第三章习题

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3.3 宽平稳和宽各态历经的概念

- 若随机过程满足

$$E[X(t)] = m_X$$

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)] = R_X(\tau), \tau = t_2 - t_1$$

$$E[X^2(t)] < \infty$$

- 则该随机过程是宽平稳随机过程

宽各态历经

$$E[X(t)] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

$$R_X(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\,dt$$

则该随机过程是各态历经的

- 值得注意的是：这两种性质不是等价的，即宽平稳随机过程不一定是宽各态历经的，但宽各态历经随机过程一般是宽平稳的

首先讨论平稳性，由题可知

$$E[X(t)] = \int_0^T S(t+\varphi)\cdot\frac{1}{T}d\varphi$$

$$= \frac{1}{T}\int_t^{t+T} S(\theta)d\theta$$

$$= const$$

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)]$$

$$= \int_0^T S(t_1+\varphi)S(t_2+\varphi)\cdot\frac{1}{T}d\varphi \quad \Rightarrow \tau = t_2 - t_1$$

$$= \frac{1}{T}\int_0^T S(t_1+\varphi)S(\tau+t_1+\varphi)d\varphi$$

$$\theta = t_1 + \varphi \Rightarrow$$

$$R_X(t_1, t_2) = \frac{1}{T}\int_{t_1}^{t_1+T} S(\tau+\theta)S(\theta)\,d\theta$$

$$= R_X(\tau)$$

$$E[X^2(t)] = \int_0^T S^2(t+\varphi)\cdot\frac{1}{T}\,d\varphi$$

$$= \frac{1}{T}\int_t^{t+T} S^2(\theta)d\theta < \infty$$

- 综合以上讨论，该随相周期过程是宽平稳的

现在讨论各态历经性：

$$\overline{X(t)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} s(t+\varphi)\,dt$$

$$= \frac{1}{T} \int_{0}^{T} s(t+\varphi)\,dt$$

$$= \frac{1}{T} \int_{0}^{T} s(\theta)\,d\theta$$

$$= E[X(t)]$$

$$
\begin{aligned}
&\overline{X(t)X(t+\tau)} \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} x(t)x(t+\tau)\,dt \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} s(t+\varphi)s(t+\tau+\varphi)\,dt \\
&= \frac{1}{T}\int_{0}^{T} s(\theta)s(\theta+\tau)\,d\theta \\
&= R_X(\tau)
\end{aligned}
$$

- 综上所讨论，该随机过程是各态历经的

- 3.4. 本题实际上是3.3题的一个特例, 可以直接引用3.3题的结论
- 该随相周期过程是各态历经的，所以有

$$
\begin{aligned}
E[X(t)] &= \overline{X(t)} \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} S(t+t_0)\,dt_0 \\
&= \frac{1}{T}\int_{0}^{T} S(\theta)\,d\theta \\
&= \frac{1}{T}\times\frac{1}{2}\times\frac{T}{4}\times a = \frac{a}{8}
\end{aligned}
$$

也可直接由定义得到

$$E[X(t)] = \int_0^T S(t+t_0) \times \frac{1}{T} dt_0$$

$$= \frac{1}{T}\int_0^T S(\theta)d\theta$$

$$= \frac{1}{T} \times \frac{1}{2} \times \frac{T}{4} \times a = \frac{a}{8}$$

3.5 仍然由各态历经的定义

$$\overline{X(t)} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} a\cos(\omega_0 t + \varphi) dt$$

$$= 0$$

$$E[X(t)] = E[A]E[\cos(\omega_0 t + \Phi)]$$

$$= E[A]\int_0^{2\pi} \cos(\omega_0 t + \varphi) \times \frac{1}{2\pi} d\varphi$$

$$= 0$$

- 显然该随机过程均值具有各态历经性

现在考察自相关函数是否具有各态历经性

$$\overline{X(t)X(t+\tau)}$$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} A^2\cos(\omega_0 t+\varphi)\cos(\omega_0 t+\omega_0\tau+\varphi)dt$$

