作业六

36, 设 $V=\mathbb{R}^6$，有标 $e_1, e_2 \cdots e_6$. 则 $(V, R, +\cdot)$. 设线性映射 σ. $\sigma(e_1)=e_1$. $\sigma(e_i)=0, 2\le i\le 6$.

则 σ 核维数等于 5,

设内积空间 $(V\ R\ +\ \cdot\ (\cdot\ \cdot))$.

则 $\operatorname{Im}(\sigma)^{\perp} = \operatorname{Ker}(\sigma) = [e_2, e_3, \cdots e_6]$

令 $\tau(e_1)=0$. $\tau(e_i)=e_i, 2\le i\le 6$. 则 $\operatorname{Ker}\tau = \operatorname{Im}\sigma$. $\operatorname{Im}\tau = \operatorname{Ker}\sigma$.

37, (1)

$$\begin{cases} \sigma(\alpha_1+\alpha_2) = \alpha_1+\alpha_2 - 2(\alpha_1+\alpha_2,\ \alpha_0)\alpha_0 = \alpha_1+\alpha_2-\left(2(\alpha_1\ \alpha_0)\alpha_0 + 2(\alpha_2\ \alpha_0)\alpha_0\right) \\ \qquad = \alpha_1 - 2(\alpha_1\ \alpha_0)\alpha_0 + \alpha_2 - 2(\alpha_2\ \alpha_0)\alpha_0 = \sigma(\alpha_1)+\sigma(\alpha_2). \\ \sigma(k\alpha_1) = k\alpha_1 - 2(k\alpha_1\ \alpha_0)\alpha_0 = k\alpha_1 - 2k(\alpha_1\ \alpha_0)\alpha_0 = k(\alpha_1 - 2(\alpha_1\ \alpha_0)\alpha_0) \end{cases}$$

(2). check. $\|\sigma(\alpha)\| = \|\alpha\|$.

$\Rightarrow$ $(\sigma(\alpha)\ \sigma(\alpha)) = (\alpha - 2(\alpha\ \alpha_0)\alpha_0,\ \alpha - 2(\alpha\ \alpha_0)\alpha_0)$.

$= (\alpha\ \alpha) - 2(\alpha\ \alpha_0)(\alpha\ \alpha_0) - 2(\alpha\ \alpha_0)(\alpha\ \alpha_0) + 2(\alpha\ \alpha_0)\cdot 2(\alpha\ \alpha_0)\cdot(\alpha_0\ \alpha_0)$

$= (\alpha\ \alpha). = \|\alpha\|$.

$\therefore$ 是正交变换.

38, 设 σ 在 V 标准正交基 $(\alpha_1 \cdots \alpha_n)$ 下矩阵为 $A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{n1} & & a_{nn} \end{bmatrix}$.

即. $\sigma(\alpha_1 \cdots \alpha_n) = (\alpha_1 \cdots \alpha_n)A$.

$(\sigma(\alpha_i), \alpha_j) = -(\alpha_i, \sigma(\alpha_j))$.

$\Rightarrow$. $\sum a_{ki}(\alpha_k, \alpha_j) = -\sum a_{kj}(\alpha_i\ \alpha_k)$.

$\Rightarrow$ $a_{ji}(\alpha_j\ \alpha_j) = -a_{ij}(\alpha_i\ \alpha_i)$.

$\Rightarrow$ $a_{ji} = -a_{ij}$. $\therefore$ A是反对称矩阵.

39. 设标准正交基 $\alpha_1\alpha_2$, 则 $\sigma(\alpha_1,\alpha_2) = (\alpha_1,\alpha_2)P$.

易知, 特征多项式. $(c-\lambda)(-c-\lambda) - s^2 = 0$

$\Rightarrow -c^2 - s^2 + \lambda^2 = 0 \Rightarrow \lambda^2 - 1 = 0$ $\therefore$ 特征根为 $+1, -1$.

则 σ 是反射变换, 其对称轴为 $y = \frac{1-c}{s}x$.

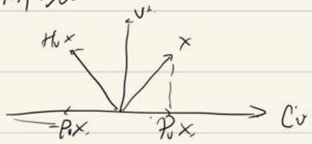
43, 有 $H = I - \frac{2vv^*}{v^*v}$

固定 $v^\perp$, $H_v(v) = v - \frac{2vv^*v}{v^*v} = -v$, $\therefore$ 是关于 $v^\perp$ 的反射, 从而正交变换.

将向量 x 按 $C^n = \mathrm{span}\{v\} \oplus v^\perp$ 正交分解为

$x = P_v x + P_v^\perp x$

则 $H_v x = P_v^\perp x - P_v x$



44, check $GG^T = I$. 令 $A = GG^T$

则 $G = \begin{bmatrix} 1 & & & & \\ & \ddots & & & \\ & & c & s & \\ & & -s & c & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}$ $G^T = \begin{bmatrix} 1 & & & & \\ & \ddots & & & \\ & & c & -s & \\ & & s & c & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix}$

$GG^T = \begin{bmatrix} 1 & & & & \\ 0 & \ddots & & & \\ & & c^2+s^2 & cs-cs & \\ & & cs-cs & c^2+s^2 & \\ 0 & & 0 & & 1 \end{bmatrix} = I.$

设 $i<j$. $Gx = x + [(c-1)x_i + sx_j]e_i + [(c-1)x_j - sx_i]e_j = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ cx_i + sx_j \\ \vdots \\ cx_j - sx_i \\ \vdots \\ x_n \end{bmatrix}$

然后, 令 $c = x_i$ $s = x_j$. 则 $cx_j - sx_i = 0$. 即消去一项. 另一项为 $x_i^2 + x_j^2$

同理可消去各项 只令某一项为 1. (因为 $\Sigma x_i^2 = 1$).

最后. $G_1G_2 \cdots G_{n-1}x = (0, 0, \ldots, 1, \cdots 0)^T$.

45, (1) $\varphi \in Hom(V,V)$. $(\varphi\alpha, \beta) = (\alpha, \varphi^{*}\beta) = (\alpha, \varphi^{\#}\beta)$.

$\Rightarrow \varphi^{*}\beta = \varphi^{\#}\beta$. $\Rightarrow \varphi^{*} = \varphi^{\#}$

(2) $\sigma(x,y) = (x,y)\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$

则 $\sigma^{*}(x,y) = (x,y)\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = (a_1x + a_2y,\ b_1x + b_2y)$

(3) 由于正交投影 $\sigma(\alpha_1 \cdots \alpha_n) = (\alpha_1 \cdots \alpha_n)A$，中 A为对称矩阵.

即 $A = A^{T}$ ∴ $\sigma^{*} = \sigma$ ∴是自伴变换.

46, (1) 定义为 $(\sigma(x), y) = (x, \sigma(y))$

(2) $\sigma\alpha = \alpha A$ 则伴随 $\sigma^{*}\alpha = \alpha A^{*}$.

性质同欧氏空间上的.

自伴变换即 $\sigma = \sigma^{*}$ 要求 $A = A^{*}$.