

(A) Prove $T(n) = 4n^3 + 2n + 4 \in \Omega(n)$

$$\begin{aligned}4n^3 + 2n + 4 &\geq cn \\n_0 &= 1, c = 10 \\4n^3 + 2n + 4 &\geq 10n \forall n \geq 1 \\ \therefore T(n) &\in \Omega(n)\end{aligned}$$

(B) Is $O(n^2)$ a tight upper bound for $T(n)$? If not, what is the tight upper bound for $T(n)$?

$$\begin{aligned}\frac{4n^3 + 2n + 4}{n^2} &\leq \frac{cn^2}{n^2} \\ \lim_{n \rightarrow \infty} 4n + \frac{2}{n} + \frac{4}{n^2} &\geq c\end{aligned}$$

$$\therefore T(n) \notin O(n^2)$$

A better tight upper bound is $O(n^3)$

$$\begin{aligned}\forall n &\geq 1 \\ T(n) &\leq 10c^3 \\ \therefore T(n) &\in O(n^2)\end{aligned}$$

(C) $T(n) \notin O(n^2) \therefore T(n) \notin \Omega(n^2)$