1、[4 points]Give examples of relations on {1, 2, 3, 4} having the properties specified

a) Reflexive, symmetric, and not transitive.

b) Not reflexive, not symmetric and transitive.

2、[10 points]Suppose A = {2, 3, 5, 6, 10, 15, 20, 30} and R is the partial order relation defined on A where xRy means x is a divisor of y

(1) Draw the Hasse diagram for R.

(2) Find all maximal elements.

(3) Find all minimal elements.

(4) Find all upper bounds for 3, 5.

(5) Find LUB({5, 10}).

(6) Find GLB({6, 15}).

(7) Is the poset(A, R) a lattice? Explain your answer.

3、[8 points]Let B = {1, 2, 3, 4, 5}, A = B x B, and define R on A as follows: (a, b)R(c, d) if and only if a - b = c - d.

(1) Prove that R is an equivalence relation.

(2) Find [(3, 5)].

(3) Compute A/R.

4、[9 points]In the questions below find the matrix that represents the given relation. Use elements in the order given to determine rows and columns of the matrix.

a) R on {-2, -1, 0, 1, 2}, where aRb means $a^2 = b^2$.

b) R^2, where R is relation on {1, 2, 3, 4} such that aRb means $|a - b| \leq 1$.

c) $\overline{R}$, where R is the relation on {w, x, y, z} such that R = {(w, w), (w, x), (x, x), (x, z), (y, y), (z, y), (z, z)}.

5、[9 points]Let R be the relation on A = {1, 2, 3, 4, 5} where R = {(1, 1), (1, 3), (1, 4), (2, 2), (2, 1), (3, 3), (3, 4), (4, 1), (4, 3), (5, 5)}.

(1) Find the reflexive closure of R.

(2) Find the symmetric closure of R.

(3) Use Warshall's algorithm to find the transitive closure of R.

6、[4 points]Let (G, *) be a group with G = {1, 2, 3, 4}. Here is an incomplete operation table for *:

$$\begin{pmatrix} * & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & ? \\ 2 & 2 & 1 & ? & ? \\ 3 & 3 & ? & 1 & ? \\ 4 & ? & ? & ? & 1 \end{pmatrix}$$

Redraw this table and fill the missing entries.

7、[20 points]Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix.

(a) Determine the (3, 6) group code e_H. (5 points)

(b) Determine the number of errors that e_H will detect and its associated decoding function will correct. (2 points)

(c) Constructing a decoding table relative to a maximum likelihood decoding function associated with e_H. (5 points)

(d) Decode the following words with the decoding table. (2 points)

a) 011001 b) 101011 c) 100101

(e) Compute the syndrome for each coset leader found in (c). (3 points)

(f) Decoding the following words with the syndromes of coset leader. (3 points)

a) 101001 b) 010011 c) 100101

8、[10 points]Let N be a normal subgroup of a group, and let R be the following relation on G: aRb if and only if $a^{-2}b \in N$.

Prove that R is a congruence relation on G and N is the equivalence class [e] relative to R, where e is the identity of G.

9、[10 points]Let R = [2 x 1 matrices, □], a, b, c, d are real number, where

$$\begin{bmatrix} a \\ b \end{bmatrix} \square \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} a+c-1 \\ b+d+1 \end{bmatrix}$$

Determine which of the following properties hold for this structure:

(a) Close

(b) Commutative

(c) Associative

(d) An identity element

(e) An inverse for every elment

10、[6 points]Given (Z, +) is an Abelian group. Prove that (Z x Z, *) is also an Abelian group. * is defined as $(a_1, b_1) * (a_2, b_2) =$(a_1 + a_2, b_1 + b_2).

11、[10 points]Consider a group Z_6, the operation table shown in following figure.

$$
\left|\begin{array}{ccccccc}
\oplus & [0] & [1] & [2] & [3] & [4] & [5] \\
[0] & [0] & [1] & [2] & [3] & [4] & [5] \\
[1] & [1] & [2] & [3] & [4] & [5] & [0] \\
[2] & [2] & [3] & [4] & [5] & [0] & [1] \\
[3] & [3] & [4] & [5] & [0] & [1] & [2] \\
[4] & [4] & [5] & [0] & [1] & [2] & [3] \\
[5] & [5] & [0] & [1] & [2] & [3] & [4]
\end{array}\right|
$$

a) Find all of the normal subgroups of Z_6. (4 points)

b) Describe a congruence relation R on Z_6 and find a corresponding normal subgroup from (a). (3 points)

c) For this congruence relation R in (b). Write the operation table of quotient group Z_6/R. (3 points)