CASTOR, LIEZELYN A.
20-03579

ME-4203
ECE 425

ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}[3-e^{-3t}+5\sin 2t]=F(s)$

SOLUTION:

$a=3 \quad ; \omega=2$

$$F(s)=3\left(\frac{1}{s}\right)-\frac{1}{s+a}+5\left(\frac{\omega}{s^2+\omega^2}\right)$$

$$F(s)=\frac{3}{s}-\frac{1}{s+3}+5\left(\frac{2}{s^2+2^2}\right)$$

$$\boxed{F(s)=\frac{3}{s}-\frac{1}{s+3}+\frac{10}{s^2+4}}$$

2. $\mathcal{L}[3+12t+42t^3-3e^{2t}]=F(s)$

SOLUTION:

$n=3 \quad ; a=-2$

$$F(s)=3\left(\frac{1}{s}\right)+12\left(\frac{1}{s^2}\right)+42\left(\frac{n!}{s^{n+1}}\right)-3\left(\frac{1}{s+a}\right)$$

$$F(s)=\frac{3}{s}+\frac{12}{s^2}+42\left(\frac{3!}{s^{3+1}}\right)-3\left(\frac{1}{s-2}\right)$$

$$F(s)=\frac{3}{s}+\frac{12}{s^2}+42\left(\frac{6}{s^4}\right)-\frac{3}{s-2}$$

$$\boxed{F(s)=\frac{3}{s}+\frac{12}{s^2}+\frac{252}{s^4}-\frac{3}{s-2}}$$

3. $\mathcal{L}[(t+1)(t+2)]=F(s)$

SOLUTION:

$n=2$

$$F(s)=\mathcal{L}\{t^2+3t+2\}$$

$$F(s)=\frac{n!}{s^{n+1}}+3\left(\frac{1}{s^2}\right)+2\left(\frac{1}{s}\right)$$

$$F(s)=\frac{2!}{s^{2+1}}+\frac{3}{s^2}+\frac{2}{s}$$

$$\boxed{F(s)=\frac{2}{s^3}+\frac{3}{s^2}+\frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left[\frac{8-3s+s^2}{s^3}\right]=f(t)$

SOLUTION:

$f(t)=\mathcal{L}^{-1}\left\{\frac{8}{s^3}-\frac{3s}{s^3}+\frac{s^2}{s^3}\right\}$

a. $f(t)=\mathcal{L}^{-1}\left(\frac{8}{s^3}\right)$

$n=2$; $n!/s^{n+1}$

$f(t)=4\left(\frac{2!}{s^{2+1}}\right)$

$f(t)=4t^n$

$f(t=4t^2$

b. $f(t)=\mathcal{L}^{-1}\left[\frac{3s}{s^3}\right]$

$f(t)=3\mathcal{L}^{-1}\left[\frac{s}{s^3}\right]$

$f(t)=3\mathcal{L}^{-1}\left[\frac{1}{s^2}\right]$

$f(t)=3t$

c. $f(t)=\mathcal{L}^{-1}\left[\frac{s^2}{s^3}\right]$

$f(t)=\mathcal{L}^{-1}\left[\frac{s^2}{s^3}\right]$

$f(t)=\mathcal{L}^{-1}\left[\frac{1}{s}\right]$

$f(t)=1$

$\boxed{\therefore f(t)=4t^2+3t+1}$

2. $\mathcal{L}^{-1}\left[\frac{5}{s-2}-\frac{4s}{s^2+9}\right]=f(t)$

SOLUTION:

a. $f(t)=\mathcal{L}^{-1}\left[\frac{5}{s-2}\right]$

$f(t)=5\mathcal{L}^{-1}\left[\frac{1}{s-2}\right]$

$a=-2$; $\frac{1}{s+a}\rightarrow e^{-at}$

$f(t)=5e^{2t}$

b. $f(t)=\mathcal{L}^{-1}\left[\frac{4s}{s^2+9}\right]$

$f(t)=4\mathcal{L}^{-1}\left[\frac{s}{s^2+9}\right]$

$\omega=\sqrt{9}=3$; $\frac{s}{s^2+\omega^2}\rightarrow \cos\omega t$

$f(t)=4\mathcal{L}^{-1}\left[\frac{s}{s^2+3^2}\right]$

$f(t)=4\cos 3t$

$\boxed{\therefore f(t)=5e^{2t}-4\cos 3t}$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right]=f(t)$

SOLUTION:

$f(t)=7\mathcal{L}^{-1}\left[\frac{1}{s^2+6}\right]$

$\omega=\sqrt{6}$; $\frac{\omega}{s^2+\omega^2}\rightarrow \sin\omega t$

$f(t)=7\mathcal{L}^{-1}\left[\frac{1}{s^2+(\sqrt{6})^2}\right]$; $1=\frac{\sqrt{6}}{\sqrt{6}}$

