Anfängerpraktikum Part II: ELE

Berke Mert[*] and Stefan Wein[†]
Technical University Munich

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In this experiment, we quantitatively examine the electrons characteristic properties. By measuring circular trajectories of emitted and accelerated electron particles, the specific charge $\frac{q}{m}$ can be determined. Making oil drops float using electric fields as the second part of the experiment allows to determine an electrons charge.

Contents

1. THEORY

1.1. Specific charge of the electron

When a particle of charge q traverses an electrical potential difference U, it gains the kinetic energy

$$E_{\mathrm{kin}} = qU_{\mathrm{B}}, \tag{1}$$

Additionally, if its initial velocity was zero, then the energy is also

$$E_{\mathrm{kin}} = \frac{1}{2}mv^2 \quad , \tag{2}$$

where m is the particles mass and v the velocity after passing U_{B}. Comparing these equations and refactoring we find that

$$v = \sqrt{\frac{2qU_{\mathrm{B}}}{m}} \quad . \tag{3}$$

Furthermore, a charged particle travelling through a magnetic field $\vec{B}$ experiences the Lorentz-force

$$\vec{F}_L = q \cdot \left(\vec{v} \times \vec{B}\right) \quad , \tag{4}$$

which acts perpendicular to the plane span by $\vec{v}$ and $\vec{B}$. Because of this, $\vec{F}_L$ doesn't change the particles kinetic energy, but forces the trajectory to be circular, hence it acts as a centripetal force. Therefore we also know that

$$|\vec{F}_L| = \frac{mv^2}{r} \quad , \tag{5}$$

where r is the radius of the circular trajectory.
These relations yield an expression for the specific charge,

$$\frac{q}{m} = \frac{v}{rB} = \frac{2U_{\mathrm{B}}}{r^2B^2} \quad . \tag{6}$$

Lastly, the magnetic field required for the experiment, generated by the coils, is approximately homogenous and constant. It can be derived to have the value

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 NI}{R} \quad , \tag{7}$$

where μ_0 is the vacuum permeability, N the coil winding number, I the current flowing through the coils and R their radius.

1.2. Millikan oil drop experiment

A small drop of oil within air in an electric field perpendicular to earth's surface experiences all of the following forces:

- The remaining buoyancy $F_b = \frac{4\pi}{3} \cdot r_0^3 \cdot g \left(\rho_{\mathrm{oil}} - \rho_{\mathrm{air}}\right)$
- Stokes'ian friction $F_R = -6\pi \cdot \eta_{\mathrm{air}} \cdot r_0 \cdot v$
- Electrical force $F_E = Q \cdot E = Q \cdot \frac{U}{d}$

While the drop is floating or moving at constant velocity, all forces must sum to zero:

$$F_b + F_R + F_E = 0 \tag{8}$$

Expanding this for both the ascending and descending motion and taking into account that during each motion the voltage is inverse of the other, we get:

$$0 = \frac{4\pi}{3} \cdot r_0^3 \cdot g \left(\rho_{\mathrm{oil}} - \rho_{\mathrm{air}}\right) - 6\pi\eta_{\mathrm{air}} \cdot r_0 \cdot v_{asc/dsc} \pm Q \cdot \frac{U}{d} \tag{9}$$

These equations can be solved for the drops radius r_0 as

$$r_0 = \frac{3}{2}\sqrt{\frac{\eta_{\mathrm{air}}\left(v_{asc} + v_{dsc}\right)}{\rho_\Delta \cdot g}} \tag{10}$$

where $\rho_\Delta = (\rho_{\mathrm{oil}} - \rho_{\mathrm{air}})$, and their charge

$$q = \frac{9\pi d}{2U} \cdot \sqrt{\frac{\eta_{\mathrm{air}}^3\left(v_{asc} + v_{dsc}\right)}{\rho_\Delta \cdot g}} \left(v_{asc} - v_{dsc}\right). \tag{11}$$

The viscosity of air at a pressure of around 1 bar can be approximated using the formula

$$\eta_{\mathrm{air}} = 2.791 \cdot 10^{-7} \cdot T^{0.7355} \; . \tag{12}$$

Additionally, the Lastly, we must use a correction term for the airs' viscosity η_{air} in the Stokes'ian friction formula, because the size of the oil drops is not small compared to the the free moving distance λ of the surrounding air molecules.

$$\eta_{\mathrm{air}} \to \eta_{\mathrm{corr}} = \eta_{\mathrm{air}} \cdot \left(1 + A \cdot \frac{\lambda}{r_0}\right)^{-1} . \tag{13}$$

