Advanced Stellar and Binary Evolution

hand-in exercise, week 4

1. Evolution of AGB stars

The luminosity of an AGB star is related to its core mass via the Paczynski relation (eq. 11.1),

$$\frac{L}{L_\odot} = 5.9 \times 10^4 \left(\frac{M_c}{M_\odot} - 0.52 \right).$$

Assume that a star enters the AGB with a luminosity of $3.0 \times 10^3\ L_\odot$, effective temperature of 3000 K and a total mass of 2 $M_\odot$.

(a) Mass is gradually added to the core by nuclear burning in the H- and He-burning shells. Assuming for the moment that both shells burn at a steady rate (i.e. ignoring thermal pulses), and assuming that the envelope of the star has a hydrogen mass fraction $X = 0.7$, show that the core mass grows at a rate of about

$$\dot{M}_c = 1.2 \times 10^{-11} \left(\frac{L}{L_\odot} \right)\ M_\odot/\mathrm{yr}.$$

(Hint: see Sect. 6.4.1-2 for the energy released by H- and He-burning. You can assume He-burning occurs by the 3α reaction.)

(b) Derive an expression for the luminosity as a function of time after the star entered the AGB phase.

(c) Assume that T_{eff} remains constant at 3000 K and derive an expression for the radius as a function of time.

(d) Derive an expression for the core mass as a function of time.

The masses of white dwarfs and the luminosity on the tip of the AGB are mostly determined by *mass loss* during the AGB phase. The mass-loss rate is uncertain, but for this exercise assume it is given by the Reimers relation, eq. (10.3),

$$\frac{\dot{M}}{M_\odot\,\mathrm{yr}^{-1}} = 4 \times 10^{-13} \eta \left(\frac{L}{L_\odot} \right) \left(\frac{R}{R_\odot} \right) \left(\frac{M_\odot}{M} \right),$$

with $\eta \approx 3$ for AGB stars.

(e) Derive an expression for the mass of the AGB star as a function of time, using $L(t)$ and $R(t)$ from the previous questions. (Hint: $-\dot{M}M = \frac{1}{2}\mathrm{d}(M^2)/\mathrm{d}t$).

(f) Use the expression from (e) and the one for $M_c(t)$ from (d) to derive:

- the time when the star leaves the AGB ($M_{\mathrm{env}} \simeq 0$),
- the luminosity at the tip of the AGB,
- the mass of the resulting white dwarf.

This requires a numerical solution of a simple equation, but you can also solve it graphically by plotting the two relations.

(g) In the above calculations, we have ignored the fact that *thermal pulses* occur during the AGB, i.e. we have assumed that the AGB star undergoes steady H- and He-shell burning. Discuss qualitatively how the answers to the above questions will change if thermal pulses are taken into account, in particular the *dredge-up* episodes that follow each thermal pulse.