Advanced Stellar and Binary Evolution

hand-in exercise, week 2

a) Because the star is undergoing H-burning and is therefore in TE, we can use

$$\frac{\mathrm{d}l}{\mathrm{d}m} = \epsilon_{\mathrm{nuc}} = \epsilon_c \left(1 - \frac{m}{0.1M}\right) \qquad \text{for } m < 0.1M.$$

Integrate over m to get $l(m)$, using the boundary condition that $l = 0$ at $m = 0$:

$$l(m) = \int_0^m \epsilon_c \left(1 - 10\frac{m'}{M}\right) \mathrm{d}m' = \epsilon_c \left(m - 5\frac{m^2}{M}\right) \qquad \text{for } m < 0.1M. \tag{1}$$

Note that for $m \geq 0.1M$, $\epsilon_{nuc} = 0$ and therefore $l(m) = \text{constant} = L$, because $l(M)$ must be equal to the total luminosity. Thus,

$$l(m) = L \qquad \text{for } m \geq 0.1M. \tag{2}$$

Combining eqs. (1) and (2) gives $L = l(0.1M) = 0.05\epsilon_c M \quad \Rightarrow$

$$\epsilon_c = 20\frac{L}{M} \tag{3}$$

Putting in the given L and M gives $\epsilon_c = 1.03 \times 10^3\,\mathrm{erg\,g^{-1}\,s^{-1}}$.

b) With radiative energy transport no mixing occurs, so changes in X are only due to nuclear reactions. We use eq. (6.43) from the lecture notes, rewriiten for this exercise as

$$\frac{\mathrm{d}X}{\mathrm{d}t} = -\frac{\epsilon_{\mathrm{nuc}}}{Q_{\mathrm{H}}}.$$

Since the right-hand side is independent of time, we can simply integrate to find

$$X(m,t) - X_0 = -\int_0^t \frac{\epsilon_{\mathrm{nuc}}}{Q_{\mathrm{H}}}\mathrm{d}t = -\frac{\epsilon_c}{Q_{\mathrm{H}}}\left(1 - 10\frac{m}{M}\right) t$$

with the initial value $X_0 = 0.7$, so

$$X(m,t) = 0.7 - \frac{\epsilon_c}{Q_{\mathrm{H}}}\left(1 - 10\frac{m}{M}\right) t.$$

With ϵ_c from question (a) and Q_{H} as given, $\epsilon_c/Q_{\mathrm{H}} = 1.64 \times 10^{-16}\,\mathrm{s^{-1}} = 5.16 \times 10^{-9}\,\mathrm{yr^{-1}}$. At $t = 100\,\mathrm{Myr}$ we then find a central value of $X_c = 0.184$, with X linearly increasing to 0.7 at $m = 0.1M$.

d) The Schwarzschild criterion (eq. 5.52 in the lecture notes) tells us convection occurs wherever

$$\nabla_{\mathrm{rad}} = \frac{3}{16\pi acG}\frac{P}{T^4}\frac{\kappa l}{m} > \nabla_{\mathrm{ad}}.$$

Noting that $P_{\mathrm{rad}} = \frac{1}{3}aT^4$ we can rewrite this as

$$\nabla_{\mathrm{rad}} = \frac{\kappa}{16\pi cG}\frac{P}{P_{\mathrm{rad}}}\frac{l}{m} > \nabla_{\mathrm{ad}}. \tag{4}$$

Applying the additional assumptions, we have $\kappa = \kappa_{\mathrm{es}} = 0.2(1+X)\,\mathrm{cm^2\,g^{-1}}$, $P/P_{\mathrm{rad}} = 500$, and $\nabla_{\mathrm{ad}} = 0.4$ for an (ionized) ideal gas. From eqs. (1–3) in exercise (a) we find:

$$\frac{l}{m} = \frac{L}{M}f(m) \qquad \text{with} \qquad f(m) = \begin{cases} 20\left(1 - 5\dfrac{m}{M}\right) & m/M < 0.1 \\ \dfrac{M}{m} & m/M \geq 0.1 \end{cases} \tag{5}$$

with $L/M = 51.5\,\mathrm{erg\,g^{-1}\,s^{-1}}$. Expressing $16\pi cG$ in cgs units, and taking $X = 0.7$, we can then write the Schwarzschild criterion (eq. 4) as

$$\nabla_{\rm rad} = 0.0871\, f(m) > 0.4.$$

Noting that $f(m) = 20$ in the centre and $f(m) = 10$ at $m/M = 0.1$, we see that the inner 10% of the mass is convective, and the Schwarzschild boundary must lie in the region $m/M > 0.1$. Using $f(m) = M/m$, we find the boundary is at $m/M = 0.218$. The star therefore has a convective core of mass $m_{\rm core} = 0.654\, M_\odot$.