1) $V = x^3 + 3xy^2 + z - 39x - 36y + 2z + 26$

$V'_x = 3x^2 + 3y^2 - 39$

$V'_y = 6xy - 36$

$V'_z = 1 + 2 = 3$

$V''_{xx} = 6x$ $\quad V''_{xz} = 0$ $\quad V''_{zx} = 0$ $\quad V''_{xy} = V''_{yx} = 6y$

$V''_{xy} = 6y$ $\quad V''_{yz} = 0$ $\quad V''_{zy} = 0$ $\quad V''_{xz} = V''_{zx} = 0$

$V''_{yx} = 6y$ $\quad V''_{yz} = V''_{zy} = 0$

$V''_{yy} = 6x$ $\quad V''_{zz} = 0$

2) $V = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$

$V'_x = -\frac{256}{x^2} + 2\frac{x}{y}$ $\quad V'_y = -\frac{x^2}{y^2} + 2\frac{y}{z}$ $\quad V'_z = -\frac{y^2}{z^2} + 2z$

$V''_{xy} = \cancel{-\frac{256}{x^2}} - 2\frac{x}{y^2}$ $\quad V''_{yx} = -2\frac{x}{y^2}$ $\quad V'_{zx} = 0$

$V''_{xz} = 0$ $\quad V''_{yz} = -2\frac{y}{z^2}$ $\quad V''_{zy} = -\frac{2y}{z^2}$

$V''_{xy} = V''_{yx} = -2\frac{x}{y^2}$

$V''_{zx} = V''_{xz} = 0$

$V''_{yz} = V''_{zy} = -\frac{2y}{z^2}$

3. $U = x^2 + y^2 + z^2$ $\quad \vec{c}(-9, 8, -12)$ $\quad M(8, -12, 9)$

$U'_x = 2x \quad U'_y = 2y \quad U'_z = 2z \quad \text{grad}\, U = (2x, 2y, 2z)$

$|\vec{c}| = \sqrt{81 + 64 + 144} = \sqrt{289} = 17$

$c_0 = \left(-\frac{9}{17}, \frac{8}{17}, -\frac{12}{17}\right)$

$\text{grad}\, U\big|_M = (16, -24, 18)$

$U'_{\vec{c}} = \left(-\frac{9 \cdot 16}{17} + \frac{8 \cdot (-24)}{17} + \left(-\frac{12 \cdot 18}{17}\right)\right) = \frac{8}{17} \cdot \left(-18 + (-24) + (-27)\right) =$

$= -\frac{24}{17} \cdot (6 + 8 + 9) = -\frac{24}{17} \cdot 23 = -\frac{552}{17}$

4. $U = e^{x^2 + y^2 + z^2}$ $\quad \vec{c} = (4, -13, -16)$ $\quad L(-16, 4, -13)$

$U'_x = 2xU \quad U'_y = 2yU \quad U'_z = 2zU \quad \text{grad}\, U = 2U \cdot (x, y, z)$

$|\vec{c}| = \sqrt{16 + 169 + 256} = \sqrt{441} = 21$

$\vec{c_0} = \frac{1}{21} \cdot (4 - 13, -16)$

$\text{grad}\, U\big|_L = 2 \cdot e^{256 + 16 + 169}(-16, 4, -13) = 2e^{441}(-16, 4, -13)$

$U'_{\vec{c}} = \frac{2e^{441}}{21}\left(4 \cdot (-16) + 4 \cdot (-13) + (-16)(-13)\right) = \frac{2e^{441}}{21}(-64 - 52 + 208) =$

$= \frac{184 \cdot e^{441}}{21}$

5*. $V = \log_{21}(x^2+y^2+z^2)$ $\qquad F(-19, 8, -4)$

$$V'_x = \frac{2x}{(x^2+y^2+z^2)\ln 21} \qquad V'_y = \frac{2y}{(x^2+y^2+z^2)\ln 21} \qquad V'_z = \frac{2z}{(x^2+y^2+z^2)\ln 21}$$

$$\operatorname{grad} V = \frac{2}{(x^2+y^2+z^2)\ln 21}\cdot(x, y, z)$$

$$\operatorname{grad} V\big|_F = \frac{2}{(361+64+16)\ln 21}(-19, 8, -4) = \frac{2}{441\ln 21}\cdot(-19, 8, -4)$$

