Measurement of the Neutrino Mass in the KATRIN experiment

Fortgeschrittenen-Praktium, Physik Department, TUM

Berke Mert
Matr. 03738893

Tianrui Xu
Matr. 03716809

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1 Introduction

The Neutrino is a fundamental particle, embedded in the Standard Model. They were first postulated by Wolfgang Pauli to explain the missing energy in the energy spectrum of the electron in radioactive β-decay. The occurrence of Neutrino oscillations proves that Neutrinos have mass, as opposed to originally believed. However, these experiments are sensitive to the difference of the squared massed of two mass states and therefore don't show the absolute mass scale of the Neutrino.
The Karlsruhe Tritium Neutrino (KATRIN) experiment attempts to measure the mass of neutrinos with precision.
This lab report attempts to replicate the results of the KATRIN experiment by first using a simplified model of the Tritium β-decay spectrum that is unique to KATRIN with Monte Carlo Data and then using basics of Data Analysis to set an upper and lower bound to the neutrino mass to evaluate the sensitivity of the experiment.

2 Concepts of the KATRIN experiment

In the β-decay of a tritium isotope, a daughter isotope, an electron and an electron antineutrino are produced.

$$\mathrm{T}_2 \rightarrow (^3\mathrm{HeT})^+ + e^- + \overline{\nu}_e + Q \tag{1}$$

The endpoint energy, E_0, is conserved in this process, meaning the surplus energy Q minus the recoil energy of the daughter isotope E_{rec} is distributed among the electron and neutrino, with the energies E and E_ν:

$$E_0 = Q - E_{rec} = E - E_\nu \tag{2}$$

This means we can deduce the energy of the neutrino, and therefor the mass of the neutrino, by determining the root of the differential rate for an allowed β-decay,

$$\frac{d\Gamma}{dE} = C \cdot F(Z', E) \cdot p \cdot (E + m_e) \cdot (E - E_0) \cdot \sqrt{(E - E_0)^2 - m_\nu^2} \cdot \Theta(E_0 - E - m_\nu) \ , \tag{3}$$

in a sub eV scale.
The important part of the KATRIN experiment happens in the MAC-E filter, which realizes the measurement of the differential rate by applying a counter voltage U, that effectively acts as a filter for electrons with energy higher than qU. Additionally, there are magnetic fields that adiabatically (meaning momentum is conserved) project the momentum of the filtered electrons in the direction of the detector, so they can reach the detector as well. The detector is a silicon-based focal-plan detector, that counts the event of an electron hitting the detector in each of its pixels.

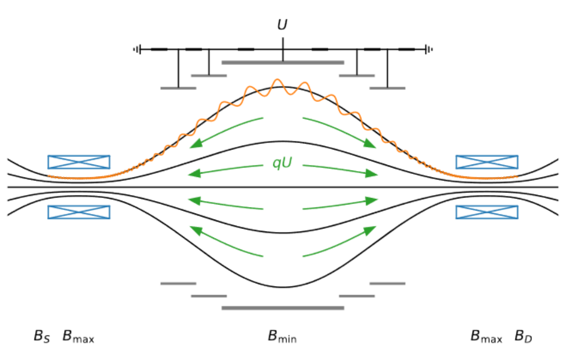


Figure 1: Principle of the MAC-E filter; Sources : [1]

3 Analysis & Results

3.1 Understanding of the Model

3.1.1 Differential spectrum

We start with a plot of the differential spectrum with equation(3), in which we set the pre-factor $C = 1.433488 \cdot 10^{-13}$ and use the approximation of the relativistic Fermi function as equation(5). Relevant variables can be obtained with the following equation.

$$\begin{aligned} E_{tot} &= E + m_e && \text{total electron energy} \\ p &= \sqrt{E_{tot}^2 - m_e^2} && \text{electron momentum} \\ \beta &= \frac{E_{tot}}{p} && \text{relativistic beta factor} \\ \eta &= \frac{2 \cdot a}{\beta} && \text{number of protons in } T_2\text{, fine structure constant } \alpha \end{aligned} \tag{4}$$

$$F(Z', E) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \cdot (1.002037 - 0.001427\beta) \tag{5}$$

As we can see in figure2, with increasing neutrino mass, the zero-point of rate decreases on the energy scale, reflecting in an overall negative energy difference from E_0. This corresponds to our estimation from the theory part.
We also have to take negative neutrino masses, $m_\nu^2 = -1\,\mathrm{eV}^2$, into consideration, because of the statistical uncertainty. We will later see that, with a 90% confidence interval, the lower bound of the squared neutrino mass reaches the negative part.