Although A and λ are normally variable, for the purpose of these experiments, it will be sufficient to assume them as constant with values of $A = 1,257$ and $\lambda = (72 \pm 2)$ nm. This also changes the oil drops radius like

$$r_0 \to r_{\mathrm{corr}} = \sqrt{r_0^2 + \frac{A^2\lambda^2}{4}} - \frac{A\lambda}{2} \; . \tag{14}$$

The charge, calculated using these new inputs, becomes

$$q \to q_{\mathrm{corr}} = \frac{3\pi d}{U} \cdot \eta_{\mathrm{corr}} \cdot r_{\mathrm{corr}} \cdot \left(v_{asc} - v_{dsc}\right) \tag{15}$$

Finally, it should be noted that the charges measured are expected to have values multiple of the elementary charge e, so

$$q_{\mathrm{corr}} = n \cdot e \qquad (n = 1, 2, 3, \ldots) \tag{16}$$

2. METHODS

2.1. Determination of the specific charge

The core element of our experimental setup to determine the specific charge is the cathode ray tube—an evacuated glass beaker with an electron canon. It consists of a heated heated oxidcathode, a Wehneltzylinder and an anode with hole. To make the electron beam visible the beaker is filled with neon gas at 1.3 Pa pressure. A homogenic magnetic field—generated by a helmholtz coil pair, with winding number N = 130 and radius R = (150 ± 2) mm—goes through the cathode ray tube to force the electron beam onto a circular trajectory. We adjust the Wehnelt voltage—the radius of the trajectory shouldn't become smaller—until we get a sharp distinct electron beam.
Now we take two sets of data, where we measure the

1. the voltage at a multimeter ten times at (30 ± 0.5) mm, (40 ± 0.5) mm and (30 ± 0.5) mm for a constant current, (1.45 ± 0.13) A, and then

2. the current at a multimeter ten times at (30 ± 0.5) mm, (40 ± 0.5) mm and (30 ± 0.5) mm for a constant acceleration voltage, (170 ± 2.2) V.

2.2. Millikan experiment

In the Millikan experiment, we spray oil drops into a capacitor, with plate distance (6.00 ± 0.05) mm and a variable voltage from 0 to 600 V, whose polarity we can reverse with an external switch. Since our capacitor operates in the dark, we can shine at our oil droplets with a light source, scattering on them and reflecting into the micrometer-ocular. We can then set up a digital grid on our software that serves as "finish lines", where we stop the time for each of twenty individual ascending and descending oil drops, and note down the voltage, the distance traveled and the time it took them.
Parallel groups and the moodle website of the experiment will give us the other sets of data.

3. PRELIMINARY CONSIDERATIONS

3.1. Classical limit

In equation (2) we implicitly assumed that we can calculate energy classically. However this assumption is not trivial, since—according to Einstein's Theory of SR—the kinetic energy is only given by

$$E_{\text{kin}} = mc^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right) \approx \frac{1}{2}mv^2 \quad ,$$

for velocities less than ten percent of the speed of light. Let's check this condition for the given acceleration U_{B} of magnitude 300 V:

$$v = \sqrt{\frac{2eU_{\text{B}}}{m_{\text{e}}}} \approx 0.03c \ll 0.1c \quad .$$

Since we are measuring voltages less than 300 V we can safely use the classical approximation of the kinetic energy in the part of the experiment where we determine the specific charge of the electron.

3.2. Initial-zero velocity of the electron

To check if the electrons in the cathode initially are resting, we first assume that the velocity of the electron leaving the cathode is one percent the speed of light, then we get

$$W_{\text{emitted}} = \frac{1}{2}m_{\text{e}}v^2 \approx 0.01 \text{ eV} \quad .$$

With the thermic energy, $W_{\text{Thermic}} = \frac{3}{2}k_{\text{B}}T$—where T is the temperature and k_{B} is Boltzmann's constant—, and the work function W_{A} of magnitude 1 eV, we get the energy conservation law,

$$W_{\text{Thermic}} = W_{\text{emitted}} + W_{\text{A}} \quad ,$$

that yields

$$T = \frac{2(W_{\text{emitted}} + W_{\text{A}})}{3k_{\text{B}}} \approx 7814 \text{ K}.$$

This temperature however is way higher than the maximum temperature of 1000K that our cathode approaches, which is why we can assume the resting of the electron before leaving the cathode.

