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ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FF:

1. $\mathcal{L}[3-e^{-3t}+5\sin 2t]=F(s)$

a) $\mathcal{L}[3] = 3\mathcal{L}\{1\} = 3(1/s) = 3/s$

b) $\mathcal{L}\{-e^{-3t}\} = -\mathcal{L}\{e^{-3t}\} = -(1/s+3)$

c) $\mathcal{L}\{5\sin 2t\} = 5\mathcal{L}\{\sin 2t\}$
$= 5(2/s^2+4)$
$= 10/s^2+4$

THEREFORE:
$$F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}$$

2. $\mathcal{L}\{3+12t+42t^3-3e^{2t}\} = F(s)$

a) $\mathcal{L}\{3\} = 3\mathcal{L}\{1\} = 3(1/s) = 3/s$

b) $\mathcal{L}\{12t\} = 12\mathcal{L}\{t\} = 12(1/s^2) = 12/s^2$

c) $\mathcal{L}\{42t^3\} = 42(3!/s^{3+1}) = 252/s^4$

d) $\mathcal{L}\{-3e^{2t}\} = -3\mathcal{L}\{e^{2t}\}$
$= -3(1/s-2) = -3/s-2$

THEREFORE:
$$F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}$$

3. $\mathcal{L}\{(t+1)(t+2)\} = F(s) \approx \mathcal{L}\{t^2+3t+2\} = F(s)$

a) $\mathcal{L}\{t^2\} = 2!/s^{2+1} = 2/s^3$

b) $\mathcal{L}\{3t\} = 3\mathcal{L}\{t\} = 3/s^2$

c) $\mathcal{L}\{2\} = 2\{1\} = 2/s$

THEREFORE:
$$F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}$$

II SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\{\frac{8-3s+s^2}{s^3}\} = f(t)$

2. $\mathcal{L}^{-1}\{\frac{5}{s-2} - \frac{4s}{s^2+9}\} = f(t)$

3. $\mathcal{L}^{-1}\{\frac{7}{s^2+6}\} = f(t)$

II (ASSIGNMENT 1)

1. $\mathcal{L}\left\{\dfrac{8-3s+s^2}{s^3}\right\} = f(t)$

SIMPLIFYING:

$\mathcal{L}^-\left\{\frac{8}{s^3}-\frac{3}{s^2}+\frac{1}{s}\right\} = f(t)$

a) $\mathcal{L}^-\{8/s^3\} = \mathcal{L}^-\{4(2)/s^{2+1}\}$
$= 4\mathcal{L}^-\{2!/s^{2+1}\}$
$= \mathcal{L}^-[8/s^3]$
$f(t) = 4t^2$

b) $\mathcal{L}^-\{-3/s^2\} = -3\mathcal{L}^-\{[1/s^2]\}$
$f(t) = -3t$

c) $\mathcal{L}^-\{1/s\} = 1$

THEREFORE:

$f(t) = 4t^2 - 3t + 1$

2. $\mathcal{L}^{-1}\left\{\dfrac{5}{s-2} - \dfrac{4s}{s^2+9}\right\} = f(t)$

a) $\mathcal{L}^-\{5/s-2\} = 5\mathcal{L}^-\{1/s-2\}$
$f(t) = 5(e^{2t})$

b) $\mathcal{L}^-\{4s/s^2+9\} = -4\mathcal{L}^-\{s/s^2+9\}$
$= -4\{s/s^2+3^2\}$
$= -4\cos 3t$

THEREFORE:

$f(t) = 5e^{2t} - 4\cos 3t$

3. $\mathcal{L}^-\left\{\dfrac{7}{s^2+6}\right\} = f(t) = 7\mathcal{L}^-\left\{\dfrac{1}{s^2+6}\right\}$; SINCE $= 1 = \sqrt{6}/\sqrt{6}$
$= 7/\sqrt{6}\,\mathcal{L}^-\{\sqrt{6}/s^2+(\sqrt{6})^2\}$
$= 7/\sqrt{6}\,(\sin\sqrt{6}t)$
$= 7/\sqrt{6}\,(\sin\sqrt{6}t)(\sqrt{6}/\sqrt{6})$
$= 7\sqrt{6}/6\,(\sin\sqrt{6}t)$

