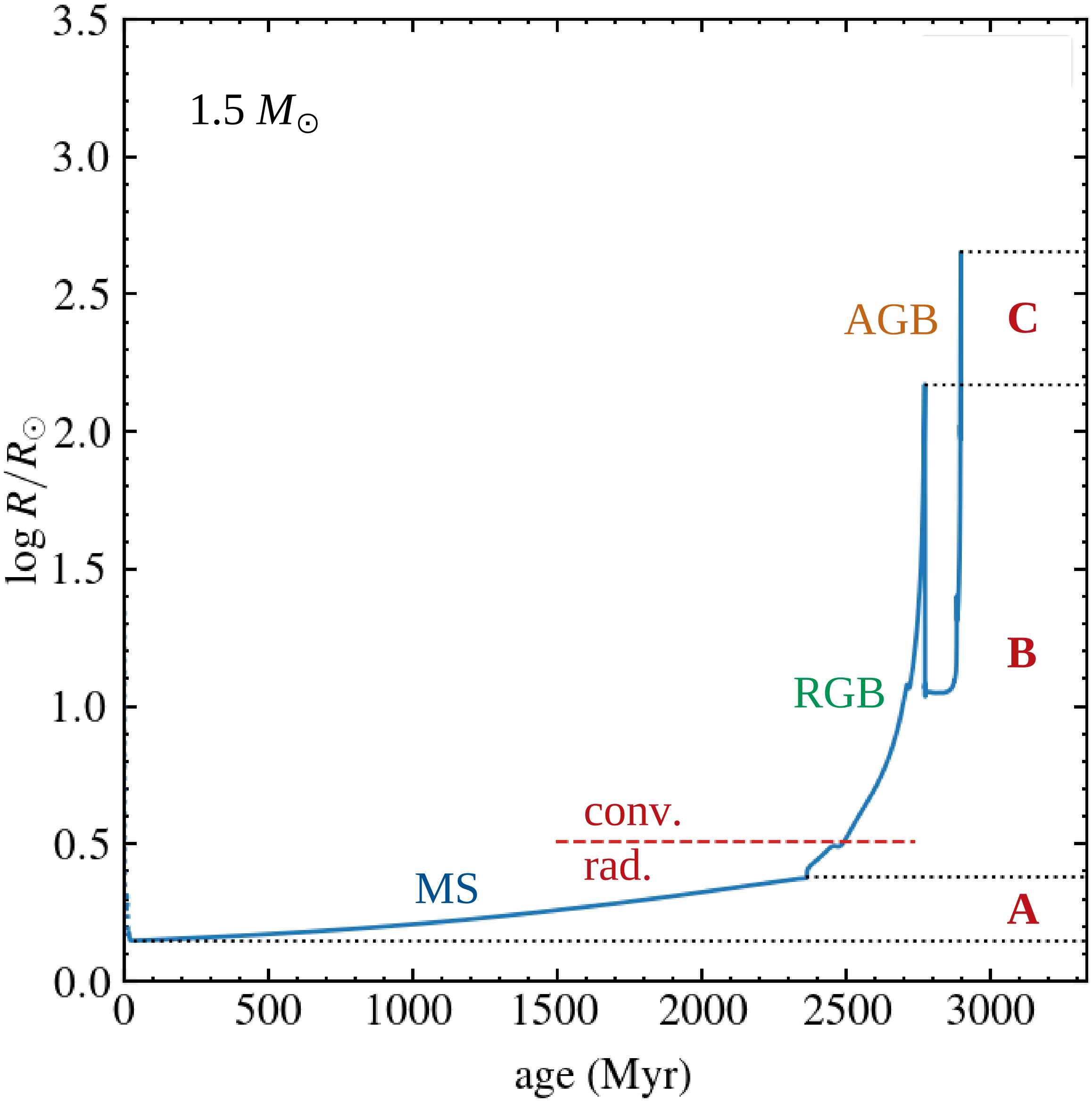


cases of binary evolution



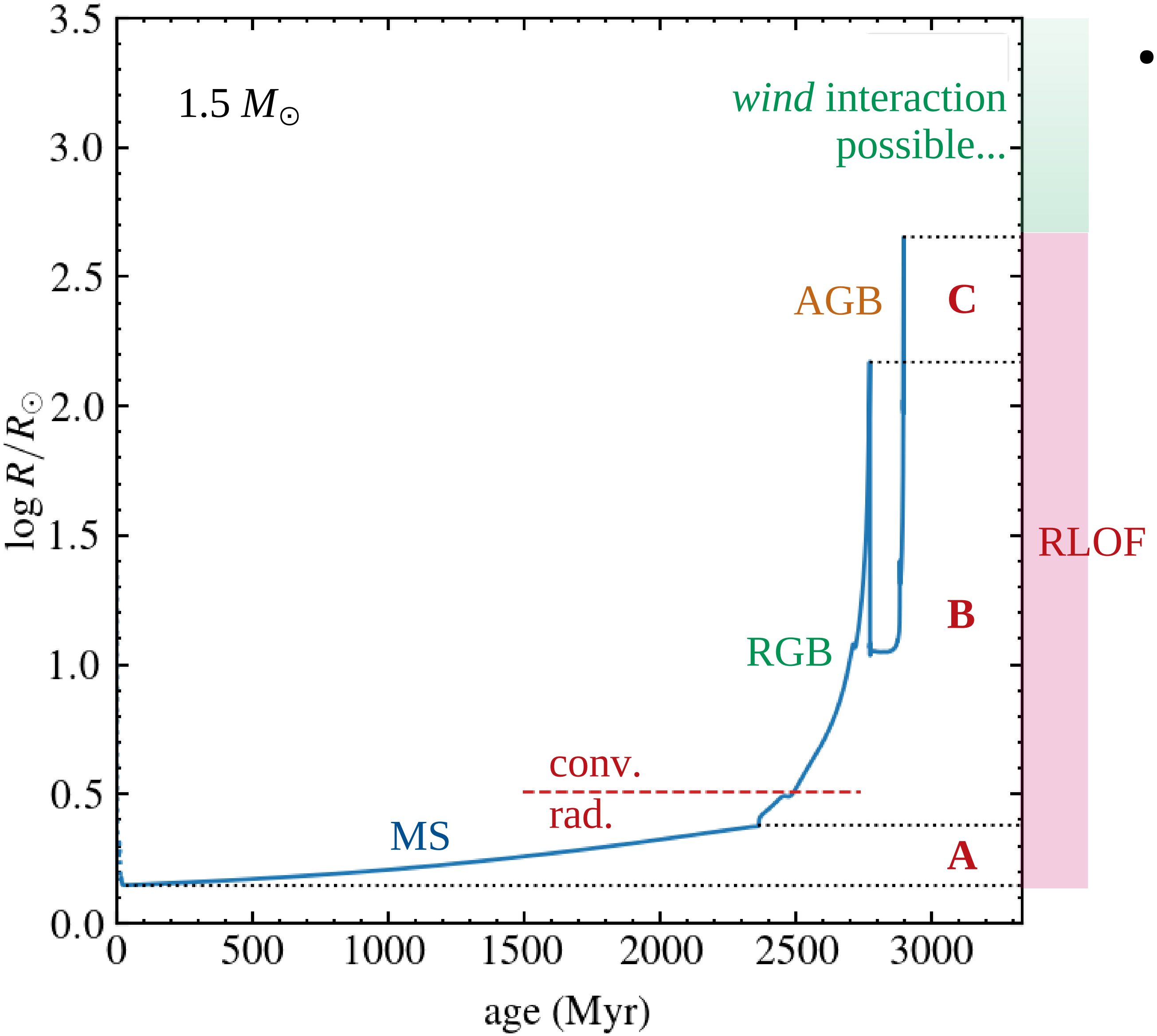
- distinguish three cases of binary evolution, depending on when a star first fills its Roche lobe
  - case A: during the main sequence
  - case B: during H-shell burning, but before He-ignition
  - case C: after He exhaustion