$f(t)=7\mathcal{L}^{-1}\left[\frac{\sqrt{6}}{\sqrt{6}(s^2+(\sqrt{6})^2}\right]$

$f(t)=\frac{7}{\sqrt{6}}\mathcal{L}^{-1}\left[\frac{\sqrt{6}}{s^2+(\sqrt{6})^2}\right]$

$f(t)=\left(\frac{7}{\sqrt{6}}\sin\sqrt{6}t\right)\cdot\frac{\sqrt{6}}{\sqrt{6}}$

$\boxed{f(t)=\frac{7\sqrt{6}}{6}\sin\sqrt{6}t}$

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ASSIGNMENT 2

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

SOLUTION:

$$f(t) = \mathcal{L}^{-1}\left[\left[\frac{1}{s(s^2+2s+2)}\right] = \left(\frac{A}{s} + \frac{Bs+C}{s^2+2s+2}\right)\right] s(s^2+2s+2)$$

$$1 = A(s^2+2s+2) + s(Bs+C)$$

if $s=0$

$$1 = A(0^2+2(0)+2) + 0(B(0)+C)$$

$$1 = 2A$$

$$\frac{1}{2} = A$$

SUBSTITUTE THE VALUE OF A TO SOLVE FOR B AND C:

$$\left[1 = \frac{1}{2}(s^2+2s+2) + s(Bs+C)\right] 2$$

$$2 = s^2+2s+2+2Bs^2+2Cs$$

$$2 = s^2(1+2B) + s(2+2C) + 2$$

$$2-2 = s^2(1+2B) + s(2+2C)$$

$$0 = s^2(1+2B) + s(2+2C)$$

$$1+2B=0 \qquad 2B=-1 \qquad B=-\frac{1}{2}$$

$$2+2C=0 \qquad 2C=-2 \qquad C=-\frac{2}{2} \qquad C=-1$$

PLUG IN THE VALUES:

$$f(t) = \mathcal{L}^{-1}\left[\frac{1/2}{s} + \frac{(-1/2\,s)+(-1)}{s^2+2s+2}\right]$$

$$f(t) = \mathcal{L}^{-1}\left[\frac{1}{2s} - \frac{1/2\,s+1}{s^2+2s+2}\right]$$

a. $f(t) = \mathcal{L}^{-1}\left[\dfrac{1}{2s}\right]$

$$f(t) = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{1}{s}\right]$$

$$f(t) = \frac{1}{2}$$

b. $f(t) = \mathcal{L}^{-1}\left[\dfrac{1/2\,s+1}{s^2+2s+2}\right]$

$$f(t) = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{s+2}{s^2+2s+2}\right]$$

$$f(t) = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{s+1+1}{s^2+2s+1+1}\right]$$

$$a=1;\ \omega=1;\ f(t) = \mathcal{L}^{-1}\left[\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right] = e^{-at}\left[\cos\omega t + \sin\omega t\right]$$

$$f(t) = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{(s+1)+1}{(s+1)^2+1^2}\right]$$

$$f(t) = \frac{1}{2}\left[e^{-t}(\cos t + \sin t)\right]$$

$$\therefore f(t) = \frac{1}{2} - \frac{1}{2}\left[e^{-t}(\cos t + \sin t)\right]$$

$$\boxed{f(t) = \frac{1}{2}\left(1 - e^{-t}\left[\cos t + \sin t\right]\right)}$$

2. $F(s)=\dfrac{5(s+2)}{s^2(s+1)(s+3)}$

SOLUTION:

$$f(t)=\mathcal{L}^{-1}\left[\left\{\frac{5(s+2)}{s^2(s+1)(s+3)}\right\}=\frac{A}{s}+\frac{B}{s^2}+\frac{C}{s+1}+\frac{D}{s+3}\right]s^2(s+1)(s+3)$$

$$5s+10=As(s+1)(s+3)+B(s+1)(s+3)+Cs^2(s+3)+Ds^2(s+1)$$

if $s=0$:

$$5(0)+10=A(0)(0+1)(0+3)+B(0+1)(0+3)+C(0^2)(0+3)+D(0^2)(0+1)$$

$$10=3B$$

$$\frac{10}{3}=B$$

if $s=-1$:

$$5(-1)+10=A(-1)(-1+1)+B(-1+1)(-1+3)+C(-1^2)(-1+3)+D(-1^2)(-1+1)$$

$$5=2C$$

$$\frac{5}{2}=C$$

if $s=-3$:

$$5(-3)+10=A(-3)(-3+1)(-3+3)+B(-3+1)(-3+3)+C(-3^2)(-3+3)+D(-3^2)(-3+1)$$

$$-5=-18D$$

$$\frac{-5}{-18}=D$$

$$\frac{5}{18}=D$$

PLUG IN VALUES TO SOLVE FOR A:

$$5s+10=As(s+1)(s+3)+\frac{10}{3}(s+1)(s+3)+\frac{5}{2}(s^2)(s+3)+\frac{5}{18}(s^2)(s+1)$$