3.3. Influence of the earth's magnetic field

Earth's magnetic field has a magnitude of 20 μT in the horizontal direction, meaning it could have an impact on the radius of the electron's circular trajectory.
However the deviation of the radius from the system without earth's magnetic field is negligably small,

$$\delta r = \sqrt{\frac{2U_{\text{B}}m_{\text{e}}}{eB^2}} - \sqrt{\frac{2U_{\text{B}}m_{\text{e}}}{eB_{\text{eff}}^2}} \approx 0.6 \text{ mm} \quad ,$$

where $B_{\text{eff}} = B + B_{\text{earth}}$. We can't really observe such a small deviation, which is why we can neglect earth's magnetic field.

4. RESULTS AND DISCUSSION

4.1. Determination of the specific charge

Together with equations (7) and (6) we can determine the specific charge of the electron; for the uncertainties we refer to equations (A8) and (A9).
With the determined six values for the specific charge, we can calculate the weighted mean average and its uncertainties according to equations (A3) and (A5); our final experimental measurement of the specific charge is

$$\left(\frac{q}{m}\right)_{\text{exp}} = (193.2 \pm 7.4)\ \frac{\text{GC}}{\text{kg}} \quad .$$

Comparing this result to the literature value, $\left(\frac{q}{m}\right)_{\text{lit}} = 175.9\ \frac{\text{GC}}{\text{kg}}$, we can see that—even though the result is close to the literature values—the experiment doesn't match within its uncertainty range with the literature value.
Throughout the experiment, we could have done multiple errors, which would explain the small deviation from the literature value. For example before every measurement we adjusted the Wehnelt voltage, so we could get a sharp and distinct circular trajectory of the electron. But the Wehnelt voltage is effectively a counter voltage against U_{B}, which then lowers the measured voltage at the multimeter and the radii of the circular trajectories, therefore rigging the measurements. Additionally when taking the measurement we didn't account for possible reading error that could explain the experimental result further.
With these points in mind the result of our experiment is still satisfactory, since our deviation from the literature value is quite small at less than one percent.

TABLE I: Specific charge measurements for constant current $I = (1.45 \pm 0.13)$ A.

Radius in [mm]	Voltage in [V][a]	Specific Charge in [GC/kg]
30.0 ± 0.5	134.1 ± 0.6	233.4 ± 42.7
40.0 ± 0.5	191.9 ± 0.7	187.9 ± 34.1
50.0 ± 0.5	287.5	180.1 ± 32.6

[a]For uncertainties, check equations (A3) and (A5).

TABLE II: Specific charge measurements for constant acceleration Voltage $U_{\text{B}} = (170.0 \pm 2.2)$ V.

Radius in [mm]	Current in [A][a]	Specific Charge in [GC/kg]
30.0 ± 0.5	1.83 ± 0.04	185.5 ± 12.0
40.0 ± 0.5	1.34 ± 0.04	195.8 ± 14.0
50.0 ± 0.5	1.05 ± 0.04	203.5 ± 16.5

[a]For uncertainties, check equations (A3) and (A5).

4.2. Millikan experiment

With our measurements recorded, the goal is to calculate the oil droplets' charges. During the experiment, the rooms air pressure was $p = (725 \pm 1)$ mmHg ≈ 0.997 bar ≈ 1 bar, which enables us to use Equation (12) to calculate the airs' viscosity. With the measured temperature of $T = (24 \pm 0.5)$ °C and propagating the measurement uncertainty according to equation (A1), the resulting value is $\eta_{\text{air}} = (18.4 \pm 0.0)$ μPas. Performing the necessary calculations to finally obtain q_{corr} together with their uncertainties as in (A1) and visualizing it in a plot yields the following: The graph should show sudden

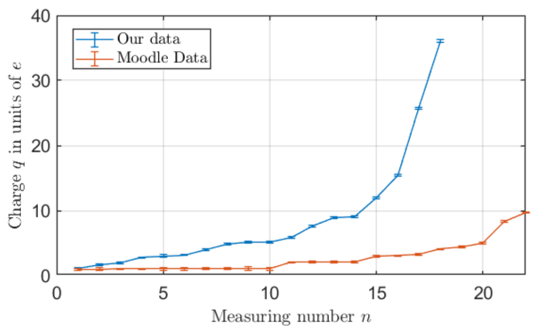

FIG. 1: Various charges of oil droplets measured in the millikan experiment.

jumps in value, indicating the quantisation of the elementary charge. Choosing a handful of measures around the value of $n \cdot e$, we can calculate the weighted mean value as in equation (A3) and its uncertainty like equation(A5) to obtain a final experimental value for the charge e:

$$\bar{q}_{exp} = (1.620 \pm 0.033) \cdot 10^{-19}\ \text{C} \quad . \tag{17}$$

This result is in uncertainty range to the literature value of $e = 1.609 \cdot 10^{-19}$ C, so this experiment produces satisfactory results.