cases of binary evolution



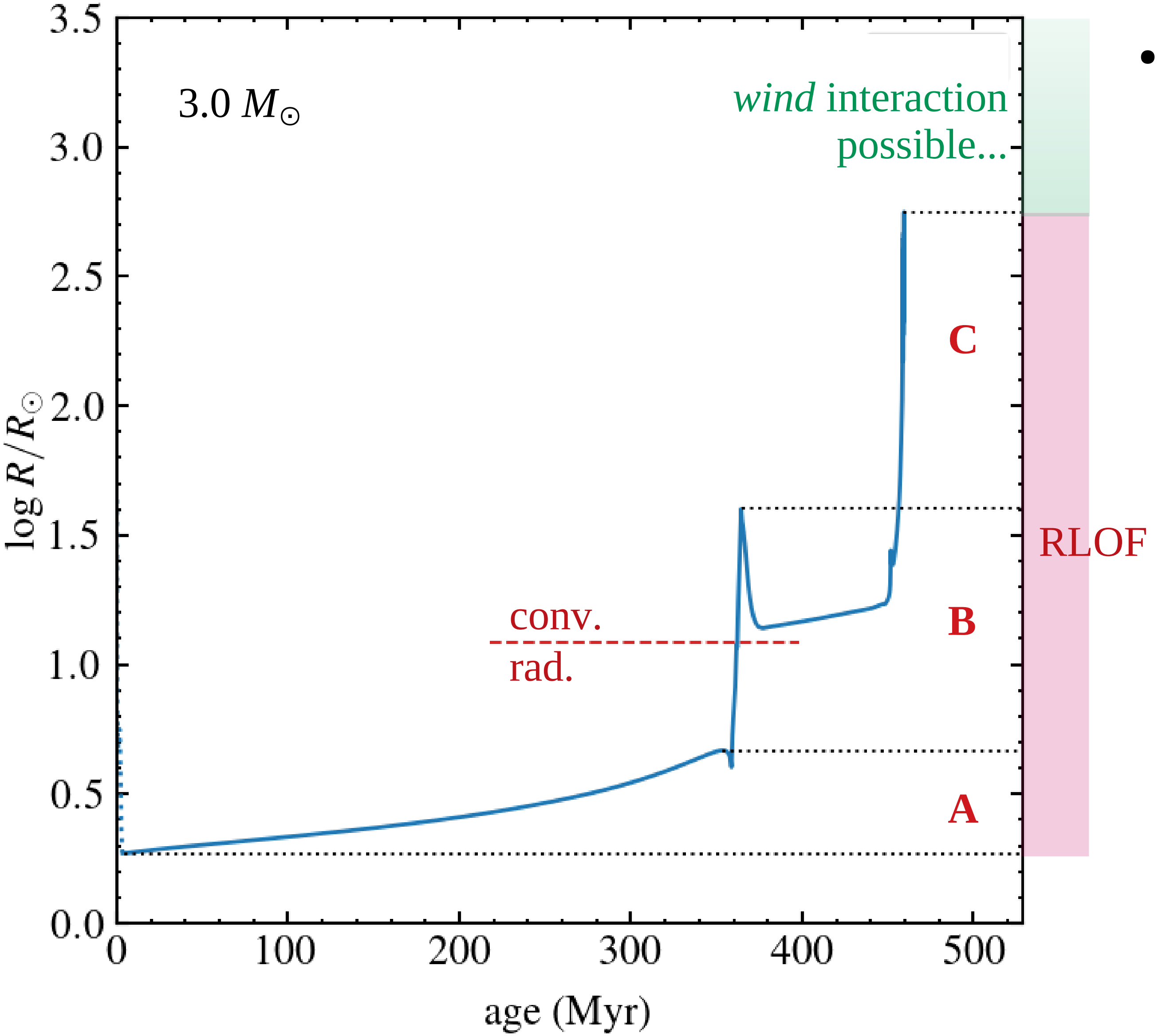
- an equally important distinction that affects how mass transfer proceeds:
  - radiative outer envelope (main sequence, Hertzsprung gap)
  - convective outer envelope (red giant, or very low-mass MS star)

