Stellar Evolution – Hints to exercises – Chapter 10

10.1 **Conceptual questions**

(a) See Section 9.3
(b) See Section 9.3.1
(c) See Section 9.3.4
(d) See Section 10.1
(e) See Section 10.1
(f) See Section 10.2.1

10.2 **Evolution of the abundace profiles**

(a) Take into account:
- the central abundances that you can read from the figures,
- the location of the H-burning shell,
- the extent of a convection zone in the core (if present).
(Hint: look at Fig. 9.10 for an earlier evolution phase).

(b) Idem.

(c) They are modified by convective dredge-up at points E (for 5 $M_\odot$) and D (for 1 $M_\odot$).

10.3 **Red giant branch stars**

(a) Use the virial theorem to calculate the total energy $E_{\rm tot} = E_{\rm gr} + E_{\rm int} = \frac{1}{2}E_{\rm gr}$.

(b) $R \leq (0.45)^2 R_\odot$.

(c) Yes.

10.4 **Core mass - luminosity relation for RGB stars**

(a) The energy release q per gram of H-burning is (see also Ch.5)

$$q = \frac{\mathrm{d}E}{\mathrm{d}M_{\rm H}} = \frac{26.73\times10^6\times1.6\times10^{-12}\,\mathrm{erg}}{4\times1.67\times10^{-24}\,\mathrm{g}} = 6.4\times10^{18}\mathrm{erg/g}$$

The envelope that provides fuel for the shel has a hydrogen mass fraction $X_{\rm env}\approx 0.7$, so a core growth $\mathrm{d}M_c$ corresponds to a $\mathrm{d}M_{\rm H} = X_{\rm env}\,\mathrm{d}M_c$.
$\Rightarrow \mathrm{d}E = X_{\rm env} q\,\mathrm{d}M_c \Rightarrow \dfrac{\mathrm{d}E}{\mathrm{d}t} = L = X_{\rm env} q\,\dfrac{\mathrm{d}M_c}{\mathrm{d}t}$.

(b) Combine (a) with eq. (9.2), $L/L_\odot = 2.3\times10^5 (M_c/M_\odot)^6$. Rewrite and integrate:

$$\int_{M_c(t_0)}^{M_c(t)} \left(\frac{M_c}{M_\odot}\right)^{-6} \mathrm{d}\left(\frac{M_c}{M_\odot}\right) = \int_{t_0}^{t} \frac{2.3\times10^5 L_\odot}{X_{\rm env} Q M_\odot}\,\mathrm{d}t = \int_{t_0}^{t} 9.9\times10^{-14}\,\mathrm{d}t$$

with dt in seconds in the last expression. Integrate to obtain an expression for $M_c(t)$.

(c)

$$(t - t_0) = 2.02 \times 10^{12}\,\text{sec}\left[\left(\frac{0.15M_i}{M_\odot}\right)^{-5} - (0.45)^{-5}\right]$$

For a 1 $M_\odot$ star: $\tau_{\text{RGB}} \approx 8.4 \times 10^8$ yrs, $\tau_{\text{RGB}}/\tau_{\text{MS}} \approx 0.08$.
For a 2 $M_\odot$ star: $\tau_{\text{RGB}} \approx 2.3 \times 10^7$ yrs, $\tau_{\text{RGB}}/\tau_{\text{MS}} \approx 0.016$.

10.5 **Jump in the composition**

(a) A chemical profile like this can be the result of convection in the core. High mass stars have convective cores.

(b) The pressure and temperature are continuous functions of the mass coordinate. A jump in composition (i.e. in μ) has to be compensated by a jump in density, if we assume that the ideal gas law holds. First express μ in terms of the discontinuous variable X.

$$\mu^{-1} = X\frac{2}{1} + Y\frac{3}{4} + Z\frac{A/2}{A} = ... = \frac{5}{4}X - \frac{1}{4}Z + \frac{3}{4}$$

Now consider the logarithm of the ideal gas $P = \rho kT/(m_u\mu)$ law, which you can rewrite as:

$$\ln\rho = \ln\mu + \ln\left(\frac{Pm_u}{kT}\right).$$

Now consider the difference Δ just above and below the jump in composition.

$$\Delta\ln\rho = \Delta\ln\mu + \Delta\ln\left(\frac{Pm_u}{kT}\right).$$

The last term is zero, because P and T are continuous and m_u and k are constants, so $\Delta\ln\rho = \Delta\ln\mu$.

$$\Delta\ln\rho = \frac{\Delta\rho}{\rho} = -\Delta\ln\left(\frac{1}{\mu}\right) = -\ln\left(\frac{5\times 0.7 - 0.02 + 3}{5\times 0.1 - 0.02 + 3}\right) = -0.622$$

(c) For **kramers opacity**: $\kappa_{bf} \sim Z(1 + X)\rho T^{-3.5}$, then

$$\Delta\ln\kappa = \Delta\ln Z + \Delta\ln(1 + X) + \Delta\ln\rho - 3.5\Delta\ln T.$$

T and Z are continuous, therefore $\Delta\ln\kappa = \Delta\ln(1 + X) + \Delta\ln\rho$.
For **electron scattering** $\kappa_e = 0.2(1 + X)$ then $\Delta\kappa/\kappa = \ln(1 + 0.7) - \ln(1 + 0.1)$