11.1

4

a) a
b) a, b, d, e, g, h, i, o
c) $c, f, j, k, l, m, n, p, q, r, s$
d) Vertex j has no children
e) d
f) p
g) g, b, a
h) e, f, g, j, k, l, m

16

Since when $n >= 2$ and $m >= 2$, there must be a simple circuit in $K_{m,n}$, indicating it is not a tree. When $n = 1$ or $m = 1$, there is no simple circuit and K is connected. Hence $K_{m,n}$ is a tree when $m = 1$ or $n = 1$.

20

$[(3-1)100+1]/3 = 67$ leaves.

28

$\sum_{k=0}^{h} m^k = \frac{m^{h+1}}{m-1}$ vertices;
m^h leaves.

44

We can follow the rule that colors vertices with the height of odd number with one color, and for those with the height of even number, colors it with another color.

11.2

6

$\lceil log_3 4 \rceil = 2$ weighings. We compare the first two coins. If one is lighter, then it is the counterfeit; otherwise if two are equal, then we compare the other two coins, and the lighter one of them is the counterfeit.

16

First we compare every two vertices to get 2^{k-1} winners, and it takes 2^{k-1} comparisons. The next round generates 2^{k-2} winners and takes 2^{k-2} comparisons, until $2^{1-1} = 1$. Hence the number of comparisons is $\sum_{i=1}^{k-1} 2^{i-1} = 2^k - 1 = n - 1$.

30

a) A: 0, B: 10, C: 11

b) The frequencies are AA: 0.64, AB: 0.152, AC = 0.008, BA = 0.152, BB = 0.0361, BC = 0.0019, CA = 0.008, CB = 0.0019, CC = 0.0001;

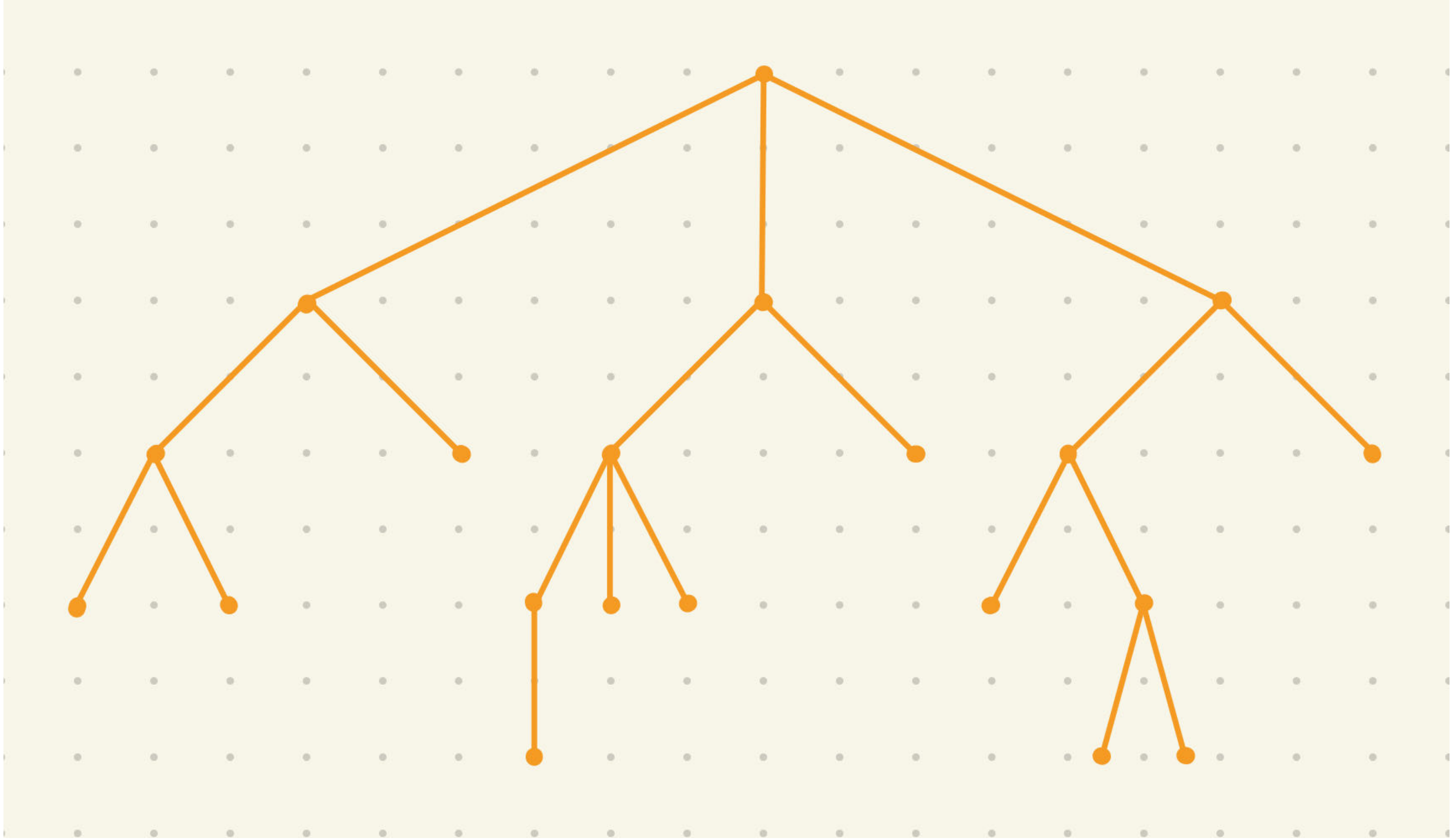
So the Huffman codes are AA: 0, AB: 11, AC: 10111, BA: 100, BB: 1010, BC: 1011011, CA: 101100, CB: 10110100, CC: 10110101