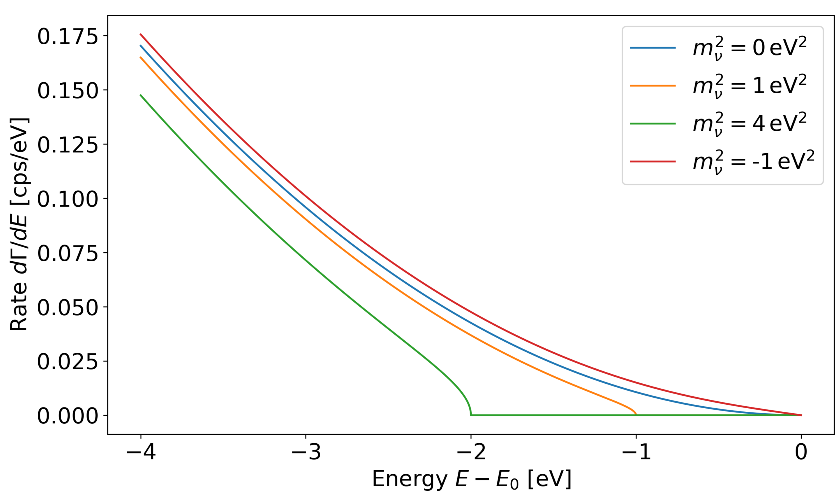


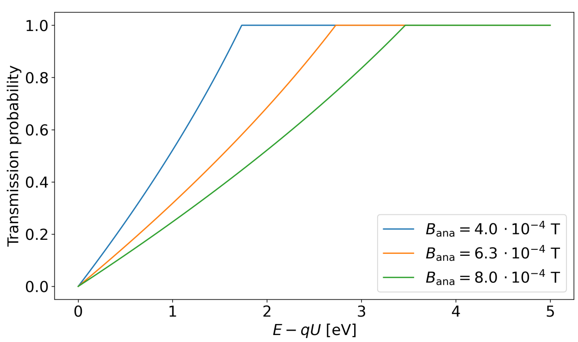
Figure 2: Differential spectrum for different neutrino masses.

3.1.2 Transmission function

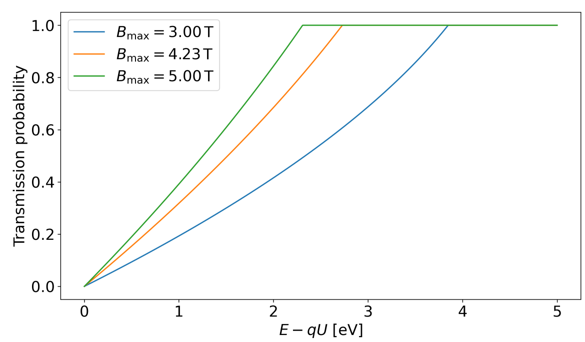
The next step is to plot the transmission probability, which can be obtained with equation(6). As reference value, we use $B_{source} = 2.52\mathrm{T}$, $B_{max} = 4.23\mathrm{T}$ and $B_{ana} = 6.3 \cdot 10^{-4}\mathrm{T}$. Figures 3a and 3b show that, the transmission probability reaches one with lower energy difference, if one increases B_{max} or decreases B_{ana}. The faster the transmission function reaches one, the smaller the energy resolution and the better the accuracy of the experiment. However, we can't simply increase B_{max}, because momentum could be lost in the process of projecting the momentum of the electron in the direction of the detector. Also, we can't increase B_{max}, since we already are at a technological limit. We could decrease B_{ana}, but that would mean the KATRIN experiment needs a larger spectrometer which adds much costs to the project.

