CS492: Probabilistic Programming

Amortised Inference

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KAIST

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(defquery biased-coin []
  (let [r (sample
             (uniform-continuous 0 1))
        a (observe (flip r) true)
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Importance sampling with proposal $q(r)$.

1. Generate $(w_1,r_1), \ldots, (w_n,r_n)$ by running prog.

2. Estimate $\mathbb{E}_{p(r|a,b,c)}[f(r)] \approx \sum_i f(r_i)*(w_i/\sum_j w_j)$.

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$w_1 = 1$

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$w_1 = 1 * p(.4)/q(.4)$

$r_1 = .4$

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$w_1 = .096 * p(.4)/q(.4)$

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$w_1 = .096 * p(.4)/q(.4)$
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$w_2 = .144 * p(.6)/q(.6)$
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How to find good q?

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How to find good q? Use amortised inference!

We often need to infer posterior of one model multiple times with different observations.

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Other examples: Finance model, captcha, brain, etc.

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Amortised inference. 1) Learn a proposal q(x; y) parameterized by obs. y via preprocessing.

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Other examples: Finance model, captcha, brain, etc.

Amortised inference. 1) Learn a proposal $q(x; y)$ parameterized by obs. y via preprocessing. 2) Use $q(x;y_0)$ for any actual observation y_0 later.

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neural nets

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Other examples: Finance model, captcha, brain, etc.