c) For (a), the average number of bits is $1 \times 0.8 + 2 \times (0.19 + 0.01) = 1.2$. For (b), it is $[1 \times 0.64 + 2 \times 0.152 + 3 \times 0.152 + 4 \times 0.0361 + 5 \times 0.008 + 6 \times 0.008 + 7 \times 0.0019 + 8 \times (0.0019 + 0.0001)]/2 = 0.83085$. Hence (b) is more efficient.

11.3

6

a)



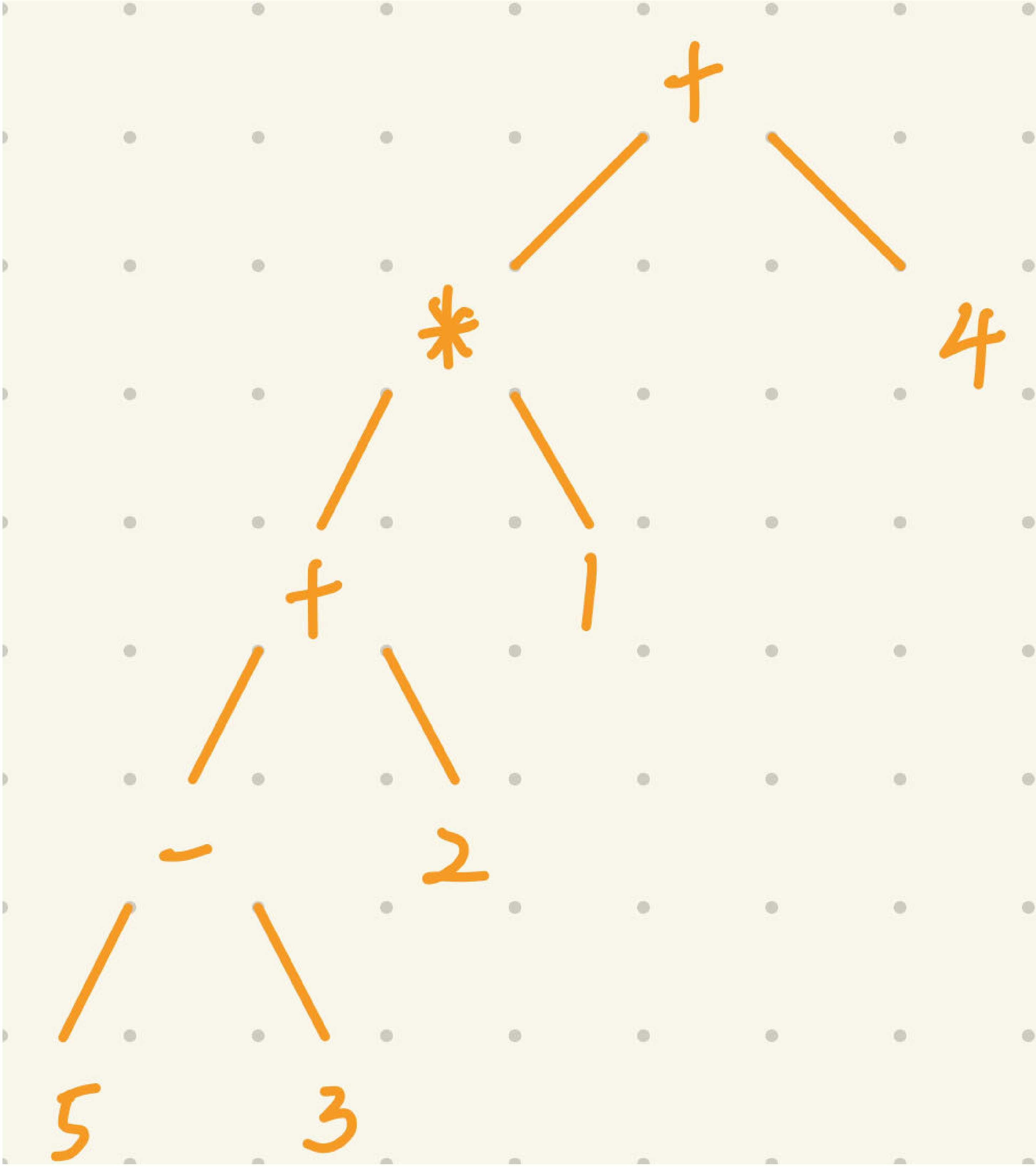
b) Since $2.4.2.1$ occurs, then there should be an address with the prefix $2.4.1$, which is not found.
c) Since $1.2.2.1$ occurs, then $1.2.2$ must not be a leaf, which is contradicted to the fact.