$$T(qU, E) = \begin{cases} 0 & E < qU \\ \frac{1-\sqrt{1-f\cdot\frac{B_{source}}{B_{ana}}\frac{E-qU}{E}}}{1-\sqrt{1-\frac{B_{source}}{B_{ana}}}} & qU \leq E \leq qU\frac{f\cdot B_{max}}{f\cdot B_{max}-B_{ana}} \\ 1 & E > qU\frac{f\cdot B_{max}}{f\cdot B_{max}-B_{ana}} \end{cases} \tag{6}$$

$$\text{with } f = \frac{\frac{E-qU}{m_e}+2}{\frac{E}{m_e}+2} \, . \tag{7}$$



(a) ...with varied B_{ana}



(b) ...with varied B_{max}

Figure 3: Transmission functions...

3.1.3 Dummy model

In this section, we look at a dummy model according to the function,

$$R(qU) = N \cdot \int_{qU}^{E_0} \frac{d\Gamma}{dE}(E; m_\nu^2, E_0) \cdot T(qU, E)\, dE + B, \tag{8}$$

where N is a signal amplitude which serves as a normalizing factor and B is a constant background rate. For the model we use,

- $N = 1$,
- $B = 0.2$ cps,

and plot the model function in the endpoint energy range for different neutrino masses in figure 4, analogous to figure 2. We observe that the difference of individual spectrum vanishes near the endpoint energy at 18 573 eV, after which only the constant background is important, since the background rate becomes more dominant at this energy range.

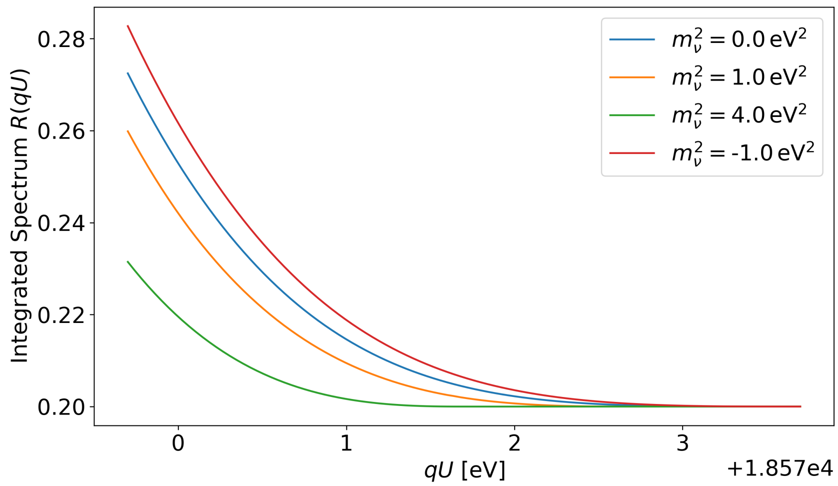


Figure 4: Integrated Spectrum for different Neutrino masses. Analogous to the differential spectrum, the total rate is lower for higher neutrino masses.

