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> Since $r^2 - c_1 r - c_2 = 0$ has only one root r_0, we have $\Delta = c_1^2 + 4c_2 = 0, \quad r_0^2 = c_1 r_0 + c_2$, then $r_0 = \frac{c_1}{2}$
>
> $$\begin{aligned} c_1 a_{n-1} + c_2 a_{n-2} = c_1(\alpha_1 r_0^{n-1} + \alpha_2(n-1)r_0^{n-1}) + c_2(\alpha_1 r_0^{n-2} + \alpha_2(n-2)r_0^{n-2}) \\ = \alpha_1 r_0^{n-2}(c_1 r_0 + c_2) + \alpha_2 r_0^{n-2}(c_1(n-1)r_0 + c_2(n-2)) \\ = \alpha_1 r_0^n + \alpha_2 r_0^{n-2}((n-1)(c_1 r_0 + c_2) - c_2) \\ = \alpha_1 r_o^n + \alpha_2 r_0^{n-2}((n-1)r_0^2 - (-\frac{(2r_0)^2}{4})) = \alpha_1 r_o^n + \alpha_2 n r_0^n \end{aligned}$$
>
> Suppose that $\{a_n\}$ is a solution of the recurrence relation, and the initial conditions $a0 = C_0$ and $a_1 = C_1$ hold.
>
> $$\begin{gathered} a_0 = C_0 = \alpha_1 \\ a_1 = C_1 = \alpha_1 r_0 + \alpha_2 r_0 \end{gathered}$$
>
> Then we solve these two equations
>
> $$\alpha_1 = C_0, \quad \alpha_2 = \frac{C_1 - r_0 C_0}{r_0}$$
>
> Since we know that the theorem is true when $c_2 \neq 0$, which shows that $(r^2 - c_1 r - c_2)|_{r=0} = -c_2 \neq 0$. Hence $r_0 \neq 0,$ with these values for α_1 and α_2, the sequence $\{a_n\}$ with $\alpha_1 r_1^n + \alpha_2 r_2^n$ satisfies the two initial conditions.

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> When $n = 1$, $a_1 = sf_{1-1} + tf_1 = t$.
> When $n = 2$, $a_2 = sf_{2-1} + tf_2 = s + t = a_1 + a_0$.
> Let $k \geq 2$. Assume $\forall 1 \leq j \leq k$, $a_j = sf_{j-1} + tf_j$ is true;
> When $n = k + 1$, we know that
>
> $$\begin{aligned} a_{k+1} = a_k + a_{k-1} = sf_{k-1} + tf_k + sf_{k-2} + tf_{k-1} \\ = s(f_{k-1} + f_{k-2}) + tf_k + f_{k-1} = sf_{k+1-1} + tf_k \end{aligned}$$
>
> is true as well.
> By mathematical induction, $\forall n \geq 1,\ a_n = sf_{n-1} + tf_n$ is true.

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a) $a_n = 2a_{n-1} + 100, (n \geq 2) \quad a_1 = 2$

b) $a_n = 2a_{n-1} + 100 = 2(2a_{n-2} + 100) + 100 = 2^2(2a_{n-3} + 100) + 2 * 100 + 100 = 2^3(2a_{n-4} + 100) + 2^2 * 100 + 2 * 100 + 100 = \cdots = 2^{n-2}(2a_1 + 100) + 100\sum_{i=0}^{n-3} 2^i = 2^n + 100\sum_{i=0}^{n-2} 2^i = 51 * 2^n - 100 \ (n \geq 1)$

c) $a_n = 2a_{n-1} - n, (n \geq 3) \quad a_2 = 4, a_1 = 2$

d) The associated linear homogeneous recurrence relation is $a_n = 2a_{n-1}$, with the solution $a_n^{(h)} = \alpha 2^n$. Since $F(n) = -n$ is a polynomial in n of degree 1, a particular solution is a linear function in n, say, $a_n = cn + d$. Substituting it into the recurrence relation implies that $(c-1)n - (2c - d) = 0$. It follows that $c = 1, d = 2$. So the particular solution is $a_n^{(p)} = n + 2$ and the general solution is $a_n = \alpha 2^n + n + 2$. Let n = 2, then we have $a_2 = \alpha 2^2 + 2 + 2 = 4$. Hence $\alpha = 0$, the recurrence relation is $\forall n \geq 2,\ a_n = n + 2, \quad a_1 = 2.$