4.3. Electron mass

Since we now measured the specific charge $\frac{e}{m}$ and the charge e of the electron, we can also calculate its mass.

$$m_e = \frac{q_{exp}}{(q/m)_{exp}} = (8.389 \pm 0.365) \cdot 10^{-31}\ \text{kg} \tag{18}$$

Compared with the literature value of $m_{e,lit} = 9.109 \cdot 10^{-31}$ kg, the deviation of about 8% is likely caused mostly by the Wehnelt voltage (see section(4.1)). Since it distorted the measurements of the specific charge, it also propagates into this result, as it is calculated using the other value.

[1] M. Sass. Hinweise zur Beurteilung von Messungen, Messergebnissen und Messunsicherheiten (ABW). Year 2021, last updated.

Appendix A: Auxiliary material

The Uncertainty calculations are done using this reference material [1].

1. Gaussian Error Propagation

We can use the Gaussian error propagation,

$$u(g)=\sqrt{\sum_{i=1}^{n}\left(\frac{\partial g}{\partial x_i}u(x_i)\right)^2} \quad , \tag{A1}$$

assuming the variables x_i , $i \in \{1,\dots,n\}$ of the function g are independent; $u(x_i)$ is the uncertainty of the variable.

2. Consideration of multiple uncertainties

Individual can have uncertainty of type A and multiple uncertainties of type B. The overall uncertainty is then given by

$$u(x_i)=\sqrt{u_A(x_i)^2+u_{B1}(x_i)^2+u_{B2}(x_i)^2+\dots} \quad . \tag{A2}$$

3. Weighted mean value

The weighted mean value can be calculated using

$$\overline{x}=\frac{\sum_{i=1}^{n}w_i x_i}{\sum_{i=1}^{n}w_i} \tag{A3}$$

whereby

$$w_i=\frac{1}{u(x_i)^2} \quad . \tag{A4}$$

The uncertainty is given by

$$\begin{aligned} u_{int}(\overline{x}) &= \sqrt{\frac{1}{\sum_{i=1}^{n}w_i}} \\ u_{ext}(\overline{x}) &= \sqrt{\frac{\sum_{i=1}^{n}w_i(x_i-\overline{x})^2}{(n-1)\sum_{i=1}^{n}w_i}} \\ u(\overline{x}) &= \max(u_{int},u_{ext}) \end{aligned} \tag{A5}$$

4. Uncertainties of the multimeter

The handbook for the used multimeter lists the measuring tolerances in a given measuring range. The uncertainties consist of an accuracy and a resolution, which gives us

$$u(U)=\sqrt{(0.8\%\cdot U+0.8)^2+\left(\frac{1\text{ V}}{2\sqrt{3}}\right)^2} \quad \text{and} \tag{A6}$$

$$u(I)=\sqrt{(2\%\cdot I+0.8)^2+\left(\frac{0.01\text{ A}}{2\sqrt{3}}\right)^2} \quad , \tag{A7}$$

where we used equation (A2).

5. Uncertainty of the magnetic field strength

We determine this uncertainty according to equation (A1),

$$u(B)=B\sqrt{\left(\frac{u(I)}{I}\right)^2+\left(\frac{u(R)}{R}\right)^2} \quad . \tag{A8}$$

6. Uncertainty of the specific charge

We determine this uncertainty according to equation (A1),

$$u\left(\frac{q}{m}\right)=\frac{q}{m}\sqrt{\left(\frac{u(U_{\mathrm{B}})}{U_{\mathrm{B}}}\right)^2+\left(\frac{2u(B)}{B}\right)^2+\left(\frac{2u(r_{\mathrm{k}})}{r_{\mathrm{k}}}\right)^2} . \tag{A9}$$

7. Uncertainties of the Millikan-experiment calculations

By Gaussian error propagation as in equation (A1), for the viscosity of air it is

$$u\left(\eta_{\mathrm{air}}\right)=\sqrt{\left(\frac{\partial\eta_{\mathrm{air}}}{\partial T}\cdot u(T)\right)^2} \tag{A10}$$

With the expression for η_{air} from equation (12) the simple derivative becomes

$$\frac{\partial\eta_{\mathrm{air}}}{\partial T}=2.053\cdot 10^{-7}\frac{1}{T^{0.2645}} \quad . \tag{A11}$$

For the uncertainty in T of $u(T)=0.5$ °C, it propagates into η_{air} as per

$$u(T)=1.026\cdot 10^{-7}\frac{1}{T^{0.2645}} \tag{A12}$$

At the measured temperature of $T=24$ °C $=297$ K it yields $u(\eta_{\mathrm{air}})=0.02$ μPas.