Anfängerpraktikum Part II: TRA

Berke Mert* and Stefan Wein†

Team 3-01

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This lab report is about the application of the bipolar junction transistor as a voltage and current amplifier in the grounded emitter circuit. Differential parameters are determined in the small signal approximation as well as the frequency response curves to quantify the BJT. It will be conlcuded that the given mathematical model for the BJT describes everything accurately but the dynamic pattern of the frequency response curves.

Contents

1. THEORY

1.1. Basics of the Bipolar-Junction-Transistor

A Bipolar-Junction-Transistor (BJT) is a semiconductor that conducts, when current flows through the base, or blocks current flow, when no current flows through the base. A semiconductor is basically a chemical substance, for example silicium, whose electrical properties are changed in the process of doping.

If we exchange a silicium atom with a donator— an atom with one valence electron, e.g. arsene—, we get an n-doped material and if we exchange silicium atom with an acceptor— an atom with seven valence electron, e.g. bor—, we get a p-doped material. When we have a material with two pn-junctions—when a p- and an n-doped material border each other—, we get a BJT. Figure (1) shows a schematic drawing of a npn-BJT.

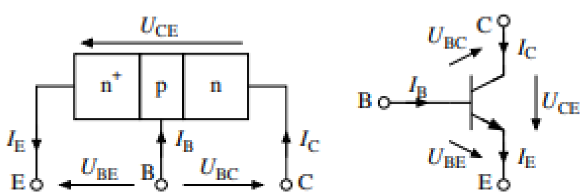

FIG. 1: Circuit symbol of a npn-BJT.

1.2. Mathematical description of the BJT in the small signal approximation

In the small signal approximation we consider the impact of a small perturbation around the working point in first order. A taylor-series of the Eber-Moll equation yields this set of equations in admittance form, if we neglect zeroth order terms,

$$\begin{pmatrix} i_{\mathrm{b}} \\ i_{\mathrm{c}} \end{pmatrix} = \begin{pmatrix} 1/r_{\mathrm{BE}} & 0 \\ S & 1/r_{\mathrm{CE}} \end{pmatrix} \begin{pmatrix} u_{\mathrm{BE}} \\ u_{\mathrm{CE}} \end{pmatrix} \quad , \tag{1}$$

with the differential input resistance, r_{BE}, as $1/r_{\mathrm{BE}} = \frac{\partial I_{\mathrm{B}}}{\partial U_{\mathrm{BE}}}|_{U_{\mathrm{CE}}}$, the differential output resistance, r_{CE}, as $1/r_{\mathrm{CE}} = \frac{\partial I_{\mathrm{C}}}{\partial U_{\mathrm{CE}}}|_{U_{\mathrm{BE}}}$, and the steepness, $S = \frac{\partial I_{\mathrm{C}}}{\partial U_{\mathrm{BE}}}|_{U_{\mathrm{CE}}}$. With the collector current

$$I_{\mathrm{C}} = I_{\mathrm{S}} \exp\left(\frac{qU_{\mathrm{BE}}}{k_{\mathrm{B}}T}\right) \quad , \tag{2}$$

we get

$$S = \frac{qI_{\mathrm{C}}}{k_{\mathrm{B}}T} \quad , \tag{3}$$

where I_{S} is the theoretical reverse current, q is the unit charge, k_{B} is Boltzmann's constant and T is the temperature of the BJT.

The basis current also takes an exponential form,

$$I_{\mathrm{B}} \propto \exp\left(\frac{qU_{\mathrm{BE}}}{k_{\mathrm{B}}T}\right) \quad . \tag{4}$$

The voltage amplification without current-feedback can be determined using

$$A = -S \cdot (R_{\mathrm{C}} \parallel r_{\mathrm{CE}} \parallel R_{\mathrm{L}}) \quad , \tag{5}$$

and the voltage amplification with current-feedback can be determined using

$$A = -\frac{\beta \cdot (R_{\mathrm{C}} \parallel R_{\mathrm{L}})}{r_{\mathrm{BE}} + (1+\beta)R_{\mathrm{E}}} \quad . \tag{6}$$

2. METHODS

2.1. Adjustment of the working point

We build a circuit according to figure (2)—without the ohmic resistor, R_L.

To set the working point, we choose R_C to be 10 kΩ, change the input voltage, u_e, to a sinusoidal signal form with an amplitude of 10 mV and a frequency of 5.5 kHz. We now maximize the amplitude of output signal, u_a, without distorting the signal form by setting the resistance of the potentiometer, R_{21}, to 7.02 kΩ.

