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ME 4203

ASSIGNMENT 1

I. SOLVE FOR THE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}[3-e^{-3t}+5\sin 2t] = F(s)$

$F(s) = \mathcal{L}(3) - \mathcal{L}(-e^{-3t}) + \mathcal{L}(5\sin 2t)$

$F(s) = \frac{3}{s} - \frac{1}{s+3} + 5\left(\frac{2}{s^2+2^2}\right)$

$\boxed{F(s) = \frac{3}{s} - \frac{1}{s+3} + \frac{10}{s^2+4}}$

2. $\mathcal{L}[3+12t+42t^3-3e^{2t}] = F(s)$

$F(s) = \mathcal{L}(3) + \mathcal{L}(12t) + \mathcal{L}(42t^3) - \mathcal{L}(3e^{2t})$

$F(s) = \frac{3}{s} + \frac{12}{s^2} + 42\left(\frac{3!}{s^{3+1}}\right) - 3\left(\frac{1}{s-2}\right)$

$\boxed{F(s) = \frac{3}{s} + \frac{12}{s^2} + \frac{252}{s^4} - \frac{3}{s-2}}$

3. $\mathcal{L}[(t+1)(t+2)] = F(s)$

$F(s) = \mathcal{L}[t^2+3t+2]$

$F(s) = \mathcal{L}(t^2) + \mathcal{L}(3t) + \mathcal{L}(2)$

$F(s) = \frac{2!}{s^{2+1}} + \frac{3}{s^2} + \frac{2}{s}$

$\boxed{F(s) = \frac{2}{s^3} + \frac{3}{s^2} + \frac{2}{s}}$

II. SOLVE FOR THE INVERSE LAPLACE TRANSFORM OF THE FOLLOWING:

1. $\mathcal{L}^{-1}\left[\frac{8-3s+s^2}{s^3}\right] = f(t)$

$f(t) = \mathcal{L}^{-1}\left[\frac{8}{s^3} - \frac{3s}{s^3} + \frac{s^2}{s^3}\right]$

$f(t) = \mathcal{L}^{-1}\left(\frac{8}{s^3}\right) - \mathcal{L}^{-1}\left(\frac{3s}{s^3}\right) + \mathcal{L}^{-1}\left(\frac{s^2}{s^3}\right)$

$f(t) = 4\left(\frac{2!}{s^{2+1}}\right) - 3t + 1$

$\boxed{f(t) = 4t^2 - 3t + 1}$

2. $\mathcal{L}^{-1}\left[\frac{5}{s-2} - \frac{4s}{s^2+9}\right] = f(t)$

$f(t) = \mathcal{L}^{-1}\left(\frac{5}{s-2}\right) - \mathcal{L}^{-1}\left(\frac{4s}{s^2+9}\right)$

$f(t) = 5\mathcal{L}^{-1}\left(\frac{1}{s-2}\right) - 4\mathcal{L}^{-1}\left(\frac{s}{s^2+9}\right)$

$\boxed{f(t) = 5e^{2t} - 4\cos 3t}$

3. $\mathcal{L}^{-1}\left[\frac{7}{s^2+6}\right] = f(t)$

$f(t) = 7\,\mathcal{L}^{-1}\left[\frac{1}{s^2+\sqrt{6}^2}\right]$

$f(t) = \frac{7}{\sqrt{6}}\,\mathcal{L}^{-1}\left[\frac{\sqrt{6}}{s^2+6}\right]$

$f(t) = \frac{7}{\sqrt{6}}\sin\sqrt{6}t$

$\boxed{f(t) = \frac{7\sqrt{6}}{6}\sin\sqrt{6}t}$

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ASSIGNMENT 2

1. $F(s) = \frac{1}{s(s^2+2s+2)}$

$= \mathcal{L}^{-1}\left[\frac{1}{s(s^2+2s+2)}\right] = \frac{A}{s} + \frac{Bs+C}{s^2+2s+2}$