evolution of stellar radius



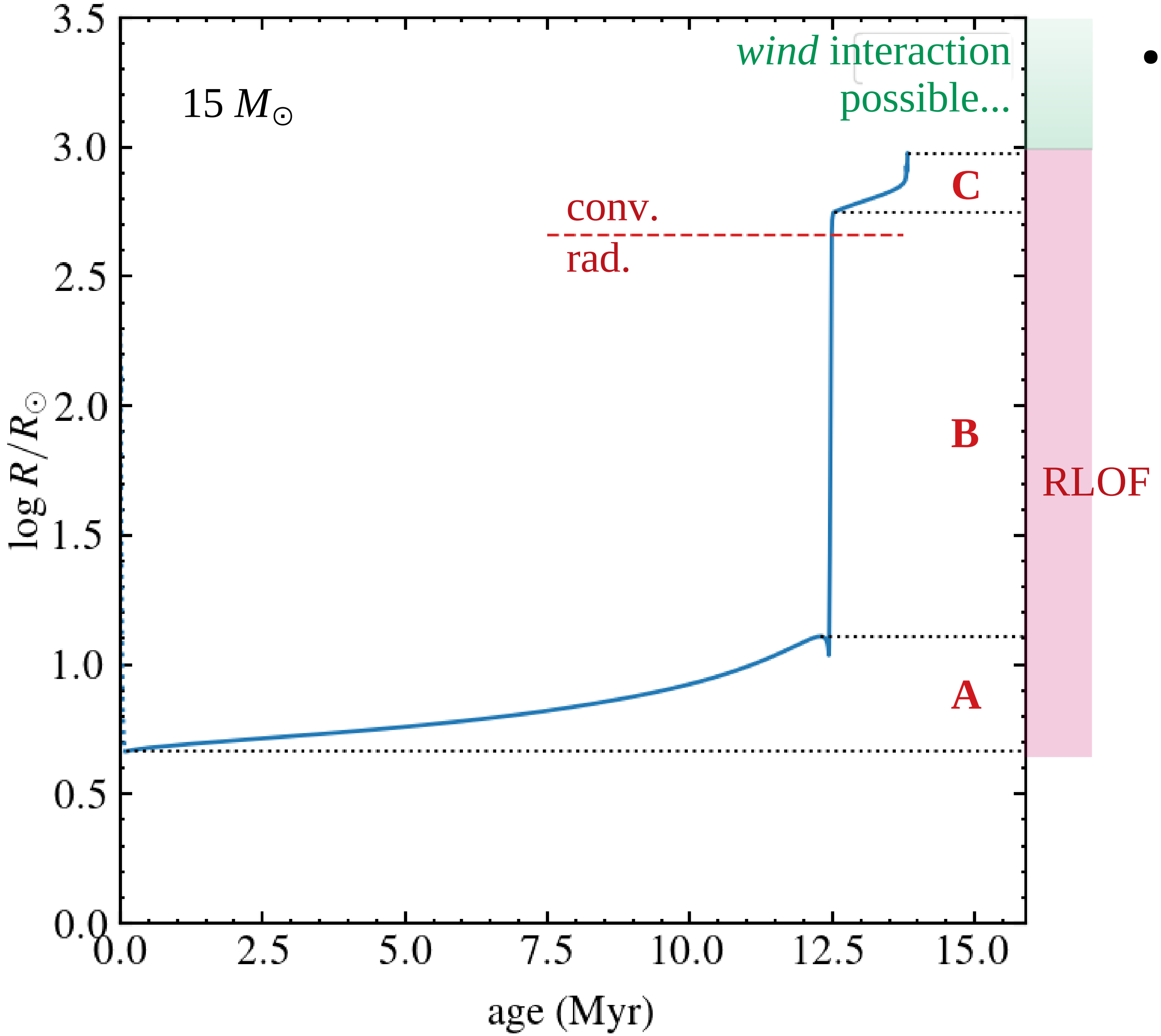
- low-mass stars:
  - most expansion occurs on the RGB
  - large range of orbital periods for case B (as red giant with *convective* envelope)
  - AGB-wind interaction in wider orbits

evolution of stellar radius



- intermediate-mass stars:
  - expansion during all phases (HG, RGB, AGB)
  - large range of orbital periods for case B and case C
  - AGB-wind interaction in wider orbits

