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ASSIGNMENT I

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}[3-e^{-3t}+5\sin 2t]=F(s)$, $u(t)\Rightarrow 1$

SOLUTION:

a. $\mathcal{L}\{3\}\rightarrow 3\mathcal{L}\{1\}=3\left(\frac{1}{s}\right)=\frac{3}{s}$

Note: $\mathcal{L}\{1\}$ or $\mathcal{L}[u(t)]=1/s$

b. $\mathcal{L}\{-e^{-3t}\}=-\mathcal{L}\{e^{-3t}\}$; $a=3$

since $\mathcal{L}\{e^{-at}u(t)\}=\frac{1}{s+a}$,

$\mathcal{L}\{-e^{-3t}\}=-\frac{1}{s+3}$

c. $\mathcal{L}[5\sin 2t]=5\mathcal{L}\{\sin 2t\}$; $\omega=2$

since $\mathcal{L}\{\sin\omega t\, u(t)\}=\frac{\omega}{s^2+\omega^2}$

$\mathcal{L}\{5\sin 2t\}=5\left(\frac{2}{s^2+(2)^2}\right)$

$\mathcal{L}\{5\sin 2t\}=\frac{10}{s^2+4}$

hence,

$F(s)=?$

$$\boxed{F(s)=\frac{3}{s}-\frac{1}{s+3}+\frac{10}{s^2+4}}$$

2. $\mathcal{L}[3+12t+42t^3-3e^{2t}]=F(s)$, $u(t)\Rightarrow 1$

SOLUTION:

a. $\mathcal{L}\{3\}=3\mathcal{L}\{1\}$

since $\mathcal{L}\{1\}$ or $\mathcal{L}[u(t)]=1/s$

$\mathcal{L}\{3\}=3\left(\frac{1}{s}\right)=\frac{3}{s}$

b. $\mathcal{L}\{12t\}=12\mathcal{L}\{(t)\}$

since $\mathcal{L}[t\,u(t)]=\frac{1}{s^2}$

$\mathcal{L}\{12t\}=12\left(\frac{1}{s^2}\right)=\frac{12}{s^2}$

c. $\mathcal{L}\{42t^3\}=42\mathcal{L}\{t^3\}$; $n=3$

since $\mathcal{L}\{t^n\}=\frac{n!}{s^{n+1}}$

$\mathcal{L}\{42t^3\}=42\left(\frac{3!}{s^{3+1}}\right)=42\left(\frac{6}{s^4}\right)$

$\mathcal{L}\{42t^3\}=\frac{252}{s^4}$

d. $\mathcal{L}\{-3e^{2t}\}=-3\mathcal{L}\{e^{2t}\}$; $a=-2$

$\mathcal{L}\{-3e^{2t}\}=-3\left(\frac{1}{s-2}\right)=-\frac{3}{s-2}$

since $\mathcal{L}[e^{-at}]=1/s+a$

then:

$$\boxed{F(s)=\frac{3}{s}+\frac{12}{s^2}+\frac{252}{s^4}-\frac{3}{s-2}}$$

3. $\mathcal{L}[(t+1)(t+2)=F(s)$

$\mathcal{L}[t^2+3t+2]=F(s)$

SOLUTION:

a. $\mathcal{L}\{t^2\}\Rightarrow n=2$, since $\mathcal{L}[t^n]=\frac{n!}{s^{n+1}}$

$\mathcal{L}\{t^2\}=\frac{2!}{s^{2+1}}$

$\mathcal{L}\{t^2\}=\frac{2}{s^3}$

b. $\mathcal{L}\{3t\}=3\mathcal{L}\{t\}$

since $\mathcal{L}\{t\}=\frac{1}{s^2}$

$\mathcal{L}\{3t\}=3\left(\frac{1}{s^2}\right)$

$\mathcal{L}\{3t\}=\frac{3}{s^2}$

c. $\mathcal{L}\{2\} = 2\mathcal{L}\{1\}$

Note: $\mathcal{L}\{1\} = 1/s$

$\mathcal{L}\{2\} = 2\left(\frac{1}{s}\right)$

$\mathcal{L}\{2\} = \frac{2}{s}$

then,

$$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-}\left[\frac{8-3s+s^2}{s^3}\right] = f(t)$

$\mathcal{L}^{-}\left[\frac{8}{s^3} - \frac{3s}{s^3} + \frac{s^2}{s^3}\right] = f(t)$

