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$$E(X)=\int_{-\infty}^{+\infty}xf(x)dx=\int_{0}^{+\infty}\frac{x^2}{\sigma^2}e^{-x^2/(2\sigma^2)}dx$$

令 $t^2=\frac{x^2}{\sigma^2}$，有

$$E(X)=\sigma\int_{0}^{+\infty}t^2e^{-t^2/2}dt=\sqrt{\frac{\pi}{2}}\sigma$$

$$E(X^2)=\int_{-\infty}^{+\infty}x^2f(x)dx=\int_{0}^{+\infty}x\ \frac{x^2}{\sigma^2}e^{-x^2/(2\sigma^2)}dx$$

令 $t=\frac{x^2}{2\sigma^2}$，有

$$E(X^2)=2\sigma^2\int_{0}^{+\infty}te^{-t}dt=2\sigma\Gamma(2)=2\sigma^2$$

$$D(X)=E(X^2)-[E(X)]^2=2\sigma^2-\frac{\pi}{2}\sigma^2=\frac{4-\pi}{2}\sigma^2$$

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$$E(X)=\sum_{k=1}^{\infty}kP\{X=k\}=\sum_{k=1}^{\infty}kp(1-p)^{k-1}=-p\sum_{k=1}^{\infty}((1-p)^k)'=\\-p(\sum_{k=1}^{\infty}(1-p)^k)'=-p(\frac{1-p}{p})'=\frac{1}{p}$$

$$E(X^2)=\sum_{k=1}^{\infty}k^2P\{X=k\}=\sum_{k=1}^{\infty}k^2p(1-p)^{k-1}=-p\sum_{k=1}^{\infty}k((1-p)^k)'=\\-p(\sum_{k=1}^{\infty}k(1-p)^k)'=p((1-p)\sum_{k=1}^{\infty}(1-p)^k)'=p(\frac{1-p}{p^2})'=\frac{2-p}{p^2}$$

> $$D(X)=E(X^2)-[E(X)]^2=\frac{2-p}{p^2}-\frac{1}{p^2}=\frac{1-p}{p^2}$$

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(1)

> 设总销售量为 $Y=X_1+X_2+X_3+X_4+X_5$，有
>
> $$E(Y)=\sum_{i=1}^{5}E(X_i)=200+240+180+260+320=1200$$
>
> 由于 $X_i\ (i=1,2,3,4,5)$ 之间相互独立，有
>
> $$D(Y)=\sum_{i=1}^{5}D(X_i)=225+240+225+265+270=1225$$
>
> 故变量 Y 满足 $Y\sim N(1200,35^2)$ 的正态分布

(2)

> 设至少需要储存 n 千克该产品，则
>
> $$P\{Y\le n\}=\Phi(\frac{n-1200}{35})>0.99=\Phi(2.33)$$
>
> 即
>
> $$\frac{n-1200}{35}>2.33$$
>
> $$n>2.33\times 35+1200=1281.55$$

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(1)

> $$P\{X_1=2\}=C_4^2(\frac{1}{2})^4=\frac{3}{8}$$
>
> $$P\{X_2=2\}=C_6^2(\frac{1}{3})^2(1-\frac{1}{3})^4=15\times\frac{2^4}{3^6}$$
>
> $$P\{X_3=5\}=C_6^5(\frac{1}{3})^5(1-\frac{1}{3})=\frac{6}{3^6}$$

由于 X_1, X_2, X_3 之间相互独立，有

$$P\{X_1 = 2, X_2 = 2, X_3 = 5\} = P\{X_1 = 2\}\, P\{X_2 = 2\}\, P\{X_3 = 5\} = \frac{20}{3^9}$$

$$E(X_1X_2X_3) = E(X_1)E(X_2)E(X_3) = (4 \times \frac{1}{2})(6 \times \frac{1}{3})(6 \times \frac{1}{3}) = 8$$

$$E(X_1 - X_2) = E(X_1) - E(X_2) = 2 - 2 = 0$$

$$E(X_1 - 2X_2) = E(X_1) - 2E(X_2) = -2$$

(2)