THEREFORE:

$f(t) = \dfrac{7\sqrt{6}}{6}(\sin\sqrt{6}t)$

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ASSIGNMENT 2

I. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $F(s) = \dfrac{1}{s(s^2+2s+2)}$

SOLUTION:

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+2)}\right\} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$$

$$1 = A(s^2+2s+2) + (Bs+C)(s)$$

IF $s=0$

$$1 = A(s^2+2s+2) + (Bs+C)(s)$$

$$1 = A(0^2+2(0)+2) + \cancel{[B(0)+C](0)}$$

$$1 = A(2)$$

$$A = 1/2$$

$$1 = (1/2)(s^2+2s+2) + (Bs+C)(s)$$

MULTIPLY BOTH SIDES BY 2

$$[1 = (1/2)(s^2+2s+2) + (Bs+C)(s)]\,2$$

$$2 = (s^2+2s+2) + (2Bs^2+2Cs)$$

COMBINING LIKE TERMS AND FACTORING

$$2-2 = s^2+2Bs^2+2s+2Cs$$

$$0 = s^2(2B+1) + 2s + 2Cs$$

$$0 = s^2(2B+1) + s(2C+2)$$

$$2B+1=0 \qquad 2C+2=0$$

$$B = -1/2 \qquad C = -1$$

SOLVING

$$\mathcal{L}^{-1}\{[(1/2)/s] + [(-1/2)s + (-1)/(s^2+2s+2)$$

$$= 1/2\,\mathcal{L}^{-1}\{1/s\} + \mathcal{L}^{-1}\{[(-1/2)s - 1]/s^2+2s+2\}$$

a.) $1/2\,\mathcal{L}^{-1}\{1/s\} = 1/2(1) = 1/2$

b.) $\mathcal{L}^{-1}\{(-1/2)s - 1/(s^2+2s+2)\}$

$$= \mathcal{L}^{-1}\left\{\frac{-1/2(s+2)}{s^2+2s+2}\right\} = \frac{-1}{2}\mathcal{L}^{-1}\left\{\frac{s+1+1}{s^2+2s+1+1}\right\}$$

$$= \frac{-1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+1}\right\} \text{ WHERE } a=1,\ \omega=1$$

$$= \frac{-1}{2}\mathcal{L}^{-1}\left\{\frac{(s+1)+1}{(s+1)^2+1}\right\} = \frac{-1}{2}e^{-t}[\cos t + \sin t]$$

THEREFORE

$$f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}[\cos t + \sin t]$$

$$= \frac{+1}{2}[1 - e^{t}(\cos t + \sin t)]$$

ASSIGNMENT 2

2. $F(s) = \dfrac{5(s+2)}{s^2(s+1)(s+3)}$

$$\frac{5(s+2)}{s^2(s+1)(s+3)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+3}$$

$$5\mathcal{L}^{-}(s+2) = A(s)(s+1)(s+3) + B(s+1)(s+3)$$
$$+ C(s^2)(s+3) + D(s^2)(s+1)$$

IF $s = 0$;

$$(0+2) = \cancel{A(0)(0+1)(0+3)} + B(0+1)(0+3)$$
$$+ \cancel{C(0^2)(0+3)} + \cancel{D(0^2)(0+1)}$$
$$2 = 3B$$
$$B = 2/3$$

THEN:

$$5\mathcal{L}^{-}(s+2) = A(s^3+4s^2+3s) + \tfrac{2}{3}(s^2+4s+3)$$
$$+ C(s^3+3s^2) + D(s^3+s^2)$$