$\mathcal{L}^{-}\left[\frac{8}{s^3} - \frac{3}{s^2} + \frac{1}{s}\right] = f(t)$

SOLUTION:

a. $\mathcal{L}^{-}\left[\frac{8}{s^3}\right] = \mathcal{L}^{-}\frac{4(2)}{s^{2+1}} = 4\mathcal{L}^{-}\frac{2!}{s^{2+1}}$

$n = 2$; FROM TABLE 2.1, $\frac{n!}{s^{n+1}} \to t^n$

$\mathcal{L}^{-}\left[\frac{8}{s^3}\right] = 4(t^2)$

$f(t) = 4t^2$

b. $\mathcal{L}^{-}\left[-\frac{3}{s^2}\right] = f(t)$

$-3\mathcal{L}^{-}\left[\frac{1}{s^2}\right] = f(t)$

BASED FROM TABLE 2.1,

$1/s^2 \to t$

$-3\mathcal{L}^{-}\left[\frac{1}{s^2}\right] = -3t$

$f(t) = -3t$

c. $\mathcal{L}^{-}\frac{1}{s} = 1$ (BASED FROM TABLE 2.1, $1/s \to 1$)

then:

$$\boxed{f(t) = 4t^2 - 3t + 1}$$

2. $\mathcal{L}^{-}\left[\frac{5}{s-2} - \frac{4s}{s^2+9}\right] = f(t)$

SOLUTION:

a. $\mathcal{L}^{-}\left[\frac{5}{s-2}\right] = 5\mathcal{L}^{-}\left[\frac{1}{s-2}\right]$

FROM TABLE 2.1, $e^{-at} = 1/s+a$

here, $a = -2$

$\mathcal{L}^{-}\left[\frac{5}{s-2}\right] = 5e^{2t}$

b. $\mathcal{L}^{-} - \frac{4s}{s^2+9} = -4\mathcal{L}^{-}\frac{s}{s^2+9}$

$= -4\mathcal{L}^{-}\frac{s}{s^2+3^2}$; $\omega = 3$

It satisfies $\frac{s}{s^2+\omega^2} \to \cos\omega t$

$\mathcal{L}^{-}\frac{4s}{s^2+9} = -4\cos 3t$

then: $\boxed{f(t) = 5e^{2t} - 4\cos 3t}$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = f(t)$

SOLUTION:

$$\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = 7\mathcal{L}^{-1}\frac{1}{s^2+6}$$

$$= 7\mathcal{L}^{-1}\frac{1}{s^2+(\sqrt{6})^2}$$

NOTE: $1 = \frac{\sqrt{6}}{\sqrt{6}}$

$$\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = 7\mathcal{L}^{-1}\frac{\frac{\sqrt{6}}{\sqrt{6}}}{s^2+(\sqrt{6})^2}$$

$$\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = \frac{7}{\sqrt{6}}\mathcal{L}^{-1}\frac{\sqrt{6}}{s^2+(\sqrt{6})^2}$$

$\omega = \sqrt{6}$, it satisfies $\frac{\omega}{s^2+\omega^2} \rightarrow \sin \omega t$

$$\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = \frac{7}{\sqrt{6}} \sin \sqrt{6}t \left[\frac{\sqrt{6}}{\sqrt{6}}\right]$$

$$\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = \frac{7\sqrt{6}}{6} \sin \sqrt{6}t = f(t)$$

$$\boxed{f(t) = \frac{7\sqrt{6}}{6} \sin \sqrt{6}t}$$

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ASSIGNMENT 2

1. $F(s) = \frac{1}{s(s^2+2s+2)}$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + (Bs+C)(s)$$

if $s=0$:

$$1 = A[(0)^2 + 2(0) + 2] + [B(0)+C](0)$$

$$1 = A(2)$$

$$A = \frac{1}{2}$$

then:

$$\left[1 = \frac{1}{2}(s^2+2s+2) + (Bs+C)s\right]2$$

$$2 = s^2 + 2s + 2 + 2Bs^2 + 2Cs$$

$$2-2 = 2Bs^2 + s^2 + 2Cs + 2s$$

$$0 = s^2(2B+1) + s(2C+2)$$

$$2B+1=0 \qquad 2C+2=0$$

$$2B=-1 \qquad 2C=-2$$

$$B=-\frac{1}{2} \qquad C=-1$$

now:

$$\mathcal{L}^{-1}\left\{\frac{\frac{1}{2}}{s} + \frac{(-\frac{1}{2})s+(-1)}{s^2+2s+2}\right\} = \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\}$$

note: from Table 2.1 Laplace Transform Table

$\mathcal{L}\{u(t)\}$ or $\mathcal{L}\{1\} = 1/s$

① $\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = \frac{1}{2}(1) = \frac{1}{2}$

