作业五.

习题二 32, $(t^2, t^2) = \int_{-1}^{1} x^4 dx = \frac{x^5}{5}\Big|_{-1}^{1} = \frac{2}{5}$. $\quad (t, t) = \int_{-1}^{1} t^2 dt = \frac{2}{3}$ $\quad (1, 1) = \int_{-1}^{1} dt = 2$.

$(t^2, t) = \int_{-1}^{1} t^3 dt = 0$ $\quad (t^2, 1) = \int_{-1}^{1} t^2 dt = \frac{2}{3}$. $\quad (t, 1) = \int_{-1}^{1} t\, dt = 0$.

$$\therefore G = \begin{bmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{5} \end{bmatrix}.$$

$$(f, g) = (1, -1, 1) \begin{bmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{5} \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix} = (1, -1, 1) \begin{bmatrix} -\frac{4}{3} \\ -\frac{8}{3} \\ -\frac{4}{3} \end{bmatrix} = 0$$

37, 有 $\beta_1 = \alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

$$\beta_2 = \alpha_2 - \frac{(\alpha_2, \beta_1)}{(\beta_1, \beta_1)} \beta_1 = (2, 1, 0, -3)^T - \frac{-1}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{7}{3} \\ 1 \\ \frac{1}{3} \\ -\frac{8}{3} \end{bmatrix}$$

$$\beta_3 = \alpha_3 - \frac{(\alpha_3, \beta_2)}{(\beta_2, \beta_2)} \beta_2 - \frac{(\alpha_3, \beta_1)}{(\beta_1, \beta_1)} \beta_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} - \frac{\frac{4}{3} + 3}{\frac{123}{9}} \begin{bmatrix} \frac{7}{3} \\ 1 \\ \frac{1}{3} \\ -\frac{8}{3} \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} - \frac{13}{123} \begin{bmatrix} 7 \\ 3 \\ 1 \\ -8 \end{bmatrix} - \frac{1}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{41} \\ -\frac{54}{41} \\ \frac{23}{41} \\ \frac{-20}{41} \end{bmatrix}.$$

$\therefore$ 作归一化, 有标准正交基为 $\frac{1}{3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$, $\frac{1}{\sqrt{123}} \begin{bmatrix} 7 \\ 3 \\ 1 \\ -8 \end{bmatrix}$, $\frac{1}{\sqrt{3854}} \begin{bmatrix} 3 \\ 54 \\ -23 \\ 20 \end{bmatrix}$.

39, $\beta_1 = \alpha_1$.

$$\beta_2 = \alpha_2 - \frac{(\alpha_2, \alpha_1)}{(\alpha_1, \alpha_1)} \alpha_1 = \alpha_2 - \frac{\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}^T \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}} \alpha_1 = \alpha_2 - \frac{4}{5} \alpha_1.$$

$\therefore$ 标准正交基为 $\eta_1 = \frac{\beta_1}{\sqrt{(\beta_1, \beta_1)}} = \frac{1}{\sqrt{5}} \alpha_1$. 过渡矩阵为 $\begin{bmatrix} \frac{1}{\sqrt{5}} & 0 \\ \frac{4}{\sqrt{45}} & -\frac{5}{\sqrt{45}} \end{bmatrix}$.

$$\eta_2 = \frac{\beta_2}{\sqrt{(\beta_2, \beta_2)}} = \frac{4\alpha_1 - 5\alpha_2}{\sqrt{45}}$$

40, (1). $(\alpha, \beta) = \left(x^T \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}, y^T \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} \right) = (x_1\alpha_1 + x_2\alpha_2 + \cdots x_n\alpha_n, \; y_1\alpha_1 + y_2\alpha_2 + \cdots y_n\alpha_n)$ $\quad \alpha_1 \alpha_1$, $\alpha_1 \alpha_2$.

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \bar{y}_j (\alpha_i, \alpha_j).$$

又 $x^T A \bar{y} = x^T \left(\sum \bar{y}_i \begin{bmatrix} (\alpha_1, \alpha_i) \\ (\alpha_2, \alpha_i) \\ \vdots \\ (\alpha_n, \alpha_i) \end{bmatrix} \right) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \bar{y}_j (\alpha_i, \alpha_j)$.

$\therefore (\alpha, \beta) = x^T A \bar{y}$.