Производная по самому быстрому направлению - это производная по направлению градиента

$$V'_{grad V} = (\operatorname{grad} V_0 \cdot \operatorname{grad} V) = |\operatorname{grad} V_0|\,|\operatorname{grad} V|\cdot\cos 0 = |\operatorname{grad} V|$$

$$V'_{grad V} = \frac{2}{441\ln 21}\sqrt{361+64+16} = \frac{2}{21\ln 21}$$

6. $V = x^2y + \frac{1}{3}y^3 + 2x^2 + y^2 + 1$

$V'_x = 2xy + 4x \qquad V'_y = x^2 + y^2 + 2y$

$$\begin{cases} 2x(y+2) = 0 \\ x^2 + y^2 + 2y = 0 \end{cases} \qquad x = 0,\ y = -2$$

$x = 0$: $\quad y^2 + 2y = 0$

$y(y+2) = 0 \qquad y = 0,\ y = -2$

6. $U = x^2 y + \frac{1}{3} y^3 + 2x^2 + 3y^2 - 1$

$U'_x = 2xy + 4x$ $\quad\quad$ $U'_y = x^2 + y^2 + 6y$

$\begin{cases} 2x(y+2) = 0 \\ x^2 + y^2 + 6y = 0 \end{cases}$ $\quad\quad$ $x = 0, \quad y = -2$

$x = 0$: $y^2 + 6y = 0$

$y(y+6) = 0$ $\quad\quad$ $y = 0, \quad y = -6$

$y = -2$: $x^2 + 4 - 12 = x^2 - 8 = 0$ $\quad\quad$ $x = \pm\sqrt{8} = \pm 2\sqrt{2}$

имеем точки: $(0;0), (0;-6)\ (2\sqrt{2}; -2), \ (-2\sqrt{2}; -2)$

$U''_{xx} = 2y + 4$ $\quad\quad$ $U''_{xy} = U''_{yx} = 2x$ $\quad\quad$ $U''_{yy} = 2y + 6$

$$\begin{pmatrix} 2y+4 & 2x \\ 2x & 2y+6 \end{pmatrix} = A$$

$\Delta_1 = 2y + 4$

$\Delta_2 = 4(y+2)(y+3) - 4x^2$

Точка $(0;0)$

$\Delta_1 = 4 > 0$ $\quad\quad$ $\Delta_2(0;0) = 4 \cdot 2 \cdot 3 - 0 = 24 > 0$

$\boxed{(0;0) \text{ — минимум ф-ции } U}$

Точка $(0;-6)$

$\Delta_1(0;-6) = -12 + 4 = -8 < 0$

$\Delta_2(0;-6) = 4 \cdot (-4)(-3) - 0 = 48 > 0$

$\boxed{(0;-6) \text{ — максимум ф-ции } U}$

Точка $(2\sqrt{2}; -2)$:

$\Delta_1 = \cancel{4\sqrt{2}+4 > 0}$ 0

$\Delta_2 = \cancel{0 - 8\sqrt{2} = -8\sqrt{2}}$ $0 - 4 \cdot 8 = -32 < 0$

$$A' = \begin{pmatrix} 2y+6 & 2x \\ 2x & 2y+4 \end{pmatrix}$$

$\Delta_1'(2\sqrt{2}; -2) = 2 > 0$

$\Delta_2'(2\sqrt{2}; -2) = -32 < 0$

$\boxed{(2\sqrt{2}; -2) \text{ — седловая точка}}$

Точка $(-2\sqrt{2}; -2)$:

$\Delta_1(-2\sqrt{2}; -2) = 0$ Так, я понял. Переходим к A'

$$A' = \begin{pmatrix} 2y+6 & 2x \\ 2x & 2y+4 \end{pmatrix}$$

$\Delta_1' = 2 > 0$

$\cancel{\Delta_2' = 4(y+}$ $\Delta_2'(-2\sqrt{2}; -2) = 0 - 4 \cdot 8 = -32 < 0$

$\boxed{(-2\sqrt{2}; -2) \text{ — седловая точка}}$