18

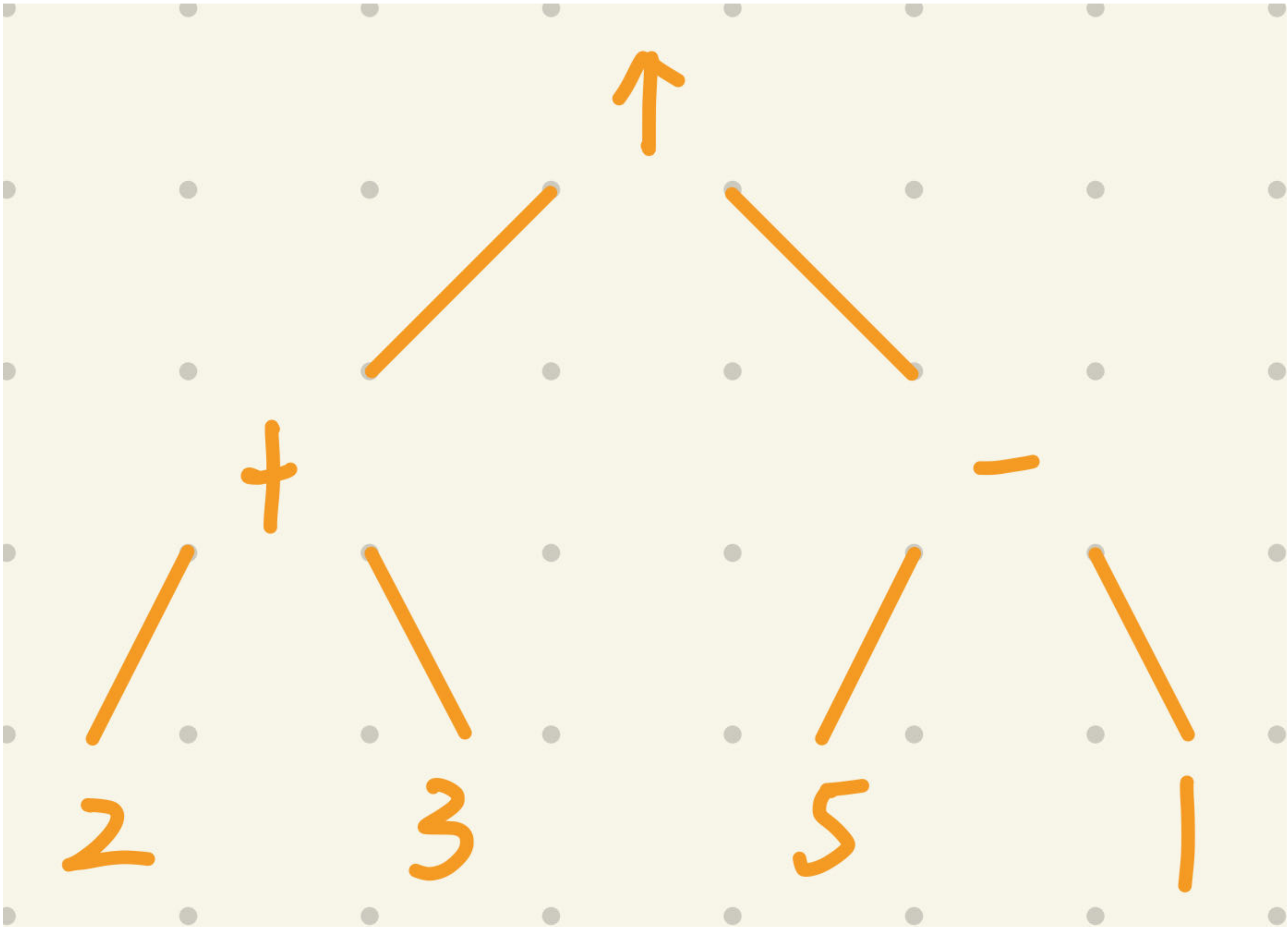
a) $\leftrightarrow \neg \wedge p\ q\ \ \vee \neg\ p \neg\ q, \vee \wedge \neg\ p\ \ \leftrightarrow\ \ q \neg\ p \neg\ q$
b) $p\ q\ \ \wedge \neg\ p \neg\ q \neg \vee \leftrightarrow, p \neg\ q\ p \neg \leftrightarrow \wedge\ q \neg \vee$
c) $((\neg(p \wedge q)) \leftrightarrow ((\neg p) \vee (\neg q))), (((\neg p) \wedge (q \leftrightarrow (\neg p))) \vee (\neg q))$