We can illustrate the influence of each parameter by plotting this function

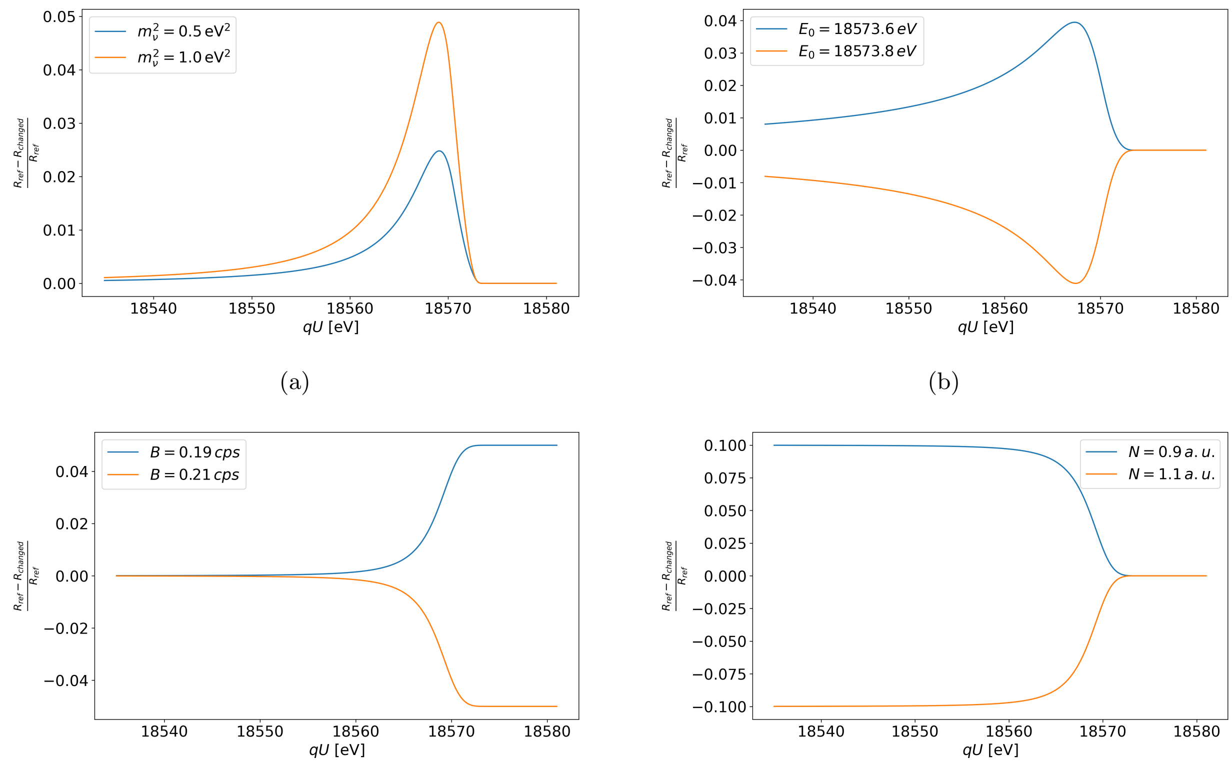
$$\frac{R_{ref}(qU) - R_{changed}(qU)}{R_{ref}(qU)} \,, \tag{9}$$

where we used

- $m_\nu^2 = 0\,\mathrm{eV}^2$,
- $E_0 = 18\,573.7\,\mathrm{eV}$,
- $N = 1$,
- $B = 0.2\,\mathrm{cps}$

as the reference model R_{ref}, and $R_{changed}$ as the same model with slightly varied parameters. The results can be seen in figures 5a – 5d. Here, only the background rate B has a big influence at higher energies. We can also observe that the influence of the neutrino mass and endpoint energy peaks at the point where the background rate is equal to the signal rate. We can determine the solution to $B/I - 1 = 0$, where I is the integral of the differential spectrum and transmission function, with a root searching algorithm, in this case signal-to-background ratio is 1 at $qU = 18\,568.53\,\mathrm{eV}$.

To conclude our discussion of the simple model, we would like to add possible improvements to it. The KATRIN experiment uses Tritium in its molecular form, therefor the daughter molecule in the β-decay can be in a rotational-vibrational or electronically excited state which lowers the effective energy of the electron and neutrino and shifts the endpoint. Tritium molecules in the source are also in thermal motion and can experience the Doppler effect and broaden the differential spectrum. Lastly, we have to account for potential energy losses in inelastic scattering process in the source, which was neglected before.



(a) (b) (c) (d)

Figure 5: Influence of the individual parameters in the dummy model.

3.2 Basics of Data Analysis

From this point on, we use the simplified model function,

$$M(E) = N \cdot \frac{d\Gamma}{dE}(E; m_\nu^2, E_0) + B \ , \tag{10}$$

where N is a signal amplitude and B is a constant background rate.