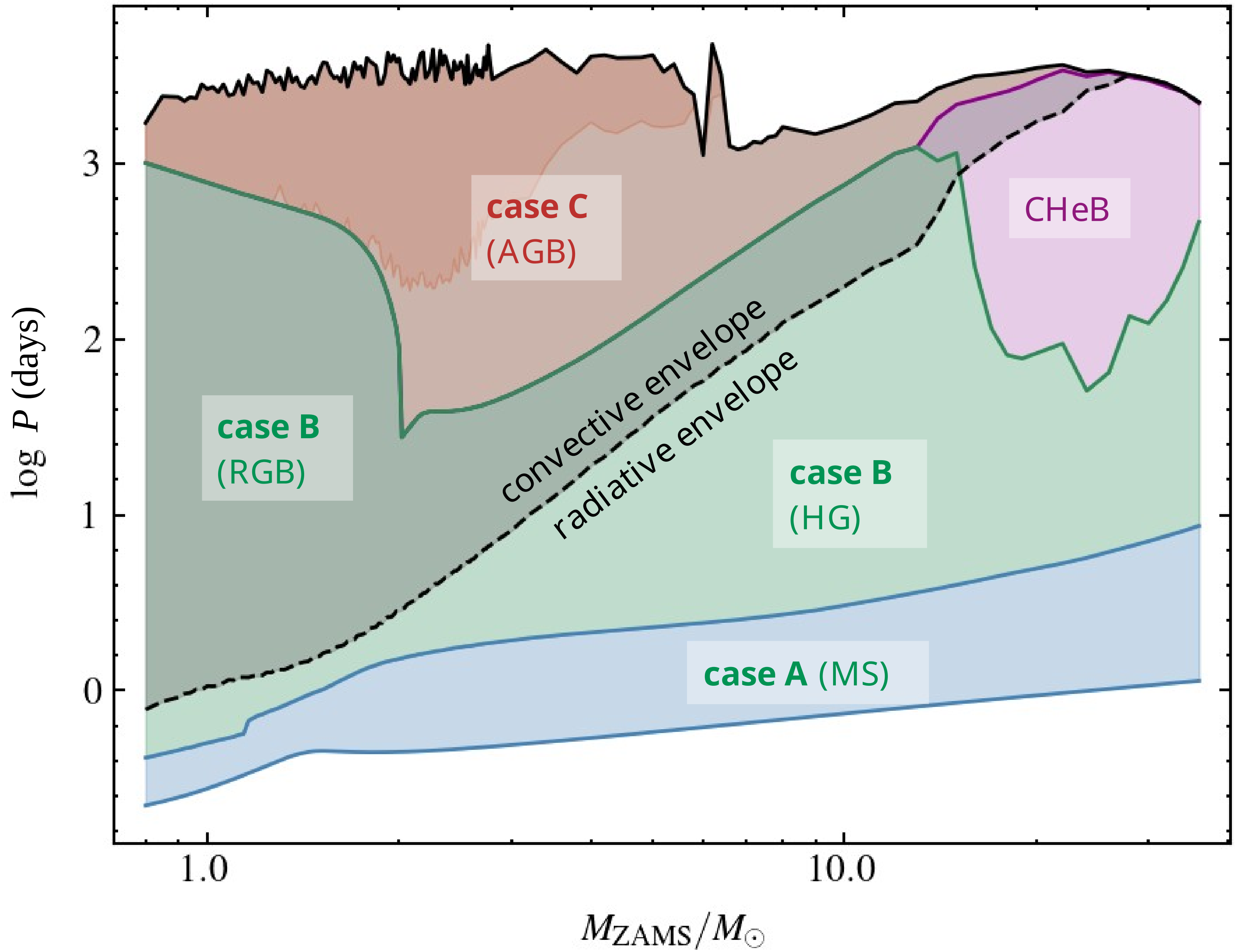
evolution of stellar radius



- high-mass stars:
  - most expansion occurs during HG
  - large range of orbital periods for case B (with *radiative* envelope)

parameter space for RLOF

- dependence on primary mass and orbital period:

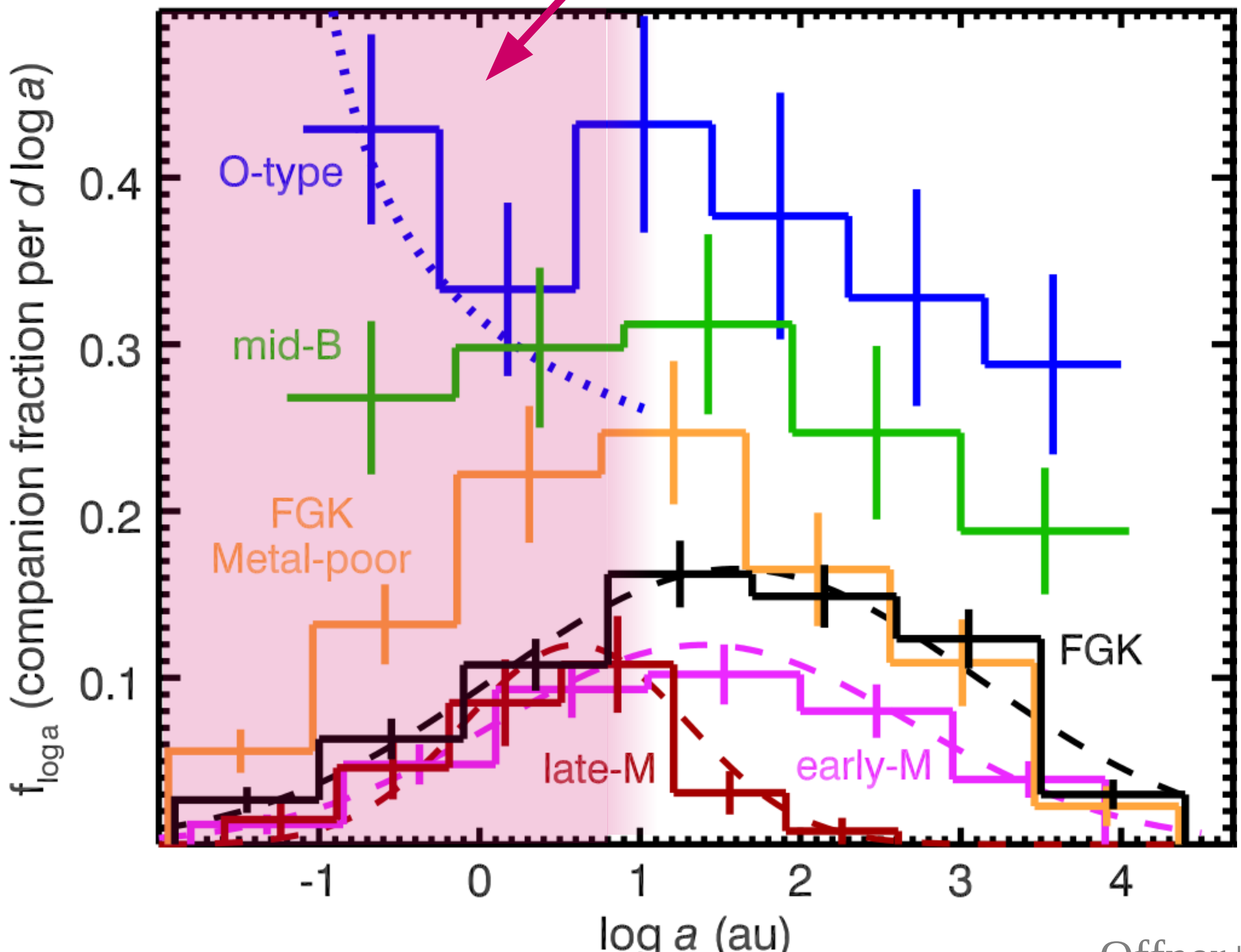


binary statistics

- orbital period distributions for different populations:

range of a for which $R_L < R_{*,\mathrm{max}}$

companion fraction
per log a interval



> 15 $M_\odot$

~5–8 $M_\odot$

0.8–1.2 $M_\odot$

0.3–0.6 $M_\odot$

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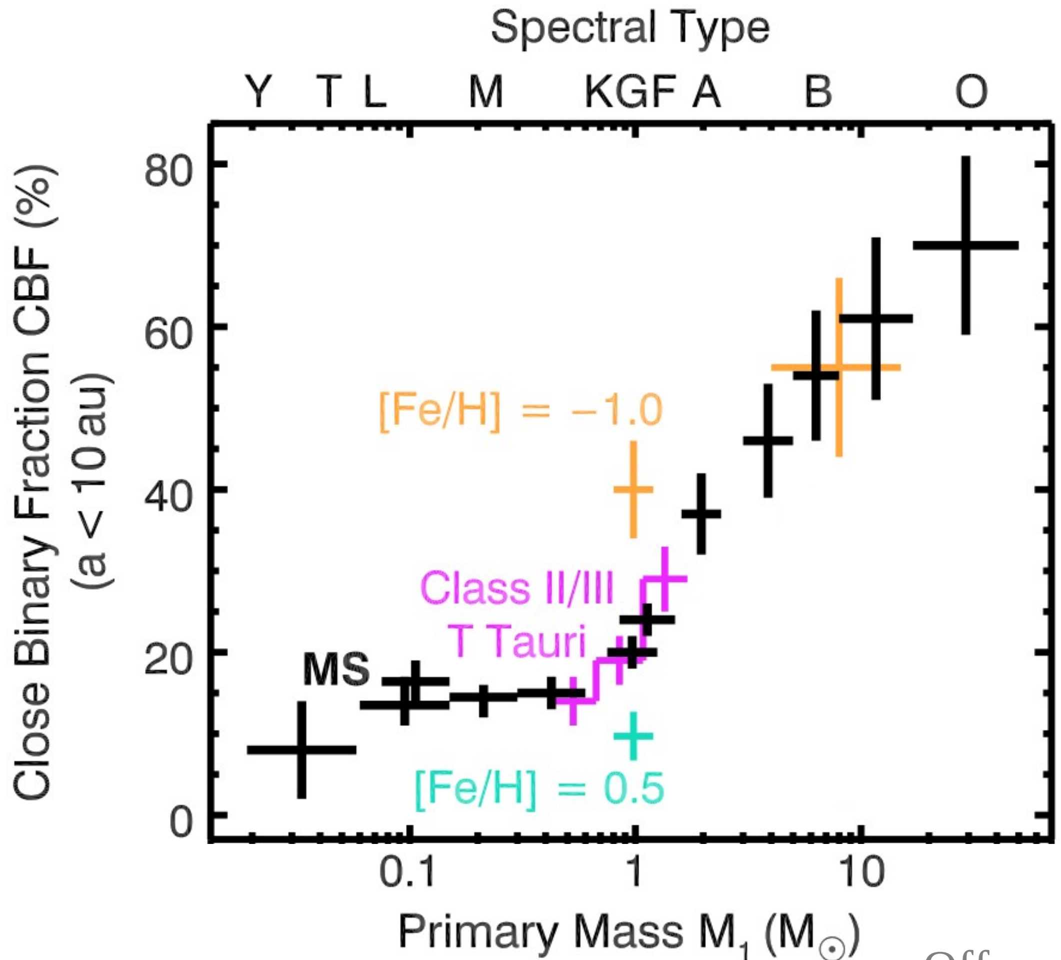
interacting binaries

- fraction of primary stars that will interact by RLOF with a companion during evolution:

~20% of solar-type stars

~70% of massive stars



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- N.B wider binaries (a > 10 AU) can also interact by means of their stellar winds