22

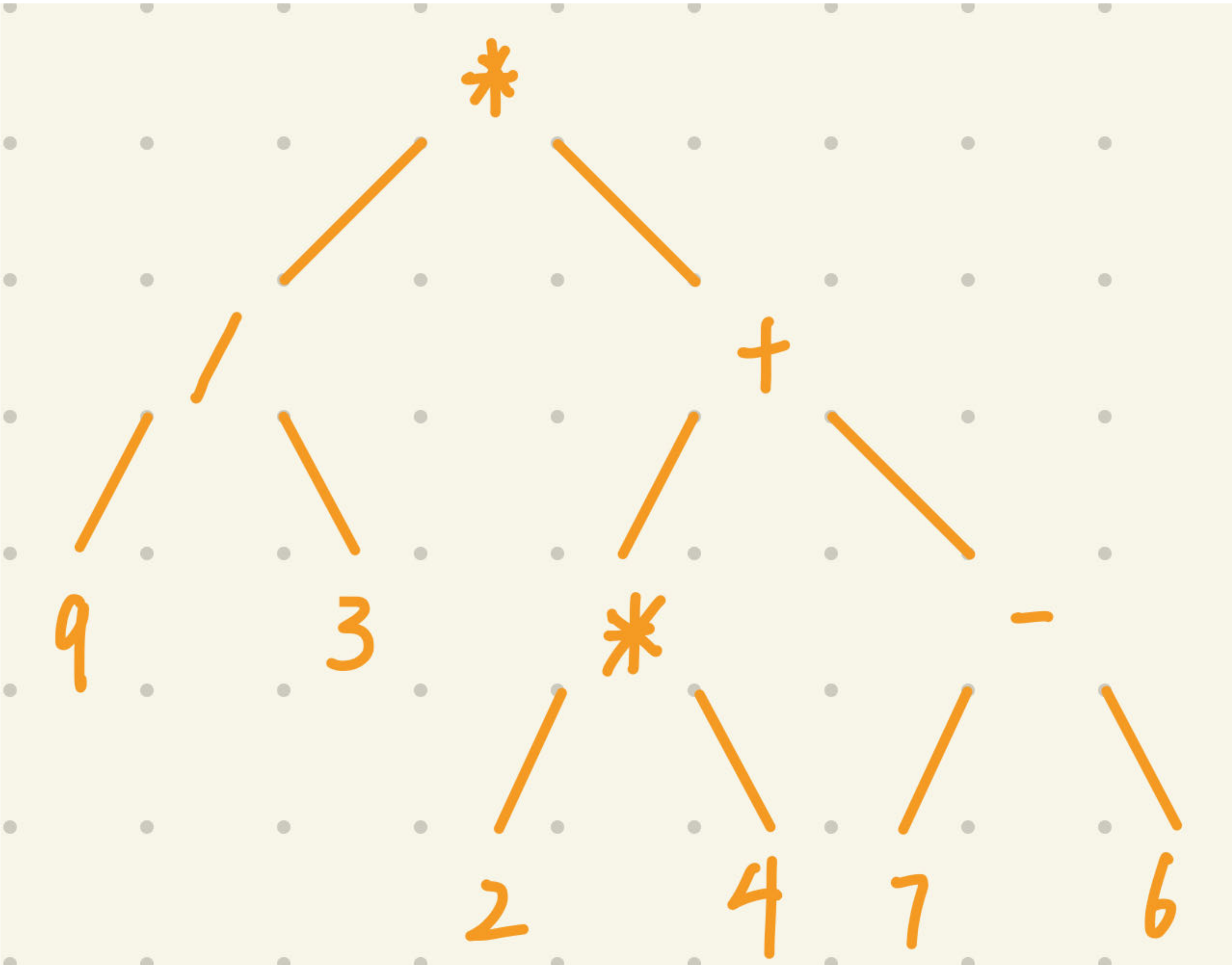
a) $((((5-3)+2)*1)+4)$



b) $((2+3) \uparrow (5-1))$



c) $(9/3) * ((2 * 4) + (7 - 6))$



34

If $X_1, X_2, \ldots, X_n$ are well formuale and * is an n-ary operator, then $*X_1X_2\ldots X_n$ is a well-formed formula.

11.4

12

a) 1
b) 2
c) 3

14

22

For the breadth-first search, we can describe it with recursive method. First when n = 0, clearly the tree is a vertex. Suppose T(n) is the tree of Q(n), then T(n+1) should be the copy of T(n), and then connect another T(n) to the root of T(n+1).

For the depth-first search, since n-cube is a Hamilton path, we will eventually get a path.

24

We can order the vertices in the order that we first meet in the search processes.

50

For the edges that are in the T, it is true obviously.
For those are not in the T, assuming uv. Suppose u is in the higher or the same level of v. If v is more than 1 level lower than u, then from the process of breadth-first search, we know that the ancestor of v that in the same level of u should take 2 iteration to reach v, while u only takes 1, which contradicts the rule of BFS. In reserve, for the situation that v is more than 1 level lower than u, it is same as well.
If v is 1 level lower than u, we know that by the process of BFS, v's parent (in the same level of u) has the chance to reach v faster than u, so it is true, in reverse, too.
If both of them are in the same level, we know that they are added to T in the same iteration, and the process of edge uv is in the next iteration, it is true as well.

54