3.2.1 Monte Carlo data

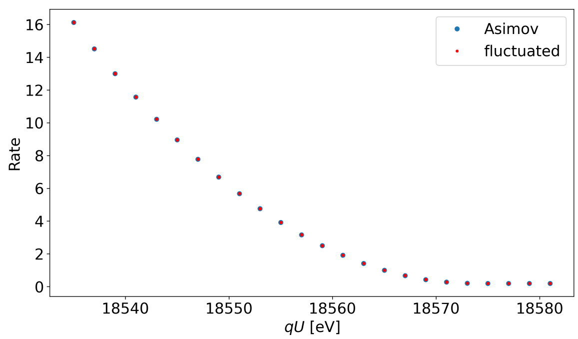
To model the KATRIN experiment, we use the measuring time distribution, where we evaluate the model in equation (10) at 24 energy points linearly spaced between 18535 eV to 18581 eV (each with a measuring time $t_{point} = \frac{750\,\mathrm{d}}{24}$).
To get the expectation of our model, we multiply the model (10) with the measuring time for each energy point to get our expected number of counts,

$$\lambda = M(E) \cdot t \ . \tag{11}$$

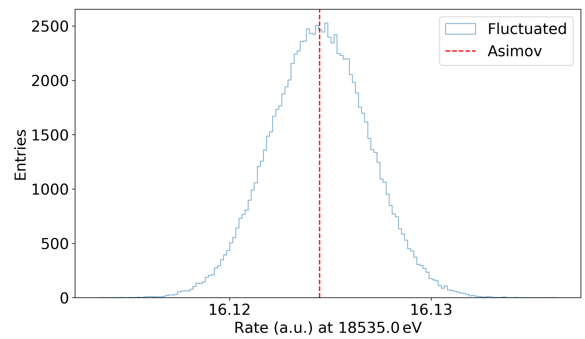
This is the Asimov spectrum and represents the expectation of our model to our best knowledge.
We can also look at a statistical representation of our model and generate 10^5 randomly generated Poisson distribution,

$$P(\lambda, M(E)) = e^{-M(E)} \frac{M^{\lambda}(E)}{\lambda!} \ , \tag{12}$$

where λ is the number of expected counts and $M(E)$ is our model prediction (10).
To compare the rates of the spectra, we plot them first and against the electron energy and then plot the rate entries at 18535 eV in a histogram, which produces figures 6a and 6b.



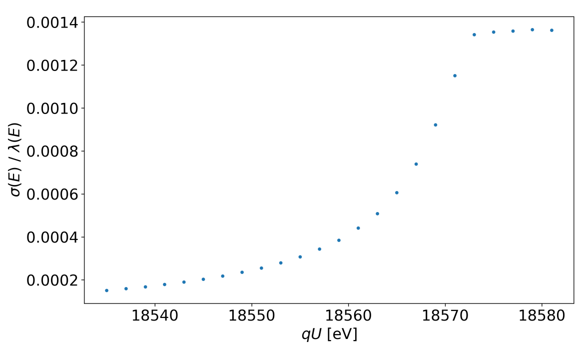
(a) Rate vs. energy plot



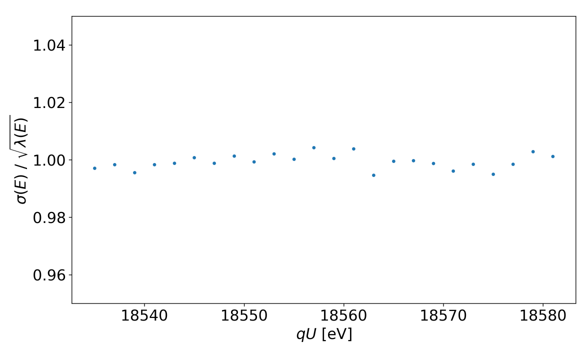
(b) Histogram of the rate entries at 18535 eV for 10^5 randomly generated Poisson distributions.

Figure 6: Fluctuated vs Asimov data

We see that the rate of the fluctuated spectra are normally distributed around the Asimov rate, with a small standard deviation from the mean value. We can also show this by plotting the coefficient of variation, $\sigma(E)/\lambda(E)$, for each energy point in figure 7a. The variation here seems to be satiated at higher energies. We can also determine the behavior of σ as a function of lambda by plotting $\sigma(E)/\sqrt{\lambda(E)}$ in figure 7b, which shows that the standard deviation is approximately the square root of the total count number with an error of less than one percent.



(a) Coefficient of variation vs. energy plot, which shows the relatively small deviation of σ from λ.



(b) Standard error vs. energy plot, which shows that $\sigma(\lambda) \approx \sqrt{\lambda}$.