$$= \frac{A^2}{2}\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}[\cos(2\omega_0 t+\omega_0\tau+\varphi)+\cos(\omega_0\tau)]dt$$

$$= \frac{A^2}{2}\cos(\omega_0\tau)$$

$$
\begin{aligned}
&R_X(t_1,t_2) \\
&= E[A^2]E[\cos(\omega_0 t+\varphi)\cos(\omega_0 t+\omega_0\tau+\varphi)] \\
&= \frac{1}{2}E[A^2]\cos(\omega_0\tau)
\end{aligned}
$$

$$A^2 \neq E[A^2]$$

- 所以该随机过程不具有各态历经性
- A不是常量而是随机变量

3.6　本题是3.5题的一个推广

$$E[X(t)X(t+\tau)] = E[A^2]E[\cos(\omega_0 t+\varphi)\cos(\omega_0 t+\omega_0\tau+\varphi)]$$

$$=\frac{1}{2}E[A^2]\cos(\omega_0\tau)$$

$$\overline{X(t)X(t+\tau)} = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}A^2\cos(\omega_0 t+\varphi)\cos(\omega_0 t+\omega_0\tau+\varphi)dt$$

$$=\frac{A^2}{2}\cos(\omega_0\tau)$$

当A不是随机变量时，有

$$\frac{1}{2}E[A^2]\cos(\omega_0\tau) = \frac{A^2}{2}\cos(\omega_0\tau)$$

3.10先复习一下平稳过程自相关函数的两条性质：

- 1. $R_X(0)=E[X^2(t)]$
- 2. 不包含任何周期分量的非周期平稳过程满足

$$\lim_{\tau\to\infty} R_X(\tau)=R_X(\infty)=m_X^2$$

$$R_X(\tau)=4e^{-|\tau|}\cos\pi\tau+\cos 3\pi\tau$$

$$E[X^2(t)]=R_X(0)=5 \quad m_X^2=\lim_{\tau\to\infty} 4e^{-|\tau|}\cos\pi\tau=0$$

$$\sigma^2=E[X^2(t)]-m_X^2=5$$

$$R_X(\tau) = 4e^{-|\tau|}\cos\pi\tau + \cos 3\pi\tau$$
$$= R_{X_1}(\tau) + R_{X_2}(\tau)$$

- 噪声分量 $P_N = R_{X_1}(0) = 4$
- 信号分量 $P_S = R_{X_2}(0) = 1$
- 功率信噪比 $SNR = \dfrac{1}{4}$

3.12　先讨论平稳性

$$E[X(t)] = E[A\cos(\omega t + \Phi)]$$
$$= E[A]E[\cos\omega t\cos\Phi - \sin\omega t\sin\Phi]$$
$$= E[A]\{E[\cos\omega t]E[\cos\Phi] - E[\sin\omega t]E[\sin\Phi]\}$$
$$= 0$$

$$E[\cos\Phi] = \int_{-\pi}^{\pi}\cos\varphi\frac{1}{2\pi}d\varphi = 0$$

$$E[\sin\Phi] = \int_{-\pi}^{\pi}\sin\varphi\frac{1}{2\pi}d\varphi = 0$$

$$E[X(t)X(t+\tau)]$$

$$=E[A^2]E[\cos(\omega t+\Phi)\cos(\omega t+\omega\tau+\varphi)]$$

$$=8\cdot\frac{1}{2}E[\cos(2\omega t+\omega\tau+2\Phi)+\cos(\omega\tau)]$$

$$=4E[\cos\omega\tau]=4\int_{-5}^{5}\cos\omega\tau\frac{1}{10}d\omega$$

$$=\frac{4\sin 5\tau}{5\tau}$$

$$E[X^2(t)]=R_X(0)=4<\infty$$

- 可知该随机过程是宽平稳的

现在讨论各态历经性：

$$\overline{X(t)} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} A\cos(\omega t+\varphi)\,dt$$