Measuring the the basis-emitter voltage, U_{BE}, the collector-emitter voltage, U_{CE}, and the collector current, I_C for different values of R_C yields table (I).

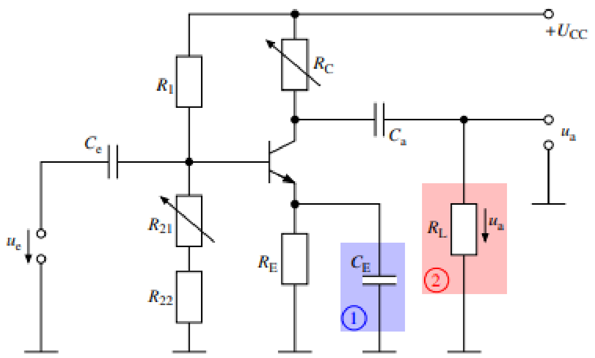

FIG. 2: Circuit to set the working point.

TABLE I: Measurements for the working point.

R_C in [kΩ]	U_{BE} in [V]	U_{CE} in [V]	I_C in [mA]
1.00	0.58	7.77	0.53
5.01	0.58	5.63	0.54
10.04	0.58	2.93	0.54

2.2. Measurement of the characteristic curves

We build a circuit according to figure (3) with the resistors

- $R_1 = 1$ kΩ,
- $R_2 = 220$ Ω,
- $R_X = 1$ kΩ.

To measure the input characteristic curve, we measure U_{CE} in range of 0 to 670 mV and note down I_B while we keep U_{CE} at a constant values of 5.63 V.

Now, to measure the output characteristic curve, we measure U_{CE} in range of 0 to 10 V and note down I_C while we keep U_{BE} at a constant values of 0.58 V.

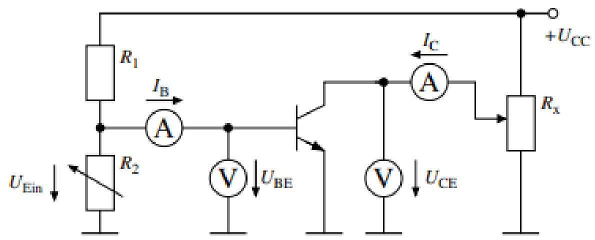

FIG. 3: Circuit to measure the characteristic curves of the grounded emitter circuit.

2.3. Amplification of the grounded emitter circuit

We determine the amplification of three different configurations of the grounded emitter circuit, see figure (2), by measuring $\hat{u}_a$ and $\hat{u}_e$ at 5.5 kHz input signal frequency for different values of R_{C}. The three different configurations are the circuit

1. with the Capacitor, C_{E}, but without the ohmic resistor, R_L,
2. without C_{E} and R_L,
3. with C_{E} and R_L.

2.4. Measurement of the frequency response curves

To determine the transfer function curve and phase-shift curve of figure (2), we measure the time difference between neighbouring peaks of the input and output voltage and measure the amplitudes $\hat{u}_{\mathrm{a}}$ and $\hat{u}_{\mathrm{e}}$ for frequencies in range of 6 Hz to 250 kHz. Here we also choose R_C to be of magnitude 10 kΩ.

3. PRELIMINARY CONSIDERATIONS

3.1. Impact of parasitic Elements in the BJT

We need to consider parasitic elements—resitances and capacities of real wires—-in the BJT, see figure (4), to have a better understanding of the dynamic patterns of the frequency response curve.

To do that we determine the values for r_{BE} and C_{BE}, r_{CE} and C_{CE} and r_{BC} and C_{BC} with a voltmeter at each component.

However this poses a problem. We practically cannot seperate the capacity from the resistance in a real wire, so we need to measure the voltage at a parallel circuit which only gives us the total resistance $\frac{1}{R} = \left|\frac{1}{Z_1} + \frac{1}{Z_2}\right|$ and not the resistance of each component.

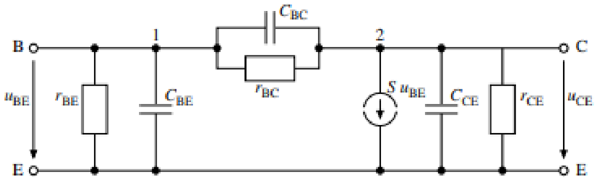

FIG. 4: A BJT with parasitic elements.