Suppose T_1 has a edges that are not in T_2, T_2 has b edges that are not in T_3. So the distance between T_1 and T_2 is $2a$, between T_2 and T_3 is $2b$. Worst of all, these $(a+b)$ edges are in the T_1 and T_2, but not in T_3. Hence the distance between T_1 and T_3 is at most 2(a+b).

11.5

4

{a, b}, {a, e}, {a, d}, {c, d}, {d, h}, {a, m}, {d, p}, {e, f}, {e, i}, {g, h}, {l, p}, {m, n}, {n, o}, {f, j}, {k, l}.
Total weight is 28.

8

{a, b}, {a, e}, {c, d}, {d, h}, {a, d}, {a, m}, {d, p}, {e, f}, {e, i}, {g, h}, {l, p}, {m, n}, {n, o}, {f, j}, {k, l}.
Total weight is 28.

10

Prim: choose a vertex to start and add the edge that is shortest and adjacent to the vertex that does not form a simple circuit repeatedly until no vertex can be added. In this case, we choose a vertex not in the constructed spanning forest and follow the steps above to generate another spanning tree. In the end, we can get minimum spanning forest.

Kruskal: choose the shortest edge in the graph that does not form a simple circuit, until no such edges remain. Then we have the minimum spanning forest.

14

{d, h}, {d, e}, {b, f}, {d, g}, {a, b}, {b, e},
{b, c}, {f, i}

18

Suppose we have a minimum
spanning tree that does not contain the minimal edge uv(and the edges that have the same weight as uv). Then we add edge uv to the spanning tree. In this case we will form a simple circuit. To avoid that, we can the minimal edge ab (except uv) to break the circuit, and we get a new spanning tree with the weight $W + W_{uv} - W_{ab} < W$, indicating that T is not a minimum spanning tree. Hence every minimum spanning tree contains the edge with the smallest weight.