sample/object duality and reverse KL

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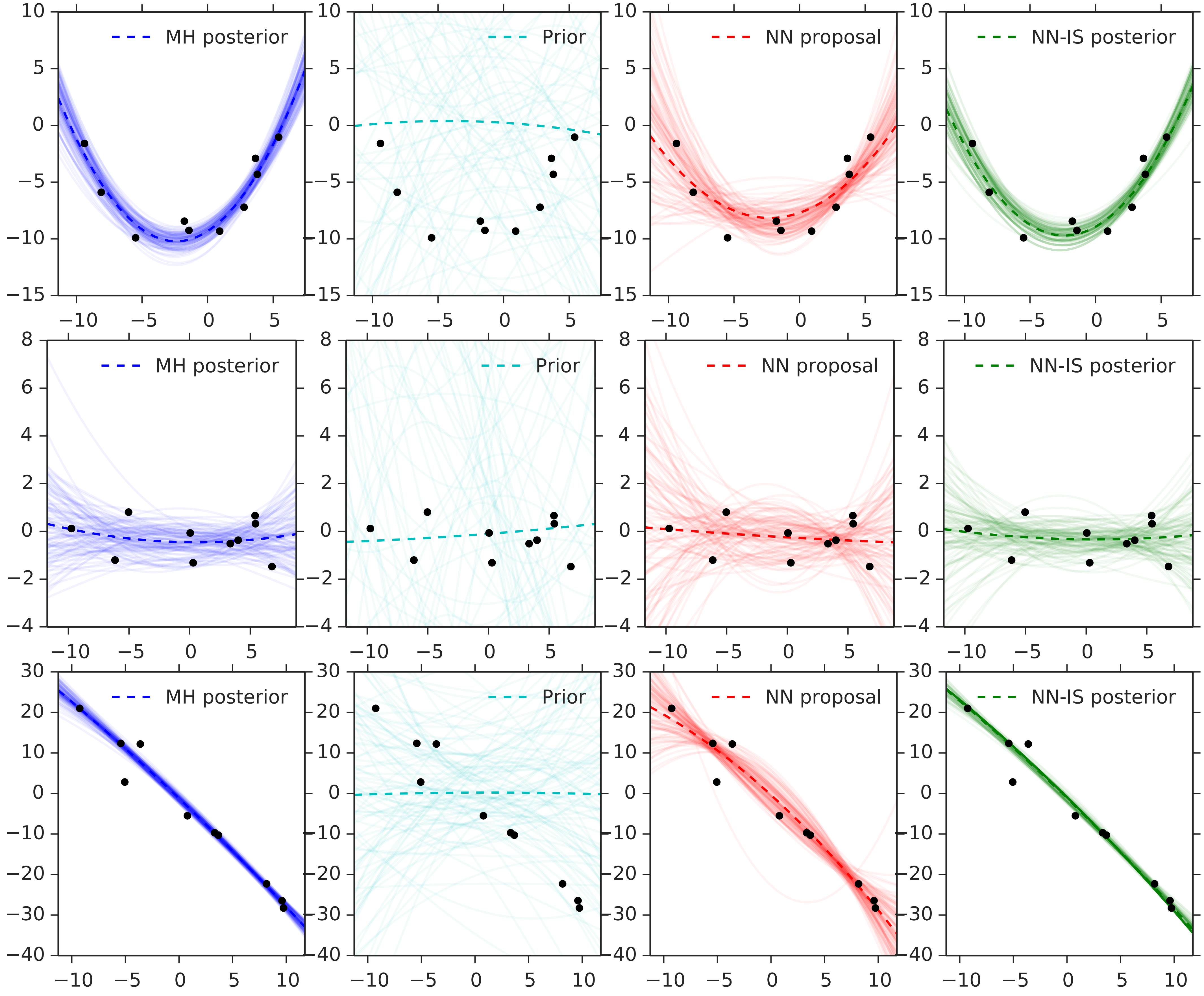
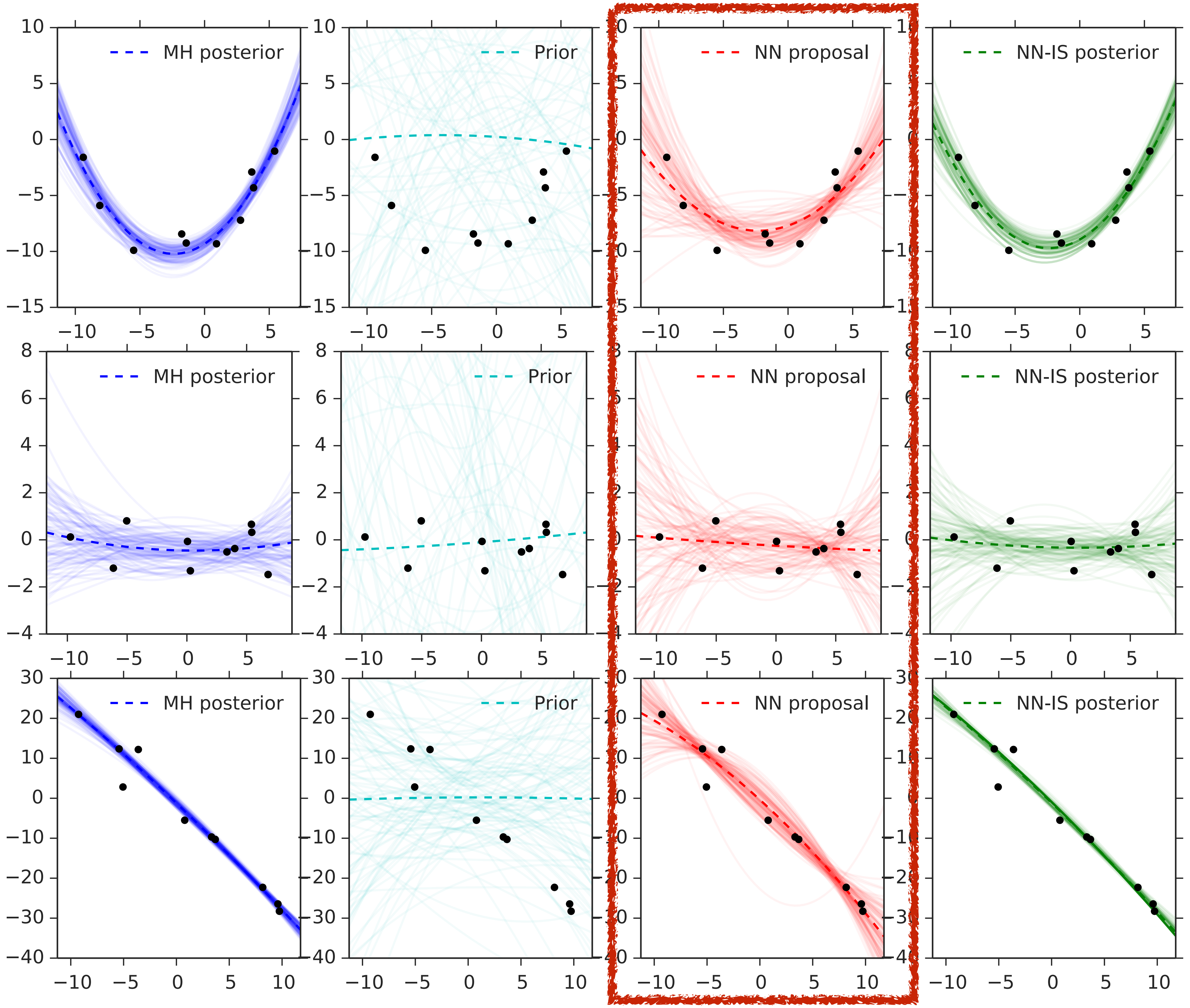
neural nets



Model for non-linear regression [Paige et al., ICML16]



Model for non-linear regression [Paige et al., ICML16]

Observed images

(W4kgvQ) (uV7FeWB) (MqhnpT)

more preprocessing

Samples

W4kgvQ	uV7EeWB	MqhnpT
WA4rjvQ	uV7FeWB	MypppT
Woxewd9	mTTEMMm	RIrpES
BKvu2Q	C9QDsoN	rS5FP2B

less preprocessing

Captcha solving [Le et al., AISTATS16]

Learning outcome

Can describe how amortised inference works for models written in math.

Can explain key ideas behind implementing amortised inference for probabilistic programs.

Proposal learning problem

Given:

1. joint dist. $p(x,y)$ for latent x and observed y,
2. proposal $q_\theta(x;y)$ parameterized by θ and y.