3.2. Hybrid form of small signal approximation

We want to determine the hybrid form of equation (1). Generally, the matrix-vector equation in admittance form is given by,

$$\begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \quad , \tag{7}$$

and in hybrid form by,

$$\begin{pmatrix} u_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} i_1 \\ u_2 \end{pmatrix} \quad . \tag{8}$$

To obtain the hybrid form of equation (1) we use the following calculation rules:

$$\begin{aligned} h_{11} &= \frac{1}{y_{11}} & h_{12} &= -\frac{y_{12}}{y_{11}} \\ h_{21} &= \frac{y_{21}}{y_{11}} & h_{12} &= \frac{y_{11}y_{22} - y_{12}y_{21}}{y_{11}} \quad . \end{aligned}$$

This gives us the small signal approximation in hybrid form

$$\begin{pmatrix} u_{\mathrm{BE}} \\ i_{\mathrm{c}} \end{pmatrix} = \begin{pmatrix} r_{\mathrm{BE}} & 0 \\ S r_{\mathrm{BE}} & 1/r_{\mathrm{CE}} \end{pmatrix} \begin{pmatrix} u_{\mathrm{BE}} \\ u_{\mathrm{CE}} \end{pmatrix} \quad . \tag{9}$$

The relation between S and β is given by

$$S = \frac{i_\mathrm{b}}{u_\mathrm{e} - u_\mathrm{re}}\beta \quad , \tag{10}$$

with the input voltage, u_e, and the voltage at R_E, u_re.

4. RESULTS AND DISCUSSION

4.1. Characteristic curves and small signal parameters

Figures (5) and (6) show the characteristic curves of the BJT. We fit the first curve using

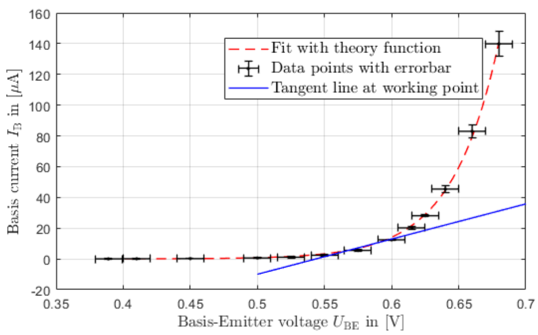

FIG. 5: Input characteristic curve.

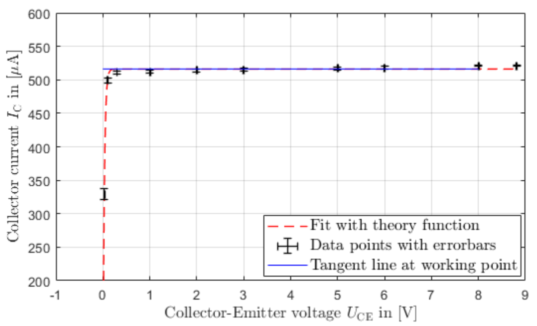

FIG. 6: Output characteristic curve.

the theory function

$$f(x) = (440.8{\pm}319.1){\cdot}10^{-9}\exp\left[(28.80 \pm 1.08)x\right]$$

acording to equation (4), and the second curve with the function

$$g(x) = (-516.2{\pm}17.9){\cdot}(\exp[(-33.83{\pm}9.22)x]-1) \quad .$$

We have to point out that the second fit is not physical—since the output characteristic curve doesn't represent an exponential function—, but still practical because we want to determine the differential output resistance.

Differentiating both functions both fit functions and evaluating them at the working point, see table (I), yields us the inverse of the differential resistances,

$$r_\mathrm{BE} = \left(\left.\frac{df}{dx}\right|_{x=0.58}\right)^{-1} = (4.38 \pm 4.46)\ \mathrm{k\Omega} \quad ,$$

$$r_\mathrm{CE} = \left(\left.\frac{dg}{dx}\right|_{x=0.58}\right)^{-1} = (20.0 \pm 3.7)\ \mathrm{k\Omega} \quad .$$

We can also calculate the working temperature of the transistor by using the fit parameter $b = (28.80 \pm 1.08)$, since the relation

$$b = \frac{q}{k_\mathrm{B}T} \quad ,$$

holds true: $T = (402.9 \pm 15.1)$ K. At last we can determine the steepness with $I_\mathrm{C} = 0.54$ mA and the previously determined working temperature according to equation (3),

$$S = (15.55 \pm 0.58)\frac{1}{\mathrm{k\Omega}} \quad .$$

4.2. Amplification in the grounded emitter circuit

We can model the voltage amplification of configuration one with the equation (5), which simplifies for big values of r_{CE} to

$$A \approx -SR_{\text{C}} = (-18.3 \pm 0.2)\frac{1}{\text{k}\Omega} \cdot R_{\text{C}} \quad .$$

We can use this because we have a high input frequency of 5.5 kHz and the capacitor effectively acts as a short-circuit, meaning we have no current feedback in this circuit.