$$5s+10=As(s^2+4s+3)+\frac{10}{3}(s^2+4s+3)+\frac{5}{2}(s^3+3s^2)+\frac{5}{18}(s^3+s^2)$$

$$0=As^3+4As^2+3As+\frac{10}{3}s^2+\frac{40}{3}s+\frac{30}{3}+\frac{5}{2}s^3+\frac{15}{2}s^2+\frac{5}{18}s^3+\frac{5}{18}s^2-5s-10$$

$$0=s^3\left(A+\frac{5}{2}+\frac{5}{18}\right)+s^2\left(4A+\frac{10}{3}+\frac{15}{2}+\frac{5}{18}\right)+s\left(3A+\frac{40}{3}-5\right)+\frac{30}{3}-10$$

$$0=s^3\left(A+\frac{25}{9}\right)+s^2\left(4A+\frac{100}{9}\right)+s\left(3A+\frac{40}{3}-5\right)+10-10$$

$$A+\frac{25}{9}=0$$

$$A=-\frac{25}{9}$$

PLUG IN VALUES:

$$f(t)=\mathcal{L}^{-1}\left[\frac{-25/9}{s}+\frac{10/3}{s^2}+\frac{5/2}{s+1}+\frac{5/18}{s+3}\right]$$

a. $f(t) = \mathcal{L}^{-1}\left[-\frac{25/9}{s}\right]$

$f(t) = -\frac{25}{9}\mathcal{L}^{-1}\left[\frac{1}{s}\right]$

$f(t) = -\frac{25}{9}$

b. $f(t) = \mathcal{L}^{-1}\left[\frac{10/3}{s^2}\right]$

$f(t) = \frac{10}{3}\mathcal{L}^{-1}\left[\frac{1}{s^2}\right]$

$f(t) = \frac{10}{3}t$

c. $f(t) = \mathcal{L}^{-1}\left[\frac{5/2}{s+1}\right]$

$f(t) = \frac{5}{2}\mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$

$a = 1;\ \frac{1}{s+a} \Rightarrow e^{-at}$

$f(t) = \frac{5}{2}e^{-t}$

d. $f(t) = \mathcal{L}^{-1}\left[\frac{5/18}{s+3}\right]$

$f(t) = \frac{5}{18}\mathcal{L}^{-1}\left[\frac{1}{s+3}\right]$

$a = 3;\ \frac{1}{s+a} \Rightarrow e^{-at}$

$f(t) = \frac{5}{18}e^{-3t}$

$$\boxed{\therefore f(t) = -\frac{25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}}$$

3. $F(s) = \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

SOLUTION:

$$\begin{array}{r} s^2 + s + 2 \\ s^2+s \overline{)\, s^4 + 2s^3 + 3s^2 + 4s + 5} \\ -\ s^4 + s^3 \\ \hline s^3 + 3s^2 + 4s + 5 \\ -\ s^3 + s^2 \\ \hline 2s^2 + 4s + 5 \\ -\ 2s^2 + 2s + 5 \\ \hline 2s + 5 \end{array}$$

$f(t) = \mathcal{L}^{-1}\left[s^2 + s + 2 + \frac{2s+5}{s^2+s}\right]$

a. $f(t) = \mathcal{L}^{-1}[s^2 + s + 2]$

$f(t) = y'' + y + 2$

$f(t) = \frac{d^2y}{dt^2} + \frac{dy}{dt} + 2\delta t = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta t$

b. $f(t) = \mathcal{L}^{-1}\left[\frac{2s+5}{s^2+5}\right]$

$\mathcal{L}^{-1}\left[\frac{2s+5}{s^2+5}\right] = \mathcal{L}^{-1}\left\{\frac{2s+5}{s(s+1)}\right\} = \frac{A}{s} + \frac{B}{s+1}$

$\left[\frac{2s+5}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}\right] s(s+1)$

$2s + 5 = A(s+1) + Bs$

if $s = 0$:

$2(0) + 5 = A(0+1) + B(0)$

$5 = A$

if $s = -1$:

$2(-1) + 5 = A(-1+1) + B(-1)$

$3 = -B$

$-3 = B$

PLUG IN VALUES:

$f(t) = \mathcal{L}^{-1}\left[\frac{5}{s} + \frac{-3}{s+1}\right]$

$f(t) = \mathcal{L}^{-1}\left[\frac{5}{s} - \frac{3}{s+1}\right]$; $a = 1$

$f(t) = 5 - 3e^{-t}$

$$\boxed{\begin{array}{l} \therefore f(t) = y'' + y + 2\delta t + 5 - 3e^{-t} \\ f(t) = \frac{d^2y}{dt^2} + \frac{dy}{dt} + 2\delta t + 5 - 3e^{-t} \end{array}}$$

OR

$$\boxed{f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta t + 5 - 3e^{-t}}$$