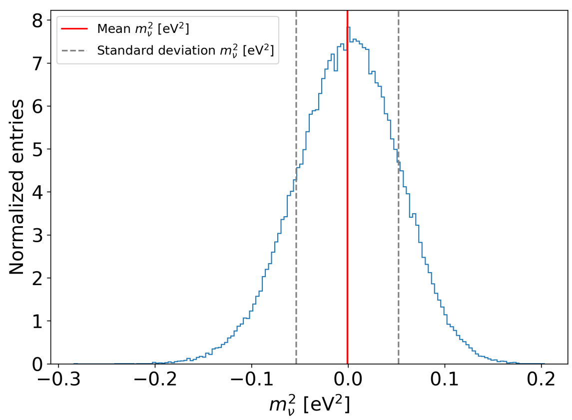
Figure 7

3.2.2 Parameter inference

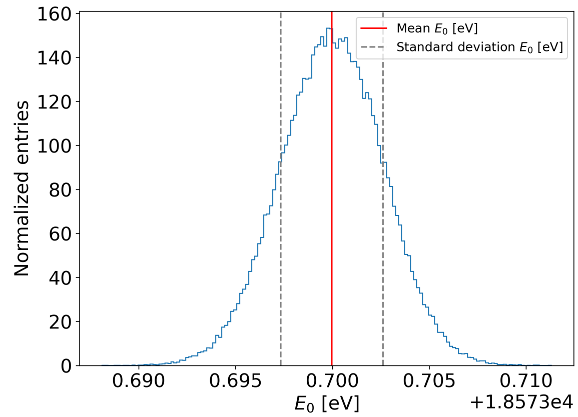
In order to retrieve the best parameters for describing the data, we use a normal distribution as an approximation to the Poisson distribution, because of the size of data. A χ^2 minimization is performed with the Asimov spectrum, and we obtained the following value for the parameters:

Table 1: Results for χ^2 minimization, which doesn't differ much from the expectation value used in the previous parts.

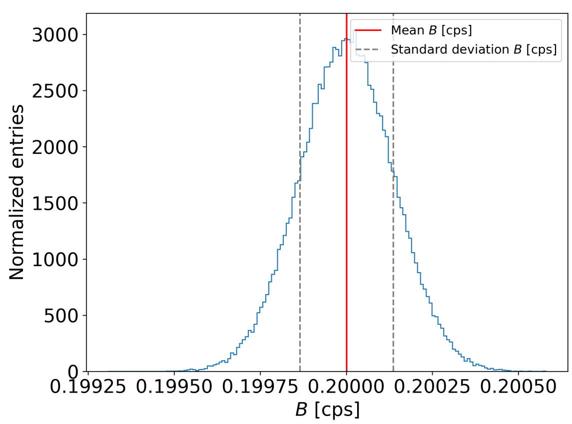
	m_ν [eV]	E_0 [eV]	B [cps]	N
Asimov	$2.05330649 \cdot 10^{-3}$	$1.85737001 \cdot 10^4$	0.200002526	0.999996313
expectation	0	18573.7	0.2	1



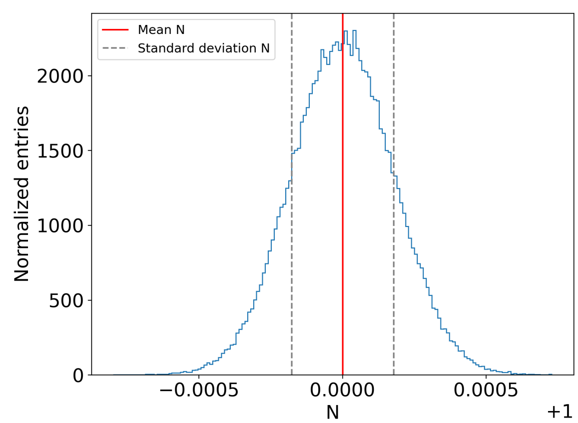
(a) $m_\nu^2 = -0.001 \pm 0.053$ eV^2



(b) $E_0 = 18573.699 \pm 0.003$ eV



(c) $B = 0.1999 \pm 0.0001$ cps



(d) $N = 1.0000 \pm 0.0001$

Figure 8: Distributions of the fit parameters for 10^5 Poisson distributed spectra with mean value and standard deviation (format: $\mu \pm \sigma$).