For the next configuration we don't have a capacitor, so we have no current feedback. The amplification can be described by equation (6) which simplifies for big values of β and R_L to

$$A \approx -\frac{R_{\text{C}}}{R_{\text{E}}} = -\frac{1}{(-1.03 \pm 0.02)\ \text{k}\Omega} R_{\text{C}} \quad .$$

For the same reason as configuration one we use equation (5) for the last configuration and get for big values of r_{CE}

$$A \approx -S \cdot (R_{\text{C}} \parallel R_{\text{L}}) = -\frac{(-18.5 \pm 0.8)\frac{1}{\text{k}\Omega}}{1/R_{\text{C}} + 1/(10.0 \pm 1.1)\text{k}\Omega}$$

We can see that the fit values of the parameters R_{L} and R_{E}, are or would be in range of the theoretical value if there was no negative sign in the equation.

We can also compare the fit value of the parameter S with S from the previous chapter. Again we have the same problem as before and additionally see that they do not match within their confidence range, which could be consequence of the 'not so physical' fit we did earlier or a missing minus sign in the calculations. Nonetheless, the results are really close to each other and therefore satisfactory.

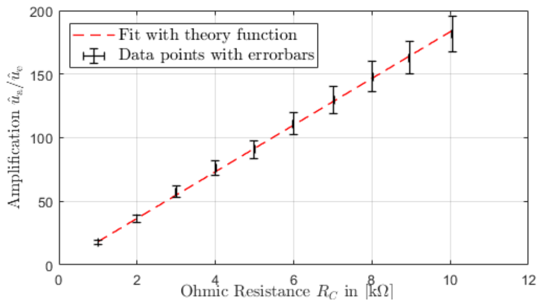

FIG. 7: Configuration 1.

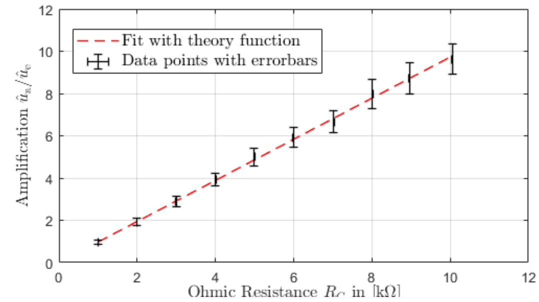

FIG. 8: Configuration 2.

4.3. Frequency response curves

Figures (10) and (11) show the frequency response curves of the BJT for frequencies from 5 Hz to 250 kHz.

We instantly recognize that for small amplitudes we get a phase-shift of $(2n+1)\frac{\pi}{2}$, where n is an integer. For big amplitudes on the other hand

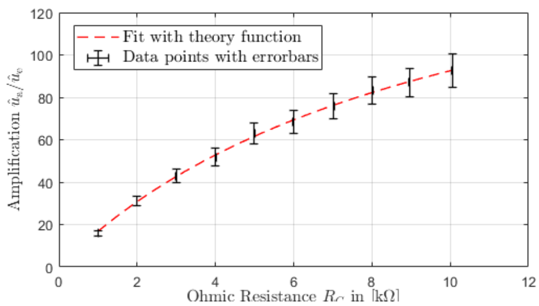

FIG. 9: Configuration 3.

we get a phase-shift of about πn.

Looking at the transfer function curve, we notice that for low frequencies in range of 5 Hz to 100 Hz we get a high-pass filter. For high frequencies in range of 10 kHz to 250kHz we get a low-pass filter.

We also find that in range of 10 Hz to 150 kHz the Amplification is higher than $\frac{A_{\max}}{\sqrt{2}}$. For a detailed explanation of the dynamic pattern of the frequency response curves we refer to chapter (3.1).