$1 = A(s^2+2s+2) + s(Bs+C)$

IF $s=0$

$1 = A(0^2+2(0)+2) + 0(B(0)+C)$

$\frac{1}{2} = \frac{2A}{2} \rightarrow A = \frac{1}{2}$

$1 = \frac{1}{2}(s^2+2s+2) + s(Bs+C)$

$1 = \frac{s^2+2s+2+2Bs^2+2Cs}{2}$

$2 = s^2+2s+2+2Bs^2+2Cs$

$2 = s^2(1+2B) + s(2+2C) + 2$

$B = \frac{-1}{2}$ $C = -1$

$= \mathcal{L}^{-1}\left[\frac{1}{s(s^2+2s+2)}\right] = \frac{\frac{1}{2}}{s} - \left[\frac{-\frac{1}{2}s-1}{s^2+2s+2}\right]$

1) $\frac{1}{2}\mathcal{L}^{-1}\left[\frac{1}{s}\right] = \frac{1}{2}(1) = \frac{1}{2}$

2) $\frac{-1}{2}\mathcal{L}^{-1}\left[\frac{s+2}{s^2+2s+2}\right]$

$\mathcal{L}^{-1}\left[\frac{(s+a)+\omega}{(s+a)^2+\omega^2}\right] = e^{-at}\left[\cos\omega t + \sin\omega t\right]$

$= \frac{-1}{2}\mathcal{L}^{-1}\left[\frac{s+1+1}{s^2+2s+1+1}\right]$; $a=1$, $\omega=1$

$= \frac{-1}{2}\mathcal{L}^{-1}\left[\frac{(s+1)+1}{(s+1)^2+1^2}\right]$

$= \frac{-1}{2}e^{-t}\left[\cos t + \sin t\right]$

$\therefore f(t) = \frac{1}{2} - \frac{1}{2}e^{-t}\left[\cos t + \sin t\right]$

$$\boxed{f(t) = \frac{1}{2}\left(1 - e^{-t}(\cos t + \sin t)\right)}$$

2. $F(s) = \frac{5(s+2)}{s^2(s+1)(s+3)}$

$= \mathcal{L}^{-1}\left[\frac{5(s+2)}{s^2(s+1)(s+3)}\right] = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+1} + \frac{D}{s+3}$

① $5(s+2) = A(s)(s+1)(s+3) + B(s+1)(s+3) + C(s^2)(s+3) + D(s^2)(s+1)$

$5s+10 = A(s)(s^2+4s+3) + B(s^2+4s+3) + C(s^3+3s^2) + Ds^3 + Ds^2$

$5s+10 = As^3 + 4As^2 + 3As + Bs^2 + 4Bs + 3B + Cs^3 + 3Cs^2 + Ds^3 + Ds^2$

$5s+10 = As^3 + Cs^3 + Ds^3 + 4As^2 + Bs^2 + 3Cs^2 + Ds^2 + 3As + 4Bs + 3B$

② $5s+10 = s^3(A+C+D) + s^2(4A+B+3C+D) + s(3A+4B) + 3B$

IF $s=0$ USING EQUATION 1

$5(0+2) = A(0)(0+1)(0+3) + B(0+1)(0+3) + C(0^2)(0+3) + D(0^2)(0+1)$

$10 = 3B \rightarrow B = \frac{10}{3}$

IF $s=-1$

$5(-1+2) = A(-1)(-1+1)(-1+3) + B(-1+1)(-1+3) + C(-1)^2(-1+3) + D(-1)^2(-1+1)$

$5(1) = C(1)(2)$

$5 = 2C \rightarrow C = \frac{5}{2}$

IF $s = -3$

$$5(-3+2) = A(-3)(-3+1)(-3+3) + B(-3+1)(-3+3) + C(-3)^2(-3+3) + D(-3)^2(-3+1)$$

$$5(-1) = D(9)(-2)$$

$$-5 = -18D$$

$$D = \frac{5}{18}$$

USING EQUATION 2

$$5s+10 = s^3\left[A + \frac{5}{2} + \frac{5}{18}\right] + s^2\left[4A + \frac{10}{3} + 3\left(\frac{5}{2}\right) + \frac{5}{18}\right] + s\left[3A + 4\left(\frac{10}{3}\right)\right] + 3\left(\frac{10}{3}\right)$$