$$= \lim_{T\to\infty} \frac{A}{2\omega T}\int_{-T}^{T} \cos(\omega t+\varphi)d(\omega t+\varphi)$$

$$= \lim_{T\to\infty} \frac{A}{2\omega T}\int_{-T}^{T} d\sin(\omega t+\varphi)$$

$$= \lim_{T\to\infty} \frac{A\cos\varphi\sin\omega T}{\omega T} = 0 = E[X(t)]$$

$$
\begin{aligned}
&\overline{X(t)X(t+\tau)} \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} A^2\cos(\omega t+\varphi)\cos(\omega t+\omega\tau+\varphi)dt \\
&= \frac{A^2}{2}\cos\omega\tau \\
&\neq \frac{4\sin 5\tau}{5\tau}
\end{aligned}
$$

- 故该随机过程不具有各态历经性

3. 13

- 已知 $$R_X(\tau) = 2e^{-2|\tau|}\cos\omega_0\tau$$
  $$R_Y(\tau) = 9 + e^{-3|\tau^2|}$$
- 可得

$$m_X^2 = R_X(\infty) = 0 \qquad m_Y^2 = R_Y(\infty) = 9$$

$$E[Z(t)] = E[VX(t)Y(t)]$$

$$= E[V]E[X(t)]E[Y(t)]$$

$$= 0$$

由方差的定义

$$D[Z(t)] = E[Z^2(t)] - E^2[Z(t)]$$

$$E[Z^2(t)] = E[V^2]E[X^2(t)]E[Y^2(t)]$$
$$= 13 \times R_X(0) \times R_Y(0)$$
$$= 13 \times 2 \times 10 = 260$$

$$D[Z(t)] = 260$$

由自相关函数的定义

$$R_Z(\tau) = E[Z(t)Z(t+\tau)]$$

$$= E[V^2 X(t)X(t+\tau)Y(t)Y(t+\tau)]$$

$$= E[V^2]E[X(t)X(t+\tau)]E[Y(t)Y(t+\tau)]$$

$$= 13 \times R_X(\tau) \times R_Y(\tau)$$

$$= 26e^{-2(\tau)}\cos\omega_0\tau(9+e^{-3\left|\tau^2\right|})$$

- 另外一种求方差的方法

$$D[Z(t)] = R_Z(0) - E^2[Z(t)] = 260$$

3.14

$$R_{XY}(\tau) = E[X(t)Y(t+\tau)]$$
$$= E[aX(t)X(t+\tau-\tau_1) + X(t)N(t+\tau)]$$
$$= aR_X(\tau-\tau_1) + R_{XN}(\tau)$$

- 当N(t)均值为零，并与X(t)相互独立时

$$R_{XY}(\tau) = aR_X(\tau-\tau_1) + E[X(t)]E[N(t+\tau)]$$
$$= aR_X(\tau-\tau_1)$$

3.25　题中RND修改为RAND，
方差递推公式修改为：

$$\sigma_X^2(k)=\frac{k-1}{k}\sigma_X^2(k-1)+\frac{1}{k}\left\{x_k-m_X(k-1)\right\}^2$$

(1)略

(2)进行三次运算，结果比较如下：

m_x	0.5089	0.5292	0.5102
均值理论值（maen(x)）	0.5089	0.5292	0.5102

v_x	0.0749	0.0855	0.0926
方差理论值（var(x)）	0.0725	0.0827	0.0898

```
%--------------------------------------------------------------%
% RSA_3_25.m                                                   %
% References     : Random Signal Analysis                      %
% CopyRight      : L                                           %
% Created        : November 2nd，08                             %
%--------------------------------------------------------------%
clc
clear all
close all

%---------------parameters  initialization--------------%
x=rand(1,200);
m_x=0;
v_x=0;

%------------------arithmetic  operation----------------%
for k=1:200
   v_x=((k-1)/k)*v_x+(1/k)*(x(k)-m_x)^2;
   m_x=m_x+(1/k)*(x(k)-m_x);
end
```