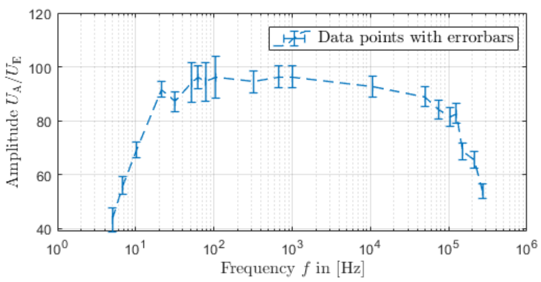

FIG. 10: Transfer function curve of the BJT.

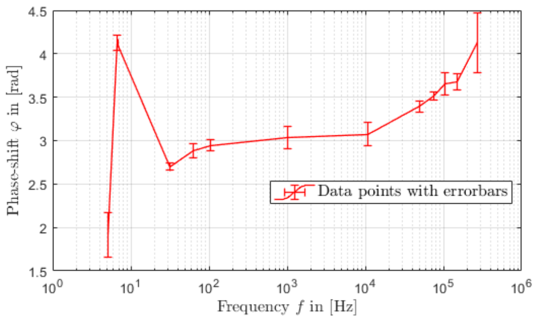

FIG. 11: Phase-shift curve of the BJT.

5. SHORT QUESTION–NETWORK OF A HIGH-PASS AND LOW-PASS FILTER

A simple two-port network, see figure (12), can be described by the equations:

$$\begin{pmatrix} \underline{I}_e \\ \underline{I}_a \end{pmatrix} = \begin{pmatrix} 1/\underline{Z}_1 & -1/\underline{Z}_1 \\ -1/\underline{Z}_1 & 1/\underline{Z}_1 + 1/\underline{Z}_2 \end{pmatrix} \begin{pmatrix} \underline{U}_e \\ \underline{U}_a \end{pmatrix} \quad .$$

We set $\underline{I}_a = 0$ and simplify these equation to

$$H(Z_1, Z_2) = \frac{\underline{U}_a}{\underline{U}_e} = \frac{Z_2}{Z_1 + Z_2}$$

which is called the transfer function.

If we want to build a high-pass filter we choose $\underline{Z}_1 = \frac{1}{i\omega C}$ and $\underline{Z}_2 = R$, with the ohmic resistor R and capacity C; for a low-pass filter we change the Impedances. This yields us the transfer functions

$$H_{\mathrm{HP}}(\omega) = \frac{R}{R + 1/i\omega C}$$
$$H_{\mathrm{LP}}(\omega) = \frac{1}{1 + 1/i\omega RC}$$

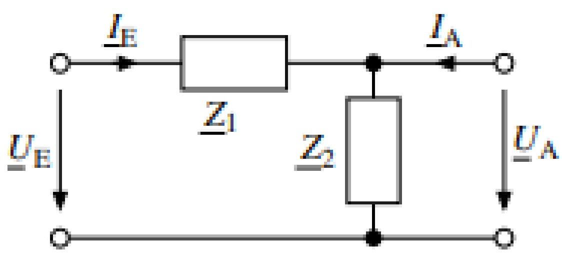

FIG. 12: A simple two-port network.

[1] Technische Universtität München. Hinweise zur Beurteilung von Messungen, Messergebnissen und Messunsicherheiten (ABW). Year 2021, last updated.

Appendix A: Auxillary material

The Uncertainty calculations are done using this reference material [1].

1. Gaussian Error Propagation

We can use the Gaussian error propagation,

$$u(g) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i} u(x_i) \right)^2} \quad , \tag{A1}$$

assuming the variables x_i , $i \in \{1, ..., n\}$ of the function g are independent; $u(x_i)$ is the uncertainty of the variable.

2. Consideration of multiple uncertainties

Individual can have uncertainty of type A and multiple uncertainties of type B. The overall uncertainty is then given by

$$u(x_i) = \sqrt{u_A(x_i)^2 + u_{B1}(x_i)^2 + u_{B2}(x_i)^2 + ...} \quad . \tag{A2}$$

3. Weighted mean value

The weighted mean value can be calculated using

$$\overline{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} \tag{A3}$$

whereby

$$w_i = \frac{1}{u(x_i)^2} \quad . \tag{A4}$$

The uncertainty is given by

$$u_{int}(\overline{x}) = \sqrt{\frac{1}{\sum_{i=1}^{n} w_i}}$$

$$u_{ext}(\overline{x}) = \sqrt{\frac{\sum_{i=1}^{n} w_i (x_i - \overline{x})^2}{(n-1) \sum_{i=1}^{n} w_i}}$$

$$u(\overline{x}) = \max(u_{int}, u_{ext}) \tag{A5}$$