② $\mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-\frac{1}{2}(2)}{s^2+2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}(s+2)}{s^2+2s+2}\right\}$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+2}{s^2+2s+2}\right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+1+1}{s^2+2s+1+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\} = -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+1}\right\} ; a=1, \omega=1$$

note: $\mathcal{L}^{-1}\left[\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right] = e^{-at}[\cos\omega t + \sin\omega t]$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\} = \left[e^{-t}[\cos t + \sin t]\right]\left(-\frac{1}{2}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right\} = -\frac{1}{2}e^{-t}[\cos t + \sin t]$$

Rechecking: $-\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+1}\right\} \rightarrow -\frac{1}{2}\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1} + \frac{1}{(s+1)^2+1}\right\}$

$\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+1}\right\}$ follows $\mathcal{L}^{-1}\frac{s^2}{s^2+\omega^2} = \cos\omega t$; since $s \rightarrow s+a, s \rightarrow s+1$

$s \rightarrow s+1, \omega^2 \rightarrow 1$ $\qquad \mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2+\omega^2}\right\} = e^{-at}\cos\omega t$

$$\mathcal{L}^{-}\left\{\frac{s+1}{(s+1)^2+1}\right\} = e^{-t}\cos t$$

while

$\mathcal{L}^{-}\left\{\frac{1}{(s+1)^2+1}\right\}$ follows $\mathcal{L}^{-}\left\{\frac{\omega}{s^2+\omega^2}\right\} = \sin \omega t$

since $s \rightarrow s+a$; $\mathcal{L}^{-}\frac{\omega}{(s+a)^2+\omega^2} = e^{-at}\sin \omega t$

$\omega \rightarrow 1,\ a \rightarrow 1$

hence :

$$\mathcal{L}^{-}\left\{\frac{1}{(s+1)^2+1}\right\} = e^{-t}\sin \omega t = e^{-t}\sin t$$

therefore :

$$\mathcal{L}^{-}\frac{-\frac{1}{2}s-1}{s^2+2s+2} = -\frac{1}{2}\left[e^{-t}\cos t + e^{-t}\sin t\right]$$

$$\mathcal{L}^{-}\frac{-\frac{1}{2}s-1}{s^2+2s+2} = -\frac{1}{2}e^{-t}\left[\cos t + \sin t\right]$$

$$\therefore\ f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\left[\cos t + \sin t\right]$$

$$\boxed{f(t) = \frac{1}{2}\left(1 - e^{-t}(\cos t + \sin t)\right]}$$

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)} \Rightarrow \mathcal{L}^{-}F(s) = f(t)$

$$\frac{5(s+2)}{s^2(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+3}$$

① $5(s+2) = A(s)(s+1)(s+3) + B(s+1)(s+3) + C(s^2)(s+3) + D(s^2)(s+1)$ ---> Equation 1

* $5s+10 = A(s)(s^2+4s+3) + B(s^2+4s+3) + C(s^3+3s^2) + D(s^3+s^2)$

* $5s+10 = As^3 + 4As^2 + 3As + Bs^2 + 4Bs + 3B + Cs^3 + 3Cs^2 + Ds^3 + Ds^2$

* $5s+10 = As^3 + Cs^3 + Ds^3 + 4As^2 + Bs^2 + 3Cs^2 + Ds^2 + 3As + 4Bs + 3B$

* $5s+10 = s^3(A+C+D) + s^2(4A+B+3C+D) + s(3A+4B) + 3B$

Using Equation 1:

if $s = 0$

$5(0+2) = A(0)(0+1)(0+3) + B(0+1)(0+3) + C(0^2)(0+3) + D(0^2)(0+1)$

$10 = 3B$

$B = \frac{10}{3}$

if $s = -1$

$5(-1+2) = A(-1)(-1+1)(-1+3) + B(-1+1)(-1+3) + C(-1)^2(-1+3) + D(-1)^2(-1+1)$

$5(1) = C(1)(2)$

$5 = 2C$

$C = \frac{5}{2}$

if $s=-3$

$$5(-3+2) = A(-3)(-3+1)(-3+3) + B(-3+1)(-3+3) + C(-3)^2(-3+3) + D(-3)^2(-3+1)$$

$$5(-1) = D(9)(-2)$$

$$-5 = -18D$$

$$D = \frac{5}{18}$$

Using Equation 2:

$$5s+10 = s^3\left[A + \frac{5}{2} + \frac{5}{18}\right] + s^2\left[4A + \frac{10}{3} + 3\left(\frac{5}{2}\right) + \frac{5}{18}\right] + s\left[3A + 4\left(\frac{10}{3}\right)\right] + 3\left(\frac{10}{3}\right)$$

for A:

$$A + \frac{5}{2} + \frac{5}{18} = 0$$

$$A + \frac{25}{9} = 0$$

$$A = -\frac{25}{9}$$

Now:

$$\mathcal{L}^{-1}\ \frac{-25/9}{s} + \frac{10/3}{s^2} + \frac{5/2}{s+1} + \frac{5/18}{s+3}$$

① $\mathcal{L}^{-1}\ \frac{-25/9}{s} = -\frac{25}{9}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = -\frac{25}{9}(1) = -\frac{25}{9}$

② $\mathcal{L}^{-1}\ \frac{10/3}{s^2} = \frac{10}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = \frac{10}{3}t$

③ $\mathcal{L}^{-1}\ \frac{5/2}{s+1} = \frac{5}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}$; $a=1$ from $e^{-at} \xrightarrow{\text{Laplace}} \frac{1}{s+a}$

$$= \frac{5}{2}e^{-t}$$

④ $\mathcal{L}^{-1}\ \frac{5/18}{s+3} = \frac{5}{18}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\}$; $a=3$ from $\mathcal{L}e^{-at} = \frac{1}{s+a}$

$$= \frac{5}{18}e^{-3t}$$

hence:

$$\boxed{f(t) = -\frac{25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}}$$

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$F(s) = \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s^2 + s}$$

$$\begin{array}{r l}
 & s^2 + s + 2 \\
s^2 + s \,\big) & s^4 + 2s^3 + 3s^2 + 4s + 5 \\
 & -\,s^4 + s^3 \\
\hline
 & \quad s^3 + 3s^2 \\
 & -\quad s^3 + s^2 \\
\hline
 & \qquad 2s^2 + 4s \\
 & -\qquad 2s^2 + 2s \\
\hline
 & \qquad\qquad 2s + 5
\end{array}$$

$$F(s) = s^2 + s + 2 + \frac{2s+5}{s^2+s}$$

$$F(s) = s^2 + s + 2 + \frac{2s+5}{s(s+1)}$$

$$\mathcal{L}^{-1}\left\{ s^2 + s + 2 + \frac{2s+5}{s(s+1)} \right\}$$

① $\mathcal{L}^{-1}\{s^2\} = \dfrac{d^2 f}{dt^2}$ from Laplace of Differential Equations

② $\mathcal{L}^{-1}\{s\} = \dfrac{df}{dt}$

③ $\mathcal{L}^{-1}(2) = 2\mathcal{L}^{-1}\{1\} = 2\,\delta(t)$ from Table 2.1

④ $\mathcal{L}^{-1} \dfrac{2s+5}{s(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$2s + 5 = A(s+1) + B(s)$

if $s = -1$

$2(-1) + 5 = A(-1+1) + B(-1)$

$3 = -B$

$B = -3$

if $s = 0$

$2(0) + 5 = A(0+1) + B(0)$

$A = 5$

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{s} + \frac{-3}{s+1}\right\}$$

① $\mathcal{L}^{-1}\left\{\dfrac{5}{s}\right\} = 5\mathcal{L}^{-1}\left\{\dfrac{1}{s}\right\} = 5(1) = 5$ note: $\mathcal{L}\{u(t)\}$ or $\mathcal{L}\{1\} = 1/s$

② $\mathcal{L}^{-1}\left\{\dfrac{-3}{s+1}\right\} = -3\,\mathcal{L}^{-1}\left\{\dfrac{1}{s+1}\right\} = -3e^{-t}$ note: $\mathcal{L}(e^{-at}) = \dfrac{1}{s+a}$, $a = -1$

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s(s+1)}\right\} = 5 - 3e^{-t}$$

hence:

$$\boxed{f(t) = \frac{d^2 f}{dt^2} + \frac{df}{dt} + 2\delta(t) + 5 - 3e^{-t}}$$