(2). 设另一组基 $\beta_1 \cdots \beta_n$. 则有度量矩阵 $G = \begin{bmatrix} (\beta_1, \beta_1) & \cdots & (\beta_1, \beta_n) \\ \vdots & \ddots & \vdots \\ (\beta_n, \beta_1) & \cdots & (\beta_n, \beta_n) \end{bmatrix}$. 设 α, β 在这组基下坐标: u, v.

设 β 与 α 间过渡矩阵为 P. 则 $x = Pu$ $y = Pv$.

则 $(\beta_i\ \beta_j) = ([\alpha_1 \cdots \alpha_n] \bar{P}_i\quad [\alpha_1 \cdots \alpha_n] \bar{P}_j)$

则 $G = P^T A \bar{P}$ $\therefore (\alpha, \beta) = x^T A \bar{y} = (Pu)^T A (\overline{Pv}) = u^T G \bar{v}$ 故形式与基无关.

42, (1). $\beta_1 = \alpha_1 - \alpha_2$. $\beta_2 = 9\alpha_1 - 3\alpha_2$. $\alpha_1 = \frac{1}{6}(\beta_2 - 3\beta_1)$ $\alpha_2 = \frac{1}{6}(\beta_2 - 9\beta_1)$

则 $(\alpha_1\ \alpha_1) = (\alpha_1\ \ \frac{1}{6}\beta_2 - \frac{1}{2}\beta_1) = \frac{1}{6} \times 15 - \frac{1}{2} \times 1 = 2$.

$(\alpha_1\ \alpha_2) = (\alpha_1\ \ \frac{1}{6}\beta_2 - \frac{9}{6}\beta_1) = \frac{1}{6} \times 15 - \frac{9}{6} \times 1 = 1$. $\therefore G_\alpha = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$(\alpha_2\ \alpha_2) = (\alpha_2\ \ \frac{1}{6}\beta_2 - \frac{9}{6}\beta_1) = \frac{1}{6} \times 3 - \frac{9}{6} \times -1 = 2$.

$(\beta_1\ \beta_1) = (\alpha_1 - \alpha_2\ \ \beta_1) = 1 - (-1) = 2$. $G_\beta = \begin{bmatrix} 2 & 12 \\ 12 & 126 \end{bmatrix}$.

$(\beta_1\ \beta_2) = (\alpha_1 - \alpha_2\ \ \beta_2) = 15 - 3 = 12$.

$(\beta_2\ \beta_2) = (9\alpha_1 - 3\alpha_2\ \ \beta_2) = 9 \times 15 - 3 \times 3 = 126$

(2) $\eta_1 = \alpha_1$

$\eta_2 = \alpha_2 - \frac{(\alpha_2\ \alpha_1)}{(\alpha_1\ \alpha_1)} \alpha_1 = \alpha_2 - \frac{1}{2}\alpha_1$.

则 $\gamma_1 = \frac{\eta_1}{\sqrt{(\eta_1, \eta_1)}} = \frac{\alpha_1}{\sqrt{2}}$

$\gamma_2 = \frac{\eta_2}{\sqrt{2 - 1 + \frac{1}{4} \cdot 2}} = \frac{\alpha_2 - \frac{1}{2}\alpha_1}{\sqrt{\frac{3}{2}}} = \frac{2\alpha_2 - \alpha_1}{\sqrt{6}}$

是两标准正交基.

43, check. 可逆矩阵A可写成正定阵和正交矩阵的乘积.

由于A可逆. 则 $A^T A$ 正定. 故有正交阵 Q. 使得 $Q^T A^T A Q = D^2$ 是对角阵且对角元素都为正.

于是有 $D^{-1} Q^T A^T A Q D^{-1} = I$. 其中 D 上对角元素都为正.

则 $R = AQD^{-1}$ 是正交矩阵. $A = RDQ^T = (RDR^T) RQ^T$ 有 RDR^T 是正定的.

RQ^T 是正交的. 所以 A 可分解成正定阵乘正交阵.

设 V的标准正交基 $\gamma_1 \cdots \gamma_n$. 则与 $\alpha_1 \cdots \alpha_n$ 有过渡矩阵 P. 有 $P = CQ$ C为正定. Q为正交.

则 $(\gamma_1 \cdots \gamma_n) = (\alpha_1 \cdots \alpha_n) P = (\alpha_1 \cdots \alpha_n) CQ$

则有 $(\beta_1 \cdots \beta_n) = (\gamma_1 \cdots \gamma_n) Q^{-1} = (\alpha_1 \cdots \alpha_n) C$. 是标准正交基.

