Advanced Stellar and Binary Evolution

hand-in exercises, week 11

1. ϕ Persei

The binary system ϕ Persei consists of a He-rich hot subdwarf and a rapidly rotating Be star in a circular orbit with $P = 126.7$ days. The masses are $1.2 \pm 0.2\,M_\odot$ and $9.6 \pm 0.3\,M_\odot$ for the helium star and the Be star, respectively[1]. This system is very likely the product of a phase of stable mass transfer in the progenitor binary.

a) Assuming that the system formed by *conservative case B* mass transfer, calculate the initial masses and orbital period of the binary. (Hint: use Section 18.2 of the lecture notes.) For now, ignore the observational errors.

b) Are these initial parameters consistent with the case B assumption? Motivate your answer.

b) This system also poses an interesting constraint on the *mass transfer efficiency*, i.e. the parameter β discussed in Chapter 17 of the lecture notes. Using the observed masses, what is the minimum value of β required to produce this system by case B mass transfer? Take into account the observational errors on the masses.

2. Spin-up by accretion

a) Consider a star that is being spun up by accreting gas from the inner edge of a Keplerian accretion disk. Write the angular momentum of the star as $J_* = r_\mathrm{g}^2 M R^2 \Omega$. Assume that initially the star has $\Omega = 0$, and that the radius of the star R and the dimensionless structure parameter r_g^2 do not change as the star gains mass and spins up. Calculate by which factor the mass M can increase until the surface reaches break-up rotation ($\Omega = \Omega_\mathrm{Kep}$), expressed in terms of r_g^2.

b) Main-sequence stars typically have $r_\mathrm{g}^2 < 0.10$. What is the maximum fraction of its original mass that a MS star can gain from an accretion disk before reaching break-up?

It is often considered that stars cannot accrete more mass after they reach break-up rotation. Is this compatible with the constraint derived for ϕ Persei in exercise 1c?

3. Darwin instability

Tidal interaction becomes unstable when the stellar angular momentum J_* becomes larger than one third of the orbital angular momentum J_orb : the so-called Darwin instability, see Chapter 16.

a) Consider a star that gradually expands in a binary with a circular orbit. Derive the condition on the binary mass ratio $q = M_1/M_2$ (where M_1 is star under consideration and M_2 is treated as a point mass), such that tidal interactions keep the star in stable co-rotation with the orbit until the star fills its Roche lobe. Using the approximation of Eq. (15.18) to the Roche-lobe radius, show that this condition can be written as:

$$q^{0.66}(1+q)^{0.6} \lesssim \frac{1.72}{r_\mathrm{g}^2}. \qquad (1)$$

b) Estimate the value of the critical mass ratio for polytropic stellar models with $n = 3$, with $r_\mathrm{g}^2 = 0.076$; and $n = 1.5$, with $r_\mathrm{g}^2 = 0.21$.

[1] Mourard et al. (2015, A&A 577, A51); see also Table 18.2 in the lecture notes for somewhat older parameter measurements.