1) $I = \int \frac{2x+3}{(x-2)(x+3)}dx$

$\frac{2x+3}{(x-2)(x+5)} = \frac{1}{x-2} + \frac{1}{x+5}$

$I = \ln|x-2| + \ln|x+5| + C$

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2) $I = \int e^{2x}\cos 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx =$

$= \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}\int e^{2x}\cos 3x\,dx$

$I = \frac{3e^{2x}\sin 3x + 2e^{2x}\cos 3x}{13} + C$

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3) $\int_0^{\ln 2} x e^{-x}dx = -xe^{-x}\Big|_0^{\ln 2} + \int_0^{\ln 2} e^{-x}dx =$

$= -\ln 2 \cdot (e^{\ln 2})^{-1} - (e^{\ln 2})^{-1} + 1 = -\frac{\ln 2 + 1}{2} + 1 = \frac{1-\ln 2}{2}$

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4) $I = \int_2^{+\infty} \frac{dx}{x^2+x-2} = \int_2^{+\infty}\left(\frac{1}{3(x-1)} - \frac{1}{3(x+2)}\right)dx =$

$= \lim_{b\to\infty}\left(\frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2|\right)\Big|_2^b =$

$= \lim_{b\to\infty}\left(\frac{1}{3}\ln|b-1| - \frac{1}{3}\ln 1 - \frac{1}{3}\ln|b+2| + \frac{1}{3}\ln 4\right) =$

$= \lim_{b\to\infty}\frac{1}{3}\left(\ln\frac{4(b-1)}{b+2}\right) = \lim_{b\to\infty}\frac{1}{3}\ln\frac{4-\frac{4}{b}}{1+\frac{2}{b}} = \frac{1}{3}\ln 4$

$5^{*})$ $I = \int_0^1 \ln x\,dx = \lim_{\alpha \to 0} \int_\alpha^1 \ln x\,dx$

$$\int_\alpha^1 \ln x\,dx = x\ln x\Big|_\alpha^1 - \int_\alpha^1 dx = \ln 1 - \alpha\ln\alpha - 1 + \alpha$$

$$\lim_{\alpha \to 0}\left(\ln\frac{1}{\alpha^\alpha} - 1 + \alpha\right) = \ln 1 - 1$$