44, (1) 设 A特征值为 λ 和,特征向量 α. 则 $A\alpha=\lambda\alpha$.

$\alpha^T A^T = \lambda\alpha^T$. $-\alpha^T A = \lambda\alpha^T$.

$-\alpha^T A\alpha = \lambda\alpha^T\alpha$. $\therefore -\lambda=\lambda$ $\lambda=0$ 或纯虚数

$-\lambda\alpha^T\alpha = \lambda\alpha^T\alpha$.

(2) 设此非零特征值为 a. 则 $A(\alpha+\beta i) = a(\alpha+\beta i)$

$A\alpha = a\alpha$ $A\beta = a\beta$. $\alpha^T A^T = a\alpha^T = -\alpha^T A$

$-\alpha^T A\beta = -\alpha^T a\beta = -a\alpha^T\beta$ 又有 $-\alpha^T A\beta = a\alpha^T\beta$,

$\therefore -a\alpha^T\beta = a\alpha^T\beta$ 由于 $a\neq 0$ 所以 $\alpha^T\beta=0$

45, 由于是 Hermite 矩阵. $A^T = \bar{A}$. 设有非零特征值 a, 对应特征向量 α.

即 $A\alpha = a\alpha$. $\alpha^T A^T = a\alpha^T$ 由于 $x^*Ax=0$

$\alpha^T \bar{A} = a\alpha^T$ $\alpha^* A = \bar{a}\alpha^T$

$\alpha^* A\alpha = \bar{a}\alpha^*\alpha = \alpha^* a\alpha$ $\therefore a\alpha^*\alpha = \bar{a}\alpha^*\alpha$.

所以 a 只能为零, 所以 A 所有特征值为零. 又A可对角化, 故 $A=0$

46, 注意到 A 对称但不可逆. 因此此内积有对称性,线性, 但没有正定性.

其长度: 有长度为零的非零向量. 角度: 有夹角为0的两线性无关向量.

平行或: 向量平行不一定线性相关, 向量垂直不一定正交.

习题二.

31. 原式可化为 check $R(A)^\perp = N(A^*)$. 设 $\alpha\in N(A^*)$ 即 $A^*\alpha=0$. 即 α 与 $(A^*)^*=A$ 的每一列都正交

即 $N(A^*)\subseteq R(A)^\perp$. 又由于. $R(A)^\perp$ 的维数与 $N(A^*)$ 维数相等,

$\therefore N(A^*) = R(A)^\perp$. $\therefore R(A) = N(A^*)^\perp$

设 $f, g \in U$. $k \in R$ 有 $f(0)=g(0)=0$.

32, (1) pf. 1° $f(0)+g(0)=0$. $\therefore f(0)+g(0) \in U$.

2° $kf(0)=0$ $\therefore kf(0) \in U$. $\therefore U$ 是 V 的子空间.

显然有 $x_1 \cdots x_n$ 是 U 的一组基.

(2). 设 $f(x)$ 取 x, x^2 设正交补为 $\{g(x)=1+ax+bx^2 \mid g(x) \in U^{\perp}\}$

$3b=-6+16$

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-2.

则有 $(f(x), g(x))=0$

$$\begin{cases} \frac{1}{2}+\frac{a}{3}+\frac{b}{4}=0 \\ \frac{1}{3}+\frac{a}{4}+\frac{b}{5}=0 \end{cases} \quad \begin{cases} 4a+3b=-6 \\ 5a+4b=-\frac{20}{3} \end{cases} \quad \begin{cases} a=-4 \\ b=\frac{10}{3} \end{cases}$$

$$\therefore U^{\perp}=[1-4x+\frac{10}{3}x^2].$$

34, Fourier 级数系数为 $a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$ $b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$.

而 $f(x)$ 与 $\cos nx$ 内积定为 $(f(x), \cos nx)=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$. 与 a_n 相同.

同理. b_n 也一样. 所以. a_n b_n 确实恰为 $f(x)$ 与 $\cos nx$, $\sin nx$ 的内积.

35, $\Rightarrow$: 显然成立, 因为最佳近似向量可知 $\beta-\alpha \perp \alpha$. 即 $\beta-\alpha \in U^{\perp}$.

$\Leftarrow$: 设 $\gamma \in U$ 则 $\beta-\gamma=\beta-\alpha+\alpha-\gamma$ 又有 $\beta-\alpha \in U^{\perp}$. $\alpha-\gamma \in U$.