Find θ such that $q_\theta(x;y)$ is good for most y.

Proposal learning problem

Specified by $p(x)$ and $p(y|x)$.
Interested in $p(x|y)$.
But specific y not given yet.

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Differentiable wrt. θ for fixed x,y.
E.g. $q_\theta(x;y) = \text{normal}(x; f_\theta(y), g_\theta(y))$ for neural nets f,g.

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Proposal learning problem tackled by amortised inf.

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y sampled from p(y)

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Small KL divergence from $p(x|y)$ to $q_\theta(x;y)$.

$KL[\, p(x|y) \,||\, q_\theta(x;y) \,] = \mathbb{E}_{p(x|y)}[\, \log(p(x|y)/q_\theta(x;y)) \,]$.

Proposal learning problem

$\text{argmin}_\theta\ \mathbb{E}_{p(y)}[\text{KL}[\ p(x|y)\ ||\ q_\theta(x;y)\ ]]$.
Solve this by stochastic gradient descent.

inf.

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argmin$_\theta$ $\mathbb{E}_{p(y)}$[KL[p(x|y) || q_θ(x;y)]].
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KL[p(x|y) || q_θ(x;y)]= $\mathbb{E}_{p(x|y)}$[log(p(x|y)/q_θ(x;y))].

KL divergence from $p_1(x)$ to $p_2(x)$

Denoted by $KL[p_1(x) \,||\, p_2(x)]$.

$KL[p_1(x) \,||\, p_2(x)] := \mathbb{E}_{p_1(x)}[\log(p_1(x)/p_2(x))]$.

Average log ratio of two probabilities.

Measures how $p_1(x)$ is close to $p_2(x)$.

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[Q] What is $KL[p(x) \,||\, p(x)]$?

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[A] 0

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[A] 0.37

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[Q] What is $KL[[tt \mapsto .4; ff \mapsto .6] \,||\, [tt \mapsto .5; ff \mapsto .5]]$?

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[Q] What is $KL[[tt \mapsto .4; ff \mapsto .6] \,||\, [tt \mapsto .5; ff \mapsto .5]]$?

[A] 0.02

Proposal learning problem

$\mathrm{argmin}_\theta\ \mathbb{E}_{p(y)}[\mathrm{KL}[\ p(x|y)\ ||\ q_\theta(x;y)\ ]]$.
Solve this by stochastic gradient descent.

inf.

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Proposal learning problem inf.

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Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

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Initialise θ

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Initialise θ

$\theta \leftarrow \theta - 0.01 * \nabla_\theta \mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

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(until θ doesn't change)

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Learning rate

Initialise θ

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Can't compute, but can approximate.

Stochastic gradient descent for $\mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]]$

Initialise θ

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Can't compute, but can approximate.

Sample (x_1,y_1), …, (x_n,y_n) from $p(x,y)$.

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$\nabla_\theta \mathbb{E}_{p(y)}[KL[p(x|y)||q_\theta(x;y)]] \approx -1/n * \sum_{i=1..n} \nabla_\theta \log q_\theta(x_i;y_i)$.

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exists since $q_\theta(x_i;y_i)$ is differentiable.

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Initialise θ

Repeat the following until θ doesn't change:

1. Sample (x_1,y_1), …, (x_n,y_n) from $p(x,y)$
2. $G \leftarrow -1/n * \sum_{i=1..n} \nabla_\theta \log q_\theta(x_i;y_i)$
3. $\theta \leftarrow \theta - 0.01 * G$

Proposal learning problem

argmin$_\theta$ $\mathbb{E}_{p(y)}$[KL[p(x|y) || q$_\theta$(x;y)]].
Solve this by stochastic gradient descent.

inf.

y sampled from p(y)

Given:

1. joint dist. p(x,y) for latent x and observed y,
2. proposal q$_\theta$(x;y) parameterized by θ and y.

Find θ such that q$_\theta$(x;y) is good for most y.

Small KL divergence from p(x|y) to q$_\theta$(x;y).
KL[p(x|y) || q$_\theta$(x;y)]= $\mathbb{E}_{p(x|y)}$[log(p(x|y)/q$_\theta$(x;y))].

What about probabilistic programs?

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How to sample y?

Sample/observe duality

To sample observations, just replace sample by observe.

```
(defquery biased-coin []
  (let [r (sample
             (uniform-continuous 0 1))
        a (observe (flip r) true)
        b (observe (flip r) true)
        c (observe (flip r) true)]
   r))
```

Sample/observe duality

To sample observations, just replace sample by observe.

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(defquery biased-coin-joint []
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        b (sample (flip r))
        c (sample (flip r))]
   [r [a b c]]))
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```

[Q] Define a sensible $q_\theta(r;a,b,c)$.

References

1. Inference networks for sequential Monte Carlo in graphical models. Paige et al. ICML'16.
2. Inference compilation and universal probabilistic programming. Let et al. AISTATS'17.