SOLVING FOR A :

$$A + \frac{5}{2} + \frac{5}{18} = 0 \rightarrow A = \frac{-25}{9}$$

$$A + \frac{25}{9} = 0$$

$$\mathcal{L}^{-1}\left[\frac{\frac{-25}{9}}{s} + \frac{\frac{10}{3}}{s^2} + \frac{\frac{5}{2}}{s+1} + \frac{\frac{5}{18}}{s+3}\right] =$$

1) $\mathcal{L}^{-1}\frac{\frac{-25}{9}}{s} = \frac{-25}{9}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = \frac{-25}{9}(1) = \frac{-25}{9}$

2) $\mathcal{L}^{-1}\frac{\frac{10}{3}}{s^2} = \frac{10}{3}\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = \frac{10}{3}t$

3) $\mathcal{L}^{-1}\frac{\frac{5}{2}}{s+1} = \frac{5}{2}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = \frac{5}{2}e^{-t}$

4) $\mathcal{L}^{-1}\frac{\frac{5}{18}}{s+3} = \frac{5}{18}\mathcal{L}^{-1}\left\{\frac{1}{s+3}\right\} = \frac{5}{18}e^{-3t}$

$$\therefore \boxed{f(t) = \frac{-25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}}$$

3. $F(s) = \dfrac{s^4+2s^3+3s^2+4s+5}{s(s+1)}$

$$F(s) = \frac{s^4+2s^3+3s^2+4s+5}{s^2+s}$$

$$\begin{array}{r|l} & s^2+s+2 \\ s^2+s & s^4+2s^3+3s^2+4s+5 \\ & -s^4 \quad s^3 \\ \hline & s^3+3s^2 \\ & -s^3+s^2 \\ \hline & 2s^2+4s \\ & -2s^2+2s \\ \hline & 2s+5 \end{array}$$

$$F(s) = s^2+s+2 + \frac{2s+5}{s^2+s}$$

$$F(s) = s^2+s+2 + \frac{2s+5}{s(s+1)}$$

$$\mathcal{L}^{-1}\left\{s^2+s+2 + \frac{2s+5}{s(s+1)}\right\}$$

1) $\mathcal{L}^{-1}\{s^2\} = \dfrac{d^2f}{dt^2}$

2) $\mathcal{L}^{-1}\{s\} = \dfrac{df}{dt}$

3) $\mathcal{L}^{-1}(2) = 2\mathcal{L}^{-1}\{1\} = 2\delta(t)$

4) $\mathcal{L}^{-1}\dfrac{2s+5}{s(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1}$

$$2s+5 = A(s+1) + B(s)$$

IF $s=-1$

$$2(-1)+5 = A(-1+1) + B(-1)$$

$$3 = -B \rightarrow B = -3$$

IF $s=0$

$$2(0)+5 = A(0+1) + B(0)$$

$$A = 5$$

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{5}{s} + \frac{-3}{s+1}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{5}{s}\right\} = 5\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 5(1) = 5$$

$$\mathcal{L}^{-1}\left\{\frac{-3}{s+1}\right\} = -3\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = -3e^{-t}$$

$$\mathcal{L}^{-1}\left\{\frac{2s+5}{s(s+1)}\right\} = 5 - 3e^{-t}$$

$$\therefore \boxed{f(t) = \frac{d^2f}{dt^2} + \frac{df}{dt} + 2\delta(t) + 5 - 3e^{-t}}$$