$\therefore \|\beta-\gamma\|^2=\|\beta-\alpha\|^2+\|\alpha-\gamma\|^2 \geq \|\beta-\alpha\|^2$. 又由于它对 $\forall \gamma$ 都成立, 所以 α 是 β 的最佳近似向量.

28, (1). $A=\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}=(\alpha_1\ \alpha_2\ \alpha_3)$

则. $\beta_1=\alpha_1=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$\beta_2=\alpha_2-\frac{(\alpha_2\alpha_1)}{(\alpha_1\alpha_1)}\alpha_1=\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}-\frac{1}{2}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}=\begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$$

$$\beta_3=\alpha_3-\frac{(\alpha_3\alpha_2)}{(\alpha_2\alpha_2)}\alpha_2-\frac{(\alpha_3\alpha_1)}{(\alpha_1\alpha_1)}\alpha_1=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}-\frac{1}{\frac{6}{4}}\begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}-0=\begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{3} \end{bmatrix}.$$

则 $\eta_1=\begin{bmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$ $\eta_2=\begin{bmatrix} \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \\ -\frac{\sqrt{6}}{6} \end{bmatrix}$ $\eta_3=\begin{bmatrix} \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix}$

$\frac{6}{4}\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\frac{\sqrt{2}}{2}$. $4+1+1$ 6.

$\frac{1}{2}$ $\frac{2}{3}\begin{bmatrix} \\ \\ \end{bmatrix}$ $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ $\frac{1}{4}$ $\frac{\sqrt{6}}{2}$

$\frac{6}{16}$ $\frac{\sqrt{6}}{4}$

$\therefore Q = \begin{bmatrix} 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} \end{bmatrix}$

$$R = Q^T A = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{6}}{3} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} \\ \frac{\sqrt{3}}{3} & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{3} \\ 0 & 0 & \frac{\sqrt{3}}{3} \end{bmatrix}.$$

$$\therefore A = QR = \begin{bmatrix} 0 & \frac{\sqrt{6}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{6}}{6} & -\frac{\sqrt{3}}{3} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{3}}{3} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{3} \\ 0 & 0 & \frac{\sqrt{3}}{3} \end{bmatrix}.$$

(2) $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix} = (\alpha_1\ \alpha_2\ \alpha_3)$.

$$\beta_1 = \alpha_1 = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

$$\beta_2 = \alpha_2 - \frac{(\alpha_2, \beta_1)}{(\beta_1, \beta_1)} \beta_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} - \frac{3}{4} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 2 \\ -\frac{1}{2} \end{bmatrix}.$$

$$\beta_3 = \alpha_3 - \frac{(\alpha_3, \beta_2)}{(\beta_2, \beta_2)} \beta_2 - \frac{(\alpha_3, \beta_1)}{(\beta_1, \beta_1)} \beta_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \frac{\frac{7}{2}}{\frac{9}{2}} \begin{bmatrix} \frac{1}{2} \\ 2 \\ -\frac{1}{2} \end{bmatrix} - \frac{6}{8} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{8}{9} \\ \frac{4}{9} \\ \frac{8}{9} \end{bmatrix}$$

则 $Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{4}{3\sqrt{2}} & \frac{1}{3} \\ \frac{1}{\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{2}{3} \end{bmatrix}$.

$$R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{3\sqrt{2}} & \frac{4}{3\sqrt{2}} & -\frac{1}{3\sqrt{2}} \\ -\frac{2}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{4}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\ 0 & \frac{1}{3\sqrt{2}} & \frac{7}{3\sqrt{2}} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}.$$

$$\therefore A = QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{3\sqrt{2}} & -\frac{2}{3} \\ 0 & \frac{4}{3\sqrt{2}} & \frac{1}{3} \\ \frac{1}{\sqrt{2}} & -\frac{1}{3\sqrt{2}} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} \frac{4}{\sqrt{2}} & \frac{2}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\ 0 & \frac{1}{3\sqrt{2}} & \frac{7}{3\sqrt{2}} \\ 0 & 0 & \frac{4}{3} \end{bmatrix}.$$

(3). $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = (\alpha_1\ \alpha_2)$ 则 $\beta_1 = \alpha_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\beta_2 = \alpha_2 - \frac{(\alpha_2, \alpha_1)}{(\alpha_1, \alpha_1)} \alpha_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$\therefore Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}$ $R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{2}{\sqrt{3}} \end{bmatrix}.$

$$\therefore A = QR = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{2}{\sqrt{3}} \end{bmatrix}.$$