Then we repeat the process for the fluctuated spectra for each parameter separately, the results are plotted in figure8ato8d. The mean value of the distribution is clearly the best fit parameters, which only differ from the expectation value with a negligibly small value. And the standard deviation gives the 68% confidence interval $[\mu - \sigma, \mu + \sigma]$, which contains 68% of true value, if we were to repeat the experiment $N \to \infty$ times.

3.2.3 Uncertainty and sensitivity

Aside of the best fit of the parameters, it is also worth determining its uncertainty and its corresponding confidence interval, namely, to find the 1-σ confidence interval for m_ν^2. To retrieve this, we perform a so-called profile likelihood analysis. For a given m_ν^2 value, the value of the negative logarithm of the likelihood function with respect to $\hat{\theta} = (E_0, N, B)$ can be obtained with the following equations,

$$D(m_\nu^2) = -ln\mathcal{L} = \frac{1}{2}\chi^2(\vec{\mu}; \vec{N}) \tag{13}$$

in which minimizing $-ln\mathcal{L}$ corresponds to χ^2 minimization. Then, build the difference from best fit value from parameter interference section.

$$\Delta D(m_\nu^2) = D(m_\nu^2) - D_{best} \geq 0 \tag{14}$$

$$\Delta D(m_\nu^2) = \frac{1}{2} \tag{15}$$

With the last step, we can obtain two values, one smaller and one larger than the best fit value.

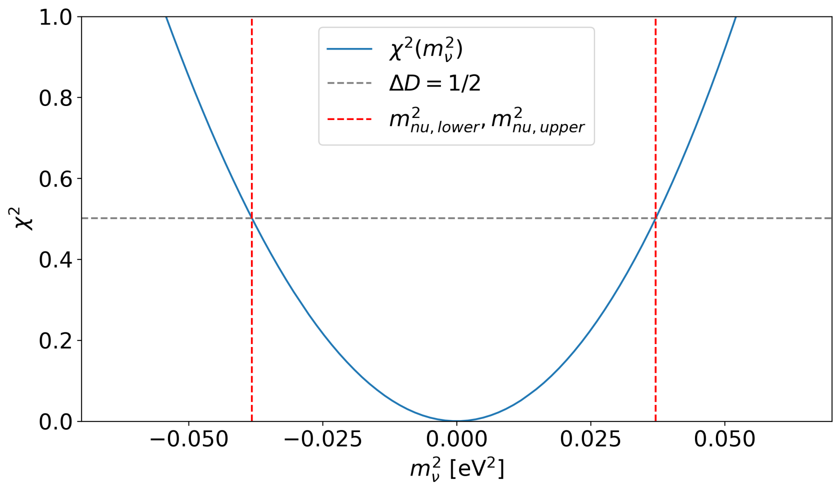


Figure 9: Profile-Likelihood function in terms of m_ν^2 with upper and lower bounds corresponding to 68.27 % of all measured events

In figure 9, we plot χ^2 against m_ν^2. The 68% confidence interval is [-0.03819, 0.037134] eV^2. The confidence interval for 90% can be obtained by multiplying with a factor 1.645, which gives [-0.06283, 0.06108] eV^2. With the upper limit of 90% confidence intervals, we can calculate the sensitivity of the experiment, which reads 0.2472 eV.

4 Summary and conclusions

To this experiment we learn the basic principles, a scratch of the experiment procedure, of the KATRIN experiment and focus on important methods of data analysis such as, Monte Carlo data, maximum Likelihood method and interval estimation. Moreover, we learned about the use of Python dealing with a rather large set of data. These methods can be applied to many experiments which require dealing with large set of data is required, such as describing ferromagnetism with Ising model in 3D with computer simulation or analyzing rough surface with spraying process using SDTrimSP.

5 Literature

References

[1] Christian Karl, Susanne Mertens, Martin Slezák, Physics department, TUM, Advanced Lab Course - Neutrino Mass Analysis with KATRIN -