(i) X, Y 相互独立

$$E(Z) = E(5X - Y + 15) = 5E(X) - E(Y) + E(15) = 5 \times 3 - 1 + 15 = 29$$

$$D(Z) = D(5X - Y + 15) = 25D(X) + D(Y) = 25 \times 4 + 9 = 109$$

(ii) X, Y 不相关

$$E(Z) = 29, \quad D(Z) = 109$$

(iii) X, Y 相关系数为 0.25

$$E(Z) = 29, \quad Cov(X, Y) = \sqrt{D(X)}\sqrt{D(Y)}\rho_{xy} = 2 \times 3 \times 0.25 = 1.5$$

$$D(Z) = 25D(X) + D(Y) - 10Cov(X, Y) = 109 - 10 \times 1.5 = 94$$

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$$P\{XY = 1\} = P\{X = 1, Y = 1\} = P(AB)$$

$$P\{XY = 0\} = 1 - P(AB)$$

故

$$E(X) = P\{X = 1\} = P(A), \quad E(Y) = P\{Y = 1\} = P(B),$$
$$E(XY) = P\{XY = 1\} = P(AB)$$

当 $\rho_{xy}=0$ 时，有 $E(XY)=E(X)E(Y)$，即

$$P(AB)=P(A)P(B)$$

$$P\{X=0,Y=0\}=P(\overline{AB})=P(\overline{A})P(\overline{B})=P\{X=0\}P\{Y=0\}$$

$$P\{X=0,Y=1\}=P(\overline{A}B)=P(\overline{A})P(B)=P\{X=0\}P\{Y=1\}$$

$$P\{X=1,Y=0\}=P(A\overline{B})=P(A)P(\overline{B})=P\{X=1\}P\{Y=0\}$$

$$P\{X=1,Y=1\}=P(AB)=P(A)P(B)=P\{X=1\}P\{Y=1\}$$

故 X,Y 相互独立

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$$\begin{gathered}Cov(Z_1,Z_2)=Cov(\alpha X+\beta Y,\alpha X-\beta Y)=\\ \alpha^2Cov(X,X)-\alpha\beta Cov(X,Y)+\alpha\beta Cov(Y,X)-\beta^2Cov(Y,Y)=\\ \alpha^2D(X)-\beta^2D(Y)=(\alpha^2-\beta^2)\sigma^2\end{gathered}$$

由于 X,Y 相互独立，有

$$D(Z_1)=D(\alpha X+\beta Y)=\alpha^2(X)+\beta^2(Y)=(\alpha^2+\beta^2)\sigma^2$$

$$D(Z_2)=D(\alpha X-\beta Y)=\alpha^2(X)+\beta^2(Y)=(\alpha^2+\beta^2)\sigma^2$$

故

$$\rho_{Z_1Z_2}=\frac{(\alpha^2-\beta^2)\sigma^2}{\sqrt{D(Z_1)}\sqrt{D(Z_2)}}=\frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}$$

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由题，$\mu_1=\mu_2=0$， $\sigma_1=\sqrt{3},\sigma_2=2$，有

$$f(x,y)=\frac{1}{4\sqrt{3}\pi\sqrt{1-1/16}}e^{-\frac{1}{2(1-1/16)}(\frac{x^2}{3}+\frac{xy}{4\sqrt{3}}+\frac{y^2}{4})}=\frac{1}{3\sqrt{5}\pi}e^{-\frac{8}{15}(\frac{x^2}{3}+\frac{xy}{4\sqrt{3}}+\frac{y^2}{4})}$$

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由于$V^2 \geq 0, W^2 \geq 0$，有 $E(V^2) \geq 0, E(W^2) \geq 0$
当 $E(W^2) = 0$ 时，有 $D(W) = E(W) = 0$，即 $P\{W = 0\} = 1$，此时有 $P\{VW = 0\} = 1$， $E(VW) = 0$，故

$$[E(VW)]^2 = 0 = E(V^2)E(W^2) = 0$$

等式成立

当 $E(W^2) \neq 0$ 时
构造函数

$$y(t) = E[(V + tW)^2] = E(V^2 + 2tVW + t^2W^2) = \\ E(V^2) + 2tE(VW) + t^2E(W^2)$$

由于 $(V + tW)^2 \geq 0$，故$E[(V + tW)^2] \geq 0$ 恒成立，即

$$y(t) = E(V^2) + 2tE(VW) + t^2E(W^2) \geq 0$$

故有

$$\Delta = 4[E(VW)]^2 - 4E(V^2)E(W^2) \leq 0$$

即

$$[E(VW)]^2 \leq E(V^2)E(W^2)$$