MULTIPLY BOTH SIDES BY 3

$$3s+6 = 3A(s^3+4s^2+3s) + 2(s^2+4s+3)$$
$$+ 3C(s^3+3s^2) + 3D(s^3+s^2)$$
$$6 = 3A(s^3+4s^2+3s) + 2(s^2+4s+3)$$
$$+ 3C(s^3+3s^2) + 3D(s^3+s^2) - 3s$$
$$6 = (3As^3+12As^2+9As) + (2s^2+8s+6) + (3Cs^3+9Cs^2)$$
$$+ (3Ds^3+3Ds^2)$$
$$0 = 3s^3(A+C+D) + s^2(12A+2+9C+3D) + s(9A+5)$$

GET A:

$$9A+5=0 \; ; \; 9A = -5 = A = -5/9$$

SUBSTITUTE

$$= 9(-5/9+C+D) + (12[-5/9) + 2 + 9C + 3D)$$
$$= (-5+9C+9D) + (-20/3 + 9C + 3D + 2)$$
$$-\; \cancel{9C} + 9D = 5$$
$$\cancel{9C} + 3D = 14/3$$
$$6D = 1/3 \; ; \; D = 1/18$$

SUBSTITUTE

$$A + C + D = 0$$
$$(-5/9) + C + (1/18) = 0$$
$$C = 1/2$$

THEN:

$$5\mathcal{L}^{-}\frac{-5/9}{s} + 5\mathcal{L}^{-}\frac{2/3}{s^2} + 5\mathcal{L}^{-}\frac{1/2}{s+1} + 5\mathcal{L}^{-}\frac{1/18}{s+3}$$

$$f(t) = \frac{-25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}$$

ASSIGNMENT 2

3. $F(s) = \dfrac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$

$$F(s) = \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s^2 + s}$$

$$\begin{array}{r l}
 & s^2 + s + 2 \\
s^2 + s \,\big) & s^4 + 2s^3 + 3s^2 + 4s + 5 \\
 & -\; s^4 + s^3 \\
\hline
 & s^3 + 3s^2 \\
 & -\; s^3 + s^2 \\
\hline
 & 2s^2 + 4s \\
 & -\; 2s^2 + 2s \\
\hline
 & 2s + 5
\end{array}$$

THEN: $F(s) = s^2 + s + 2 + [(2s+5)/(s^2+s)]$

$$F(s) = s^2 + s + 2 + \frac{2s+5}{s(s+1)}$$

SOLVING:

$$\mathcal{L}^{-}\left\{ s^2 + s + 2 + \left[\frac{2s+5}{s(s+1)}\right]\right.$$

a) $\mathcal{L}^{-}\{s^2\} = \dfrac{d^2 f}{dt^2}$ b) $\mathcal{L}^{-}\{s\} = \dfrac{df}{dt}$

c) $\mathcal{L}^{-}\{2\} = 2\mathcal{L}\{1\}$
$= 2\delta(t)$

d) $\mathcal{L}^{-}\left\{\dfrac{2s+5}{s(s+1)}\right\} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$2s+5 = A(s+1) + B(s)$

IF $s=0$; $2(0) + 5 = A(0+1) + \cancel{B(0)}$

$A = 5$

SUBSTITUTE = $2s + 5 = 5(s+1) + B(s)$

$2s + 5 = 5s + 5 + B(s)$

$2s - 5s + \cancel{5 - 5} = B(s)$

$-3s = B(s)$

$B = -3$

THEN = $\mathcal{L}^{-}\left\{\dfrac{2s+5}{s(s+1)}\right. = \mathcal{L}^{-}\left\{\dfrac{5}{s} + \dfrac{-3}{s+1}\right\}$

a) $\mathcal{L}^{-}\{5/s\} = 5\mathcal{L}^{-}\{1/s\} = 5(1) = 5$

b) $\mathcal{L}^{-}\{-3/s+1\} = -3\,\mathcal{L}^{-}\{1/s+1\} = -3e^{-t}$

THEREFORE: $\mathcal{L}^{-}\left\{\dfrac{2s+5}{s(s+1)}\right\} = 5 - 3e^{-t}$

THEREFORE:

$$f(t) = \frac{d^2 f}{dt^2} + \frac{df}{dt} + 2\delta(t) + 5 - 3e^{-t}$$