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InterQR.fn(DAT, tau, method, tstar)

ARGUMENTS

DAT Matrix of data with (P+2) columns and n rows, where p is the dimension of predictors, and n is the total observation. The first column is all ones for the intercept, and the last column in DAT is the response vector.

tau Vector of quantiles we are interested in.

method "FAL" or "FAS", respresenting the Fused Adaptive Lasso and Fused Adaptive Sup-norm methods we consider in the paper.

tstar Tuning parameter controlling the degree of penalization

VALUE Return estimated coefficient matrix, with (p+1) rows, and K columns, where K is the total number of desired quantiles. Each row contains estimated quantile coefficients for one predictor at K quantile levels.

Tun.Sel(DAT, tstar, coef.matrix, dec=4, tau)

ARGUMENTS

coef.matrix Matrix of estimated quantile coefficients, which is the return value

from InterQR.fn

dec How many decimal places of quantile coefficients to round when count

degree of freedom.

VALUE Return AIC (or BIC) value corresponding to one fixed tuning parameter

```
Examples
#genearate data matrix
dat1<- function(n,p,gamma,beta)</pre>
{
X<- cbind(1,matrix(runif(p*n,0,1),byrow=F,nrow=n)) #X includes intercept
y<- X%*%beta + (1+gamma*X[,p+1])*rnorm(n,0,1) #last predictor vaies
return(cbind(X,y)) #first 1:(p+1)columns are X, column (p+2) is Y
}
library(quantreg)
n<- 200
p<- 1 #univariate case
# when gamma=0, slope coefficients are constant; gamma=2, slope coefficnets vary across quantiles
gamma<- 2
tau < -seq(from = 0.1, to = 0.9, by = 0.1)
beta<- c(1,rep(3,p)) #true intercept and slope coefficients
tgrid<- 100 #grid search points
set.seed(1220901)
DAT<- dat1(n=n,p=p,gamma=gamma,beta=beta) #X:1+p columns, Y:p+2 column
#use FAL method
coef.FAL<- InterQR.fn(DAT, tau, method="FAL", tstar=0.5)
#use FAS method
coef.FAS<- InterQR.fn(DAT, tau, method="FAS", tstar=0.5)
```

```
# Simulations of Example 3 from paper, show FAL method only here, FAS is very similar

# Note: in this example, coefficients for the first predictor always remain constant across quantiles.
```

#Coefficients for the second predictor may vary, or stay constant across quantiles, depending on

#different choices of gamma.

```
dat1<- function(n,p,gamma,beta)</pre>
{
X<- cbind(1,matrix(runif(p*n,0,1),byrow=F,nrow=n)) #X includes intercept
y<- X%*%beta + (1+gamma*X[,p+1])*rnorm(n,0,1)
                         #first 1:(p+1)columns are X, column (p+2) is Y
return(cbind(X,y))
}
library(quantreg)
n<- 200
p<- 2
          #Bivariate case
#gamma=2: coefficients for the first predictor are constant across quantiles, but vary
#across quantiles for the second predictor
#gamma=0: coefficients are constant for both predictors across quantiles
gamma<- 2
tau < -seq(from = 0.1, to = 0.9, by = 0.1)
k<- length(tau)
beta<- c(1,rep(3,p))
nsim<- 500 #number of simulations
tgrid<- 100 #number of grid search points
```

```
COEF.rq = COEF.FAL = array(0,dim=c((p+1),k,nsim))
DAT.save = array(0,dim=c(n,p+2,nsim))
                                          #save the simulated data matrix
set.seed(1220901)
for (i in 1:nsim)
{
DAT<- dat1(n=n,p=p,gamma=gamma,beta=beta) #X:1+p columns, Y:p+2 column
X.dat<- DAT[,1:(p+1)] #design matrix including intercept 1
y.dat<- DAT[,(p+2)]
DAT.save[,,i]<- DAT
#conventional RQ method
X.rq<- X.dat[,-1] #don't need intercept for rq regression!
rq.coef<- coef(rq(y.dat~X.rq,tau=tau))
COEF.rq[,,i]<- rq.coef
#FAL method
####tuning parameter
tmax <- p*(k-1)
#grid search for t from 0 to tmax
 t<- seq(0,1,length=tgrid)*tmax #grid search for unadaptive lasso
aic.FAL<- aic.FAS<- rep(0, tgrid)
coef.mat<- array(0, dim=c(p+1, k, tgrid))</pre>
```

```
#find t with the smallest AIC
for (q in 1:tgrid)
 {
  coef.mat[,,q]<- InterQR.fn(DAT, tau, method="FAL", tstar=t[q]) #return estimated coefficient matrix
  aic.FAL[q]<- Tun.Sel(DAT=DAT, tstar=t[q], coef.matrix=coef.mat[,,q], dec=4, tau=tau) #return AIC
values
}
idx<- which.min(aic.FAL) #find the index of tuning parameter that returns the smallest AIC value
t.select<- t[idx]
                    #select the tuning parameter which returns the smallest AIC value
COEF.FAL[,,i]<- coef.mat[,,idx] #save the estimated coefficient matrix
print(i)
}
#True slope and intercepts
if (p==1) {true.beta<- rbind(beta[1]+qnorm(tau),beta[2]+gamma*qnorm(tau))} else {
true.beta<- rbind(beta[1]+qnorm(tau),
matrix(rep(beta[2:p],each=k),byrow=T,ncol=k),beta[p+1]+gamma*qnorm(tau))}
# Get Integrated MSE
IMSE.FAL = IMSE.RQ = matrix(0,nsim,k)
for (j in 1:nsim)
 X<- DAT.save[,1:(p+1),j]
```

```
IMSE.FAL[j,]<- apply((X%*%true.beta-X%*%COEF.FAL[,,j])^2,2,mean) ##why not y.true-?
IMSE.RQ[j,]<- apply((X%*%true.beta-X%*%COEF.RQ[,,j])^2,2,mean)
}

sfnc.sim.IMSE.FAL<- apply(IMSE.FAL,2,mean)

sfnc.se.IMSE.FAL<- apply(IMSE.FAL,2,sd)/sqrt(nsim)

sfnc.sim.IMSE.RQ<- apply(IMSE.RQ,2,mean)

sfnc.se.IMSE.RQ<- apply(IMSE.RQ,2,sd)/sqrt(nsim)
```